

STOCHASTIC PROPERTIES OF A MODEL OF THE  
INTERNATIONAL WOOL MARKET

E. Philip Howrey

William H. Witherell

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Princeton University  
Econometric Research Program

207 Dickinson Hall  
Princeton, New Jersey

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## ABSTRACT

Commodity markets in general and the wool market in particular are often thought to be characterized by extreme price instability. Price variations in the wool market have had significant effects on exporting countries for which, in most cases, wool is one of the major exports, and on the textile industries in consuming countries because of the high proportion of the gross product of these industries that is accounted for by the cost of raw materials. In order to introduce sensible policy changes, it is necessary first to isolate and to quantify the determinants of price variability. Having accomplished this, it is then possible to examine the dynamic properties of the resulting system of equations with a view toward introducing corrective measures if this seems desirable.

The econometric model that is analyzed in this paper consists of twenty-four behavioral (stochastic) equations and four identities in seventy-three variables. Although no one equation in the model is very complex, it is not at all clear a priori how the system will respond to random disturbances. Using a technique of systems analysis called "analytical simulation," it is found that while the system is deterministically stable, the random shocks

are an important source of variation in the endogenous variables. While random disturbances were found to have mainly a long-run effect on wool production, causing low-frequency variation in the series, short-run variation is particularly predominant in the price level generated by the model and in some of the consumption and stock variables.

These results correspond reasonably well with the actual behavior of the wool market during the sample period and this lends some credibility to the model. The importance of the stochastic component of the solution of the model is not without practical significance for it indicates that price stabilization schemes should be based on an analysis of stochastic rather than deterministic systems. This suggests that some of the techniques of engineering control theory for stochastic systems may be useful in the design of economic stabilization policies.

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E. Philip Howrey

William H. Witherell\*

## 1. Introduction

Econometric models are often considered to be primarily tools for economic forecasting. While it may be true that forecasting precision is the ultimate test of an econometric model, there are a number of other important uses for econometric systems. For example, an econometric model can be used for purely descriptive purposes in which case the model provides a convenient framework within which to organize and summarize historical events. For the policy maker, the quantitative information about the speeds of adjustment of the variables in the econometric model is (conceptually, at least) extremely useful. Finally, econometric models often provide interesting insights into the structure and the dynamic properties of an economic system.

This paper is concerned with the determination of the dynamic properties of a stochastic model of the wool market. The wool market model that is specified and estimated consists of a set of stochastic linear difference equations with constant coefficients. Using the point estimates of the coefficients, the dynamic properties of the model can be summarized by referring to the

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\*The authors are respectively, Assistant Professor of Economics, Princeton University and Senior Economic Analyst, Standard Oil Company (New Jersey). This paper was presented at the Fourth Monmouth Conference on Statistics and Quality Assurance and the research was conducted in the Econometric Research Program at Princeton University with support from the National Science Foundation (GS 1840) and the Textile Research Institute.

solution of the system of equations. The stochastic component of this solution, characterized by its spectrum matrix, is often ignored in dynamic systems analysis. Yet, if the disturbance terms are regarded as a characteristic of economic behavior and not due solely to specification or measurement error, suppression of the stochastic aspects of the model provides an incomplete and perhaps misleading description of the market.

The outline of this paper is as follows. In the next section the model of the wool market is introduced. The spectrum-matrix approach to stochastic systems analysis is outlined in Section 3. In Section 4 some of the properties of the spectrum matrix of the wool market model are described. The results are summarized in the final section of the paper.

## 2. An Econometric Model of the World Wool Market

The model analyzed in this study is a moderately large simultaneous equation model of the world market for a major agricultural commodity, raw wool<sup>1</sup>. There are a number of aspects of this market that make it an interesting subject for analysis. Since wool is predominantly a Southern Hemisphere product for which the largest consumers are located in the Northern Hemisphere, over half of the world production enters international trade. Although government influence or participation in wool production and marketing occurs to varying degrees in some of the producing

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<sup>1</sup> This model was developed in Witherell [14].

countries, the international price of raw wool is determined essentially at open market auctions in Australia, New Zealand, and South Africa. Wool prices in the three other major producing countries, Argentina, United States and Uruguay, where the prevailing system is sale by private treaty to market agents, tend to follow the prices determined in the auctions.

Wool prices were highly variable during the period considered in this study, which includes seasons from 1948-1949 to 1964-1965. These price variations have had significant effects both on the exporting countries for which, in most cases, wool is one of their major exports and on the textile industries in the consuming countries due to the high proportion of the gross product of these industries that is accounted for by the cost of raw materials. Therefore, a major purpose in developing the model was to determine the dynamic structure of the wool market, and, in particular, the causes of this high variability of wool prices.

The econometric model representing the world wool market consists of 24 linear behavioral equations and four identities in 73 variables. The model explains seasonal wool production in six major producing countries and a "rest of the world" sector, annual wool consumption in eight consuming countries and a "rest of the world" sector, producer stocks in five countries, consumer stocks in three countries, and wool prices. A listing of the equations and variables of the model is given in the appendix. The parameter estimates were obtained by fitting the model to data for the seasons 1948-1949 to 1964-1965. After a brief discussion

of the data and estimation techniques used in this study, the general characteristics of the various sectors of the model are outlined<sup>2</sup>.

## 2.1 The Data and Estimation Techniques

The relationships of the model are expressed, insofar as is possible, in terms of comparable real economic variables. The wool and synthetic fiber data are in millions of pounds. The price and income data were converted to U. S. dollars at the then current exchange rates. The income data were deflated by the wholesale price indices of the respective countries, and the various price data were deflated by various relevant commodity price indices as is indicated in the list of variables. While admittedly not perfect (due to the lack of complete flexibility of exchange rates), this method should provide an adequate transformation of the data into comparable form.

Another important aspect of the wool market is that wool is a seasonal product, the season for most countries running from September to June. Thus, the data for wool production are expressed in terms of the seasonal, rather than the calendar year outputs. Data on wool stocks are for stocks existing at the end of the season, around June 30, for most countries. Also, the wool price variable is seasonal. However, consumption data are available on only a calendar year basis for most countries. This

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<sup>2</sup> For a detailed explanation of the model, see Witherell [14].

difference did not cause a problem with the model, however, for there is a substantial lag between production and consumption of wool, from four to ten months. Thus, the model was estimated on the assumption that production in the season covering parts of years  $t$  and  $t+1$  and consumption in year  $t+1$  occurred in the same observation period.

Finally, a word should be said about the seasonal wool price variable. The "world wool price" variable used is that of a composite quality wool. Specifically a weighted average of Dominion wool prices for Merino and crossbred quality wools (64s and 46s quality counts) was calculated, the weights being determined by the percentage of Merino and crossbred quality wools in the total world wool clip (production). The Dominion price series consist of average prices of wool sold in London, Australia, South Africa and New Zealand markets and have been found to be very good indices of the whole spectrum of world wool prices<sup>3</sup>. The resulting price series was deflated by the Sauerbeck index of world commodity prices.

Due to the temporal structure of the wool market and the assumption of no serial correlation in the disturbances, the model is recursive with respect to some but not all of its components. Therefore, the model separates into two blocks of equations. One block (consisting of equations 9 through 14, and

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<sup>3</sup> See Witherell [14, Chapter 8].



equations 16, 17, 18, 20, 21, and 22) together with identities 26, 27, and 28 is a simultaneous system of 15 linear equations. This block was estimated as a system, and the method used was two-stage least squares with principal components of all predetermined variables used as instrumental variables<sup>4</sup>. Use of this principal components method was necessary due to the large number of variables in the system and the limited number of observations on the data.

The other block of equations in the model, consisting of all the wool production equations, 2 through 8, and some consumption equations (15, 19, 23, and 24) is recursive with respect to the simultaneous block (when the market-clearing identity, equation 25 is added). That is, these equations contain no current endogenous variables as explanatory variables. All of the equations in this block were first estimated by the ordinary single-equation least-squares method. However, there appeared to be a significant problem of serially correlated residuals in the estimates of four of these equations (2, 3, 6, and 8). In order to correct for this serial correlation, an approximate form of generalized least squares suggested by Cochrane and Orcutt was used<sup>5</sup>.

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<sup>4</sup> Kloeck and Mennes [9, pp.53-55].

<sup>5</sup> Cochrane and Orcutt [1]. See also Johnston [8, p.179].

Specifically, it was assumed that the disturbances are generated by a first order Markov process i.e.,

$$u_t = \rho u_{t-1} + e_t$$

where  $e_t$  satisfies

$$E(e_t, e_{t+s}) = 0 \text{ for all } t \text{ and for all } s = 0$$

An estimate of  $\rho$ ,  $\hat{\rho}$ , was obtained by regressing the residuals from the ordinary least squares estimate of the equations on the lagged values of the residuals. Then the equations were re-estimated by ordinary least squares, using the transformed data,  $y_t - \hat{\rho}y_{t-1}$ ,  $x_{i_t} - \hat{\rho}x_{i_{t-1}}$ , where  $y$  is the dependent variable and the  $x_i$  are the explanatory variables. To the degree that the assumption about the Markov scheme is correct and the estimates of the serial correlation coefficients are accurate, this method provides estimates which are very close to the generalized least squares estimates. In this study, the method does appear to improve the statistical quality of the estimates.

In the appendix the estimated standard errors of all the coefficients are given beneath their coefficients, and the estimated values for the multiple correlation coefficients,  $R^2$ , the Durbin-Watson statistics, D.W. and the F-test statistics are given for each estimated equation. The identities (equations 25, 26, 27, and 28) were, of course, not estimated. Let us turn now to a brief discussion of the main sectors of the model.

## 2.2 The Production Equations

The first sector of the model, equations (2 through 8), consists of equations explaining seasonal production of wool (on a clean basis) in the five major exporting countries, Australia, New Zealand, South Africa, Argentina and Uruguay, and also in the United States, and in a "rest of the world" sector. These countries account for about 78 percent of the total wool production in the non-Communist countries and 63 percent of world production. The essential dynamic aspect of wool production is that the level of a country's wool output cannot be changed rapidly in response to changing economic conditions. To varying degrees this is true of most agricultural products. In building this dynamic aspect into the production equations, the lagged adjustment approach developed by Nerlove in his studies of agricultural supply functions for the United States was used<sup>6</sup>. In general terms, if  $W_t$  and  $W_{t-1}$  are the production of wool in seasons  $t$  and  $t-1$  and  $W_t^*$  is the desired level of production for season  $t$ , then the increase in actual production in season  $t$ , is assumed to be only a fraction,  $\beta$ , of the desired adjustment. That is,

$$(2.1) \quad W_t - W_{t-1} = \beta (W_t^* - W_{t-1})$$

The desired or equilibrium level of production is assumed to depend linearly on wool prices and a number of other variables which vary from country to country such as the price of lambs,

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<sup>6</sup>Nerlove [11].

wheat, beef, and hides. A typical equation explaining desired production would be

$$(2.2) \quad W_t^* = a + bPw_{t-2} + cPl_{t-2}$$

where  $Pw_{t-2}$  is the wool price lagged two years and  $Pl_{t-2}$  is the lamb price lagged two years. Various lags were tried and the two-year lag for wool prices seemed to work the best for all countries except Argentina and Uruguay where a one-year lag was used. Combining equations (2.1) and (2.2) yields equations of the form

$$(2.3) \quad W_t = (1-\beta)W_{t-1} + \beta bPw_{t-2} + \beta cPl_{t-2} + \beta a$$

which are in terms of observable variables and can be estimated. In the case of Australia, South Africa and Argentina, indices of seasonal rainfall were added to (2.1) and hence to (2.3) as a factor affecting the adjustment of actual production to desired. In the resulting equations, the estimates of  $\beta$  are quite small, indicating that wool production is indeed quite stable, varying only slightly and after a long lag to economic factors. The estimates of the short-run (two year) elasticity of wool production to wool prices vary from + .04 to + .21.

### 2.3 The Consumption Equations

The consumption sector of the model consists of equations 13, 15, 17, and 19 through 24. While wool products are produced in most developed and developing countries in the world, in this study we concentrate on the following eight principal

wool consuming countries, which account for about 50 percent of the total world consumption of wool: United States, United Kingdom, Japan, France, Italy, West Germany, Belgium, and the Netherlands, and a "rest of the world" sector. For each of the nine consumption equations in the model, the dependent variable is the net consumption of raw wool in that country, this variable being measured by the mill consumption of wool (on a clean basis) in the country, plus the net imports of semifinished and finished wool products in terms of their wool fiber content.

The consumption equations were first estimated using the concept of a lagged adjustment of actual to desired consumption levels similar to that used on the supply side. That is, if  $C_t$  and  $C_{t-1}$  are actual consumption in years  $t$  and  $t-1$  and  $C_t^*$  is the desired or equilibrium level of consumption in year  $t$ , and

$$(2.4) \quad C_t - C_{t-1} = \beta(C_t^* - C_{t-1})$$

where  $\beta$ , the coefficient of adjustment, is a positive fraction less than one. It was thought that because of technological rigidities, contractual rigidities, consumer uncertainty and general behavioral inertia, the effect of a change in an explanatory variable such as wool prices or income on wool consumption is not immediate but is instead accumulated over time. Therefore, the adjustment in any one year is incomplete. Desired wool consumption was specified to be a linear function of the price of wool,  $Pw_t$ , income and the change in income over the previous year,  $Y_t$  and  $\Delta Y_t$ , and a variable accounting for the

impact of synthetic fibers on wool consumption,  $NS_t$ . For this study, this latter variable was taken to be the net consumption of non-cellulosic fibers (staple and yarns) in the country in year  $t$ . While it would have been preferable to have this variable explained also in the model by relative price changes, advertising expenses and technological differences, we were not able to obtain the necessary data. Thus, the desired consumption equations are generally of the form

$$(2.5) \quad C_t^* = a + bPw_t + cY_t + d\Delta Y_t + eNS_t.$$

In some cases, some of these variables were dropped or lagged in order to obtain more reasonable results. Dummy variables accounting for the effects of the Korean and Algerian War were added for some countries. Substituting (2.5) into (2.4) yields the basic form of consumption equation that was estimated:

$$(2.6) \quad C_t = (1-\beta)C_{t-1} + \beta bPw_t + \beta cY_t + \beta d\Delta Y_t + \beta eNS_t + \beta a.$$

The results are rather heterogeneous for the countries studied. For five of the nine countries, the lagged or incomplete adjustment assumption did not yield reasonable results and hence the lagged consumption term was dropped. The estimated short-run price elasticities of demand, as was expected, are low for most countries, (- .097 to - .386), an exception being the estimate of - .932 for the United States. Estimated short-run income elasticities vary much more, from unexpected negative values for four countries to a high of + 1.515 for the United Kingdom. The introduction of synthetic fibers in the mid 1950's was estimated

to have had a significant negative effect on wool consumption in all the countries except France, Italy, and the Netherlands. Finally, what is of major importance for the dynamic aspects of the model, the estimated one year adjustment coefficients vary from a low of .118 for the "rest of the world" to a high of 1.000 for the five countries listed above for which the adjustment process was found to be completed in a year.

#### 2.4 The Producer Stock Equations

A small amount of wool, from two to eight percent of the world clip, is carried over at the end of the wool season in producing countries, mainly for speculative reasons. Equations 9 through 12 in the model explain these carry-over stocks for Australia, New Zealand, South Africa, Argentina and Uruguay. For the first three countries, the carry-over stocks at the end of season  $t$ ,  $S_t$  are assumed to be a linear function of the wool price in season  $t$ ,  $Pw_t$ , and the expected wool price in season  $t+1$ ,  $Pw_{t+1}^e$  that is

$$(2.7) \quad S_t = a + b Pw_t + c Pw_{t+1}^e$$

Expected prices are not observed, but they must be formed on the basis of data available in season  $t$ . It is assumed that all of the available information is contained in the current and lagged wool price data and that the expected wool price is a geometrically weighted distribution of current and past wool prices.

That is

$$(2.8) \quad Pw_{t+1}^e = \beta \sum_{i=0}^{\infty} (1-\beta)^i Pw_{t-i} \quad , \quad 0 \leq \beta < 1$$

This specification implies that the current price has the greatest effect on the price expected in the next period and that the effect of past prices declines exponentially with time. This would be the result, for example, if expectations were "adaptive." Equations (2.8) and (2.7) can be combined to yield

$$(2.9) \quad S_t = (1-\beta)S_{t-1} + (b + c\beta) Pw_t - (1-\beta)b Pw_{t-1}$$

Equation (2.9) was estimated for Australia, New Zealand, and South Africa. The results suggest that producers in New Zealand and South Africa place relatively more emphasis on the most recent price in forming price expectations than do Australian producers. This result, indicating a more conservative behavior by Australian producers, is probably due to the fact that it is the only country of the three which does not have a government wool price support scheme. Thus the risks are greater.

Using equation (2.9) for Argentina and Uruguay yielded very unsatisfactory results. One reason is that a very important factor affecting producer stocks in these countries has been the variations in the economic policies towards wool exports, that is variations in export taxes, multiple exchange rates and export license policies. To account for these effects in a rough way, dummy variables were constructed for each country,  $D_t$ , which are equal to one for seasons in which the government policy was "unfavorable" for exports and equal to zero in other years. The



model used for these countries is

$$(2.10) \quad S_t = a + b Pw_t + c\Delta Pw_t + dD_t$$

The coefficients of the government policy variables were found to be positive and significant for both countries. Negative coefficients were estimated for the change-in-price variable suggesting that producers assume that if prices changed in one direction in the current year, they will change in the opposite direction in the following year, which would not have been a bad assumption for the postwar years.

## 2.5 Commercial Stock Equations

The stocks of raw wool held in consumer countries by wool product manufacturers and wool dealers are denoted in this study as "commercial stocks" and are held mainly for present and expected future mill consumption but also, in some cases, for speculation or price hedging. Adequate data on commercial stocks were available for only the United States, the United Kingdom, and Japan, and the corresponding equations are 14, 16 and 18. The approach followed was to develop a rather complex buffer stock inventory model following Lovell's work, in which inventories are determined by the flexible accelerator theory taking account of errors in anticipated future consumption of wool and the possibility of speculation on wool prices<sup>7</sup>. The resulting equations

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<sup>7</sup> Lovell [10].

state that commercial stocks held in year  $t$ ,  $SC_t$ , are a function of lagged stocks,  $SC_{t-1}$ , current mill consumption,  $MC_t$ , the change in mill consumption,  $\Delta MC_t$ , and the current price of wool,  $Pw_t$ . That is,

$$(2.11) \quad SC_t = a + b SC_{t-1} + cMC_t + d\Delta MC_t + ePw_t.$$

However, in the case of the United States and the United Kingdom, the price variable was not found to be significant, that is, no evidence of price speculation could be obtained with annual data. Also, for the United States, it was found necessary to drop the lagged stocks term. There was very slight evidence of price speculation in the case of Japan. However, the current mill consumption variable was highly insignificant. In general, the results do indicate that commercial stocks are determined by either the level or the rate of change of mill consumption in both.

## 2.6 The Price Equation

With the equations explaining production, consumption, changes in inventories, the market clearing identity (25), which states that production minus changes in producer's stocks equals consumption plus all other changes in stocks, and the identities defining mill consumption (26, 27, and 28), we have a complete system of simultaneous equations. Two approaches can be followed to develop an equation indicating how prices are determined in the market. The first is by specifying a dynamic equation

explicitly indicating how prices are determined by the interaction of demand, production, and changes in stocks, that is, by some concept of excess demand. This approach was followed first but with little success. The second approach is to assume that prices change to completely clear the market. This is tantamount to normalizing one of the equations for consumption or for wool stocks on the wool price variable. This latter approach was followed using the New Zealand producer stocks equation, for it appeared to be one of the "better" equations (statistically) that contained current wool prices. The reduced form equation for wool prices indicates how all of the many variables interact in the model to determine wool prices while clearing the market.

## 2.7 Summary Comments on the Model

When all of these equations are put together, the resulting model describes the world wool market as a highly damped system with respect to production and most of the consumption equations. The main simultaneous link is the world wool price variable. Since the price elasticity of both production and consumption are generally very low, most of the market adjustments to exogenous shocks, such as a drought in the producing countries or a sharp drop in income in the consuming countries, and to stochastic disturbances, takes place through changes in wool prices and stocks.

The statistical properties of the twenty-four estimated equations vary quite widely with  $R^2$  values ranging from .57 to

.99, and Durbin-Watson statistics ranging from 1.46 to 2.51. The F-tests are significant at the five percent level for all equations and at the one percent level for all equations except equations 5, 7, and 23. These tests are not strictly valid for structural estimates of simultaneous equations models, especially when lagged values of the dependent variables are used as explanatory variables. Yet these statistics were included as a systematic means of evaluating the estimated equations. Let us now turn to a discussion of the methods of evaluating the stochastic properties of models such as this. This analysis yields insights both into the plausibility of the model itself and hopefully also into the stochastic characteristics of the world wool market.

### 3. System Dynamics

We now have a model of the wool market that consists of twenty-four stochastic equations and four identities. While no single equation is very complicated, it is not at all clear how the different variables interact to determine the evolutionary path of the system. In order to gain some insight into the operation of this model, it is necessary to characterize the dynamic properties of the system of equations. This may be accomplished in several different ways which can be summarized conveniently by referring to the complete solution of the system.

The dynamic linear econometric model to be analyzed can be written abstractly in matrix form as

$$(3.1) \quad \sum_{i=0}^p A_i y_{t-i} + Bx_t = u_t$$

where  $A_0$  is an  $n \times n$  nonsingular matrix of coefficients of the current endogenous variables;  $A_i$  ( $i=1,2,\dots,p$ ) are  $n \times n$  matrices of coefficients of lagged endogenous variables with a maximum lag of  $p$ ; and  $B$  is an  $n \times m$  matrix of coefficients of the exogenous (or independent) variables. The dependent variables of the system are represented by the vector  $y_t$ , the independent variables by  $x_t$ , and  $u_t$  is a vector of random errors. The solution of this system can be obtained quite easily by introducing the lag operator  $L$  defined by  $L y_t = y_{t-1}$ . Then (3.1) can be rewritten as

$$(3.2) \quad A(L) y_t = -Bx_t + u_t$$

where

$$A(L) = \sum_{i=0}^p A_i L^i$$

is an  $n \times n$  matrix of polynomials in the lag operator  $L$ .

Now let  $a(\lambda)$  denote the adjoint of the  $\lambda$ -matrix  $A(\lambda)$  so that  $a(\lambda) A(\lambda) = \Delta(\lambda) I_n$  where  $I_n$  is the  $n \times n$  identity matrix and  $\Delta(\lambda) = |A(\lambda)|$  is the determinantal polynomial of  $A(\lambda)$ <sup>8</sup>. Pre-multiplying both sides of (3.2) by  $a(L)$  yields

$$(3.3) \quad \Delta(L) I_n y_t = -a(L) Bx_t + a(L) u_t .$$

This is conventionally referred to as the final form of the

<sup>8</sup> A good introduction to  $\lambda$ -matrices and their use in dynamic systems analysis is contained in Frazer, Duncan, and Collar [4].

econometric model<sup>9</sup>. Written in this form, the  $i^{\text{th}}$  equation of the system contains the current and lagged values of the  $i^{\text{th}}$  endogenous variable but no other endogenous variables.

The formal solution of the system is now given by

$$(3.4) \quad I_n Y_t = K \Lambda^t - \frac{a(L)}{\Delta(L)} Bx_t + \frac{a(L)}{\Delta(L)} u_t$$

where  $K$  is an  $n \times l$  matrix of constants and  $\Lambda$  is a vector containing the  $l$  characteristic roots of the determinantal equation  $\Delta(\lambda) \equiv \delta_0 + \delta_1 \lambda + \dots + \delta_l \lambda^l = 0$ . The characteristic roots of the system indicate the internal dynamics of the model. Complex roots introduce a sinusoidal component to the solution while a real root contributes a monotonic component if it is positive and an oscillating component if it is negative. While each of the endogenous variables is generated by the same autoregressive structure as (3.3) shows, each of the variables need not exhibit the same dynamic behavior. It is possible, for example, to choose initial conditions so that some of the elements in the matrix  $K$  are zero, in which case distinctly different behavior could be exhibited by different endogenous variables. The characteristic roots of the determinantal equation of the wool market model were estimated and the model was found to be deterministically stable<sup>10</sup>. Therefore, we would expect that any large fluctuations in prices or

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<sup>9</sup> For a further discussion of the reduced form, see Theil and Boot [13].

<sup>10</sup> Witherell [14, pp.194-201].

quantities that do occur would not be generated by the internal dynamics of the model.

It is clear from the solution (3.4) that the characteristic roots themselves do not reveal the complete dynamics of the system. The component of the solution containing the exogenous variables,  $-\frac{a(L)}{\Delta(L)} Bx_t$ , also partially determines the actual movement of the system over time. The elements in the  $n \times n$  operator matrix  $-\frac{a(L)}{\Delta(L)} B$  are rational functions in  $L$ ; a typical element is of the form

$$-\frac{\sum_{j=1}^n a_{ij}(L) b_{jk}}{\delta_0 + \delta_1 L + \dots + \delta_n L^n} = c_{ik}(L) = c_{ik}^0 + c_{ik}^1 L + c_{ik}^2 L^2 + \dots$$

The polynomial  $c_{ik}(L)$  is generally not of finite degree but, provided the system is stable, the coefficients of the polynomial form an absolutely convergent series.

The matrix of polynomials  $c(L) = [c_{ij}(L)]$  is used to derive the impact and dynamic multipliers of the system<sup>11</sup>. Suppose that the  $j^{\text{th}}$  exogenous variable undergoes a unit step at time  $t = 0$ . The immediate impact on the  $i^{\text{th}}$  endogenous variable of this change is  $c_{ij}^0$ . After  $k$  periods have elapsed, the  $i^{\text{th}}$  endogenous variable will have changed by

$$\sum_{\ell=0}^k c_{ij}^{\ell}$$

<sup>11</sup> Impact and dynamic multipliers are discussed in detail in Goldberger [5].

and the total, long-run response of the  $i^{\text{th}}$  endogenous variable to a unit change in the  $j^{\text{th}}$  exogenous variable is the limit of this sum as  $k \rightarrow \infty$ . Provided the system is stable, this equilibrium multiplier will be finite as might be expected. A study of the impulse response functions, i.e., the sequences  $(c_{ij}^s; s = 0, 1, \dots)$  is often helpful in understanding the dynamics of the system. In the case of the wool market model, estimates of the impact multipliers and eight year dynamic multipliers indicate that the response of the wool market to changes in exogenous variables are distributed over a number of years and in some cases this response is oscillatory<sup>12</sup>.

The final term in the solution is the vector of random variables  $\frac{a(L)}{\Delta(L)} u_t$ . This stochastic component of the solution is frequently ignored in the analysis of the dynamic properties of econometric models. However, a number of authors have suggested, at least implicitly, that it is the stochastic element that lends realism to the solution of the model<sup>13</sup>. Moreover, the impulse response functions describe the expected response of the endogenous variables; the properties of the deviations from the expected response depends on the stochastic component in the solution. Thus the properties of the stochastic response are important for a complete description of the dynamic properties of the system.

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<sup>12</sup> See Witherell [14, pp.174-194].

<sup>13</sup> See, for example, Fisher [2] and Haavelmo [6].



The method used here to describe the stochastic component might be referred to as analytic simulation<sup>14</sup>. Returning to (3.1), suppose that the vector of exogenous variables  $x_t$  is held constant and initial values for  $y_{t-1}, y_{t-2}, \dots, y_{t-p}$  are specified. Then the current value of  $y_t$  is given by

$$(3.5) \quad y_t = A_0^{-1} \left[ \sum_{j=1}^p A_j y_{t-j} \right] - A_0^{-1} B x_t + v_t$$

where  $v_t = A_0^{-1} u_t$  clearly depends on the random vector  $u_t$ . With  $x_t$  given, a sequence of random vectors ( $u_t; t = 1, 2, \dots$ ) can be fed into (3.5) and the system can be simulated. The simulation paths of the endogenous variables could then be analyzed to determine the properties of the stochastic response of the system. Since, provided the system is stable, the vector  $y_t$  is generated by a multivariate stationary stochastic process, it would be natural to compute the spectra and perhaps the cross-spectra of the output series.

Using analytic simulation the spectra and cross-spectra can be obtained directly. Inserting the spectrum representation for the random vector  $u_t$  in the solution (3.4), the stochastic component of the solution can be written as<sup>15</sup>

$$(3.6) \quad y_t = \frac{a(L)}{\Delta(L)} \int_{-\pi}^{\pi} e^{i\omega t} dU(\omega)$$

where  $dU(\omega)$  is the kernel of the vector process  $u_t$ . Interchanging the order of operations on the right-hand side of (3.6) leads to

<sup>14</sup> For a more complete discussion of analytic simulation, the reader is referred to Howrey [7].

<sup>15</sup> A good introduction to the spectrum representation of a stochastic process is given in Yaglom [15].

$$(3.7) \quad y_t = \int_{-\pi}^{\pi} e^{i\omega t} T(\omega) dU(\omega)$$

where  $T(\omega)$  is the transfer matrix obtained by operating on  $e^{i\omega t}$  with  $\frac{a(L)}{\Delta(L)}$ . From (3.7), it is clear that the kernel of the  $y_t$  process is  $dY(\omega) = T(\omega) dU(\omega)$  so that the spectrum matrix of the multivariate process  $y_t$  is

$$(3.8) \quad F(\omega) = T(\omega) f(\omega) T^*(\omega) \quad -\pi \leq \omega \leq \pi$$

where  $f(\omega) = E[dU(\omega) dU^*(\omega)]$  is the spectrum matrix of the disturbance process and  $T^*(\omega)$  denotes the conjugate transpose of  $T(\omega)$ . The spectrum matrix contains the power spectra of the endogenous variables on the main diagonal and the cross-spectra in the off-diagonal elements.

It is frequently assumed that the disturbance process is serially uncorrelated with a contemporaneous variance-covariance matrix  $\Sigma$ . In this case the spectrum matrix of the disturbance process is

$$f(\omega) = \frac{1}{2\pi} \Sigma$$

and hence the spectrum matrix of the endogenous variables is

$$(3.8') \quad F(\omega) = \frac{1}{2\pi} T(\omega) \Sigma T^*(\omega) .$$

The elements of  $F(\omega) = [F_{ij}(\omega)]$  are the expected values of the corresponding power spectra that would be estimated from a simulation of the system. We now turn to a discussion of the power spectra implied by the econometric model discussed in the previous section.

#### 4. Properties of the Spectrum Matrix

In order to compute the spectrum matrix of endogenous variables, the contemporaneous variance-covariance matrix  $\Sigma$  was estimated from the residuals of the system of equations. It was assumed that the disturbance vector is serially uncorrelated so we are describing the response of the system to white noise. Inserting the covariance matrix  $\Sigma$ , into (3.8'), the spectrum matrix of the endogenous variables was derived.

The first somewhat surprising result that emerges from the calculations of the power spectra implied by this model is the great diversity of shapes that are observed. Thus, even though the autoregressive structure is the same for each variable as shown by (3.3), the averaging that is effected by the adjoint operator  $a(L)$  is sufficient to create the differences. This diversity is not at all apparent in aggregate econometric models; for macroeconomic models, the power spectra usually have quite similar shapes. In order to explore this diversity it will be convenient to characterize the different spectral shapes.

Six general spectrum patterns that gradually shade into one another emerges from this transfer function analysis.

- (1) The "Typical spectral shape" exemplified by South African production, shown in Figure 1, in which power decreases monotonically with frequency.
- (2) The typical spectral shape with a relative maximum near the frequency corresponding to a 2.5 year cycle. Australian wool production, Figure 2, provides an example of this spectrum shape.

- (3) The typical spectral shape with a relative minimum near  $\frac{1}{4}$  cycle per year (New Zealand wool production, Figure 3).
- (4) A U-shaped spectrum with relative concentrations of power in both the low- and high-frequency ends of the spectrum. A number of the variables in this model fall into this category; United States wool consumption (Figure 4) provides a good example.
- (5) The inverted typical spectral shape with a relative peak near the 2.5 year cycle. Argentine wool stocks (Figure 5) provide the only example of this type of spectrum.
- (6) The inverted typical spectral shape in which power increases monotonically (except possibly at the very low frequencies) with frequency. The price of wool (Figure 6) provides a good example of this spectrum shape.

Table I provides a grouping of the variables according to the shape of the power spectrum. This table shows, for example, that three endogenous variables have the typical spectral shape: wool production in South Africa, Argentina, and the United States. One might suppose that a grouping by type of spectrum might not be too dissimilar from a grouping according to type of variable. And this expectation is confirmed, at least partially, by the entries in this table. Six of the seven wool production variables fall in the spectrum-shape categories 1, 2, and 3. It will be recalled that for these three categories low-frequency power dominates the power spectrum. This is not too surprising since the production equations, taken in isolation, are first-order autoregressive specifications. What is somewhat surprising is

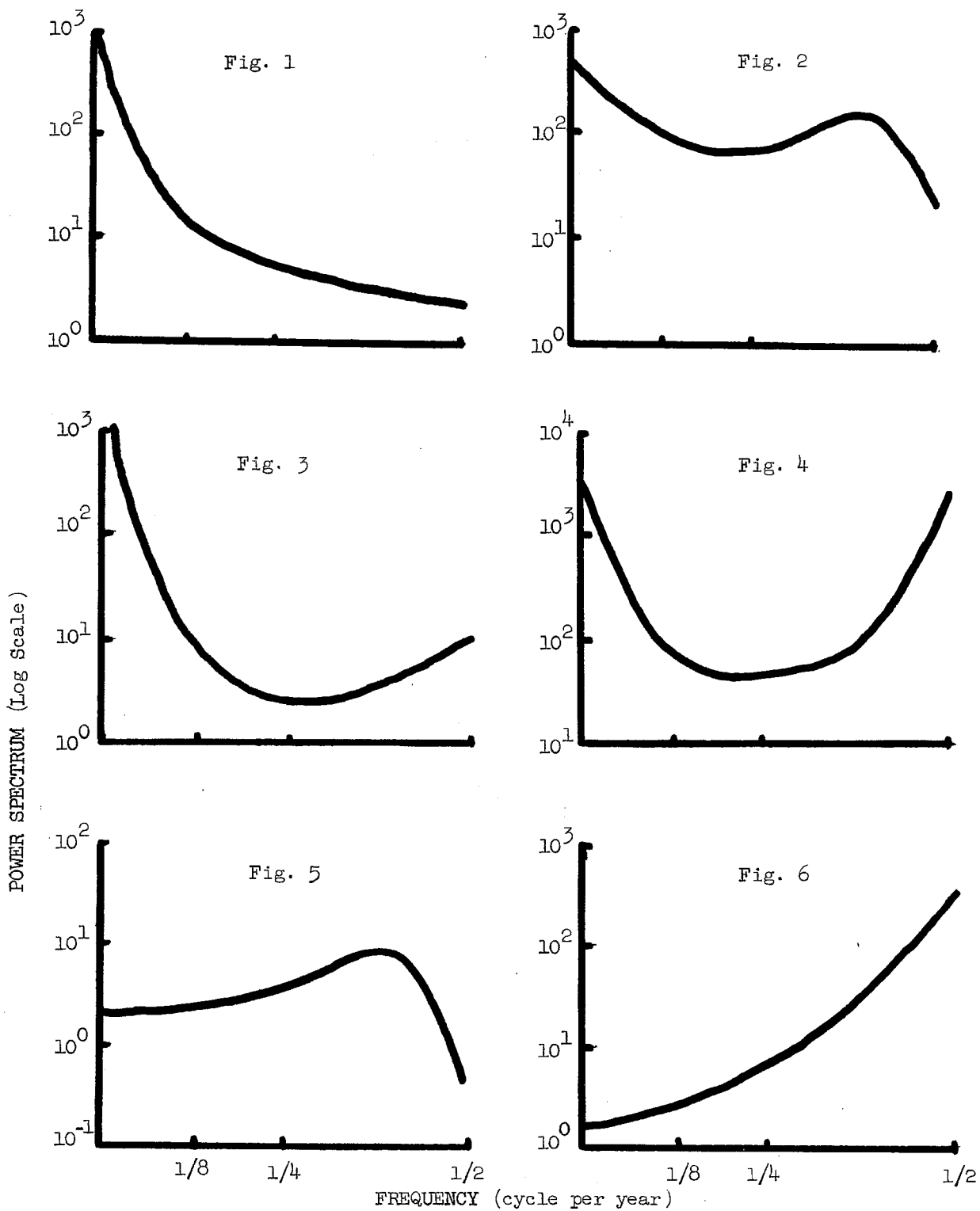


Figure 1. South African Production  
 Figure 3. New Zealand Production  
 Figure 5. Argentinian Stocks

Figure 2. Australian Production  
 Figure 4. United States Consumption  
 Figure 6. World Price

that the system interaction produces pronounced fluctuations centered on 2.5 years per cycle in some production series but not in others<sup>16</sup>.

TABLE I. Endogenous Variables Grouped by Type of Spectrum (See Text for Explanation)

Type of Spectrum	1	2	3	4	5	6
Variable	WSA	WAU	WNZ	WUR	SAG	P
	WAG	WRW	SAU	CUS		SSA
	WUS	SCUS	SCJ	CI		SUR
				CN		CUK
				CRW		SCUK
						CJ
						CF
						CG
						CB
						SNZ

The diversity of the production spectra may be somewhat overstated by these simple comparisons because they do not take into consideration the fact that neither the magnitude nor the variability of production is the same in all countries. The power spectrum of world wool production implied by this

<sup>16</sup> The fact that an interior peak emerges in the spectrum is not too surprising in view of Fishman's analysis of price behavior in commodity markets [3].

disaggregated model is shown in Figure 7. World production has the typical shape with an interior peak. This means that production in Australia and the "Rest of the World" dominate the production in the other five countries included in the model in the sense that the peak frequency exhibited by world production is contributed by peaks in Australian and Rest-of-World production. In the 1964-65 season Australia and the "Rest of the World" sector accounted for 69 percent of the total world wool production.

Turning now to the consumption side of the model, four of the eight consumption variables have the inverted typical spectral shape. The other four have a U-shaped spectrum. This diversity of results corresponds to the heterogenous nature of the estimated consumption equations. It is interesting to note that the four U-shaped spectra correspond to the consumption equations that include a lagged dependent variable. The spectrum of world consumption, shown in Figure 8, has the typical shape with a relative minimum near  $\frac{1}{4}$  cycle per year. So while the consumption equations for half of the countries in the model exhibit an inverted typical shape, the time path of world consumption implied by the model has a dominant low-frequency component just as does the world production series implied by the model.

POWER SPECTRUM (log Scale)

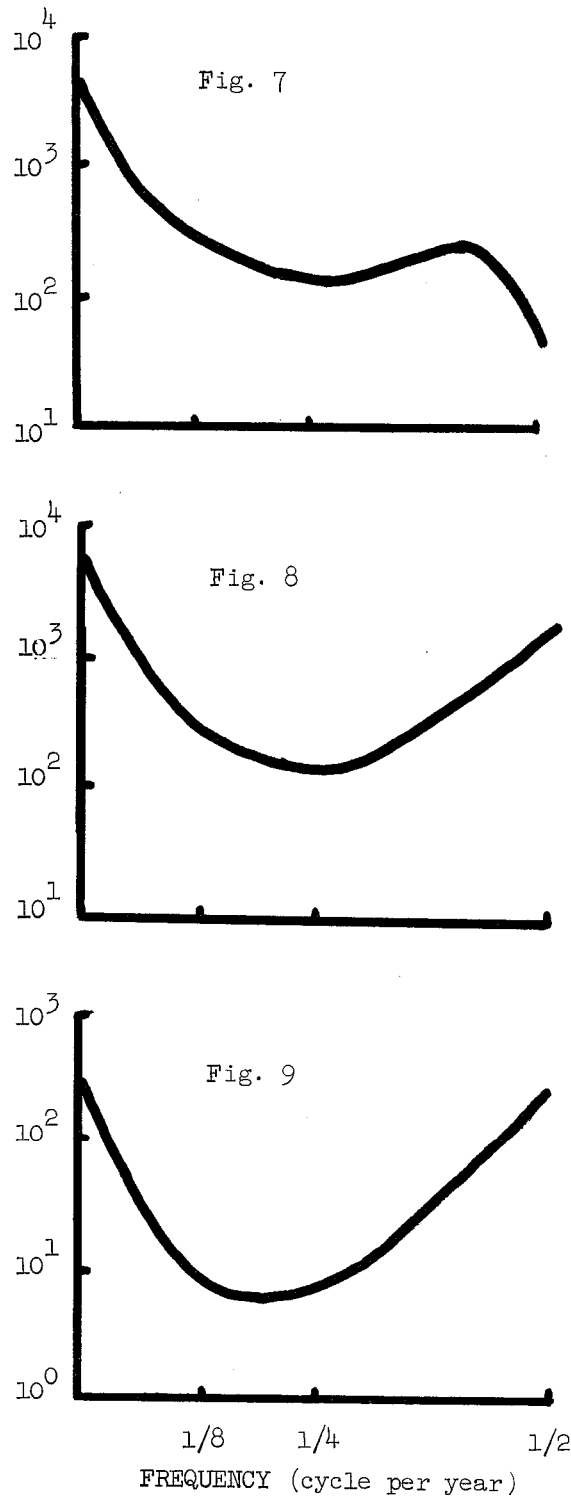


Figure 7. World Production  
Figure 8. World Consumption  
Figure 9. World Stocks



The inventory variables in this model exhibit a considerable amount of diversity. This is to be expected because the equations are for producer and consumer stocks and the forms of the equations are quite different. Four of the eight stock variables have the inverted typical spectral shape. In the case of producer stocks for Uruguay and Argentina, this result is quite reasonable for their stocks are highly volatile. In the case of New Zealand producer stocks, the result, while also reasonable, may be due to the fact that the New Zealand stock equation was normalized on the price variable and New Zealand stocks are determined by the market clearing identity. Two of the stock variables, producers stocks in Argentina and consumers stocks in the United States, exhibit relative peaks near the 2.5-year cycle. The result for the United States is an indication of the "textile cycle" that has been noted in other studies<sup>17</sup>. However, no such cycle was detected in the spectrum for United States consumption. The two remaining stock variables have the typical spectral shape with a relative minimum near  $\frac{1}{4}$  cycle per year. Given this diversity it is not surprising to find that the spectrum of world stocks, shown in Figure 9 has the typical spectral shape with a relative minimum near  $\frac{1}{4}$  cycle per year.

The final variable in the model which serves to equilibrate the system is the price of wool. The power spectrum of price, shown in Figure 6, indicates that the variance of the world wool

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<sup>17</sup> See, for example, T. M. Stanback [12].

price is composed predominantly of high-frequency variation. Of the important aggregate variables; namely, production, consumption, stocks, and price, this is the only one that exhibits the inverted typical spectral shape. The relative absence of low-frequency power implies that with respect to the stochastic aspects of the wool market, the price series are considerably less stable than the aggregate production series and relatively less stable than the aggregate stock and consumption series.

##### 5. Concluding Remarks

This paper has been concerned with an application of the spectrum-matrix approach to stochastic systems analysis. This method of "analytical simulation" was applied to a stochastic model of the wool market in order to determine the dynamic properties of the response of the endogenous variables to random disturbances. While the model of the world wool market was found to be stable in a deterministic sense, a consideration of the stochastic properties of the model revealed that random disturbances are an important source of short-run variation in the endogenous variables, particularly with respect to world wool prices and some consumption and stocks series. Stochastic disturbances were found to have mainly a long-run effect on wool production, causing low-frequency fluctuations in these series.

These results correspond reasonably well with the actual behavior of the wool market during the observation period and

this lends some credibility to the model. The importance of the stochastic component of the solution of the model is not without practical significance; it indicates that price stabilization schemes should be based on an analysis of stochastic rather than deterministic systems. This suggests that some of the techniques of engineering control theory for stochastic systems may be useful in the design of economic stabilization policies.

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APPENDIX: THE WORLD WOOL MARKET MODEL

$$(1) \quad Pw_t = 33.676 + 1.333 \text{ Snz}_t - .976 \text{ Snz}_{t-1} + .605 \text{ Pw}_{t-1}$$

(15.899) (.165) (.308) (.195)

$$R^2 = .88, \text{ D.W.} = 1.82, \text{ F} = 29.69$$

$$(2) \quad \text{Wau}_t = .695 \text{ Wau}_{t-1} + .660 \text{ Pw}_{t-2} - 1.749 \text{ Pwhau}_{t-3}$$

(.141) (.399) (1.360)

$$+ .381 \text{ Fau}_{t-1} + .569 \text{ Rau}_t$$

(.208) (.387)

$$R^2 = .96, \text{ D.W.} = 2.40, \text{ F} = 88.00$$

$$(3) \quad \text{Wnz}_t = .956 \text{ Wnz}_{t-1} + .148 \text{ RP}_{t-1} + .340 \text{ Plnz}_{t-2}$$

(.031) (.074) (.143)

$$R^2 = .99, \text{ D.W.} = 2.52, \text{ F} = 594.00$$

$$(4) \quad \text{Wsa}_t = .900 \text{ Wsa}_{t-1} + .108 \text{ Pw}_{t-2} - .079 \text{ Pwhsa}_{t-2} + .141 \text{ Rsa}_{t-1}$$

(.047) (.062) (.068) (.074)

$$R^2 = .92, \text{ D.W.} = 2.20, \text{ F} = 49.83$$

$$(5) \quad \text{Wag}_t = .791 \text{ Wag}_{t-1} + .101 \text{ Pw}_{t-1} - 11.974 \text{ Pwhag}_{t-1}$$

(.178) (.168) (20.490)

$$+ 1.769 \text{ Pbag}_{t-1} + .305 \text{ Rag}_t$$

(1.883) (.227)

$$R^2 = .62, \text{ D.W.} = 1.51, \text{ F} = 4.50$$

$$(6) \quad \text{Wur}_t = .561 \text{ Wur}_{t-1} + .251 \text{ Pw}_{t-1} + .103 \text{ Pbur}_{t-1} + .194 \text{ Phur}_{t-1}$$

(.123) (.109) (.093) (.087)

$$R^2 = .66, \text{ D.W.} = 1.86, \text{ F} = 7.77$$

$$(7) \quad \text{Wus}_t = 83.592 + .606 \text{ Wus}_{t-1} + .294 \text{ Pwus}_{t-2} - .211 \text{ Pwhus}_{t-2}$$

(77.348) (.309) (.180) (.168)

$$- .695 \text{ Pbus}_{t-2} + .297 \text{ Plus}_{t-2}$$

(.768) (1.327)

$$R^2 = .67, \text{ D.W.} = 1.54, \text{ F} = 4.87$$

$$(8) \quad \text{Wrw}_t = .950 \text{Wrw}_{t-1} + .550 \text{Pw}_{t-2} - .156 \text{Pwhus}_{t-2} \\ \quad \quad \quad (.023) \quad \quad \quad (.253) \quad \quad \quad (.091) \\ \quad \quad \quad + 1.127 \text{Plnz}_{t-2} \\ \quad \quad \quad \quad \quad \quad (.859)$$

$$R^2 = .99, \text{D.W.} = 2.15, F = 396.00$$

$$(9) \quad \text{Sau}_t = .978 \text{Sau}_{t-1} + .075 \text{Pw}_t - .054 \text{Pw}_{t-1} \\ \quad \quad \quad (.132) \quad \quad \quad (.047) \quad \quad \quad (.048)$$

$$R^2 = .72, \text{D.W.} = 1.92, F = 16.34$$

$$(10) \quad \text{Ssa}_t = .594 \text{Ssa}_{t-1} + .029 \text{Pw}_t - .023 \text{Pw}_{t-1} \\ \quad \quad \quad (.261) \quad \quad \quad (.004) \quad \quad \quad (.007)$$

$$R^2 = .77, \text{D.W.} = 1.46, F = 21.44$$

$$(11) \quad \text{Sag}_t = -53.700 + .927 \text{Pw}_t - .475 (\text{Pw}_t - \text{Pw}_{t-1}) + 98.479 \text{Dag}_t \\ \quad \quad \quad (19.905) \quad (.196) \quad \quad \quad (.172) \quad \quad \quad (11.146)$$

$$R^2 = .95, \text{D.W.} = 2.10, F = 70.59$$

$$(12) \quad \text{Sur}_t = .131 \text{Pw}_t - .206 (\text{Pw}_t - \text{Pw}_{t-1}) + 22.342 \text{Dur}_t \\ \quad \quad \quad (.028) \quad \quad \quad (.080) \quad \quad \quad (5.601)$$

$$R^2 = .73, \text{D.W.} = 1.95, F = 17.65$$

$$(13) \quad \text{Cus}_t = 419.931 + .897 \text{Cus}_{t-1} - 4.250 \text{Pw}_t + .167 \text{Yus}_t \\ \quad \quad \quad (388.944) \quad (.228) \quad \quad \quad (1.176) \quad \quad \quad (1.105) \\ \quad \quad \quad + 9.298 \Delta \text{Yus}_t - .210 \text{NSns}_t + 266.329 \text{KWD}_t \\ \quad \quad \quad (1.716) \quad \quad \quad (.150) \quad \quad \quad (94.401)$$

$$R^2 = .85, \text{D.W.} = 2.12, F = 8.29$$

$$(14) \quad SCus_t = .334 MCus_t - .162 (MCus_t - MCus_{t-1})$$

(.010)
(.053)

$$R^2 = .63, D.W. = 1.76, F = 24.07$$

$$(15) \quad Cuk_t = - .649 Pw_{t-1} + 9.109 Yuk_t - 1.896 \Delta Yuk_t$$

(.173)
(.560)
(.954)

$$- .878 NSuk_t - 40.910 KWD_t$$

(.106)
(16.935)

$$R^2 = .77, D.W. = 1.87, F = 9.01$$

$$(16) \quad SCuk_t = .186 SCuk_{t-1} + .300 MCuk_t + .074 (MCuk_t - MCuk_{t-1})$$

(.171)
(.065)
(.119)

$$R^2 = .59, D.W. = 1.93, F = 9.43$$

$$(17) \quad Cj_t = - .198 Pw_t + 8.374 Yj_t + 11.424 \Delta Yj_t - .335 NSj_t$$

(.231)
(1.807)
(4.008)
(.130)

$$R^2 = .91, D.W. = 2.30, F = 41.95$$

$$(18) \quad SCj_t = .812 SCj_{t-1} + .176 (MCj_t - MCj_{t-1}) + .034 Pw_t$$

(.096)
(.090)
(.031)

$$R^2 = .73, D.W. = 1.93, F = 17.47$$

$$(19) \quad Cf_t = 224.250 - .235 Pw_{t-1} - 1.135 Yf_t + 40.688 Df_t$$

(32.160)
(.137)
(.515)
(9.044)

$$- 40.641 KWD_t$$

(13.112)

$$R^2 = .75, D.W. = 2.23, F = 8.01$$



$$(20) \quad C_{i_t} = .793 C_{i_{t-1}} - .403 (P_{w_t} - P_{w_{t-1}}) + .696 Y_{i_t}$$

(.102)
(.125)
(.399)

$$R^2 = .57, \text{ D.W.} = 1.84, \text{ F} = 8.71$$

$$(21) \quad C_{g_t} = 86.400 - .300 P_{w_t} + 6.067 Y_{g_t} - 6.058 \Delta Y_{g_t}$$

(51.313)
(.251)
(1.070)
(2.744)

$$- .832 N_{sg_t}$$

(.237)

$$R^2 = .92, \text{ D.W.} = 2.48, \text{ F} = 33.09$$

$$(22) \quad C_{b_t} = 7.709 - .133 P_{w_t} - 2.535 Y_{b_t} - .674 N_{Sb_t}$$

(3.100)
(.110)
(3.283)
(.460)

$$R^2 = .73, \text{ D.W.} = 1.89, \text{ F} = 10.74$$

$$(23) \quad C_{n_t} = 38.546 + .742 C_{n_{t-1}} - .122 P_{w_{t-1}} - 1.237 Y_{n_t}$$

(20.286)
(.226)
(.096)
(.958)

$$- 38.058 K_{WD_t}$$

(10.019)

$$R^2 = .64, \text{ D.W.} = 2.11, \text{ F} = 4.98$$

$$(24) \quad C_{rw_t} = .882 C_{rw_{t-1}} - 1.294 P_{w_{t-1}} + 4.290 Y_{rw_t}$$

(.161)
(.40)
(2.588)

$$- 3.016 \Delta Y_{rw_t} - .446 N_{Srw_t}$$

(2.998)
(.196)

$$R^2 = .96, \text{ D.W.} = 1.79, \text{ F} = 85.143$$

$$\begin{aligned}
(25) \quad \text{Snz}_t &= \text{Wau}_t + \text{Wnz}_t + \text{Wsa}_t + \text{Wag}_t + \text{Wur}_t + \text{Wus}_t + \text{Wrw}_t \\
&\quad - \text{Sau}_t - \text{Ssa}_t - \text{Sag}_t - \text{Sur}_t + \text{Sau}_{t-1} + \text{Snz}_{t-1} \\
&\quad + \text{Ssa}_{t-1} + \text{Sag}_{t-1} + \text{Sur}_{t-1} - \text{Cus}_t - \text{Cuk}_t - \text{Cj}_t \\
&\quad - \text{Cf}_t - \text{Ci}_t - \text{Cg}_t - \text{Cb}_t - \text{Cn}_t - \text{Crw}_t \\
&\quad - \text{SCus}_t - \text{SCuk}_t - \text{SCj}_t + \text{SCus}_{t-1} + \text{SCuk}_{t-1} + \text{SCj}_{t-1} \\
&\quad - \Delta \text{SNC}_t - \Delta \text{SCRw}_t
\end{aligned}$$

$$(26) \quad \text{MCus}_t = \text{Cus}_t - \text{NIus}_t$$

$$(27) \quad \text{MCuk}_t = \text{Cuk}_t - \text{NIuk}_t$$

$$(28) \quad \text{MCj}_t = \text{Cj}_t - \text{NIj}_t$$

## LIST OF VARIABLES

The endogenous variables determined by the world wool market

model are:

1.  $Pw_t$  = Deflated world wool price, season t-1, t, cents per pound
2.  $Wau_t$  = Australian wool production, season t-1, t, millions of pounds
3.  $Wnz_t$  = New Zealand wool production, season t-1, t, millions of pounds
4.  $Wsa_t$  = South African wool production, season t-1, t, millions of pounds
5.  $Wag_t$  = Argentine wool production, season t-1, t, millions of pounds
6.  $Wur_t$  = Uruguayan wool production, season t-1, t, millions of pounds
7.  $Wus_t$  = United States wool production, season t-1, t, millions of pounds
8.  $Wrw_t$  = "Rest-of-the-World" wool production, season t-1, t, millions of pounds
9.  $Sau_t$  = Australian producer stocks, end of season t-1, t, millions of pounds
10.  $Snz_t$  = New Zealand producer stocks, end of season, t-1, t, millions of pounds
11.  $Ssa_t$  = South African producer stocks, end of season t-1, t, millions of pounds
12.  $Sag_t$  = Argentine producer stocks, end of season t-1, t, millions of pounds
13.  $Sur_t$  = Uruguayan producer stocks, end of season t-1, t, millions of pounds
14.  $Cus_t$  = United States net wool consumption, year t, millions of pounds
15.  $SCus_t$  = United States commercial wool stocks, year t, millions of pounds
16.  $Cuk_t$  = United Kingdom net wool consumption, year t, millions of pounds
17.  $SCuk_t$  = United Kingdom commercial wool stocks, year t, millions of pounds
18.  $Cj_t$  = Japanese net wool consumption, year t, millions of pounds
19.  $SCj_t$  = Japanese commercial wool stocks, year t, millions of pounds

20.  $Cf_t$  = French net wool consumption, year  $t$ , millions of pounds
21.  $Ci_t$  = Italian net wool consumption, year  $t$ , millions of pounds
22.  $Cg_t$  = German net wool consumption, year  $t$ , millions of pounds
23.  $Cb_t$  = Belgian net wool consumption, year  $t$ , millions of pounds
24.  $Cn_t$  = Netherlands net wool consumption, year  $t$ , millions of pounds
25.  $Crw_t$  = "Rest-of-the-World" net wool consumption, year  $t$ , millions of pounds
26.  $MCus_t$  = United States mill consumption of wool, year  $t$ , millions of pounds
27.  $MCur_t$  = United Kingdom mill consumption of wool, year  $t$ , millions of pounds
28.  $MCj_t$  = Japanese mill consumption of wool, year  $t$ , millions of pounds

The exogenous variables in the world wool market model are, in order of their appearance in the model.

- (1)  $Pwhau_{t-2}$  = Australian deflated wheat price, first year of season  $t-3$ ,  $t-2$ , dollars per metric ton (Deflator: Sauerbeck Price Index)
- (2)  $Fau_{t-1}$  = Australian use of super-phosphate fertilizers, season  $t-2$ ,  $t-1$ , thousands of metric tons
- (3)  $Rau_t$  = Australian rainfall index, season  $t-1$ ,  $t$
- (4)  $RP_{t-1}$  = New Zealand deflated reserve price for wool, season  $t-2$ ,  $t-1$ , cents per pound (Deflator: Sauerbeck Price Index)
- (5)  $Plnz_{t-2}$  = New Zealand deflated lamb price, first year of season  $t-3$ ,  $t-2$ , cents per kilogram (Deflator: Sauerbeck Price Index)
- (6)  $Pwhsa_{t-2}$  = South African deflated wheat price, first year of season  $t-3$ ,  $t-2$ , dollars per metric ton (Deflator: Sauerbeck Price Index)
- (7)  $Rsa_{t-1}$  = South African rainfall index, season  $t-2$ ,  $t-1$
- (8)  $Pwhag_{t-1}$  = Argentine deflated wheat price, first year of season  $t-2$ ,  $t-1$ , dollars per bushel (Deflator: Export Price Index)
- (9)  $Pbag_{t-1}$  = Argentine deflated beef price, first year of season  $t-2$ ,  $t-1$ , dollars per hundred pounds (Deflator: Export Price Index)
- (10)  $Rag_t$  = Argentine rainfall index, season  $t-1$ ,  $t$

- (11)  $P_{bur,t-1}$  = Uruguayan deflated beef price index, second year of season t-2, t-1 (Deflator: Export Price Index)
- (12)  $Phur_{t-1}$  = Uruguayan deflated hide index, second year of season t-2, t-1 (Deflator: Export Price Index)
- (13)  $P_{wus,t-2}$  = United States deflated wool support price (for 1948-54 = the average price received by farmers; for 1955-65 = the National Wool Act Incentive price), year t-2, cents per pound (Deflator: Wholesale Price Index for Farm Products.)
- (14)  $P_{whus,t-2}$  = United States deflated wheat price, year t-2, dollars per bushel (Deflator: Wholesale Price Index for Farm Products)
- (15)  $P_{bus,t-2}$  = United States deflated beef price, year t-2, dollars per hundred pounds (Deflator: Wholesale Price Index for Farm Products)
- (16)  $P_{lus,t-2}$  = United States deflated lamb price, year t-2, dollars per hundred pounds
- (17)  $Dag_t$  = Argentine export policy dummy variable, season t-1, t (= 1.0 in 1948-49, 1951-52, 1957-58)
- (18)  $Dur_t$  = Uruguay export policy dummy variable, season t-1, t (= 1.0 in 1948-49, 1951-52, 1954-55, 1957-58)
- (19)  $Yus_t$  = United States deflated disposable income, year t, billions of dollars
- (20)  $\Delta Yus_t$  =  $Yus_t - Yus_{t-1}$
- (21)  $NSus_t$  = United States net consumption of synthetic fibers, year t, millions of pounds
- (22)  $KWD_t$  = Korean War Dummy variable (=1.0 in 1951)
- (23)  $Yuk_t$  = United Kingdom deflated disposable income, year t, billions of dollars
- (24)  $\Delta Yuk_t$  =  $Yuk_t - Yuk_{t-1}$
- (25)  $NSuk_t$  = United Kingdom net consumption of synthetic fibers, year t, millions of pounds
- (26)  $Yj_t$  = Japanese deflated national income, year t, billions of dollars
- (27)  $\Delta Yj_t$  =  $Yj_t - Yj_{t-1}$
- (28)  $NSj_t$  = Japanese net consumption of synthetic fibers, year t, millions of pounds
- (29)  $Yf_t$  = French deflated national income, year t, billions of dollars
- (30)  $Df_t$  = French Algerian War Dummy variable (=1 in 1956, 1957)

- (31)  $Yi_t$  = Italian deflated national income, year t, billions of dollars
- (32)  $Yg_t$  = German deflated national income, year t, billions of dollars
- (33)  $\Delta Yg_t$  =  $Yg_t - Yg_{t-1}$
- (34)  $NSg_t$  = German net consumption of synthetic fibers, year t, millions of pounds
- (35)  $Yb_t$  = Belgian deflated national income, year t, billions of dollars
- (36)  $NSb_t$  = Belgian net consumption of synthetic fibers, year t, millions of pounds
- (37)  $Yn_t$  = Netherlands deflated national income, year t, billions of dollars
- (38)  $Yrw_t$  = Index of deflated world income, year t
- (39)  $\Delta Yrw_t$  =  $Yrw_t - Yrw_{t-1}$
- (40)  $NSrw_t$  = "Rest-of-the-World" net consumption of synthetic fibers, year t, millions of pounds
- (41)  $\Delta SNC_t$  = Change in world non-commercial stocks, end-of-season t-1, t, millions of pounds
- (42)  $\Delta SCrw_t$  = Change in world residual stocks, end-of-season t-1, t, millions of pounds
- (43)  $NIus_t$  = United States net imports of wool products, year t, millions of pounds
- (44)  $NIuk_t$  = United Kingdom net imports of wool products, year t, millions of pounds
- (45)  $NIj_t$  = Japanese net imports of wool products, year t, millions of pounds

All wool and synthetic fiber data are in millions of pounds, clean basis. All income data are in billions of U. S. dollars, converted at the then current exchange rates. The seasonal wool price variable is a weighted average of the prices of Merino and crossbred quality wools, deflated by the Sauerbeck commodity price index and converted into U. S. cents per pound, clean basis. All other prices are converted to U. S. prices.