Robustness Appendix
for "Urban Structure and Growth"

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ROBUSTNESS

Diversified Cities

In the simple version of the model presented in "Urban Structure and Growth", we made assumptions designed to ensure that cities would perfectly specialize in terms of the industries producing in them. This assumption was for the purposes of simplicity, but also served to capture the idea that there are substantial differences in the industry composition across cities. Nevertheless, cities are not perfectly specialized, and in this subsection we demonstrate two methods via which the above analysis could be extended to allow for multi-industry cities.

Industry Spillovers.—

One way to make cities diversify in this model is to allow for cross industry production spillovers that are urban in scope. As a simple example, suppose than within each sector, industries can be grouped according to the existence of spillovers. We might motivate this in terms of spillovers between the production of automobile bodies, and automobile parts, which are distinct SIC industries, but are often found to be located in the same MSAs.

In particular, suppose that for a set of industries j^{φ}, $\varphi = 1, ..., G_j$, across which there are spillovers, the productivity level for firms in these industries in a city is defined as

$$\tilde{A}_{tj} = A_{tj} \sum_{\varphi=1}^{G_j} \left(\tilde{H}_{tj^{\varphi}}^{\gamma_j} \tilde{N}_{tj^{\varphi}}^{\varepsilon_j} \right)^{1/G_j},$$

and note that by the assumption that all industries are in the same sector (or industry group), the parameters γ_j and ε_j are the same across all industries. Suppose further that all these industries begin time with the same stocks of industry specific factors.

Under these assumptions, industry pairs will covary perfectly, and it is as if we have redefined the notion of an industry to consist of a set of industries with spillovers. All of the results for cities derived above, now carry over for these industry groups. In particular, employment in one of the industries of each group in a city will be $1/G$ the level derived above, or

$$\frac{N_{tj}}{\mu_{tj}} = \frac{1}{G} \left[\frac{2\left(\gamma_j + \varepsilon_j\right)}{b} \frac{Y_{tj}}{N_{tj}} \right]^2,$$

so that the total size of such a city is exactly equal to the level found above. Note that this result applies for different sizes of industry groups across sectors. Importantly, in each case the dynamics of city sizes, as well as their predicted invariant distributions, are unchanged.

Non-traded Consumption Goods.—

Another way in which cities are naturally diversified is as a result of the importance of the consumption of non-traded goods, like some types of services. Every city must employ some people in the production of these goods as they cannot be transported across cities. To capture this in our model, we assume that preferences are now given by

$$\max(1-\delta) E_0 \left[\sum_{t=0}^{\infty} \delta^t N_t \left(\sum_{i=1}^{J} \ln \frac{C_{ti}}{N_t} + \ln \frac{C_{tNT}}{N_t} \right) \right].$$

where denotes consumption of the non-traded good. Non-traded goods are produced using a constant returns to scale production technology that uses raw labor alone, and can be produced anywhere in the city (that is, they do not have to be produced at the CBD). For simplicity we assume that one unit of labor devoted to non-tradeables production produces one unit of these non-traded goods, so $C_{tNT} = N_{tNT}$. In terms of city structure, this means non-tradeable goods providers will be spread uniformly throughout a city and will live where they produce, much like the pattern observed for small service providers. The new labor market equilibrium condition then becomes,

$$N_t = \sum_{j=1}^{J} N_{tj} + N_{tNT}.$$

It is then straightforward to show that the proportion of the labor force allocated to producing non-tradeable goods is a constant proportion of the aggregate labor force n_{NT}, where

$$N_{tNT}^{*} = \frac{N_t}{1 + \sum_{j=1}^{J} \left[\left(1 - \hat{\alpha}_j - \hat{\beta}_j\right) \left(\delta D_j^K \left(1 - \omega_j\right) + \left(1 - \delta\right)\right) \right]} \equiv n_{NT} N_t,$$

and D_j^K is the same constant that we solved for in the case without non-tradable goods (see appendix). This means that total commuting costs are now

$$TCC = b \left(\frac{1}{1 - n_{NT}} \right)^{\frac{1}{2}} \left(\frac{N_{tj}}{\mu_{tj}} \right)^{\frac{3}{2}}$$

and so the size of a city in industry j must now be

$$\left(\frac{1}{1 - n_{NT}} \right) \frac{N_{tj}}{\mu_{tj}} = \left[\frac{2\left(\gamma_j + \varepsilon_j\right)}{b} \frac{Y_{tj}}{N_{tj}} \right]^2 .$$

All cities are of the same size as before once we count workers in the non-tradable industry. Workers in the non-traded industry do not commute, but occupy space, thereby increasing total commuting costs, which reduces the number of agents working in industry j in the city. This decrease exactly compensates the increase of workers in the non-tradable sector, leaving the total size of the city unchanged. The key results is, however, that city size is proportional to the expression we derived without non-traded goods, and so all our results go through.

Sub-optimal Cities

In "Urban Structure and Growth", we assumed the existence of competitive property developers that own city sites and that can subsidize factor employment at a site. As these city developers are large, relative to the scope of the externalities at a site, and as they are able to subsidize the employment of both labor and human capital in a city, they can act to internalize externalities in the urban level. As a result, each city operates at a socially optimal scale, and the equilibrium allocation is efficient (as we prove in Proposition 1).

However, it is also important to stress that this efficiency result is not essential to the ability of our model to match the growth and city facts mentioned in the introduction to the paper. In particular, it is possible to model cities in other ways that lead to an inefficient allocation of resources while still preserving the main implications of the theory for growth and the size distribution of cities. As one example, suppose that city developers exist, but that they are unable to subsidize the employment of human capital in the city. We can motivate this restriction by the idea that it is difficult to isolate human capital from raw labor through subsidies, as emphasized by Black and Henderson (1999). Under these assumptions, with free entry of city developers, the competitive equilibrium size of cities in industry j will be given by

$$\frac{N_{tj}}{\mu_{tj}} = \left(\frac{\varepsilon_j}{\gamma_j + \varepsilon_j}\right)^2 \left[\frac{2\left(\gamma_j + \varepsilon_j\right)}{b} \frac{Y_{tj}}{N_{tj}}\right]^2,$$

which is smaller than the socially optimal city size because developers have not internalized the human capital externality in production.

Note that with suboptimal cities, we cannot use a social planners problem to characterize the competitive equilibrium allocation of the model. However, for our model it is possible to use a pseudo-planner problem that is equivalent to the equilibrium allocation and has a unique solution. For example, if we do not allow developers to subsidize human capital, the pseudo-planner problem would be identical to the one above but with a resource constraint given by

$$C_{tj} + X_{tj} + b\tilde{N}_{tj}^{\frac{3}{2}}\mu_{tj} \leq \tilde{A}_{tj}\tilde{K}_{tj}^{\beta_j}\tilde{H}_{tj}^{\alpha_j}\tilde{N}_{tj}^{1-\alpha_j-\beta_j+\varepsilon_j}u_{tj}^{1-\alpha_j-\beta_j}\mu_{tj}$$

where $\tilde{A}_{tj}$ is given for the planner and equal to $\tilde{A}_{tj} = A\tilde{H}_{tj}^{\gamma_j}$. Hence in this way one can show that, independently of the instruments that the developers can use to internalize the externalities, an equilibrium allocation exists and is unique, although in this case it is not efficient. Moreover, it is easy to show that all the results for the efficient version of the model are retained with only a simple adjustment to the constant terms in the model. In particular, the qualitative behavior of the growth rates of cities and their invariant distribution are unaffected. As stated before, what is important is the coordination role that developers play.

The idea that there is some force that acts to partially, but not perfectly, internalize externalities at an urban level has other interesting implications. One possibility is that these sorts of inefficiencies may be important in explaining cross country income differences. To see this, suppose that cities are systematically organized at a suboptimal size, either too large or too small, captured by a parameter $\kappa_j \neq 1$, such that

$$\frac{N_{tj}}{\mu_{tj}} = \kappa_j \left[\frac{2\left(\gamma_j + \varepsilon_j\right)}{b}\frac{Y_{tj}}{N_{tj}}\right]^2.$$

Obviously, for the case in which developers cannot subsidize human capital employment in the city, $\kappa_j = \varepsilon_j^2 / \left(\gamma_j + \varepsilon_j\right)^2$. With suboptimal cities, a development accounting exercise would estimate the level of total factor productivity at an aggregate level to be

$$F_j = \left[\sqrt{\kappa_j}\frac{2\left(\gamma_j + \varepsilon_j\right)}{b}\right]^{\frac{2\left(\gamma_j+\varepsilon_j\right)}{1-2\left(\gamma_j+\varepsilon_j\right)}}.$$

Interestingly, because the socially optimal size of cities maximizes output *net of* commuting costs, output itself is not maximized. Hence, a distortion to optimal city sizes can have first order effects on the estimated level of total factor productivity. Indeed, countries with inefficiently large cities will be estimated to have higher levels of total factor productivity.