

AN ECONOMETRIC MODEL  
OF THE  
FLIGHT TO THE SUBURBS

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Econometric Research Program  
Research Memorandum No. 116

October 1970

This research was supported in part by the Urban Economics Group at Princeton University which is financed by the Ford Foundation and directed by William J. Baumol, and also in part by the National Science Foundation Grant 2799. Bradford's participation, computer time, research assistance, and typing facilities were supported by the Ford Foundation; Kelejian's participation and the distribution of this report were supported by the National Science Foundation.

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I. Introduction

The problems of U. S. central cities--rising crime rates, demoralization of public school systems, increasing dirt and ugliness, etc.--are traced by many observers to the decline in the number of higher income residents. This "worsening" of the central city income distribution is attributed to the movement of wealthier families to the suburbs, a process which is, in turn, induced or hastened by deteriorating conditions in the central cities. If this description is true, it is not far-fetched to say that the problem of U. S. cities is the self-feeding flight of the middle classes to the suburbs.<sup>1</sup> In this paper we present an econometric model of residential choice by high and low income families which enables us, among other things, to test for the existence of such feed-back relationships.

Our model takes as given the number of families in each of 87 large metropolitan areas in 1960, classified by income, and predicts the residential division of rich and poor between central

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city and suburbs. We use two-stage regression techniques to fit our two-equation model and obtain unusually good fits and highly significant coefficients. Our results are without exception consistent with our a priori expectations and, in particular, they strikingly confirm the existence and quantitative significance of the flight-from-blight phenomenon just described. In brief, the principal findings are:

1. A middle class family of given income is more likely to reside in the suburbs the higher was the percentage of central city families ten years earlier who were poor, the higher is the median family income of the urbanized area (a measure of rent gradient), the lower is the fiscal surplus generated for middle class families by the central city budget (an estimate of the net dollar value of the central city budget to a middle class family).
2. A poor family is more likely to live in the suburbs the greater is the proportion of the housing there that is old, the lower is the proportion of the central city housing that is old, and the lower is the fiscal surplus of poor families in the central city.
3. For neither the middle class nor the poor does the racial composition of the central city population appear to affect residential location once the other variables have been accounted for.

The strong, statistically significant influence of the percentage poor in the central city on the middle class clearly supports the

view that central cities are caught in a vicious cycle, whereby the more rapidly the middle class families move to the suburbs, the greater is the incentive for the exodus of those remaining. This feed-back relationship is both direct--the location of the middle class depends directly on the fraction poor in the central city--and indirect through the fiscal system--the fewer the middle class in the city the heavier the tax burden on all remaining families, especially the remaining middle class families. The relative power of the two effects is illustrated by the results of one of our simulation experiments in which we make fifty percent of the poor of an urbanized area into non-poor via, say, a negative income tax. The effect of this policy on the middle class population of an average city turns out to be an increase of about 4%<sup>in that population</sup>/if interaction through the fiscal variables in the model alone is considered; when direct interaction effects are accounted for the predicted increase in central city middle class population is nearly 24%.

There are some obvious policy implications of our results. For example, a national program to alleviate poverty, such as a negative income tax, is an important instrument for improving central cities. As the above illustration makes clear, the benefit to central cities is not simply in the form of a reduced poor population. The model predicts that reducing poverty generally will lead to a greatly increased concentration of all families in the central cities, especially middle class families, with associated social and economic advantages. Secondly, the sensitivity of both rich and poor to the fiscal surplus obtainable by locating in the central city confirms

the view that the attempt by a central city to maintain a redistributive budget may be self-defeating.<sup>2</sup> Thirdly, our simulation results suggest that increasing transfers of revenues from higher levels of government to central cities is likely to increase their populations, with the increase consisting predominantly of non-poor families. In an illustrative calculation, the effect of increasing such transfers by 150% (to about fifty percent of the central city budget) would be a 12% increase in central city population; the fraction middle class would increase from 47% to 49%, while the fraction poor would decrease from 23% to 21%.

These issues and others are dealt with in somewhat more detail in later sections of the paper. In the following sections we develop the complete model. In section II we discuss the data and the variables to be explained; in section III, the model itself; in section IV, the independent variables. Section V contains the results of estimation and section VI a set of quantitative implications of hypothetical policy variations. An appendix describes a number of experiments with alternative independent variables, as summarized in section V.

## II. The Data and the Dependent Variables

The model to be described is designed for fitting to cross-sectional data on 87 urbanized areas in 1950 and 1960. Essentially, these were the 87 most populous standard metropolitan statistical areas (SMSA's) in 1960.<sup>3</sup> The data, assembled from published U. S. Census sources, include income distributions, racial composition, educational attainment, housing stock characteristics, etc. for central city, SMSA

and urbanized area, as these are defined by the U. S. Census. Central cities are defined by political boundaries, and these are single fiscal units.<sup>4</sup> SMSA's are also defined by political boundaries, specifically, county lines. However, generally counties are of minor functional importance in the United States. For our purposes the urbanized area was deemed the appropriate overall unit. This is defined by the Census on the basis of a minimum population density required for the inclusion of a Census tract within the area. Hence, in principal, the population density at the outer perimeter of all urbanized areas is the same.

The fundamental "accounting" relationships of our model embody the exhaustive distribution of family populations (POP) into three income classes. Our attention is focussed on those at the two ends of the income distribution, the poor (P), and what we for convenience call the middle class (M), though this group includes the rich and very rich as well. Families neither poor nor middle class are labelled simply residual (R). Letting the suffix C stand for central city, S for urban fringe (suburbs), and U for urbanized area (for example PC = number of poor families in the central city), we have the following identities:

$$PC + RC + MC = POPC$$

$$PS + RS + MS = POPS$$

$$PU + RU + MU = POPU$$

(1)

$$PC + PS = PU$$

$$RC + RS = RU$$

$$MC + MS = MU$$

Taking PU, RU, and MU as given, our model attempts to predict PC and MC. It consists of two interrelated equations which explain MC/MU and PC/PU, the fractions of total urbanized area middle and poor classes residing in the central city. Given the known totals of these classes, the model enables us to calculate the absolute numbers of each residing in central city and urban fringe.

The income levels by which our classes are defined are derived from the total sample of families. The income defining poverty is the 20th percentile income of the sample, a level chosen to correspond roughly with the official definitions of poverty employed by the U. S. Government. The 20th percentile family income in our sample was approximately \$2100 in 1950, and \$3800 in 1960. Although we decided to work with a single definition of poverty we were interested in exploring the effects of different definitions of the middle class. Hence, we use a set of middle class variables, consisting of families above the 50th, 60th, 70th, 80th, and 90th percentile levels (of the entire sample of families). To prevent confusion, we stress that the definitions of income classes are sample-wide: a particular urbanized area might have 40 percent of its population classified poor (with income below the 20th percentile level of the entire sample).

### III. Model Specification

The first difficulty we confronted in using our cross-section data was controlling for the large range of "relative geographies." In some cases the central city essentially is the urbanized area;

in others, it covers only a small portion. In explaining what fraction of any sub-group of the urbanized area lives in the central city, obviously the percentage of area contained in the central city is highly relevant. It is generally accepted that the population density (resident families per unit land surface) declines with distance from the city center.<sup>5</sup> If we were to vary the boundaries of a given central city and plot the fraction of any population sub-group, say the poor,  $PC/PU$ , living there as a function of the fraction,  $F$ , of the urbanized area's surface enclosed within the varying central city boundaries, we should observe a relationship like curve b in figure 1. The curve should pass through the origin (a zero-area city has no population) and through the point  $F = 1$ ,  $PC/PU = 1$  (since if the central city encompasses all of the urbanized area's surface it also has all of the population). Because of the declining density, the curve should have a continually decreasing positive slope.

The precise character of the relationship thus traced will vary from area to area, and from population sub-class to sub-class. For a population perfectly evenly spread out over the entire urbanized area the curve would look like the straight line labelled a in Figure 1. For a population completely concentrated at the mid-point of the central city, the graph would look like "curve" c, with the fraction of the population living in the central city equal to 1 for all variations in central city boundaries. The position of the curve is a measure, then, of the density gradient of the population involved. Curve a corresponds to a zero decline in density as a function of distance from the city center while curve c corresponds to an infinite rate of density



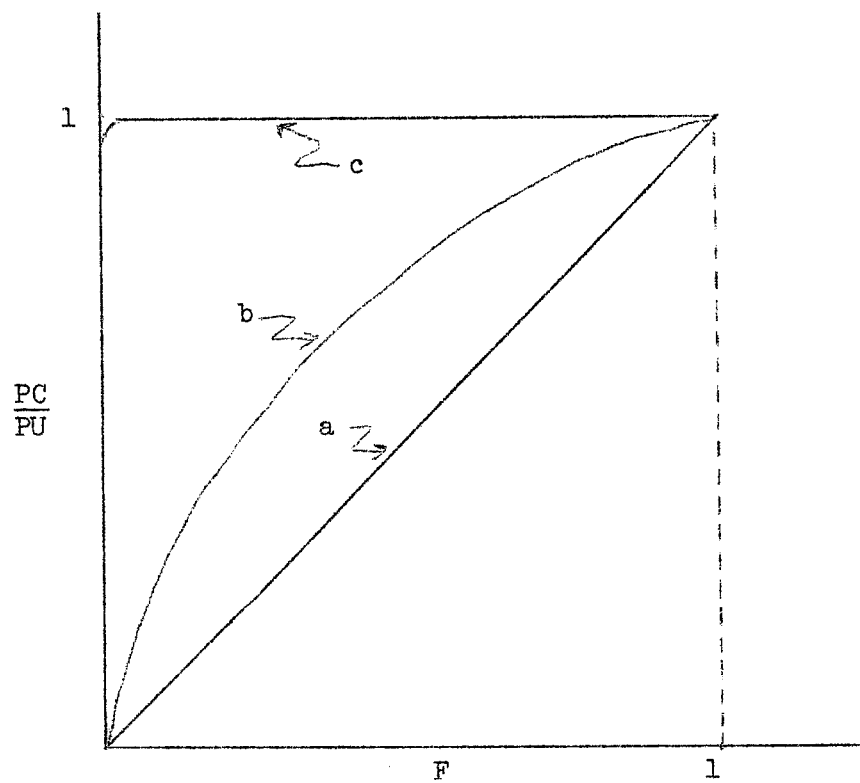


Figure 1

decline at the center.<sup>6</sup>

We assume therefore that both the fraction of the urbanized area's poor population living in the central city,  $PC/PU$ , and the fraction of its middle class population living in the central city,  $MC/MU$ , can be written as functions of the fraction of the urbanized area's surface enclosed by the central city boundaries,  $F$ . The parameters of these functions depend upon such factors as the area's income level, the character of its housing stock, etc. For any two areas alike in these respects, the difference in, say,  $PC/PU$  will be said to be attributable entirely to differences in  $F$ .

Because its members have the desired property of passing through the origin and the point (1,1) with positive but decreasing slope, we chose to work with the one-parameter family:

$$(2) \quad (\text{fraction of sub-population living in central city}) = F^\alpha,$$

$$0 \leq \alpha \leq 1 .$$

The parameter  $\alpha$  is a measure of dispersion of the population subgroup represented, and varies from area to area depending upon the characteristics of the particular city and urbanized area.<sup>7</sup>

This can be illustrated in terms of the flight from blight hypothesis. Let us suppose that in the absence of blight the middle class would be highly concentrated near the middle of all central cities. Let  $B$  be some index of blight. If there were no blight, the curve relating  $MC/MU$  to  $F$  would be the same for all cities, and approximated by curve  $c$  in Figure 1. This corresponds to parameter value  $\alpha = 0$ . For cities with a high blight index,

however, the middle class is more dispersed; the higher B, the flatter the curve relating MC/MU to F and so the higher the corresponding value of the parameter  $\alpha$ . Knowing MC/MU, F, and B for each city we could trace out the relationship between city characteristic (degree of blight) and parameter value from observed data. This, in simplified form, is the procedure we follow here.

Let MC50+, MC60+, etc., stand for the middle class family population of the central city by the 50th percentile definition, 60th percentile definition, etc., and let the notational convention be extended in the obvious way to other groups and locations.<sup>8</sup>

Then the basic structure of our model can be represented by:

$$\frac{MC_i}{MU_i} = \pi_i F_i^{\alpha_i}, \quad i = 50+, 60+, 70+, 80+, 90+,$$

(3)

$$\frac{PC20-}{PU20-} = \gamma F^\beta .$$

Here the multiplicative parameters  $\pi_i$  and  $\gamma$  are assumed the same for all cities (and should be unity), while the parameters  $\alpha_i$  and  $\beta$  vary according to conditions in each. In cities where conditions favor concentration of the given population sub-group, these parameters will be relatively low. Alternatively,  $\alpha_i$  and  $\beta$  are direct measures

of dispersion.

We can divide the factors we expect to explain the dispersion of any of our groups at a point in time into three broad classes: the "historical concentration" of the urbanized area, the cost differential between central city and fringe residence, and the net fiscal advantage of central city residence. For the middle classes we expect a measure of central city "blight" to enter as well. Symbolically,

$$\alpha_i = G_i \text{ (Historical Concentration, Cost Differential, Fiscal Advantage, Blight), } i = 50+, \dots, 90+ \quad (4)$$

$$\beta = H \text{ (Historical Concentration, Cost Differential, Fiscal Advantage).}$$

Again note that variables tending to increase the concentration of a group will have negative partial derivatives in the functions given in (4). The variables in our sample embodying these factors are different for middle and poor classes, and some variables measure aspects of more than one factor. Nevertheless, this provides a framework for presentation of the independent variable we have used.

#### IV. The Independent Variables

##### The Matter of Employment

Before turning to the development of the empirical variables to represent the forces of natural concentration, cost differential and net fiscal advantage, we should explain the absence from the list of factors influencing residential location one which is likely

to occur to many people immediately: the location of employment. As will be evident from the discussion below, what we call historical concentration is intended, among other things, to represent the degree of centralization of industry and employment. Furthermore, the a priori signs of the partial derivatives of all the independent variables used would be the same if employment location (and not residential location) had been the dependent variable. For example, if a high concentration of poor people leads to crime, insecurity, etc., we would predict that jobs of middle class people would be more likely to be located in the suburbs, the higher the fraction of central city population that is poor. What this means is that our model predicts residential location assuming that employment location is allowed to vary in response to the same independent variables.

#### Measures of Historical Concentration

We might almost call this factor in residential location the dead hand of the past. To some extent geography is likely to affect the density gradient of residential location. Two cities alike in all demographic and economic respects would surely be expected to differ in degree of concentration if one is located on a coastline with steeply rising shores, the other on a plain. However, in addition to the genuine natural geography there is at any moment in time a geography embodied in the capital structures of the urbanized area, especially office and industrial buildings, housing and transportation systems. Although strictly speaking endogenous, this built geography is so slow to change that it would be

uninstructive to attempt to explain it in our model. Thus there are two types of geographies we wish to account for.

We experimented with three methods of dealing with this problem. Probably the most important changing influence on the built geography over the past fifty years has been the rise of the automobile. This has made economical lower absolute densities and lower density gradients of residence and employment. Cities which grew large in the past should, it would therefore seem, be more concentrated than those which have developed more recently. Hence, we experimented with the "age" of urbanized areas (AGE), measured by the number of decades before 1950 that the central city attained a population of 50,000. However, age is obviously a crude indicator of the effect we sought to capture, and only by accident would it embody the natural geography. Hence, although it generally "worked" in the right direction, we looked for a better measure of concentration.

A second measure of ~~historical~~ concentration tried was the percentage of housing units as of 1960 in the urbanized areas which were built before 1940 (HB4U). The year 1940 was chosen rather arbitrarily as a date before which the transportation technology embodied in structures was still pre-automobile in character. This variable also generally worked in the right direction, although, like age, it does not capture the true natural geography. However, partly because we wished to use a close relative of this variable to measure another phenomenon (with opposite a priori effect), we finally elected to use a direct measure of historical concentration.

Define the variable CONPOP to represent the degree of concentration of the population as a whole:

$$(5a) \quad \frac{\text{POPC}}{\text{POPU}} = F^{\text{CONPOP}}, \quad \text{and therefore}$$

$$(5b) \quad \text{CONPOP} = \text{LOG} \left( \frac{\text{POPC}}{\text{POPU}} \right) / \text{LOG}(F) .$$

Our measure of historical concentration is CONPOP lagged ten years. This variable is designed to hold constant or account for the accumulated effects in the past in order to explain the present.

From the discussion in the previous section it will be recognized that CONPOP should be between zero and one; the lower it is, the greater the concentration of the urbanized area. Hence, since an increase in this variable should be associated with a decrease in concentration, the partial derivative with respect to it is predicted to be positive. Furthermore, since we expect inertia to play a smaller role in the location decision of higher family income groups, the coefficient of CONPOP is expected to decline as we use higher cut-off income levels to define the middle class.

Although we expect to, and do, find that in areas which developed relatively recently populations are less concentrated, there is a somewhat paradoxical-sounding effect of growth which works in the opposite direction. This is most easily explained by example. Suppose two urbanized areas to be identical in all respects in 1950. One area experiences considerable growth in area during the next decade, the other does not. Since growth in area takes the form of expanding suburbs at relatively low density it has a comparatively minor influence on, say, PC/PU; to take an extreme case, suppose the effect to be negligible. Then in the two areas PC/PU is the same in 1960 as it was in 1950. However F is no longer the same in both;

the growing area shows a decline in  $F$ , since the suburbs have expanded while the central city boundaries remain fixed. Since  $F$  is smaller, while  $PC/PU$  remains the same, there is an apparent increase in the concentration of the poor in the growing area. To summarize, of two areas with the same  $CONPOP_{t-10}$ , the one for which  $F$  declines most (increases least) over the decade will show, ceteris paribus, the greatest increase in concentration of populations. To register this effect we introduce the variable  $F\Delta MF5$ , the change in  $F$  between 1950 and 1960:  $F_{1960} - F_{1950}$ . Since an increase in this variable should be associated with a decrease in concentration, the partial derivative with respect to it is predicted to be positive.

#### Measures of Cost Differential

As is suggested by a priori theoretical considerations and confirmed statistically, different variables are appropriate for measurement of the city-suburb cost differential for the middle and poor classes. Although this differential involves other important elements, we considered a particularly important element of it to be relative rental costs of housing of the quality most likely to be purchased by families with specified income. Our sample does provide some information on such relative rentals, in the form of statistics on median rental and median home value, and, as will be discussed, the differential between median central city and suburban rentals performs well statistically in the equations for the middle class. However, since the housing price gradient is clearly endogenous we felt it would be more satisfying to introduce some measure of its underlying determinants.



In the case of the poor class we reasoned that comparative advantage in producing cheap housing, a supply response, would influence significantly the relative city-suburb housing costs for the poor. While some low-rental housing is built, probably much the greater proportion is simply old housing, whose most economical use is to provide low-quality housing services. Our sample contains statistics on the age structure of housing stocks. Rather arbitrarily, we picked twenty years as the appropriate age at which to call housing "old." Two variables measure the city-suburb differential: housing built before 1940 in the central city (HB4C), and housing built before 1940 in the urban fringe (HB4S). The higher the former and the lower the latter, the greater should be the concentration of poor in the central city. Thus our equation for the poor incorporates a version of the "filtering hypothesis" of housing stock evolution.<sup>9</sup>

We reasoned that the general level of prosperity would be systematically related to the cost gradient of the housing consumed by the middle class. We can develop this relationship in a simple model. Let  $P(s)$  be the price per unit housing at distance  $s$  from the city center and let  $T(s,y)$  be the total transportation expense incurred by a family with income  $y$ , living at  $s$ . Let  $H(s,y)$  be the amount of housing purchased at  $s$  by a family with income  $y$ . It must be true in equilibrium that the change in the cost of  $H$  units of housing obtainable by moving nearer or farther away from the city center exactly offsets the change in transportation cost-- otherwise the household would move:

$$(6a) \quad H(s,y)P'(s) + T_s(s,y) = 0 \quad ,$$

or

$$(6b) \quad -P'(s) = \frac{T_s(s,y)}{H(s,y)} \quad .$$

Assume now that (a) all families have the same income and (b) all locations are occupied. Then (6b) expresses the slope of the equilibrium housing cost-distance function at  $s$  as a function of average family income. Let  $\theta_g$  be the elasticity of that slope with respect to income, and let  $\theta_T$  and  $\theta_H$  be the corresponding income elasticities of transportation cost and housing demand. Then, taking logarithms in (6b), differentiating with respect to income, and then multiplying through by income, we obtain

$$(7) \quad \theta_g = \theta_T - \theta_H \quad .$$

The same reasoning would appear to hold, if we drop the assumption that all families have the same income, and interpret (6b) as the equilibrium condition for families living at  $s$ , whatever their income may be. If we assume that changes in the level of the area income distribution can be thought of as proportional changes in the incomes of families at each location, (7) describes the elasticity of the housing-cost function's slope with respect to changes in the area's median family income.

The sign of  $\theta_g$  depends upon the relative magnitudes of  $\theta_T$  and  $\theta_H$ .<sup>10</sup> For our purposes we are less interested in its sign or size than in the plausibility of the hypothesis that  $\theta_T - \theta_H$

is non-zero and its sign is uniform across cities. Assuming these conditions, seeing no compelling reason to expect otherwise, we have included the median family income of the urbanized area (MFI) as a variable measuring the relative cost of living in the central city. To illustrate the effect, if (as our evidence implies) higher values of MFI correspond to steeper housing cost-distance functions, then we should find that middle class families of given income are more likely to live in the suburbs the higher are the incomes of the other families in the urban area. Furthermore, because richer families tend to demand more space and a greater number of rooms, the dispersing effect of an increase in the rent gradient should be greater the higher the income level of the group involved. Our prediction in this case would be that the fraction of the middle class living in the central cities will be negatively related to MFI (the partial derivative with respect to MFI should be positive); furthermore, this derivative should increase as we move up the income scale.

#### Measures of Net Fiscal Advantage

The factors determining the net fiscal advantage of central city or urban fringe location will also be different for poor and middle classes. There is a priori reason to proceed on the assumption that populations tend to group by income class in suburban communities.<sup>11</sup> Hence, it is assumed that the suburban fiscal alternatives are roughly similar across cities, and that interaction of poor and middle classes in suburban fiscal units is a relatively unimportant phenomenon. That these assumptions are plausible is fortunate, as

fiscal data for the urban fringe do not exist (since the urban fringe is not defined by political boundaries, it would doubtless be very difficult to gather such statistics) and, in any event, it is not clear how one would wish to aggregate data on the set of fiscal units composing the suburbs.

To determine the influence of governmental action on residential locations it would obviously be desirable to have available detailed statistics on the output of public services, especially schooling, and to have measures of the degree of progressivity of city taxes. However, our sample only offers summary statistics on central city government budgets for 1957, including general expenditure totals, totals of revenues raised by the city government, and transfers of revenues to the city government from higher levels of government, including especially state government support for schools and state and federal government support for welfare programs.

Using Gillespie's estimates of the incidence of state and local government expenditures and taxes by income class we constructed separate measures of fiscal surplus---the value of government services received minus the local taxes paid---for the middle and poor classes. Although the precise incidence found by Gillespie varied with the assumptions made at a number of points, there is no strong reason for assuming other than an equal per family valuation of city government expenditures, regardless of family income. Taxes, on the other hand, while not progressive, do advance with income, and inspection of Gillespie's results suggested that an average non-poor family pays roughly 2.5 times as much in local taxes as the average poor

family by our definitions. Let GEXP represent the total city government general expenditure, TOTREV equal the total amount of revenue raised by the city government, and TRANS equal the amount it receives from higher levels of governments. Then our two variables representing fiscal surplus to central city middle class residents (MFISC) and poor residents (PFISC) are:

$$(8) \quad MFISC = \frac{GEXP}{PC + RC + MC} - \frac{2.5(TOTREV-TRANS)}{PC + 2.5(RC + MC)}$$

$$PFISC = \frac{GEXP}{PC + RC + MC} - \frac{(TOTREV-TRANS)}{PC + 2.5(RC + MC)}$$

There is some reason to expect that these fiscal surplus variables will have more predictive power in explaining middle class than poor location. The assumption that suburban alternatives are the same across urban areas is probably fulfilled better in the case of the middle class than of the poor. Benefits to the poor in the form of welfare programs and educational subsidies tend to vary considerably across states, and often minimal levels for such support are set by states. Hence there is some basis for expecting a positive correlation between central city and suburban fiscal surplus of poor families. Since we are unable to control for suburban fiscal surplus, this may be expected to weaken the explanatory power of PFISC.

For either the poor or the middle class, an increase in central city fiscal surplus should tend to increase concentration. Hence the sign of the partial derivatives with respect to these variables should be negative.

### Flight From Blight

Although we have included explicit fiscal variables in the middle class equation, our particular interest is in measuring the effect on middle class residential location of central city income distribution, an effect which works partially through the fiscal system. Our measure of this distribution variable is the percentage of the central city families who have incomes below the sample 20th percentile, PC20-/POPC. Given MFISC, the higher is PC20-/POPC, the more public resources will (should) be devoted to welfare, security, public hospitals, remedial schooling, etc. There are direct effects as well, of course, crime, dirt, school troubles, etc. In short, the greater PC20-/POPC, the greater the incentive for the middle class families to move to the suburbs and the partial derivative with respect to this variable should thus be positive in the equations for the middle class. Furthermore, the higher is the income level used to define middle class, the more sensitive should that group be to this variable, and hence the larger the value of the derivative.

### Other Influences

As will be discussed in the sections devoted to estimation of the model, a number of other variables seemed to us of possible relevance, including the absolute population of the urbanized area. We gave particular attention also to the possibility that the racial composition of central cities would affect the residential locational decisions. None of these variables proved statistically significant.

### V. Empirical Results

The empirical formulation of the model described in sections III and IV concerning the residential location decisions of the middle class is, with time subscript  $t$  denoting single years,

$$(9) \quad \left( \frac{MCI}{MU_i} \right)_t = \pi_i F_t^{\alpha_{it}}, \quad i = 50+, \dots, 90+$$

where

$$(10) \quad \alpha_{it} = A_{i0} + A_{i1} MFI_t + A_{i2} \left( \frac{PC20-}{POPC} \right)_{t-10} \\ + A_{i3} CONPOP_{t-10} + A_{i4} MFISC_{t-3} \\ + A_{i5} F6MF5 \quad ;$$

the corresponding formulation for the poor is

$$(11) \quad \left( \frac{PC20-}{PU20-} \right)_t = \gamma F_t^{\beta_t},$$

where

$$(12) \quad \beta_t = C_0 + C_1 HB4S_t + C_2 HB4C_t + C_3 CONPOP_{t-10} \\ + C_4 PFISC_{t-3} + C_5 F6MF5 \quad .$$

For convenience we list the definitions of the variables appearing in (9)-(12) which were described in section IV.

- MFI = median family income in the urbanized area
- CONPOP = a measure of the concentration of all population groups in the urbanized area

- MFISC = a measure of the net fiscal surplus received by a middle class family residing in the central city
- F6MF5 = the change in F between 1950 and 1960
- HB4S = the percentage of all housing units in the urban fringe built before 1940
- HB4C = the percentage of all housing units in the central city built before 1940
- PFISC = a measure of the net fiscal surplus received by a poor family residing in the central city

The lag structure in (9)-(12) is essentially dictated by the data. Fortunately, however, the lags involved may not be unreasonable. For example, decisions concerning residential location probably respond with a lag to both the benefits and burdens of taxation and so the three year lags in the fiscal variables in (10) and (12) may be a reasonable approximation. Similarly, the concentration variable, CONPOP, is supposed to represent the "dead hand of history" as it relates to natural concentration due to geographical considerations, technological considerations due to the rise of the automobile, and concentration of both employment opportunities and industry. In a sense, therefore, the ten year lag in this variable enables us to hold constant (or account for) the past and thus observe the behavior of the present. Finally, the ten year lag in the income distribution variable, PC20-/POPC, in equation (10) was determined empirically. That is, when  $(PC20-/POPC)_t$  and  $(PC20-/POPC)_{t-10}$  were jointly considered only the lagged variable proved to be significant.



We now outline the implications of the arguments given in section IV concerning the sign expectations of the parameters in (9)-(12). First, note, that if  $F = 1$ , the dependent variable must be unity and so we expect  $\pi_i = \gamma = 1$ . Now recall that because the range of  $F$  is  $0 \leq F \leq 1$ , a positive partial derivative of  $\alpha_{it}$  or  $B_t$  with respect to a particular independent variable implies that the corresponding dependent variable is negatively related to that particular independent variable. For instance, a large net fiscal surplus received by a particular income group (a high value of MFISC or PFISC) should, ceteris paribus, draw members of that income group to the central city and so we expect both  $A_{i4}$  and  $C_4$  to be negative. Similarly, if the income distribution variable (PC20-/POPC) in (10) is large, the associated social and fiscal problems should drive the middle class from the central city and so  $A_{i2}$  should be positive. The expectations concerning the coefficients of the housing variables in (12), and the growth variable  $F6MF5$  in both (10) and (12) follow, respectively, from the filtering hypothesis, and the assumption that the recent growth of urbanized areas has centered primarily in the suburbs. Thus, we expect  $C_1$ ,  $C_5$  and  $A_{i5}$  to be positive and  $C_2$  to be negative. Finally, recalling that CONPOP varies inversely with concentration, we expect  $A_{i3}$  and  $C_3$  to be positive since, ceteris paribus, a high concentration of the general population, and therefore of employment and industry, suggests a high concentration of a particular subgroup. To summarize, we expect

$$\begin{aligned}
 & \pi_i = \gamma = 1, \quad A_{i2} > 0, \quad A_{i3} > 0, \quad A_{i4} < 0, \\
 (13) \quad & A_{i5} > 0, \quad C_1 > 0, \quad C_2 < 0, \quad C_3 > 0, \quad C_4 < 0, \quad C_5 \neq 0.
 \end{aligned}$$

As demonstrated in section IV, the elasticity,  $\theta_g$ , of the price gradient for housing of a given quality with respect to average or median income of the urbanized area may be expressed as  $\theta_g = \theta_t - \theta_h$ , where  $\theta_t$  and  $\theta_h$  are the elasticities with respect to income of transportation cost, and the demand for housing of a given quality. If we assume that transportation costs are proportional to income,  $\theta_t = 1$  and so  $\theta_g > 0$  if  $\theta_h < 1$ . Therefore, returning to the empirical formulation of our model, the expectation is that

$$(14) \quad A_{i1} \gtrless 0 \quad \text{if} \quad \theta_h \lesseqgtr 1.$$

For example, assume  $\theta_h < 1$ . Then, ceteris paribus, as median family income of an area increases, the rent differentials between the central city and the suburbs should increase and therefore a middle class family of given income would be more likely to locate in the suburbs.

Our further expectations concern the manner in which the coefficients of MFI, (PC20-/POPC), and CONPOP in (10) vary as we go up the income scale--i.e., vary with respect to  $i$ . For example, if we assume that higher income families demand more living space, we would expect these families to be more sensitive to the price gradient for housing of a given quality. Thus, since our measure of that price gradient is the median family income variable, we

expect its coefficient,  $A_{i1}$ , to increase in absolute value as we go up the income scale. That is, we expect

$$(15) \quad |A_{50+,1}| < \dots < |A_{90+,1}| .$$

In a somewhat similar light, we expect

$$(16) \quad A_{50+,2} < \dots < A_{90+,2}$$

for the reasons outlined in section IV concerning the percentage of public resources devoted to welfare, security, and other considerations that are usually associated with large numbers of poor families.

Finally, whether we consider CONPOP to be an index of job concentration or a measure of historical geography in general, we expect

$$(17) \quad A_{50+,3} > \dots > A_{90+,3}$$

since the residential location decisions of higher income groups should be less dependent on such considerations.

Before turning to the empirical results, a few words might be mentioned about the estimation technique. If logs are taken in (9) and (11), the resulting equations are linear in the parameters with the exception that they contain  $\log \pi_i$  and  $\log \gamma$ . In light of (13), we therefore expect the constant terms in these equations to be zero.

As mentioned in section I, our model is conditional on the total urban population components PU, RU, and MU, and also on certain characteristics of the population as a whole---such as the median

family income. In our two-stage least squares estimation of the model (9)-(12), the fiscal advantage variables, MFISC and PFISC, and the two housing variables were treated as current endogenous variables, while MFI and the variables lagged 10 years were treated as predetermined.<sup>12</sup>

The empirical results for the "middle class" equations are given in Table 1; the corresponding results for the percentage poor are given in (18),

$$(18) \quad \hat{\beta}_t = .808 + .190 \text{ HB4S} - .674 \text{ HB4C} + .511 \text{ CONPOP} \\
\begin{array}{cccc}
(6.04) & (1.65) & (4.83) & (4.70)
\end{array} \\
- .282 \text{ PFISC} + .773 \text{ F6DF5}; \log \hat{\gamma} = .043, R^2 = .7711 \\
\begin{array}{ccc}
(2.50) & (4.98) & (1.59)
\end{array}$$

where the figures in parentheses beneath the parameter estimates are t-ratios.

The results in Table 1 and in equation (18) are certainly encouraging. First, it is noted that all of the hypotheses described in (13) are accepted at the 5% level. Concerning (14), we note that the estimate of  $A_{i1}$  is positive for all income groups, suggesting that  $\theta_h < 1$ . Finally, we see that the hypotheses described in (14)-(16) are consistent with the data in the sense that the unrestricted estimates of  $A_{i1}$ ,  $A_{i2}$ , and  $A_{i3}$  increase or decrease, as expected, in a monotonic fashion.

A number of extensions of the basic model were considered in the sense that other variables of interest were added to (10) and (12). In each case the added variable proved not to be statistically

TABLE 1

## EQUATIONS FOR THE MIDDLE CLASS\*

$$MC/MU = \pi_i F_i^{\alpha_i}$$

$$\alpha_i = A_{i0} + A_{i1}MFI + A_{i2}(PC20-/POPC)_{t-10} \\ + A_{i3}CONPOP_{t-10} + A_{i4}MFISC_{t-3} + A_{i5}F6MF5$$

| PARAMETER      | INCOME GROUP     |                  |                  |                  |                  |
|----------------|------------------|------------------|------------------|------------------|------------------|
|                | 50+              | 60+              | 70+              | 80+              | 90+              |
| Log( $\pi_i$ ) | -0.061<br>(1.79) | -0.057<br>(1.60) | -0.050<br>(1.37) | -0.038<br>(1.03) | -0.023<br>(0.56) |
| $A_{i0}$       | -0.915<br>(3.00) | -1.003<br>(3.16) | -1.113<br>(3.41) | -1.236<br>(3.71) | -1.389<br>(3.74) |
| $A_{i1}$       | 0.142<br>(3.91)  | 0.155<br>(4.10)  | 0.171<br>(4.40)  | 0.192<br>(4.83)  | 0.215<br>(4.85)  |
| $A_{i2}$       | 1.966<br>(5.32)  | 2.123<br>(5.53)  | 2.285<br>(5.79)  | 2.451<br>(6.09)  | 2.738<br>(6.09)  |
| $A_{i3}$       | 0.559<br>(4.09)  | 0.536<br>(3.77)  | 0.515<br>(3.53)  | 0.469<br>(3.15)  | 0.376<br>(2.26)  |
| $A_{i4}$       | -0.996<br>(2.48) | -1.067<br>(2.55) | -1.101<br>(2.57) | -1.048<br>(2.40) | -0.915<br>(1.87) |
| $A_{i5}$       | 0.445<br>(2.42)  | 0.445<br>(2.33)  | 0.447<br>(2.25)  | 0.503<br>(2.43)  | 0.530<br>(2.23)  |
| RSQD           | .8416            | .8397            | .8424            | .8502            | .8359            |

\* The figures in parentheses are t-ratios.

significant. The results of these experiments are given in Tables A1 and A2 in the appendix. It is worth singling out for special attention that one of the variables added to the equation for the middle class, (10), was the percentage non-white of the central city; this variable was considered with and without a ten year lag. Our expectation concerning the effect of the percentage non-white on the residential location of the middle class is obvious. However, the question is whether this variable will contribute anything to the analysis once the income distribution of the central city is accounted for via the variable (PC20-/POPC). Are the problems associated with central city residence due primarily to social and fiscal problems associated with poverty in general or to race in particular? As shown in Table A2, the race variable proved not to be significant; the suggestion is that perhaps racial considerations influence residential location decisions of middle class families only at the scale of the neighborhood.

Another experiment was to reestimate the basic model by ordinary least squares (OLSQ) in order to determine whether or not the results are sensitive to a system bias. The coefficient estimates obtained by OLSQ and the two-stage least squares technique (TSLS) are compared in Tables A3 and A4 in the appendix. Essentially, all of the OLSQ estimates differ from the corresponding TSLS estimates; however, the major difference occurs in the estimated coefficients of the fiscal variables. In contrast to the TSLS estimates, the OLSQ estimates of the coefficients of both MFISC and PFISC are practically zero; the  $t$  ratios of these OLSQ estimates are both .01. One possible explanation of these results involves the

positive feedback relation between tax revenues, TR, and therefore general expenditures, and the percentage of middle class in the central city. For instance, ceteris paribus, TR will be high if the percentage of middle class in the central city is high. If the systems estimation procedure, TSLS, is not used, this positive relationship tends to cancel the negative relationship involving the sensitivity of middle class location to high central city taxes: it therefore appears that there is no relationship if the OLSQ procedure is used. The suggestion, obviously, is that a systems bias must be considered when estimating urban models.

#### VI. Policy Experiments

A number of policies have been suggested for alleviating problems of cities. Many of these policies are intended to increase the percentage of middle and upper middle class families living in the central city. Assuming, as our empirical results suggest, that our model is an adequate description of the determinants of residential location, we are now in a position to analyze the implications of such policies as they relate to the income distribution of the central city. Before proceeding it should be noted that we are not evaluating such policies but merely analyzing them. For example, it may be desirable to de-populate central cities!

The policies we consider fall into three broad categories:

- (1) variation in the central city budget financed by central city,
- (2) changes in the amount of funds transferred to the central city government from higher levels of government, and
- (3) income

redistribution by higher levels of government. Our technique was to construct the "average" city and conduct "experiments" on it. That is, all of the variables other than the policy variables appearing in the empirical formulation of the model were replaced by their sample averages. We then postulated various changes in the policy variables and derived the corresponding changes in the dependent variables.<sup>13</sup> Because the residual families, those neither poor nor middle class, appear as taxpayers and beneficiaries of central city governments, we required a further relationship to predict the location of this group. This we acquired by simple interpolation via the least squares regression of  $\frac{RC}{RU}$  on  $\frac{PC}{PU}$  and  $\frac{MC}{MU}$ . In effect we assume that rich and poor lie at the extremes of a continuous spectrum, so that the behavior of those in between will be some sort of average of the two.

The results of the experiments are displayed in Tables 2-6. As these tables are virtually self-explanatory, only brief commentary is required. In Table 2 are shown the effects of central city budget variations, from a fifteen per cent decrease to a fifteen per cent increase in expenditure. All of the expenditure change is considered to be financed out of central city taxes of the same incidence assumed in the construction of the fiscal advantage variables: non-poor families pay 2.5 times as much in taxes as poor families. As might be expected, both the percentages and numbers of poor and middle class families residing in the central city vary with the change in



TABLE 2  
 POLICY III: KEEP TRANSFERS CONSTANT. INCREASE AND  
 DECREASE TAXES AND EXPENDITURES BY  
 EQUIVALENT AMOUNTS.

| PERCENT<br>CHANGE<br>IN BUDGET | POPC   | PC    | RC    | MC    | PC/PU | MC /MU | PC/POPC | MC/POPC |
|--------------------------------|--------|-------|-------|-------|-------|--------|---------|---------|
| -15                            | 152826 | 33974 | 46243 | 72609 | 0.710 | 0.607  | 0.223   | 0.476   |
| -10                            | 152647 | 34125 | 46255 | 72267 | 0.714 | 0.605  | 0.224   | 0.474   |
| -5                             | 152559 | 34278 | 46288 | 71993 | 0.717 | 0.602  | 0.225   | 0.472   |
| 0                              | 152285 | 34432 | 46279 | 71575 | 0.720 | 0.599  | 0.227   | 0.471   |
| 5                              | 152099 | 34586 | 46290 | 71222 | 0.723 | 0.596  | 0.228   | 0.469   |
| 10                             | 151909 | 34742 | 46301 | 70866 | 0.726 | 0.593  | 0.229   | 0.467   |
| 15                             | 151715 | 34898 | 46311 | 70506 | 0.730 | 0.590  | 0.231   | 0.465   |

TABLE 3  
 POLICY I: INCREASE TRANSFERS WHILE HOLDING GENERAL  
 EXPENDITURES CONSTANT (THEREBY REDUCING OWN TAXES).

| PERCENT<br>INCREASE<br>IN<br>TRANSFERS | POPC   | PC    | RC    | MC    | PC/PU | MC/MU | PC/POPC | MC/POPC |
|--|--------|-------|-------|-------|-------|-------|---------|---------|
| 0                                      | 152285 | 34432 | 46279 | 71575 | 0.720 | 0.599 | 0.227   | 0.471   |
| 25                                     | 155668 | 34493 | 47084 | 74091 | 0.721 | 0.620 | 0.222   | 0.476   |
| 50                                     | 158994 | 34552 | 47874 | 76567 | 0.723 | 0.640 | 0.218   | 0.482   |
| 75                                     | 162261 | 34609 | 48650 | 79001 | 0.724 | 0.661 | 0.214   | 0.487   |
| 100                                    | 165481 | 34665 | 49415 | 81400 | 0.725 | 0.681 | 0.210   | 0.492   |
| 125                                    | 168648 | 34718 | 50167 | 83763 | 0.726 | 0.701 | 0.206   | 0.497   |
| 150                                    | 171772 | 34770 | 50909 | 86093 | 0.727 | 0.720 | 0.203   | 0.502   |

TABLE 4

POLICY II: INCREASE TRANSFERS, TOTAL REVENUES, AND GENERAL EXPENDITURES BY SAME AMOUNT (THEREBY HOLDING OWN TAXES CONSTANT).

| PERCENT<br>INCREASE<br>IN<br>TRANSFERS | POPC   | PC    | RC    | MC    | PC/PU | MC/MU | PC/POPC | MC/POPC |
|--|--------|-------|-------|-------|-------|-------|---------|---------|
| 0                                      | 152285 | 34432 | 46279 | 71575 | 0.720 | 0.599 | 0.227   | 0.471   |
| 25                                     | 155479 | 34654 | 47097 | 73728 | 0.725 | 0.617 | 0.223   | 0.475   |
| 50                                     | 158626 | 34871 | 47901 | 75854 | 0.729 | 0.634 | 0.220   | 0.479   |
| 75                                     | 161730 | 35081 | 48694 | 77955 | 0.734 | 0.652 | 0.217   | 0.483   |
| 100                                    | 164792 | 35286 | 49474 | 80032 | 0.738 | 0.669 | 0.215   | 0.486   |
| 125                                    | 167816 | 35484 | 50244 | 82087 | 0.742 | 0.687 | 0.212   | 0.490   |
| 150                                    | 170800 | 35678 | 51003 | 84119 | 0.746 | 0.704 | 0.209   | 0.493   |

TABLE 5

POLICY IV: NEGATIVE INCOME TAX PROGRAM WITH NO FEEDBACK VIA INCOME DISTRIBUTION.

| PERCENT<br>DECREASE<br>IN AREA<br>POOR | POPC   | PC    | RC    | MC    | PC/PU | MC/MU | PC/POPC | MC/POPC |
|--|--------|-------|-------|-------|-------|-------|---------|---------|
| 0                                      | 152285 | 34432 | 46279 | 71575 | 0.720 | 0.599 | 0.227   | 0.471   |
| 25                                     | 153285 | 25867 | 54511 | 72908 | 0.721 | 0.610 | 0.169   | 0.476   |
| 50                                     | 154256 | 17267 | 62841 | 74148 | 0.722 | 0.620 | 0.112   | 0.481   |
| 75                                     | 155192 | 8643  | 71257 | 75291 | 0.723 | 0.630 | 0.056   | 0.486   |

TABLE 6

POLICY V: NEGATIVE INCOME TAX PROGRAM WITH FEEDBACK VIA INCOME DISTRIBUTION.

| PERCENT<br>DECREASE<br>IN AREA<br>POOR | POPC   | PC    | RC    | MC    | PC/PU | MC/MU | PC/POPC | MC/POPC |
|--|--------|-------|-------|-------|-------|-------|---------|---------|
| 0                                      | 152285 | 34432 | 46279 | 71575 | 0.720 | 0.599 | 0.227   | 0.471   |
| 25                                     | 162795 | 25713 | 56889 | 80193 | 0.717 | 0.671 | 0.158   | 0.493   |
| 50                                     | 174812 | 17053 | 68483 | 89276 | 0.713 | 0.747 | 0.098   | 0.511   |
| 75                                     | 188523 | 8478  | 81163 | 98882 | 0.709 | 0.827 | 0.045   | 0.525   |

the city budget. The last two columns of Table 2 show the effect of the budget changes on the central city income distribution, with the percentage poor ranging from a low of 22.3 to a high of 23.1 as the percentage middle class varies from 47.6 to 46.5. As shown in column one of the table, the central city population grows slightly with budget reduction. While qualitatively the changes tend to be of the sort expected, quantitatively none of the changes shown in Table 2 is likely to strike the reader as very significant, though budget variations over a 30 percentage point range are involved.

The effect of increasing the amounts transferred to the central city government from higher levels of government is shown in Table 3. The assumption underlying the table is that transfers are used to reduce taxes; in Table 4 the assumption is that transfers are used to increase expenditures. Since such transfers accounted for an average of approximately 20% of expenditures for the cities we considered, the total range in Tables 3 and 4, in terms of the dollar changes involved, is comparable to that in Table 2. The differences between Tables 3 and 4 are consistent with our expectations: the middle classes "prefer" to have the transfers used to reduce taxes, and the poor "prefer" the money to be used to expand expenditures. In both cases the increased transfers act as a strong attractive force to the middle classes, an effect which would be reinforced over time by feedbacks from the reduction in the percentage poor in the central city resulting from the change. When the transfers are used to reduce taxes there is virtually no effect on the central city poor population; the advantage of the transfer

is dissipated for the poor by the sharing of the fixed expenditure budget with larger numbers of middle class families. When expenditures are increased the central city poor population increases somewhat. Primarily as a result of the increasing concentration of the non-poor, as measured by changes in RC/RU and MC/MU, the effect of the increased transfers from higher levels of government is a marked increase in central city population.

That transfers to central city governments are likely to increase urbanized area population concentration is perhaps obvious. Not quite so obvious is the influence, described in Tables 5 and 6, of a program of income redistribution throughout the urbanized area, financed by a higher level of government. Income redistribution takes the form of transforming various proportions of the poor class in the whole urban area into residual class families (non-poor, non-middle class). Table 5 presents the effects of the program on the assumption that the income redistribution affects only 1960 population, not 1950 population. Hence it influences the locations of populations only through the central city fiscal system. There is no effect on "urban blight." The increase in numbers of non-poor reduces the per-family taxes of both poor and non-poor, leading to an increase in urbanized area concentration, especially for the middle class. When the income redistribution is assumed to apply to the 1950 population as well, so that  $(PC_{20-}/POPC_{20-})_{t-10}$  is appropriately reduced in our equations, the effect is much more

pronounced. Now the income redistribution program affects the residential location decisions of the middle class through the direct interactions among income groups as well as through the fiscal system. The reader should note the extreme sensitivity of middle class location to such an income redistribution program. For instance, in Table 6 we see that the percentage of the middle class living in the central city predicted by our model increases approximately 23 percentage points to 83% when a negative income tax program varies from a zero to a 75% reduction in the numbers of poor families.<sup>14</sup> The implication appears to be that an effective anti-poverty program would also "save" the cities!

FOOTNOTES

1. See W. Baumol for an analysis of the implications of such dynamic processes for policy toward cities.
2. For a discussion of this issue see G. Stigler.
3. For technical reasons minor deviations were made from the list of the 87 most populous SMSA's. A complete list of the cities involved and a description of the method used in selecting the sample is available from the Urban Economics Group, Department of Economics, Princeton University, Princeton, New Jersey 08540.
4. Actually, some "central cities" consist of two or more political units, for example, Minneapolis - St. Paul, Minnesota. In this paper we treat these as single political units, a procedure which should, if anything, work against the flight from blight hypothesis, since rich families might be able to segregate themselves within one of the two or more central city political units.
5. Colin Clark was early to point out that a negative exponential in distance from the city center gives a good approximation to observed densities. Richard Muth (1970) confirmed this finding and offered a theoretical justification for it.
6. This relationship can be derived analytically for the case of negative exponential density decline. Let  $D(r)$ , the population

density at distance  $r$  from the city center, be given by

$$D(r) = D(o)e^{-gr}$$

where the parameter  $g$  is called the density gradient. The population enclosed by a circle of radius  $r$  is given by the integral

$$2\pi \int_0^r xD(o)e^{-gx} dx$$

Let  $POP(A)$  denote the population of a city with area  $A$ .

Assuming a circular city, the ratio of the population in the central city of area  $AC$  to the population of the urbanized area, with surface area  $AU$ , is given by

$$POP(AC)/POP(AU) = \frac{\int_0^{\sqrt{AC}} \pi x e^{-gx} dx}{\int_0^{\sqrt{AU}} \pi x e^{-gx} dx}$$

Notice that this ratio is independent of the area's average density, indexed by  $D(o)$ . The statement in the text is that

$$\frac{\partial}{\partial g} \left( \frac{POP(AC)}{POP(AU)} \right) > 0$$

To save space the bulk of the derivation of this result will be omitted. Upon taking derivatives and rearranging, the above inequality is seen to be implied by

$$\frac{\partial}{\partial t} \left[ \frac{\int_0^t x e^{-gx} dx}{\int_0^t x^2 e^{-gx} dx} \right] < 0$$

which is, in turn, implied by

$$\left[ t \cdot \int_0^t x^2 e^{-gx} dx \right] - \left[ t^2 \cdot \int_0^t x e^{-gx} dx \right] < 0 .$$

This inequality is always true if  $t > 0$ .

7. For statistical fitting we allowed for a margin of error in the precise specification of the model by introducing a multiplicative constant. Thus the estimated equation describing the residential location of the middle class has the form  $MC/MU = \pi F^\alpha$ , where, if the model were perfect, we should have  $\pi = 1$ .
8. Thus, the percentage "poor" in the urbanized area expressed as  $PU20-/POPU$ .
9. For a discussion of filtering and references, see I. Lowry.
10. For a discussion of the relationship between the relationship between income and the value of travel time see J. Nelson. See Muth (1970) for a discussion of the demand for housing.
11. For a theoretical justification for this proposition see C. Tiebout's analysis of the determinants of local government budgets.
12. We considered our two equation model as a component of a still larger conditional model and so following Zellner and Black and Kelejian we drew upon that larger model for additional instruments. The instruments used were: all of the predetermined variables in (10) and (11); the ten year lagged values of the dependent variable and the housing variables (comparably defined); the age and total population of the



urbanized area; the fraction of the central city in 1950 that was non-white; differential between median rents of rental units in the central city and median rent for the urbanized area in 1950; and the percentage of unemployment of the civilian labor force in the central city in 1950.

13. Since MU50+ and PU20- are given to our model, the unknowns become MC50+ and PC20-. We chose MC50+ as our middle class variable so that when the "residual" class is added the entire population is accounted for.
14. Note the substantial absolute increase in concentration of urbanized area population which results from attracting the middle class back to the central city, a result supporting the view (though of course not demonstrating it) that, with settlement patterns strongly influenced by population interaction, urbanized area populations are currently too dispersed.

APPENDIX

As stated in the text, a number of extensions of the basic model were considered in the sense that additional variables of interest were added to (10) and (12). The variables added to equation (7), for the poor, were:

Log(POPU): The log of the population of the urbanized area. As an urbanized area grows, it is certainly possible that certain of its components grow disproportionately. Log (POPU) was considered in order to capture the effects of any resulting systematic relationships.

AGE: The number of decades since the central city reached a population of 50,000. This variable is considered as an additional concentration-type variable and so its coefficient should be negative.

ADC/RFCC: Welfare payments (aid to dependent children) per recipient family in the central city, 1950. This variable was also considered for 1955 and for 1960 but the results were not significantly different. Evidently, this variable should act as an attraction to the poor and so its coefficient should be negative.

MFI: The median family income of the urbanized area in 1960. As described in the text, MFI is a measure of the price gradient for housing. Assuming  $\theta_h < 1$ , its coefficient should be positive.

UNEMCC: The number of unemployed residents of the central city divided by the central city populations in 1950. This variable is a particular measure of job opportunities in the central city, and so its coefficient should be positive.

The variables added to the equation for the middle class (5), were:

Log(POPU): Same discussion as above.

AGE: Same as above.

RENTDIFF: The median rent of central city rental units minus the median rent of all urbanized area rental units in 1960. The variable was considered as an additional measure of the relative cost of central city life. Our expectation was that since MFI is a determinant of RENTDIFF, it would be unnecessary to consider both of these variables at the same time. That is, both estimated coefficients should be positive but their corresponding standard errors should be large.

%NWCC: The percentage non-white in the central city as of 1960. This variable was also considered as of 1950 but the results were essentially identical. Our expectations concerning this variable have been discussed in the text.

(PC20-/POPC): The percentage of the central city population whose income is below the 20th percentile. This variable was added in order to determine the possible lag structure relating the residential location decisions of the middle class to the income distribution of the central city.

As mentioned in the text, the other experiment considered was to reestimate the equations of the model by the OLSQ technique. Because the results were similar for all of our definitions of the middle class, only those corresponding to the 70th percentile definition are given.

TABLE A1

## EXPERIMENTS ON THE EQUATION FOR THE POOR\*

|        | Log( $\gamma$ ) | Basic Variables |                 |                  |                 |                  | Test Variables  |                           |
|--------|-----------------|-----------------|-----------------|------------------|-----------------|------------------|-----------------|---------------------------|
|        |                 | $c_0$           | $c_1$           | $c_2$            | $c_3$           | $c_4$            | $c_5$           |                           |
| TEST 1 | 0.043<br>(1.54) | 0.807<br>(4.48) | 0.190<br>(1.63) | -0.674<br>(4.69) | 0.511<br>(4.43) | -0.283<br>(1.59) | 0.773<br>(4.94) | 0.000 Log(POPU)<br>(0.01) |
| TEST 2 | 0.034<br>(1.20) | 0.737<br>(4.63) | 0.206<br>(1.87) | -0.604<br>(3.69) | 0.492<br>(4.68) | -0.195<br>(1.20) | 0.755<br>(5.12) | -0.004 AGE<br>(0.71)      |
| TEST 3 | 0.041<br>(1.54) | 0.762<br>(5.39) | 0.151<br>(1.23) | -0.660<br>(4.73) | 0.533<br>(4.82) | -0.294<br>(2.60) | 0.789<br>(5.07) | 0.005 ADC/RFCC<br>(0.93)  |
| TEST 4 | 0.043<br>(1.58) | 0.813<br>(5.13) | 0.188<br>(1.57) | -0.672<br>(4.63) | 0.510<br>(4.68) | -0.280<br>(2.36) | 0.773<br>(4.96) | -0.001 MFI<br>(0.06)      |
| TEST 5 | 0.038<br>(1.39) | 0.742<br>(4.79) | 0.176<br>(1.52) | -0.659<br>(4.72) | 0.516<br>(4.78) | -0.276<br>(2.46) | 0.763<br>(4.94) | 0.830 UNEMCC<br>(0.81)    |

\*The figures in parentheses are t-ratios.

TABLE A2

EXPERIMENTS ON THE EQUATION FOR THE MIDDLE CLASS: 70+ PERCENTILE\*

|        | $\text{Log}(\pi_i)$ | Basic Variables  |                 |                 |                 |                  | Test Variables  |  |
|--------|---------------------|------------------|-----------------|-----------------|-----------------|------------------|-----------------|--|
|        |                     | $A_0$            | $A_1$           | $A_2$           | $A_3$           | $A_4$            |                 | $A_5$                                      |
| TEST 1 | -0.051<br>(1.31)    | -1.129<br>(3.05) | 0.170<br>(3.99) | 2.277<br>(5.54) | 0.518<br>(3.40) | -1.131<br>(2.11) | 0.441<br>(2.09) | 0.002 Log(POPU)<br>(0.10)                  |
| TEST 2 | -0.042<br>(1.12)    | -1.087<br>(3.46) | 0.174<br>(4.65) | 2.288<br>(6.07) | 0.494<br>(3.46) | -0.979<br>(2.20) | 0.488<br>(2.46) | -0.004 AGE<br>(0.70)                       |
| TEST 3 | -0.056<br>(1.35)    | -1.219<br>(3.24) | 0.086<br>(1.13) | 2.304<br>(5.17) | 0.535<br>(3.23) | -1.404<br>(2.64) | 0.422<br>(1.88) | 0.906 RENT<br>(1.36)                       |
| TEST 4 | -0.044<br>(1.21)    | -1.013<br>(2.93) | 0.160<br>(3.94) | 2.088<br>(4.48) | 0.513<br>(3.62) | -1.022<br>(2.38) | 0.476<br>(2.42) | 0.159 %NWCC<br>(0.74)                      |
| TEST 5 | -0.057<br>(1.79)    | -1.417<br>(4.08) | 0.199<br>(5.17) | 1.457<br>(2.25) | 0.521<br>(4.12) | -0.827<br>(2.00) | 0.433<br>(2.52) | 1.073 PC20.../<br>(1.51) POPC <sub>t</sub> |

\*The figures in parentheses are t-ratios.

A COMPARISON OF OLSQ AND TSLS ESTIMATES  
OF THE EQUATION FOR THE POOR

TABLE A4

|      | Log( $\gamma$ ) | $c_0$           | $c_1$           | $c_2$            | $c_3$           | $c_4$            | $c_5$           |
|------|-----------------|-----------------|-----------------|------------------|-----------------|------------------|-----------------|
| TSLS | 0.043<br>(1.59) | 0.808<br>(6.04) | 0.190<br>(1.65) | -0.674<br>(4.83) | 0.511<br>(4.70) | -0.282<br>(2.50) | 0.773<br>(4.98) |
| OLSQ | 0.008<br>(0.38) | 0.474<br>(5.45) | 0.069<br>(0.96) | -0.410<br>(4.50) | 0.516<br>(6.27) | -0.001<br>(0.01) | 0.652<br>(6.39) |

TABLE A3  
 A COMPARISON OF OLSQ AND TSLS ESTIMATES  
 OF THE EQUATION FOR THE MIDDLE CLASS, 70+ PERCENTILE

|      | $\text{Log}(\pi_i)$ | $A_{i0}$         | $A_{i1}$        | $A_{i2}$        | $A_{i3}$        | $A_{i4}$         | $A_{i5}$        |
|------|---------------------|------------------|-----------------|-----------------|-----------------|------------------|-----------------|
| TSLS | -0.050<br>(1.37)    | -1.113<br>(3.41) | 0.171<br>(4.40) | 2.285<br>(5.79) | 0.515<br>(3.53) | -1.101<br>(2.57) | 0.447<br>(2.25) |
| OLSQ | -0.031<br>(1.14)    | -1.154<br>(4.77) | 0.172<br>(5.94) | 2.436<br>(8.39) | 0.362<br>(3.59) | -0.002<br>(0.01) | 0.444<br>(3.41) |



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