

A BARGAINING MODEL FOR
INDUSTRIAL WAGE DISPUTES

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Introduction:

Consider a firm F seeking to negotiate the settlement of a strike against itself by a labor union U . Suppose the union to be made up of many members, all of whom do the same job and receive the same hourly wage. And assume that hourly wage to be the only item of contention, so that "fringe benefits" may be disregarded. For then the negotiations take a particularly simple form: At time t , the union makes a wage demand $\delta(t)$ and F makes a wage offer $w(t)$. And if $\delta(t) \leq w(t)$ the strike is over and the union members go back to work. Or if $w(t) < \delta(t)$, the strike continues until the first instant $t = T$ for which $\delta(t) \leq w(t)$. There will be such a first instant if the functions $\delta(t)$ and $w(t)$ are suitably restricted.

1. Behavioral Assumptions. To motivate the restrictions we wish to place on $w(t)$, let us suppose that U has, unknown to F , unlimited strike funds. Then in theory, the union leaders have only to present their demand $\delta(t) \equiv \Delta$, and wait until the management of F agrees to it; the members of U will never object since U can always provide satisfactory strike wages. But in fact there are some demands that F will never agree to. Let W be the least such demand; we may think of W as a "break even"

point for F . And by a proper choice of the monetary unit, we may assume $W = 1$.

If U chooses its non-negotiable demand $\delta(t) \equiv \Delta$ to be no less than 1, the strike will never end, and F will go out of business. Such things have happened. But if U demands $\Delta < 1$, we would expect the strike to be settled eventually. It is our basic assumption that, if Δ is almost 1, the strike will last a very long time.

To make this notion precise, we shall assume $w(0)$, the firm's initial wage offer, to be an institutionally given number $a < 1$. It is equal to the last offer made by F before the walkout. Also, we shall assume that $w(t)$ is the solution of a differential equation of the form

$$(1.1) \quad \frac{dw(t)}{dt} = c(t)(1-w(t))^2;$$

where $c(t)$ is some moderately regular function of t , whose value is chosen by F , at each instant $t \geq 0$, subject to the constraints

$$(1.2) \quad 0 \leq c(t) \leq C.$$

Hereafter, we shall refer to any such function $c(t)$ as a "concession by F ", or more simply, "a concession".

If we allow $c(t)$ to vary over all possible concessions defined on the infinite interval $0 \leq t < \infty$, we generate all the allowable functions $w(t)$. And these fill a region $A(a)$, depending on a , in the (t, w) -plane. A typical set $A(a)$ is indicated in figure 1.

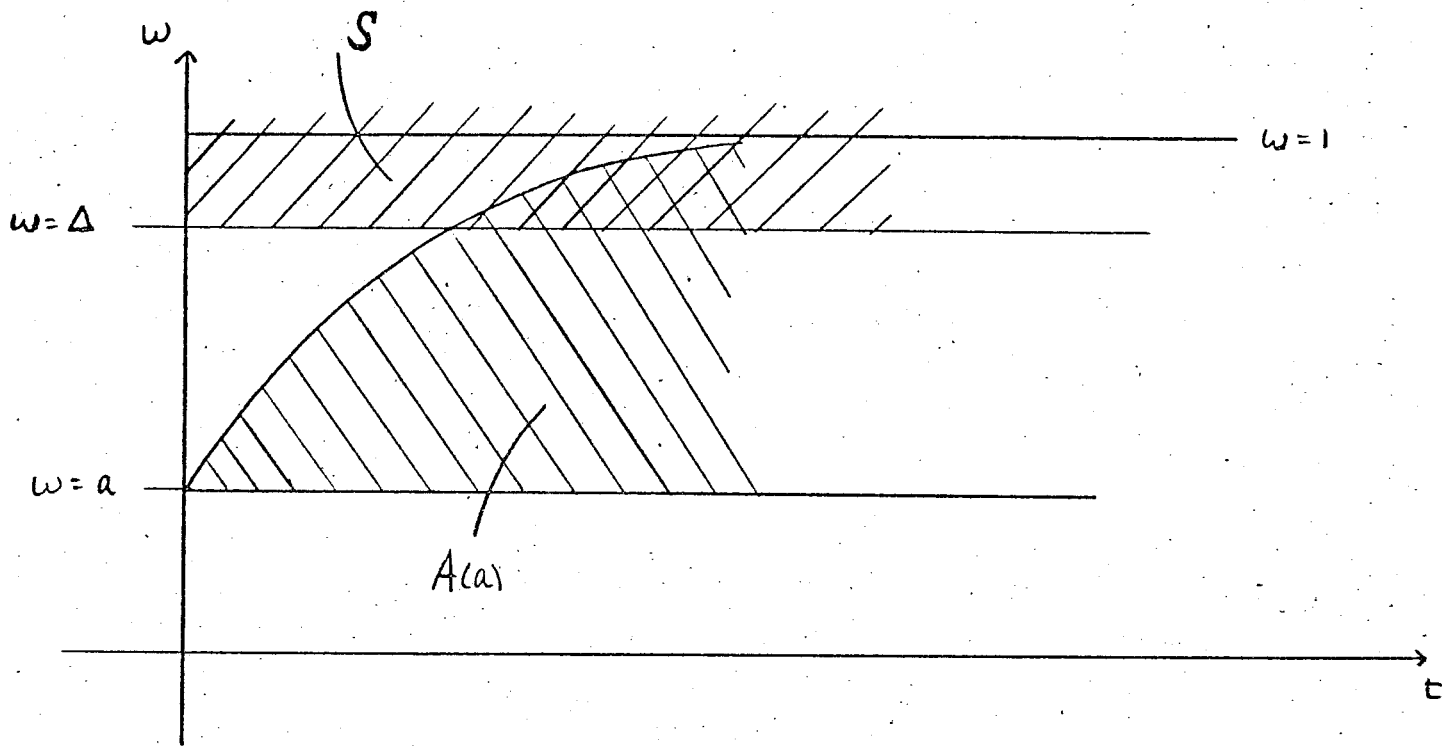


Figure 1

It is bounded below by the line $w(t) \equiv a$ and above by the curve

$$(3) \quad w(t) = 1 - (Ct + (1-a)^{-1})^{-1},$$

corresponding to the concession $c(t) \equiv C$. A point $P = P(t, w)$ of $A(a)$ is called an "attainable outcome", and $A(a)$ itself will be called the "attainable set".

Next, assume that F can rank all the possible outcomes P in $A(a)$, and indeed, in the entire region

$$(1.4) \quad R = \{P = P(t, w) : 0 \leq w \leq 1 \text{ \& } t \geq 0\}$$

by means of a cost function $K(P) = K(t, w)$ which is well and smoothly defined there. Presumably, both $\frac{\partial K}{\partial w}$ and $\frac{\partial K}{\partial t}$ are positive in R , so that the level curves of K have negative slope. Also, we assume $K(t, 0)$ to be unbounded.

If the leaders of U should now announce that they will agree to any outcome P in some closed subset S of R , but to no others, it is clear what F 's behavior should be; F should determine a point P^* in $S \cap A(a)$, at which K attains its minimum, and choose a concession which drives the system (1.1) to P^* . And if S is the upper half plane $w \geq \Delta$, as shown in figure 1, the minimum of K over $S \cap A(a)$ occurs in the lower left-hand corner of that intersection. So F should choose $c(t) \equiv C$. In short, when confronted by a sufficiently wealthy or obdurate union, a firm has no choice but to meet any demand $\Delta < 1$, and the quicker they do it, the better.

Our point is that, even with the deck so stacked against it, a firm will fight! It will not agree immediately to any demand $\Delta < 1$, but will force the union to "show its strength" by holding out for a certain length of time first. And the length of time will be great if Δ is nearly 1.

In our simple model, we may even calculate the length of time required for F to increase its offer from w_0 to Δ . It is

$$(1.5) \quad f(w_0, \Delta) = \frac{\Delta - w_0}{C(1-\Delta)(1-w_0)}$$

We note in particular that $f(a, \Delta)$ becomes infinite as Δ nears unity, and is a decreasing function of C .

Happily for the general business outlook, few unions are either as wealthy or as intransigent as the one we have been discussing. For in practice, when strikes last too long, the union members become

restive and force their leaders to reduce the demand. But to understand clearly how this may come about, we must inquire as to the motives of the union leadership. The view we shall express here leans heavily on the position of A. M. Ross [4], and more directly, on that of O. Ashenfelter and G.E. Johnson [1]. In essence, we shall assert that the objectives of the union leaders are distinct from those of the members of U , but are accomplished more often than not, by satisfying the expectations of the union rank and file. The distinction will be essential for all that follows.

At any time t , during the strike, the workers have a median expectation $e(t)$, which represents the wage they expect the strike to win for them. It is related, we shall assume, to its initial value $e(0) = e_0$ and to F's current offer $w(t)$ by

$$(1.6) \quad e(t) = e_0 e^{-\rho t} + (1 - e^{-\rho t})w(t)$$

Thus, regardless of the members' initial expectation e_0 , their current expectations decrease (for we assume, naturally, that $e_0 > w(0) = a$) at an exponential rate to the company's current offer. And their "disappointment" in any outcome $P_f(T, w(T))$, which is the name we shall give to the quantity

$$(1.7) \quad e(T) - w(T) = (e_0 - w(T))e^{-\rho T}$$

may be made arbitrarily small at any w , just by choosing T large enough. But it vanishes only if $w(T) = e_0$. Naturally the leaders of U will try to obtain an outcome which the members will not find too disappointing. And we will adopt the convention that $\delta(t) \leq e_0$ always, so the strike ends automatically if ever $w(t) = e_0$. It is clear that, without loss of generality, we may choose the unit of time so that $\rho = 1$.

The union leaders dare not hold out too long in their effort to make (1.7) small. For if after a certain time t_0 , the strike wrests no further concessions from F (-i.e. $c(t) \equiv 0$ for $t \geq t_0$), or only small ones, the union members will regret the failure of their leaders to sign the contract offered them at time $t = t_0$. And they will be responsive to a candidate for the union presidency who, before the next election, looks back over the history $w(t)$, $0 \leq t \leq T$ of the negotiation, and campaigns on the platform that he would have agreed to the contract offered at the instant τ for which the quantity $w(\tau)(T-\tau)$ is largest, thus save the workers that amount in lost wages. So the union leaders must endeavor to limit not only the members' disappointment term (1.7), but also their "regret".

$$(1.8) \quad \max_{0 \leq \tau \leq T} w(\tau)(T-\tau)$$

We emphasize that regret is not simply a function of the final outcome $(T, w(T))$, but depends on the entire negotiation history. The fact that it can be computed only thru hindsight does not mean that the union leaders may ignore it.

Now suppose that, at the end of the strike, there were a point $P_0(t_0, w(t_0))$ on the graph of the history $w(t)$; $0 \leq t \leq T$, at which both the disappointment (1.7) and the regret (1.8) had smaller values than at the final outcome $P_f(T, w(T))$. Then a candidate in the next election could claim with great effect that he would have signed at about t_0 , and so have improved on the incumbent's performance by either index. For if the difference $T-t_0$ is at all large, that claim becomes entirely plausible.

But had the union leaders been able to choose the termination time T so that, for some $m > 0$, the function

$$(1.9) \quad J(t, w(t)) = m(e_0 - w(t))e^{-t} + \max_{\tau} w(\tau)(t-\tau)$$

had a minimum at $t = T$, along the graph G of $w(t)$; $0 \leq t \leq T$ in R , there could be no points P_0 on G which are preferred to P_f by both indices. So challenging candidates would be able to attack only the incumbent's choice of the weighting number m , a less damaging kind of attack by far.

So let us assume, for the moment, that the union leaders have selected a weighting m . Then U and F are the players in a differential game.¹ A strategy for F is a function $\mathcal{L}(t, \delta)$ defined on R and taking values in the interval $[0, C]$. And a strategy for U is a pair (S, \mathcal{D}) consisting of a closed subset S of R , called the set of "acceptable outcomes", and a function $\mathcal{D}(t, w)$ from T into $[0, 1]$.

To play the game, U observes $w(t)$ at each instant t , and demands $\delta(t) = \mathcal{D}(t, w(t))$. And F responds by choosing $c(t) = \mathcal{L}(t, \delta(t)) = \mathcal{L}(t, \mathcal{D}(t, w(t)))$, so that the negotiation history $w(t)$; $0 \leq t < \infty$ is the solution of the initial value problem

$$(1.10) \quad \frac{d}{dt} w(t) = \mathcal{L}(t, \mathcal{D}(t, w(t)))(1-w(t))^2; \quad w(0) = a.$$

Play continues in this manner until the first instant T for which $P_f(T, w(T))$ lies in S . Then, as the outcome P_f is acceptable

¹A theory of differential games does exist, and is described in [2] and [3]. But it is inapplicable here for a host of reasons, so we shall be forced to proceed by ad hoc methods.

to U , it signs the contract offered by F at time T , and the members of U go back to work for the hourly wage $w(T)$.

We shall require that U always choose a demand policy which is monotone in the sense that $\mathcal{D}(t,w) \leq \mathcal{D}(t',w')$ when $t \geq t'$ and $w \geq w'$. For then $\delta(t) = \mathcal{D}(t,w(t))$ can never increase along any solution $w(t)$ of (1.10), and we may interpret the demand $\delta(t_0) = \delta_0$ as an offer by U to go back to work, at any time $t \geq t_0$, for the wage δ_0 . The offer cannot be withdrawn at a later date.

Indeed, since U knows from (1.5) that the demand δ_0 can be met at $t = t_0 + f(w(t_0), \delta_0)$, F may conclude from $\delta(t_0) = \delta_0$ that S contains at least those points $P(t,w)$ for which

$$(1.11) \quad t \geq t_0 + f(w(t_0), \delta_0) \quad \text{and} \quad w \geq \delta_0.$$

The gamble always is, of course, that S may also contain points $P'(t',w')$ such that $K(P') < K(P)$, but which U has not chosen to reveal thru its demands $\delta(\tau)$, $0 \leq \tau \leq t$.

If the offer to go back to work at time t_0 were good only until $t_0 + h$, it would reveal less about the contents of S . In fact, if $h < f(w(t_0), \delta_0)$, the offer reveals nothing about S , as the demand cannot be met by the deadline h . And though we have eliminated them here by assuming \mathcal{D} monotone, we doubt that deadlines $h < f(w(t_0), \delta_0)$ are unheard of in practice.

We shall require also that \mathcal{D} "progressively reveal" S , by which we shall mean that $\mathcal{D}(t,w)$ is the least number δ such that

$P(t + f(w, \delta), \delta)$ lies in S . If \mathcal{D} progressively reveals S , then $\mathcal{D}(t, w) - w$ is the largest wage increase U can still hope to win, given that U must agree to any outcome P_f in S , and that F has increased its offer only to w by time t . Clearly the set S determines the function \mathcal{D} which progressively reveals it in a unique way. And the only sets S which may be revealed by monotone functions \mathcal{D} are the epigraphs¹ of non-increasing functions.

By way of a solution for the game between U and F , we shall seek a pair $(\mathcal{L}^*, (S^*, \mathcal{D}))$, as well as the solution $w^*(t)$ of (1.10) and an outcome $P_f^* = P_f^*(T^*, w^*(T^*))$ which correspond thereto, such that

$$(1.12) \quad K(P_f^*) \leq K(P_f)$$

for every point P_f in S^* , and

$$(1.13) \quad J(T^*, w^*(T^*)) \leq J(t, w^*(t))$$

for every point $P(t, w^*(t))$ on the graph of $w^*(t)$. In short, we seek a Nash equilibrium point in the class of admissible pairs (S, \mathcal{D}) and functions \mathcal{L} .

The reasons why U and F should seek an outcome P_f^* with the properties (1.12) and (1.13) have already been discussed. By that discussion, we have reduced a situation involving several distinct "coalitions" namely the firm's management, the union's incumbent leadership, would-be leadership, and rank and file membership (and possibly the stockholders in the firm as well), to a two player game.

¹The epigraph of a function $f(x)$ is the set $\{(x, y): y \geq f(x)\}$.

But we do not assert that the eliminated players have no role; it is they who determine the type of solution which the remaining players must endeavor to find.

2. The Nash Equilibria: In view of our earlier discussion of wealthy unions, one might expect equilibrium points of the form $\mathcal{L}^*(t, \delta) \equiv C$, $\mathcal{S}^*(t, w) \equiv \Delta$, and $S^* = \{(t, w) \in R: w \geq \Delta\}$ ¹ to exist. And indeed if the minimum of the function (1.9) along the curve (1.3) is located at (t_0, w_0) , the choice $\Delta = w_0$ yields such an equilibrium point. For, as the reader may readily verify, the conditions (1.12) and (1.13) are then satisfied. So let us determine the location of the minimum of (1.9) along a curve of the type

$$(2.1) \quad w(t) = 1 - (ct + \alpha)^{-1},$$

of which (1.3) is the special case $\alpha = (1-a)^{-1}$.

On such a curve, both $e_0 - w(t)$ and e^{-t} are positive and decreasing, so that (1.7) is too, while (1.8) increases without bound. Hence the graph of the function (1.9) should have roughly the shape indicated in figure 2.

If we are to minimize (1.9), we must first ask where, for a given T , the quantity $A(T, \tau) = w(\tau)(T-\tau)$ attains its maximum.

¹It is clear that \mathcal{L}^* reveals S^* progressively.

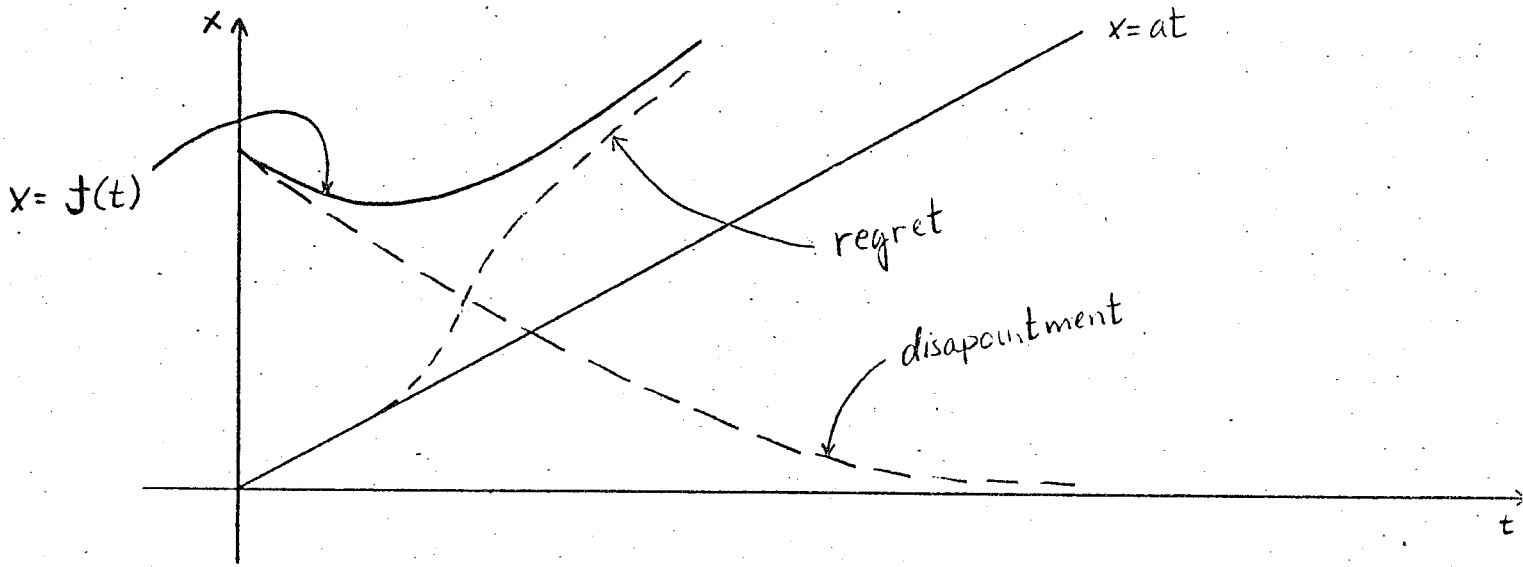


Figure 2.

$$\begin{aligned}
 A_{\tau}(T, \tau) &= \frac{\partial A(T, \tau)}{\partial \tau} = \frac{dw(\tau)}{d\tau} (T - \tau) - w(\tau) \\
 &= C(1 - w(\tau))^2(T - \tau) - w(\tau) = \frac{(C\tau + \alpha) - (C\tau + \alpha)^2}{C(\tau + \alpha)^2}
 \end{aligned}$$

is a decreasing function of τ on the interval $0 \leq \tau \leq T$, and is equal to $-w(T) < 0$ when $\tau = T$. So $A(T, \tau)$ attains its maximum over that interval when $\tau = 0$, if $A_{\tau}(T, 0) = (CT + \alpha - \alpha^2)/\alpha^2 \leq 0$, and in the interior otherwise. Thus $A_{\tau}(T, 0)$ is a linear function of T with positive slope, and there is a first $T = T_{\alpha}$ for which it is non-negative. $P_{\alpha}(T_{\alpha}, w(T_{\alpha}))$ is the last outcome on the history (2.1) for which, if the contract is signed there, the workers' regret will date from the day of the walkout, rather than from a time during the strike.

To determine the locus of the outcomes $P_{\alpha}(T_{\alpha}, w(T_{\alpha}))$, we suppose that the negotiations have followed the course (2.1) to an outcome $P_f(T, w(T))$ in R . Then, solving (2.1) for α yields

$$(2.3) \quad \alpha = \frac{1 - CT(1-w(T))}{1-w(T)},$$

so that

$$(2.4) \quad CT + \alpha - \alpha^2 = \frac{1 - (2CT+1)(1-w(T)) + C^2T^2(1-w(T))^2}{(1-w(T))^2} \geq 0$$

if and only if

$$(2.5) \quad \frac{2CT+1 - \sqrt{4CT+1}}{2C^2T^2} \leq 1-w(T) \leq \frac{2CT+1 + \sqrt{4CT+1}}{2C^2T^2}$$

That is, $A(T, \tau)$ attains its maximum over the interval $0 \leq \tau \leq T$ at $\tau = 0$ only if $P_f(T, w(T))$ lies above the graph of

$$(2.6) \quad w = 1 - \frac{2CT+1 - \sqrt{4CT+1}}{2C^2T^2}$$

in \mathcal{R} . This situation is indicated in Figure 3, where the area under (2.6) is shaded over.

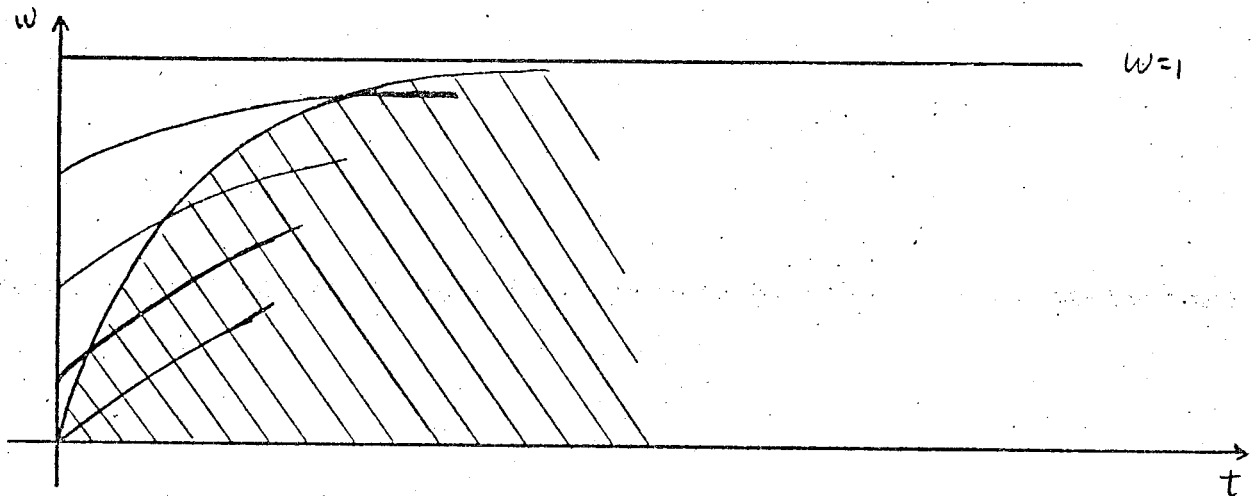


Figure 3

It is clear from the linearity of $A_\tau(T,0)$, that the curves (2.1) must pass into the shaded zone and remain there.

Next, let us suppose that (2.1) passes under (2.6) so quickly that the payoff (1.9) is still decreasing at the time of passage. Then $A(T,\tau)$ attains its maximum at the instant $\tau = \varphi(T)$ for which $(C\tau + \alpha)^2 = CT + \alpha$ or, equivalently, when

$$(2.7) \quad w(\tau) = 1 - (1-w(T))^{1/2}$$

And (1.9) becomes

$$(2.8) \quad J(T, w(T)) = m(e_0 - w(T))e^{-T} + A(T, \varphi(T)) = J_1(T),$$

a function of T alone. So (2.8) attains its minimum when

$$\begin{aligned} \frac{d}{dT} J_1(T) &= m \frac{d}{dT} (e_0 - w(T))e^{-T} + \frac{d}{dT} A(T, \varphi(T)) \\ &= A_T(T, \varphi(T)) + A_\tau(T, \varphi(T)) \frac{d\varphi(T)}{dT} - m(e_0 - w(T))e^{-T} - m \frac{dw(T)}{dT} e^{-T} \\ (2.9) \quad &= w(\varphi(T)) - m[e_0 - w(T) + C(1-w(T))^2] e^{-T} \\ &= 1 - \sqrt{1-w(T)} - m[C(1-w(T))^2 + e_0 - w(T)]e^{-T} \end{aligned}$$

changes sign. The locus of points $P_f(T, w(T))$ where this takes place is a curve $\Gamma_1 = \Gamma_1(m)$ which crosses the line $w=1$ at $(\log m(e_0 - 1), 1)$ if $e_0 > 1$ (but does not if $e_0 \leq 1$), slopes downward to the right, and approaches the line $w=0$ asymptotically. Γ_1 is

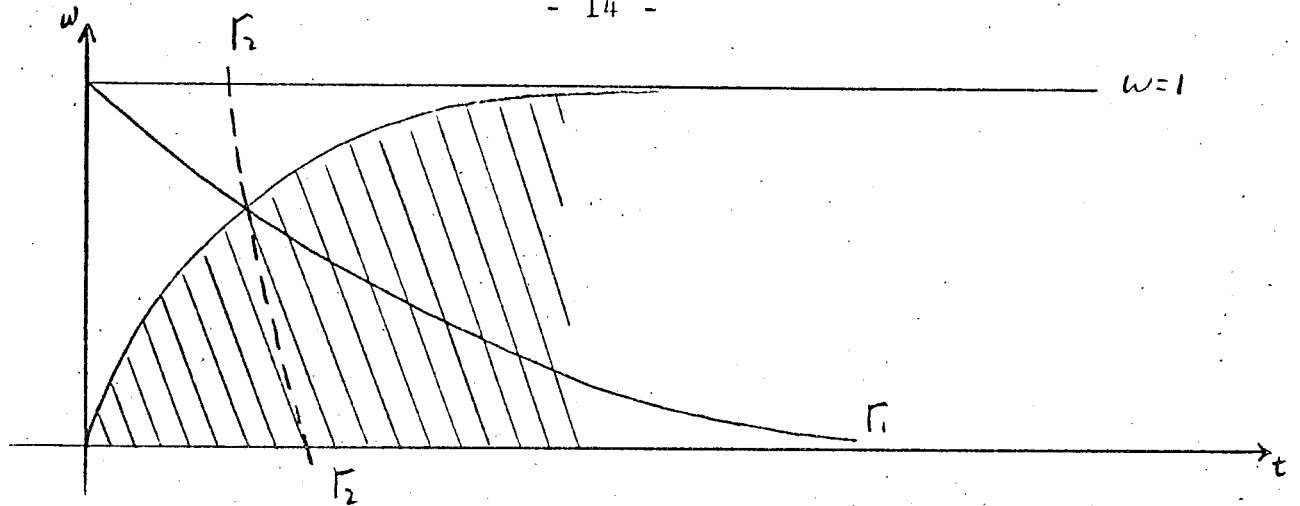


Figure 4

shown for the case $e_0 = 1+1/m > 1$ in figure 4. The broken curve Γ_2 will be explained presently.

Now recall that the curve (2.1) thru the origin corresponds to the parameter value $\alpha=1$, and the curves for which $0 < w(0) < 1$ correspond to parameters $\alpha > 1$. Let α_0 be the parameter of the curve (2.1) which passes thru the intersection of Γ_1 with the graph of (2.6). Then for $1 \leq \alpha \leq \alpha_0$, the minimum of (1.9) along the path (2.1) occurs at the intersection of that path with Γ_1 . But along paths for which $\alpha > \alpha_0$, (1.9) has the form

$$(2.10) \quad J(T, w(T)) = m(e_0 - w(T))e^{-T} + aT = J_2(T)$$

rather than the form (2.8). So the minimum is attained at the instant when

$$(2.11) \quad \frac{d}{dT} J_2(T) = a - m[C(1-w(T))^2 + e_0 - w(T)]e^{-T}$$

changes sign. And the locus of points $P_f(T, w(T))$ where this takes

place is the curve $\Gamma_2 = \Gamma_2(m)$ shown in figure 4. Γ_2 crosses $w=1$ at $(\log m(e_0 - 1)/a, 1)$, lies above Γ_1 in the unshaded portion of R , and below Γ_1 in the shaded portion. With the help of Γ_1 and Γ_2 , it is easy to construct equilibrium points of the desired form.

If $a > a_0 > 1 - 1/\alpha_0$, we take for S^* the intersection of all those closed upper half-planes which contain the point $P^*(m)$ at which (1.3) meets $\Gamma_2 = \Gamma_2(m)$. And if $a \leq a_0$, we take $P^*(m)$ to be the intersection of (1.3) with Γ_1 , then form S^* as before. We leave it for the reader to verify that there are no other such equilibrium points.

Here, a few words on the proper choice of the weighting number m are in order. For it is not necessary that the U-leaders choose m exactly on the day of the walkout. Rather, they may decide on an upper bound $M \geq m$, and make their demand equal to $\Delta^*(M)$, the ordinate of $P^*(M)$. Then if, during the course of the game, it becomes apparent that they should have chosen $m < M$, they may simply reduce their demand to $\Delta^*(m)$ as the point $(t, 1 - (Ct + (1-a)^{-1})^{-1})$ nears $P^*(m)$, since the latter is always reached before $P^*(M)$ along (1.3).

If the coordinates of $P^*(m_1)$ and $P^*(m_2)$ are (t_1, w_1) and (t_2, w_2) respectively, U can decide whether m_1 is a better choice than $m_2 > m_1$ or not by asking the members "would you rather go back to work at time t_1 at wage w_1 , or hold out for another $t_2 - t_1$ days to obtain a further increase of $w_2 - w_1$ dollars an hour?" And if enough pairs (m_1, m_2) are compared in this way, the union leaders

will be able to make a very educated guess as to the proper value of the weighting number m .

But it seems unlikely that U could ever, in practice, enforce an outcome of the type $P^*(m)$, however well m were chosen. For if unions can be intransigent, so can management. Indeed they may even seek to enforce an outcome Q_f lying on the line $w=a$, for there is an equilibrium of this sort as well. To find it, observe that if F puts $c(t) \equiv 0$, then $w(t) \equiv a$, and (1.9) takes the form

$$(2.12) \quad J(T, w(T)) = m(e_0 - a)e^{-T} + aT = J_3(T).$$

So the function $J_3(T)$ also has roughly the form indicated in Figure 2, and attains its minimum when

$$(2.13) \quad \frac{d}{dT} J_3(T) = a - m(e_0 - a)e^{-T} = J'_3(T, a)$$

changes sign. And the locus of points at which this takes place is the curve $\Gamma_3 \doteq \Gamma_3(m)$ shown in Figure 5, again for the case $e_0 = 1 + 1/m$.

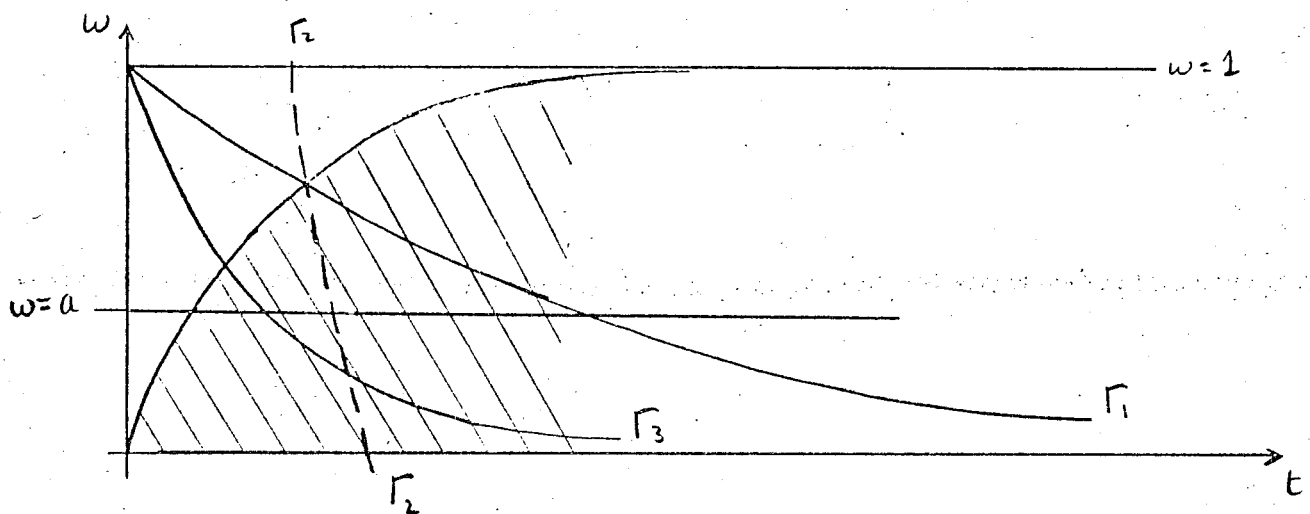


Figure 5

The verification that Γ_3 really does lie beneath Γ_1 , as shown, will be omitted. It is clear that both Γ_1 and Γ_3 meet the line $w=a=1$ at $(\log m(e_0-1), 1)$. And it is also clear that, if F really does put $c(t) \equiv 0; 0 \leq t < \infty$, U can do no better than to sign the contract offered when the point (t, a) reaches $Q^* = Q^*(m)$ where $w=a$ crosses $\Gamma_3(m)$.

Whether or not such behavior is optimal for F depends on the nature of the set S which U has chosen. For instance $c(t) \equiv 0; 0 \leq t < \infty$ is not optimal against the set S shown (shaded) in Figure 6, because S contains points which lie below Λ_Q^* , the level curve of K which passes thru Q^* .

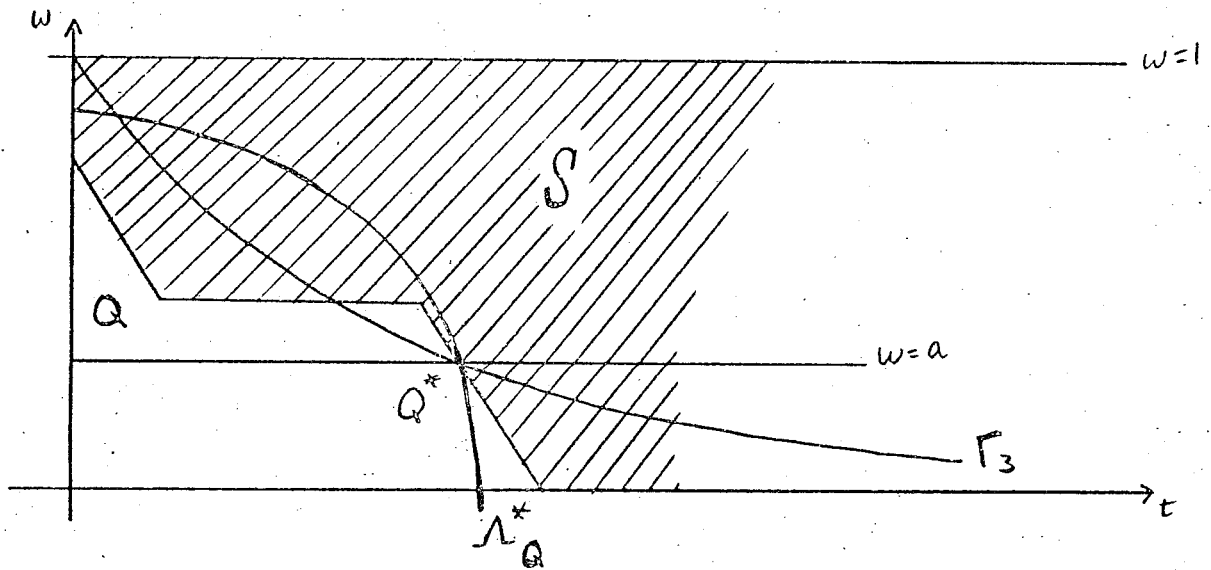


Figure 6

So if F had made only a small concession at the beginning of the strike, the strike could have been settled at the cost $K(Q) < K(Q^*)$.

Indeed, it seems likely that many firms enter contract negotiations believing that Q^* is the worst outcome that can result. So they choose a level curve Λ_Q inside Λ_Q^* , and wait for U to reveal thru its demand policy \mathcal{D} , a point Q in S which lies on or inside Λ_Q . And presumably they inform U (perhaps thru secret channels) of the particular Λ_Q they have chosen, on the theory that U will then try to persuade them to settle at some Λ' lying between Λ_Q and Λ_Q^* , etc. In short, F would probably feel that its ability to enforce the outcome Q^* guarantees an outcome lying on or below Λ_Q^* .

And it seems almost equally certain that unions have entered strikes feeling that they could enforce an outcome of the type $P^* = P^*(m)$, and so should earn the wage $\Delta^*(m)$. But in fact both are wrong; neither party can enforce either type of outcome!

To see why not, suppose that U actually does open negotiations by offering to go back to work at any time for the wage $w = \Delta^*(m)$, but for no lesser figure. And suppose that F responds by refusing to make any concessions at all until U reveals, thru its demands, the presence of a point Q in S which lies beneath Λ_Q^* . Then after a few days, U 's demand will still be $\delta(t_0) = \Delta(m)$ and F 's offer will still be just $w(t_0) = a$. And the still attainable outcomes will no longer fill the entire set $A(a)$, but only the part of it lying beneath the curve (2.1) which passes thru (t_0, a) . So even if F should switch, when $t = t_0$, from $c(t) \equiv 0$ to $c(t) \equiv C$, U can no longer hold out until $w(t) = \Delta^*(m)$, or even until $(t, w(t))$ reaches Γ_1 . Rather the union leaders must agree to terms when the functional (1.9) attains its minimum.

So let us minimize (1.9) along a history $w(t)$; $0 \leq t < \infty$ of the form shown in figure 7. It is clear that to maximize $w(\tau)(T-\tau)$, τ must be zero for a large range of values T . And for T in that range, (1.9) has the form

$$(2.14) \quad J(T, w(T)) = m(e_0 - w(T))e^{-T} + aT = J_2(T),$$

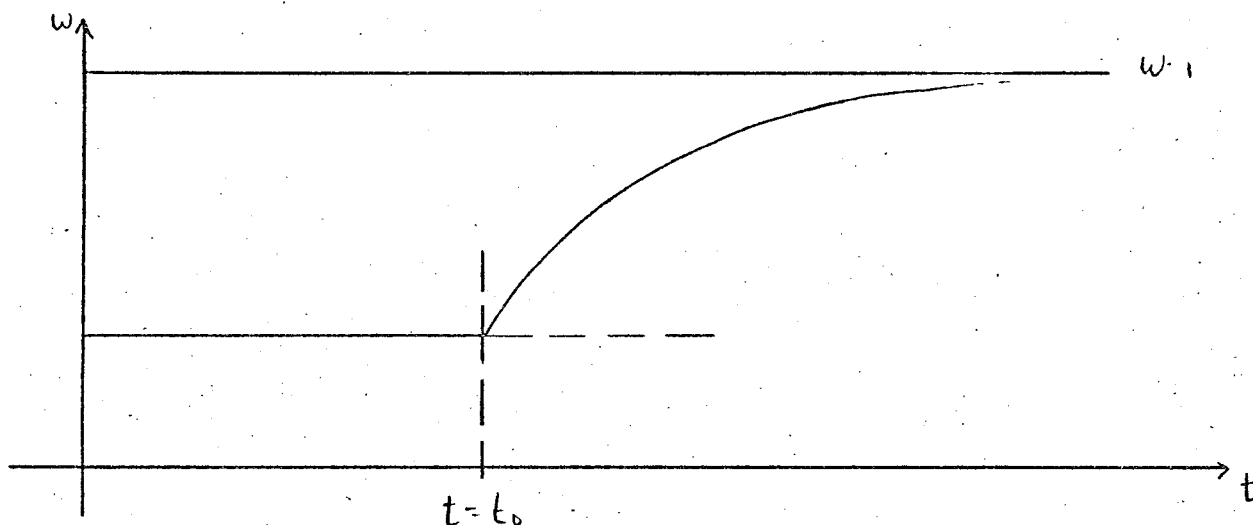


Figure 7

which is identical with (2.10). Hence (2.14) attains its minimum where the graph shown in figure 7 crosses $\Gamma_2 = \Gamma_2(m)$. We note that Γ_1, Γ_2 , and Γ_3 meet the line $w = a$ when

$$T_1 = \log m (e_0 - a + c(1-a)^2) / (1 - \sqrt{1-a}),$$

$$T_2 = \log m (e_0 - a + c(1-a)^2) / a, \quad \text{and}$$

$$T_3 = \log m (e_0 - a) / a,$$

respectively, so that their relative positions are as indicated in Figure 5, and again in Figure 8.

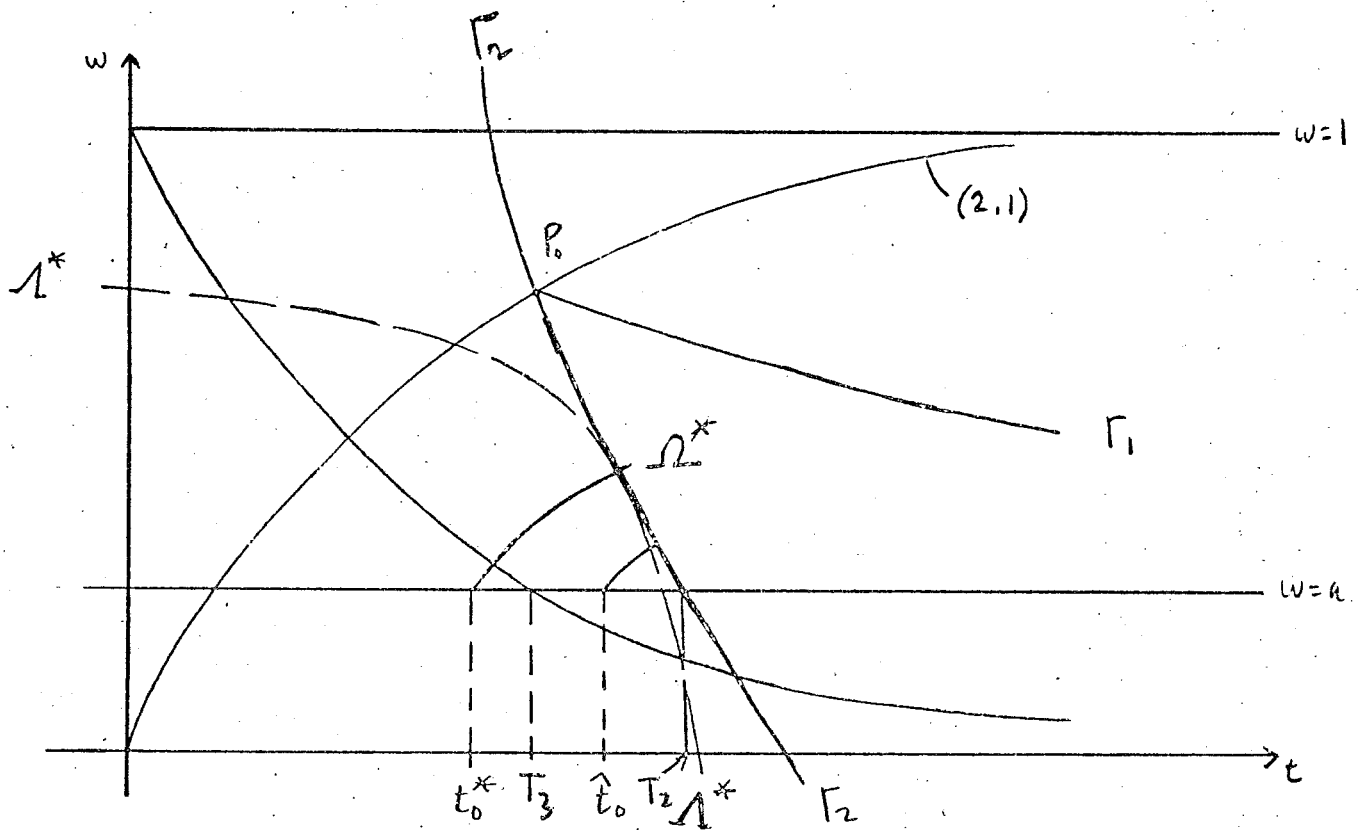


Figure 8

We interpret the figure as follows: If t_0 is not too much smaller than T_2 , and if $P_{t_0}(T(t_0), w(T(t_0)))$ is the point of intersection of the graph of $w(t)$ with Γ_2 , then τ must vanish to maximize $w(\tau)(T(t_0) - \tau)$. So (1.9) has the form (2.14) along the history $w(t)$, and has a local minimum at P_{t_0} . That is, $w(T(t_0)) - a$ is the largest wage increase U can still hope for at time t_0 , if $w(t_0) = a$. And if U wants that wage increase, it should certainly inform F that P_{t_0} is in S by setting $\delta(t_0) = w(T(t_0))$. To determine the desirability of P_{t_0} to U , it is helpful to sketch the graph of the payoff (1.9) along the history $w(t)$ shown in Figure 7. This has been done in Figure 9, for the case $T_3 < t_0 < T_2$,

which is the case of interest. On $0 \leq t \leq t_0$, (1.9) has the form (2.12) and takes its minimum at $t=T_3$. But on $t \geq t_0$, (1.9) is of the form (2.14) and has a minimum at $t = T(t_0)$, the instant at which $(t,w(t))$ crosses r_2 . Also, (1.9) is everywhere continuous along $w(t)$ and has a continuous derivative everywhere except at $t=t_0$, where there is a jump discontinuity of $m C(1-a)^2 e^{-t_0}$

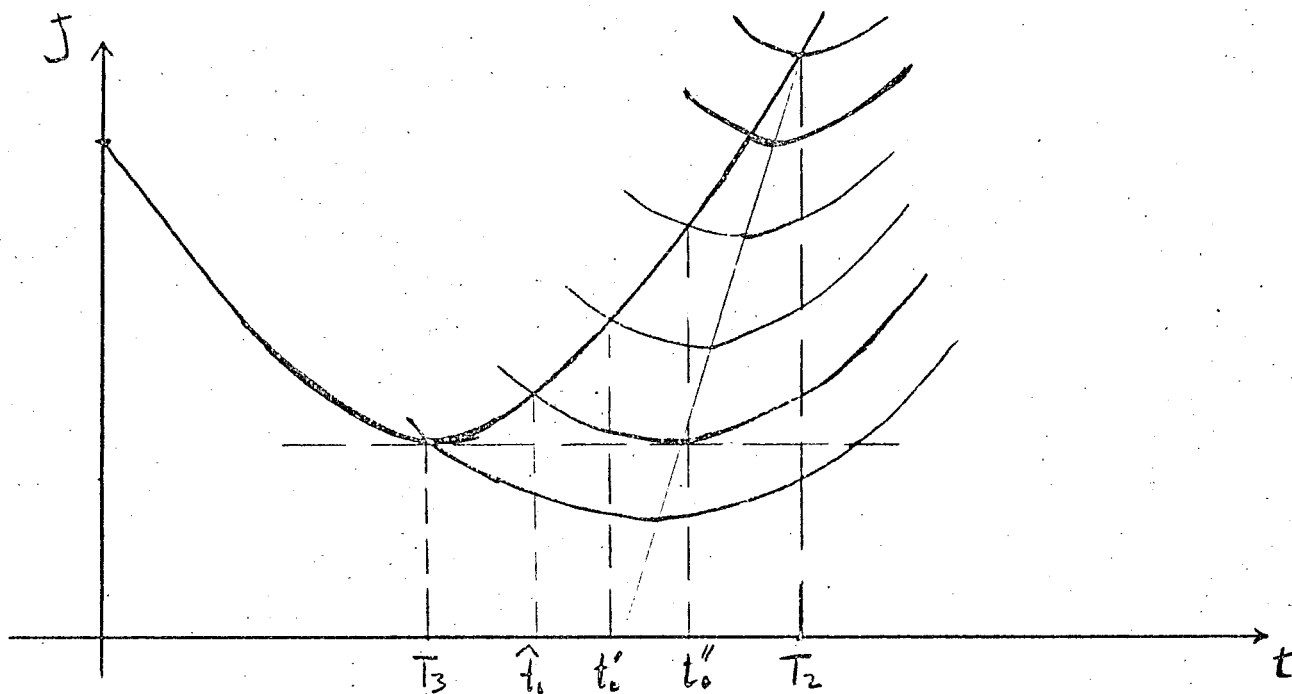


Figure 9

Thus if $T_3 < t_0 < T_2$, U's payoff has local minima at $t = T_3$ and $t = T(t_0)$. If t_0 is near T_3 , $T(t_0)$ is the better stopping time. But if t_0 is too large, T_3 is the absolute minimum. And there is a \hat{t}_0 (shown above) for which the minima are equivalent.

Thus U may sometimes allow the game point (t, a) to continue right on past the point Q^* shown in Figure 6, even though Q^* is optimal for U along $w=a$. For U has reason to hope that F may be moved, at some time $T_3 < t_0 < \hat{t}_0$, to switch from the tactic $c(t) = 0$ to $c(t) \equiv C$, at least until $t = T(t_0)$. Indeed, U can usually offer F an inducement to do this.

The nature of that inducement is indicated in Figure 8. There is an innermost level curve Λ^* of K which meets Γ_2 , say at $\Omega^* = \Omega^*(m)$. And there is a last instant $t=t_0^*$ at which F may abandon the tactic $c(t) = 0$, and still reach Ω^* . So if $t_0^* < \hat{t}_0$, Ω^* will be an optimal outcome for U , as well as for F . That is, if U chooses S^* to contain all those outcomes $P(t, w)$ lying above $w = \Delta^*(m)$ or $\Gamma_2(m)$, and chooses $\mathcal{J}^*(t, w)$ to reveal S^* , while F chooses $\mathcal{L}^*(t, \delta)$ to guarantee $K(\Omega^*)$, and to secure a better outcome if one is revealed by U , the pair $(\mathcal{L}^*, (S^*, \mathcal{J}^*))$ will satisfy the conditions (1.12) and (1.13). Only P^* , Q^* , and Ω^* are equilibrium outcomes in this sense.

Next observe that, in Figure 8, the position of Ω^* depends precipitously on the shape of the level curves Λ of K . For if they are too flat, Ω^* will lie low on Γ_2 (perhaps even under $w=a$) and Ω^* will not be the absolute minimum of J along $w(t)$. Or if they are very steep (i.e. if F is very vulnerable to strikes), $\Lambda_p^* = \{P \in R: K(P) = K(P^*)\}$ inside Λ^* , so F will do best to put $\mathcal{L}(t, w) \equiv C$ and give in to the initial demand Δ^* . In this case, F 's opening offer $w(0) = a$ was too low. And in the former case U should never have gone out on strike at all.

The most striking of our conclusions is that F cannot enforce the outcome Q^* . For U simply will not sign a contract for $w=a$; there will always be a period $t_0 \leq t \leq T$ at the end of the strike (however brief) in which $c(t) \equiv C$, so that Γ_2 is the relevant terminal curve, rather than Γ_3 . In short, there will be a concession. And given that F must concede something, it should try to obtain the least costly concession outcome Ω^* (or conceivably P^*) to which U will agree.

Finally, we should point out that there is no need that the negotiations should approach the outcome Ω^* along a path like that shown in Figure 7. For as long as $w(t) \leq a T_{\Omega}^*/(T_{\Omega}^*-t)$ for every $0 \leq t \leq T_{\Omega}^*$, the functional (1.9) has the form (2.14), and attains its minimum at $\Omega^* = \Omega^*(T_{\Omega}^*, w^*(T_{\Omega}^*))$. So F can, if it likes, make its major concessions earlier in the game, and still achieve exactly the same outcome Ω^* . But if F makes $c(t)$ too large at the very outset, there may be a time when $w(t) > a T_{\Omega}^*/(T_{\Omega}^*-t)$, so that (1.8) exceeds at along $w(t)$, in which case U will be able to hold out for an outcome Ω' which is slightly more costly to F and more satisfying to the members of U . But, as the reader may verify, this possibility exists only during the very early moments of play. Therefore it seems most unlikely that F would ever err in such fashion.

Nor is there any need, in reality, for U to make demands $\delta(t)$ which faithfully reveal S . Indeed it is far more likely that U will perturb its demand strategy \mathcal{D} so as to make m seem larger and ρ smaller than they really are. But such efforts are unlikely to be of

much avail, if F makes judicious use of the so-called "Bullware tactics", whereby management seeks to infiltrate U . For if this can be done, F may ask the same questions for the determination of m that we earlier suggested the union leaders should ask. And in practice, it is likely often the case that F has a better knowledge of the parameters of (1.9) than does U .

Numerous improvements on the model are of course possible. For in reality, fringe benefits often play an important role in strike settlements, and there are usually several salary scales (apprentice, craftsman, master craftsman, etc.) to be considered. So $\delta(t)$ and $w(t)$ should be vectors rather than numbers. Also, (1.1) and (1.2) should be replaced by probabilistic assumptions, so that only the mean time (1.5) required for F to increase its offer from w_0 to Δ could be calculated, along with certain of the moments about that mean. But we feel that even the present very simple model suffices to demonstrate that, even if the negotiators are entirely rational (and quite well informed), strikes may well last a long time.

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13. ABSTRACT <p>A (differential) game-theoretic model for industrial wage disputes is presented and discussed in some detail. Three Nash equilibrium solutions are then obtained by elementary means, and two discarded because they are not enforceable. It is then argued that the remaining equilibrium point should be regarded as the "solution" of the game.</p>			