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UNCERTAINTY AND THE EXPANSION OF BANK CREDIT

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I. The exposition of the manner in which a single bank within the banking system expands credit as compared with the way this is done by the banking system as a whole has remained virtually unchanged since it was first given by Professor C. A. Phillips in 1921.[1] This exposition is based on a static and deterministic analysis. Recently, important new results have been obtained in other areas of economic theory through the incorporation of uncertainty into decision models: in this paper we study the effects of uncertainty on the expansion of bank credit, using a simple, static model. Our approach is an application of methods successfully employed in the analysis of optimal inventory policies for business enterprises.[2]

The approach we take was first suggested by Edgeworth.[3] In an important but now seldom read paper, he described the problem of holding bank reserves as follows:

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1 Bank Credit, New York, 1921, Part I.

2 Cf. Kenneth J. Arrow, Theodore Harris, and Jacob Marschak, "Optimal Inventory Policy", Econometrica, Vol. 19, No. 3, July, 1951, pp. 250-272, especially section 3.

3 F. Y. Edgeworth, "The Mathematical Theory of Banking", Journal of the Royal Statistical Society, Vol. LI (1888), pp. 113-127. The paper is cited by Irving Fisher, The Purchasing Power of Money, New York, 1931, p. 167; and by W. J. Baumol, "The Transactions Demand for Cash: An Inventory Theoretic Approach", Quarterly Journal of Economics, Vol. LXVI, November, 1952, pp. 545-556: the emphasis in both of these references is on the aggregate demand of the public for cash and not on the expansion of bank credit.

> I have imagined a new game of chance, which is played in this manner: each player receives a disposable fund of 100 counters, part of which he may invest in securities not immediately realizable, bearing say 5 per cent per ten minutes; another portion of the 100 may be held at call, bearing interest at 2 per cent per ten minutes; the remainder is kept in the hands of the player as a reserve against certain liabilities. The demand which he has to meet is thus regulated. From time to time....there are taken a certain number say 22 digits at random.....The sum of these digits constitutes the demand which the player has to meet. We need not consider the provision which is made to meet the average amount of (99) this demand. The special object of the reserve above mentioned is to provide against demands which exceed that average. If the player can meet this excess of demand with his funds in hand, well; but if not he must call in part, or all, of the sum placed at call, incurring a forfeit of 10 percent on the amount called in. But if the demand is so great that he cannot even thus meet it, then he incurs an enormous forfeit, say 100 pounds or 1000 pounds.....The player who wins the most interest wins the game, and is entitled to as many pence, or it may be shillings, as the number of counters he has won.[4]

In the following pages we shall develop the method of treatment suggested by Edgeworth and explore its extension to the banking system as a whole. When the results of this analysis are compared with those obtained in the traditional Phillips formulation, it will be shown that the expansion of bank credit, though appreciably affected, is surprisingly insensitive to the presence of uncertainty, at least of the type we visualize. This result is hardly obvious a priori in view of the substantial differences between stochastic and deterministic analyses of the inventory problem.

II. Consider first the problem faced by the individual bank in deciding how much it should expand credit. A member of the Federal Reserve is subject to a legal reserve requirement which is at any time a certain percentage of average deposits over some period: failure to meet this requirement results in a money penalty, but perhaps more importantly, repeated failures are regarded as evidence of unsound banking practices.[5] Violations can be avoided by depositing the

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4 Ibid, p. 120. The citation is a clear example of "operational gaming".

5 Our formulation throughout assumes that banks are members of the Federal Reserve, but can be easily adapted to state banks or members of a banking system which operates with conventional reserves (Great Britain).

necessary reserves at the Federal Reserve: these may come from vault cash or correspondents' balances: or (at some out-of-pocket cost) from borrowing in the Federal Funds Market, discounting at the Federal Reserve, or liquidating assets. Every dollar of reserves or near reserves held in excess of the reserve requirement incurs an opportunity cost since the reserve balances earn no interest, nor does cash, and the excess reserves might have been profitably invested. The excess reserves necessary to protect the bank against uncertainty may be held in the form of short term governments, but again, even though some interest earnings will be realized, these will be lower than those from longer-term investments.

The bank is subject to a series of random withdrawals from and additions to its reserve and cash balances. Additions will arise from the deposit of checks drawn on other banks, Federal Reserve "float", certain transactions with the Federal Reserve and Federal Government, and new cash deposits by the public. Outflows arising from similar transactions will also be experienced. (Although treated here as random, these additions and withdrawals will naturally be affected by the bank's location, its pattern of assets and liabilities, seasonal variation, and will be subject to abrupt change in panics or depressions.)

The problem which faces the profit-maximizing bank is thus, how far should it expand credit, given the nature of the demands upon it and the legal requirement it must meet. The simple static [6] model which is

6 Here "static" means "one-period". Quantitative inventory decisions are frequently based on these static models: in addition to greatly simplifying computation, this approach is not vulnerable, as it might first appear, to the charge that it controls events in this period while ignoring the critical factor of the reserve-deposit ratio at the start of the next period. The penalties imposed on excess lending or the holding of excess reserves within the current period assures the banker that he will not be regularly confronted at the beginning of the next period by an unfavorable initial position: cf., Thomson M. Whitin, The Theory of Inventory Management, Princeton, 1953.

developed in this section enables us to isolate and analyze the effects of uncertainty within these demands, while maintaining the desirable property of computational simplicity. We follow the traditional deterministic analysis in visualizing a bank which is initially in equilibrium [7] and then receives an injection of new reserves through a cash deposit. The effect of these new reserves on the volume of loans made by the bank will depend on the way in which cash drains from the bank vary with its level of deposits. We visualize this relationship as a) an expected outflow which is linearly dependent on the level of <u>new loans</u> made in response to the increase in reserves; [8] and b) an inflow-and-outflow variance which is linearly dependent on the total volume of deposit liabilities in the bank during the period. The formulation of the variance reflects the assumptions that the cash drain events are random and independent; [9] and that the volume of activity within the bank is largely generated by the level of its deposits.

The relevant variables of the model are:

- R the dollar volume of legal reserves at the beginning of the evaluation period
- D the dollar volume of deposit liabilities at the beginning of the period
- L the dollar volume of new loans made during the period

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7 "Equilibrium" here means that the mean volume of cash flow into and out of the bank is zero.

8 This is the exact parallel to the assumption of the traditional analysis that a certain percentage of new loans will flow from the bank.

9 The assumptions of stationarity and independence which underlie our formulation are, of course, quite unrealistic, even for short periods: we have noted the presence of seasonal variations for example. However, since our concern is with the effect of random movements in C, it seems permissable, as a first approximation, to abstract from whatever regular movements it may contain.

C the net cash drain of cash and reserves from the bank during the period (-C represents a net addition)

ρ the legal reserve ratio $(0 < \rho < 1)$

Reserves are held to be legally sufficient if the following inequality holds at the end of the evaluation period:

$$(1) \quad R - C \geq \rho\ (D + L - C)$$ [10]

We have assumed that the C are mutually independent and have a known stationary distribution $\Phi(C)$.

In attempting to maximize its profits, the bank faces a two part objective function. The first of these, the positive return, consists of the net interest earned on the new loans made during the evaluation period as a result of the extra reserves which the bank has acquired: this return is given simply by IL, where I is the average return per dollar loaned.[11] The second part is the loss component, which takes into account the fact that an increase in the volume of loans will increase the probability that the bank will fall below its reserve requirement. This in turn has two components; a lump sum penalty, M, representing costs of paperwork and administration which are incurred when-

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10 Actually, the Federal Reserve evaluates a bank's reserve position on the basis of average end-of-day reserves as compared to average end-of-day deposit liabilities over a period of one or two weeks. The valid criterion of adequacy is thus not (1), but

$$R - \sum_{i=1}^{N} \frac{(N + 1 - i)\ Ci}{N} \geq \rho \left(D + \sum_{i=1}^{N} \frac{(N + 1 - i)\ (Li - Ci)}{N}\right)$$

where the i's denote days within the averaging period. (1) was chosen as an approximation since it greatly simplifies computation.

11 Which involves two simplifications: that all loans earn a uniform rate of interest; and that no provision is made for the fact that a dollar loaned at the beginning of the averaging period will earn a larger return than one loaned at the end of the period. As we are interested in the optimal way in which to expand loans, we may properly neglect the effect of interest earned during the period on loans outstanding at the beginning of the period in formulating (2).

ever the reserve requirement is violated; and a penalty, r, on each dollar of reserve deficit which exactly offsets the interest income on those loans which lead to a violation of the reserve requirement.[12] The expected size of these costs is given by:

$$(2) \quad M\int_{v_c}^{\infty} d\,\Phi\,(C) + r\int_{v_c}^{\infty} C\,d\,\Phi\,(C)$$

where $v_c = \frac{1}{1-\rho} R - \frac{\rho}{1-\rho}(D+L)$ is the maximum level of cash outflow which can occur before a reserve deficit is incurred, obtained by solving (1) for C. Hence, the entire objective function is written

$$(3) \quad \mathcal{P} = IL - M\int_{v_c}^{\infty} d\,\Phi\,(C) - r\int_{v_c}^{\infty} C\,d\,\Phi\,(C);$$

a necessary condition for the optimality of a lending policy is that $\frac{\partial \mathcal{P}}{\partial L} = 0$.

The dependence of the optimal volume of loans on the variance of C is most clearly brought out through the use of a numerical example: for this it is necessary to know explicitly the form of the function $\Phi(C)$. In the paper cited, Edgeworth discussed the application of the "laws of chance" to prediction of the cash flows to and from an individual bank. At that time, the normal distribution was held to be synonymous with these "laws", and his paper reflects the almost metaphysical properties then attributed to the normal function. Even without this bias in its favor, the normal distribution still has a great deal of a priori appeal as a description of these flows. Assuming $\Phi(C)$ is normal will give us numerical results with a minimum of effort.

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12 The unit penalty is set equal to I so that the model will not indicate the optimality of expanding loans indefinitely. For if the unit penalty were less than I, a minimum return of I - r would be realized by the bank on every dollar of new loans, given our linearity assumption. Conversely, making r so large as to more than offset I makes the cost of violations unrealistically prohibitive. By setting I = r, we make loans which lead to a violation of the reserve requirement non-profitable, without overpenalizing them.

As it stands, the objective function (3) states that the costs of violating the reserve requirement will be independent of the size of the deficit, but no profits will be realized on loans which lead to the deficiency in reserves, since the return I will be offset by the penalty r. The problem is thus to adjust loans so as to balance the unit return I against the penalty M. We define ν as the probability level which satisfies the equality

$$(4) \quad \nu M = I.$$

At this level, further lending is unprofitable: expected marginal cost is equal to marginal revenue. We suppose, as indicated above, that the cash flow distribution $\Phi(C)$ is normal, with parameters

$$(5) \quad \begin{cases} E(C) = L\mu_i \\ Var(C) = (D + L)\sigma_i^2 \end{cases}$$

where μ_i and σ_i^2 are the relations between deposits, new loans and cash drains in bank i, and E(C) and Var(C) are the mean and variance of drains for any period, dependent on the volume of deposit liabilities at the beginning and the new loans made within the period.

The probability level ν from (4) identifies a unique k in the expression $P(X \geq \mu + k\sigma) = \nu$, which will hold for any values of the parameters μ and σ in any normal distribution. With k thus determined, the optimal loan policy is found by solving the following expression for L:

$$(6) \quad L_{ui} + (D + L)^{\frac{1}{2}} k \sigma_i = \frac{1}{1-\rho} R - \frac{\rho}{1-\rho} (D + L);$$

the right hand side is the volume of cash drains which results in a shortage of reserves as previously discussed, and the left is the optimal probability of staying below that level in balancing M against I.

In giving numerical examples, we follow the traditional practice of studying the effect of an injection of new reserves into a bank which has neither deficient or excessive reserves. In Table I (p. 9) we give the effect of a $400 increase in reserves through a cash deposit when various combinations of initial deposits and coefficients of drain variance obtain. First, we assume the ratio $I/M = 1/740.75$ which from a table of the normal distribution means $k = 3$.[13] Further, μ_i, the coefficient of the mean cash drain, is assumed to be .9 (which implies that 90 % of new loans will drain to other banks); and ρ, the legal reserve ratio, is set equal to .2 of deposits. By substitution into (6), the following values are calculated.[14]

From Table I, we see that the extension of new loans by the individual bank is only very slightly reduced from the volume given by the Phillips formulation when we include the effect of the extra reserves which must be held against uncertainty: this is especially true when the deposit occurs at a bank with a large initial volume of deposits.[15]

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13 This ratio is equal to .00135, the value of the normal distribution function three standard deviations above the mean: the numbers in the example are chosen in such a way as to simplify computation and are merely illustrative.

14 A sample of the calculations follows. With initial deposits of \$10,000 and $\sigma_i^2 = .0625$, then $k\,\sigma_i = .75$, and the initial equilibrium reserve is .75 (100) = 1.25 R - 2500 (from (6); L = 0), or R = 2,060. The injection of \$400 in new reserves via a cash deposit will raise D to \$10,400 and R to \$2460: again substituting in (6) the optimal level of new loans is .9L + .75 $(10,400 + L)^{\frac{1}{2}} = 1.25(2460) - .25(10,400 + L)$. After transposing and squaring, we obtain the expression $1.3225L - 1093.0625L + 219,775 = 0$. The root which satisfies the equation is L = 345.5445.

15 The magnitude of the effect of random drains on the lending policy, which, as we previously noted, at first seems surprisingly small when compared to the results in the usual inventory models, is in part explained by the dependence of drains on loans. This is analogous to an inventory problem in which customer demands are directly dependent on the rate of production; clearly in such a case, regardless of the amount produced (loaned) extremely large changes in inventory (reserves) will never be possible.

TABLE I

D Initial deposits $	Cash drain variance parameter σ_i^2	R Equilibrium initial reserves $	L Optimal volume of new loans $
----	0	------	347.83 (Phillips)
1024	.0625	224	341.29
1024	.25	243.2	334.86
1024	1.0	281.6	322.29
10,000	.0625	2,060	345.54 *
10,000	.25	2,120	343.07
10,000	1.0	2,240	338.34
1,000,000	.0625	200,600	347.37
1,000,000	.25	201,200	347.34
1,000,000	1.0	202,400	346.85

* See footnote 14.

The figure indicated by (Phillips) above is the optimal expansion of loans for a deterministic situation which is parallel to our stochastic formulation: 10 % of new loans remain in the bank $(\mu_i = .9)$, cash drain to the public is zero, but no consideration is made of the variance of cash drains. Professor Phillips' formula, which is independent of the initial level of deposits is

$$L = \frac{(R - \rho D)\ (1 - \rho)}{(1 - \mu_i)\ \ \rho + \mu_i}$$

(<u>Bank Credit</u>, op. cit., n. 1, p. 57.)

III. The interest of the economist in this problem has, however, never been centered on the expansion of credit by the individual bank, but rather on the expansion of credit by the banking system as a whole - on the analysis of the deposit creation multiplier. As might be expected, these rather slight deviations from the traditional analysis given in Table I are magnified in their effect on aggregate system credit expansion.

In the traditional deterministic treatment, systemwide credit expansion is given by $(R - \rho D)/\rho$ if cash drain to the public is assumed to be zero, and credit expansion is defined as the <u>addition</u> to the public's money supply. The volume of credit created is here independent of the amount of the loan which remains in the first bank, and, in fact, of all inter-bank reserve flows. We may calculate the decrease in the system-wide volume of credit extension, which results from the fact that each bank must hold extra reserves to cover uncertainty, in the following three steps.

1. Determine the ratio of the volume of new loans under uncertainty to the loan volume in a deterministic model: this we call η.
2. If this coefficient is the same for all banks, the aggregate volume of expansion within the system will be

$$(7) \quad \frac{(R - \rho D)}{[1 - \eta(1 - \rho)]}$$

This expression tells us that the banks effectively expand credit by only $\eta(1 - \rho)(R - \rho D)$ instead of effectively[16] expanding by $(1 - \rho)(R - \rho D)$; and the initial reserve excess is compounded in accordance with the series $\sum_{i=0}^{\infty} [\eta(1 - \rho)^i (R - \rho D)]$. The familiar result for these infinite series

16 Effective expansion for the individual bank is the volume of credit expansion which, occuring in the presence of new loans which drain completely from the bank, would lead to the actual final aggregate expansion. Thus, the final effect of expansion in the first bank described in Table I is the same as would have occurred had all the loan drained out of the bank and the loan been \$320, rather than \$347.83.

$$(8) \quad \sum_{i=o}^{\infty} A^i = \frac{1}{1-A}$$

then gives us (7).

3. The percentage decrease ($1 - \eta$) in the expansion of credit by the individual bank as a result of uncertainty is amplified by a factor $(1 - \rho)/\rho$ in its effect on systemwide credit expansion. For under certainty the coefficient of expansion is $1/\rho$; under uncertainty it is $\frac{1}{[1- \eta (1-\rho)]}$. Thus, the percentage change in the amount of credit extended is

$$\frac{1/\rho}{1/[1- \eta (1-\rho)]} = \frac{1- \eta (1-\rho)}{\rho} = \frac{1}{\rho} - \frac{\eta(1-\rho)}{\rho} .$$

The percentage decrease in credit expansion within the system is then $(1-\rho)/\rho$ times as great as in the individual bank, when uncertainty is considered, assuming η is the same for all banks.[17]

In practice, the likelihood of uniform η for all banks is very small. We remember from Table I that the result under uncertainty differs least from the deterministic (Phillips) result, when the new reserves are received by banks with large deposits and thus high values of η. Since the uncertainty reserve held by the initial bank which receives the cash deposit is permanently withdrawn from/the deposit multiplier process, there may be enough difference in these coefficients between a large city bank and a small country bank to affect the

17 Again, we remind the reader that there is no necessity that the uncertainty reserve be held as idle cash, as our illustrative model assumes. But the necessity for holding the reserve, even in the form of short-term governments, reduces the bank's earnings and the amount of credit which it can extend. Given, however, the fact that certain banks in the system may expand credit to the point where they sometimes fall below their reserve requirement, the more prudent bank which holds an uncertainty reserve of the type we visualize can lend these funds in the Federal Funds Market, which will yield the bank a moderate return: to the extent that this occurs, the holding of such reserves, since it finances the credit expansion of the first group of banks, does not result in the reduction of system credit.

value of the deposit creation multiplier, depending on which bank receives the deposit initially.[18]

We have seen that the effect of providing an extra reserve against uncertainty, while perhaps very large in dollar terms in an economy with over $110 billion of bank credit, is relatively slight in percentage terms. Whether this effect is important enough to be explicitly included in calculating the size of the deposit creation multiplier is an empirical question which our analysis cannot resolve. It may, however, shed some light on the surprisingly low level of excess reserves found in the United States banking system in periods of prosperity.

18 Which should serve to reinforce the differential effect of the larger bank's presumably more sophisticated lending policy: both of these are, of course, modified by the extent to which the smaller banks entrust their investing policies to their larger city correspondents.

PREVIOUS RESEARCH MEMORANDA

"Beneficiation of United States Manganese Reserves", Research Memorandum No. 1, 15 January 1957.

"Programming the Supply of Strategic Materials", Research Memorandum No. 2, 18 September 1957.

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