ANALYSIS AND CONTROL OF DYNAMIC ECONOMIC PROCESSES

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1. Introduction and Background

For the period from Sept. 1, 1969 to Sept. 1, 1971, the National Science Foundation has been supporting research undertaken by the Econometric Research Program at Princeton University under the title "An Analysis of Stochastic Processes in Economics," under grant No. GS-2799. The research under this grant falls into three related areas: (1) non-linear estimation problems, (2) analysis of dynamic properties of econometric models, and (3) time-series analysis. We are pleased to report that important progress has been made in each of these three areas, and that the new findings not only justify a continuation of researches into these areas but can serve as a basis for extension of our research into a new and important area, namely, that of optimal control of dynamic economic processes.

In the first area, a book (Goldfeld and Quandt, 1971) is near completion. This book contains significant contributions to solving non-linear estimation problems; it provides numerical methods for maximizing the likelihood functions for possibly non-linear econometric models and it applies these to a variety of econometric problems. Computer codes for these algorithms have been made freely available to interested users. Other estimation problems solved include that of the use of two-stage least squares in non-linear models (Kelejians, 1971), a Bayesian approach to information loss in aggregation (Kelejian, 1972), the estimation of Cobb-Douglas type functions with multiplicative and additive errors (Goldfeld and Quandt, 1970), methods of estimation for markets in disequilibrium (Fair and Jaffee, 1972), and the estimation of linear equation systems with auto-regressive residuals, both by limited-information, instrumental-variable methods (Fair, 1970a), and by the method of full-information maximum likelihood (Chow and Fair, 1970).

In the second area, we have pioneered the use of analytical methods to study dynamic properties of econometric models. The work includes a theoretical study of the spectral and auto-covariance properties of linear stochastic difference equations used in business cycle theory (Chow, 1968), a related empirical study (Chow and Levitan, 1969b) applying the above theoretical results to a simple macroeconomic model of the United States (Chow, 1967), analyses of spectral properties of the Klein-Goldberger Model (Howrey, 1967) and of a condensed version of the Wharton model (Howrey, 1971), the definition and the derivation of spectral properties for explosive (thus non-stationary) linear stochastic systems (Chow and Levitan, 1969a), and a study of simulation versus analytical methods for studying dynamic properties of econometric models (Howrey and Kelejian, 1969). Although the studies by Chow just cited were not financed by the NSF grant, they provide a background for further study of dynamic economic processes.

In the third area, we are pleased to report the publication, in 1970, of a book on the Predictability of Stock Market Prices (Granger and Morgenstern, 1970). This study explores in detail the theoretical and statistical support for the random-walk model, besides giving an excellent summary of the application of spectral techniques in time-series analysis. This study illustrates a basic methodological position of the Econometric Research Program at Princeton, that econometric techniques can be pursued effectively in the solution of relevant economic problems, thus making empirical studies a most important part of our work.

The related empirical studies completed include books on A Short-Run Forecasting Model of the United States Economy (Fair, 1971a), on The Commercial Loan Market and Credit Rationing (Jaffee, 1971), and on Savings Deposits, Mortgages, and Residential Construction (Jaffee and Gramlich, 1971) as well as papers on aggregate price changes (Fair, 1970b), the determination of yield differentials between debt instruments of the same maturity (Fair and Malkiel, 1971), labor force participation, wage rates, and money illusion (Fair, 1971b), the determinants of deposit-rate setting by Savings and Loan Associations (Goldfeld and Jaffee, 1970), a macro-model of the United States labor market (Kelejian and Black, 1970), and an econometric model of the flight to the suburbs hypothesis (Kelejian and Bradford, 1970).

A major theme which underlies many of the above studies, and will underlie many of our future studies, is that the dynamic aspects of economic processes should be taken seriously. Although the pioneering work in systems of econometric equations, Statistical Inference in Dynamic Economic Models edited by Koopmans, recognized that economic processes are not only stochastic but dynamic, the emphasis was on the simultaneity or interdependence of economic relations rather than on their dynamic characteristics. For example, it was ten years after the publication of this study in econometric method that we saw the first attempt to study the dynamic properties of an econometric model by Adelman and Adelman. It was about another decade later that we saw analytical studies of dynamic characteristics of econometric systems in terms of spectral and cross-spectral densities, as exemplified by some of the references in the second area of our research quoted above. The field of stochastic dynamics in economics is maturing. It is timely and important to investigate comparative stochastic dynamics, and to study the optimal control of stochastic systems in economics for improving the dynamic performance of the system.

2. Proposed Research

a. Optimal Control of Stochastic Systems in Economics.

Since optimal control of stochastic systems in economics is our new area of research and since some of the previous areas that we propose to continue studying are related to it, we will describe our proposed research in this area first.

Consider the reduced form of an econometric system that has already been estimated, relating a vector y_t of dependent variables to lagged dependent variables, say y_{t-1} and y_{t-2}, and to some exogenous variables x_t, which are subject to the control of the policy maker, and its lagged values x_{t-1}. Let the policy maker choose the vector x_t of control variables in the current period according to y_{t-1}, y_{t-2} and

x_{t-1}, the relation between x_t and these other three vector variables being the feedback control equation. If the objective is to minimize a linear combination of the variances of the economic variables and/or to achieve certain target rates of change for the economic variables, one can seek an optimal control policy, i.e., an optimal feedback control equation. If the econometric model is linear, an analytical solution to the optimal feedback control equations (the optimal coefficients in the linear control equations) can be obtained by the elementary technique of Lagrange multipliers, both for the case of the steady state when the process under control becomes covariance-stationary (Chow, 1970a), and the case of minimizing a quadratic loss function for a finite time horizon (Chow, 1970b), the optimal coefficients in the feedback control equation being functions of time in the latter case. In view of the current interest in such simple feedback control equation as $x_{1,t} = 1.05x_{1,t-1}$, where x_1 denotes money supply and the time t is in years, and of the important problem of achieving stability and growth of the American economy, it seems urgent that optimal macroeconomic policies be studied rigorously in the setting of comparative stochastic dynamics.

After obtaining analytical solutions to optimal control in the case of linear models by elementary methods as stated above, we propose to study the following problems. First, it will be of interest to obtain the optimal feedback control equations in connection with a simple linear econometric model such as (Chow, 1967) or (Chow and Moore, 1971) for a welfare function which depends mainly on the variance and the rate of change of real GNP, the unemployment rate, and the rate of change of the price level, and to compare the optimal solution with a simple policy rule of thumb such as increasing the money supply by a constant percentage rate. In such an empirical study, one has to make sure that the econometric model used is a fairly accurate representation of the real world; this will require further testing or modification of the econometric model to be used. In any empirical study of optimal control, it will be of interest to examine the sensitivities of the optimal control equation (and of optimal welfare) to parameters in the econometric model and/or in the welfare function. Our framework thus provides a means of measuring the qualitative differences of econometric models in terms of the implicit optimal control equations.

Advocates of a particular control rule (such as expansion of money supply at a constant rate) to stabilize the economy often consider the steady state only; it would be important to study the transient as well as the steady state behavior of the economy when such a rule is introduced and to compare the transient and the steady state solutions to optimal control. Another topic worthy of study is the coordination of economic policies. Let x_1 denote monetary variables and x_2

denote fiscal variables. The question is how to choose x_1 under different assumptions about x_2. The optimal control solution is the solution of joint maximization. It could be compared with the solution when the pattern of response of x_2 is given, and thus the gain from coordination can be measured. One can also consider a minimax solution.

Our framework also provides a means for evaluating economic policies in a certain historical period. If the behavior of certain policy makers can be described by a feedback equation - an issue which can be studied empirically by statistical testing and estimation of such a relationship - then the stability of the economy (in terms of the welfare function) under the empirical feedback relationship can be compared with that under the optimal feedback relationship. One could find out how much better the policy makers could have done and how they should have done it. Once we are willing to explain empirically the control variables, we raise the question of specification of "exogenous variables" in econometric models. What is the advantage of treating some variables as "exogenous," as compared with explaining them by some auto-regressive equations in the system? Can one provide statistical tests for alternative specifications of econometric models in terms of classification of certain variables as exogenous or endogenous?

Another related area of research is the estimation of economic parameters in the light of control. In the above, we have assumed that the parameters in the econometric model employed are given. Such an assumption appears to be adequate for the study of optimal stabilization policies in the current state of the art, since existing econometric models are probably not accurate enough for fine tuning the economy but may be accurate enough for the purpose of analyzing the general characteristics of good control policy. Moreover, the study of optimal control based on the assumption that the parameters of econometric models are given can shed light on the qualitative differences among these econometric models. In the future, when sufficient knowledge of the dynamic economic processes shall have been accumulated, one will have to revise his parameter estimates while setting optimal control policies continuously. Bayesian methods can be applied to the revision of parameter estimates. Some general results are known, e.g., in M. Aoki, Optimization of Stochastic Systems (Academic Press, 1967). However, solutions, either analytical or numerical, for particular interesting cases still have to be worked out. In this regard, the situation is not unlike the application of Bayesian methods to econometrics - while some general results on optimal control theory for stochastic systems are known, specific solutions relevant to economic problems are still to be worked out.

b. Theoretical Problems of Model Construction and Estimation

The preceding paragraph has suggested that estimation problems and control problems are closely related, and that the direction of research in estimation will be influenced by the purpose at hand or the viewpoint of the researcher. In particular, there are four areas in connection with model construction and estimation that we propose to pursue, besides what was described in the last paragraph.

The first area is that of aggregation problems. Specifically, we are interested in aggregation over time, aggregation over commodities, as well as aggregation over individual economic units. In connection with the econometric models that we plan to develop and employ for optimal-control purposes (to be referred to in section c below) we propose to make theoretical and empirical comparisons of quarterly, annual, and possibly monthly versions of the models. This will enable us to study also the question of specifying the welfare function in terms of the variances of annual changes, quarterly changes, or monthly changes of relevant economic variables. Concerning aggregation over commodities or sectors, we plan to compare the performances of the small models (including our own) with larger models - see also section c below - and to study theoretically the question of optimal aggregation and of measuring the possible gain from disaggregation. Concerning aggregation over individual economic units, we have had some success with the following approach and are prepared to engage in a more comprehensive study of this problem.

The approach can be illustrated by the aggregation of production functions of firms in the derivation of a production function for the industry (Cornwall, 1971). Given the firm's production function with a vector a of parameters and given a vector p of prices of output and inputs, maximization of profits yields a profit function of p and a for the firm. If the frequency distribution of a is specified, then the total profit of the industry becomes a function of p, after integrating over a, using the above frequency distribution. The industry's production function is deduced from the above profit function for the industry. This approach has been used in (Cornwall, 1971) to derive the result of the well-known paper by H. S. Houthakker, "The Pareto Distribution and the Cobb-Douglas Production Function in Activity Analysis," Review of Economic Studies, 1955-56. We propose not only to derive industry production functions from specific mathematical descriptions of the firm's technologies, but also to obtain general results on the derived demand and supply functions of the industry in relation to the properties of the production functions of the component firms, including homogeneity and returns to scale (Cornwall, 1970), e.g., whether the industry can have non-increasing returns to scale when each firm has increasing returns.

The second area of our intended research in model construction and estimation is a set of yet unsolved non-linear estimation problems related to the book (Goldfeld and Quandt, 1971). They include the evaluation of the accuracy of linear covariance approximations (Cramer-Rao bound) for non-linear estimators, the design of algorithms to produce better covariance approximations for non-linear estimators, analysis of the effects of aggregation over vintages on the estimation of production relations and on production (Smallwood, 1970), estimation problems for markets in disequilibrium, and methods to detect and to estimate structural shifts. The last is related to the problem of optimal control, since the introduction of control rules might lead to structural shifts and it would be of practical importance for stabilization policy to be able to detect such shifts if they occur.

The third area is the integration of simultaneous-equation models with auto-regressive models. Recently, there has been much interest in comparing the forecasting abilities of simultaneous-equation models with the so-called naïve auto-regressive models, but there is no reason why the two types of models should be treated as alternatives, or why the auto-regressive properties of economic time-series cannot be built into the simultaneous-equation models. We have made some progress in this area by providing maximum likelihood estimates of linear equation systems with auto-regressive residuals (Chow and Fair, 1970), but the way in which economic theory should be combined with standard time-series models in the specification of dynamic economic processes should be studied systematically, both from the theoretical and the empirical points of view.

The fourth area, like the third, is motivated by our serious consideration of the dynamic or time-dependent aspects of econometric models. It is the design of optimal estimators of parameters for forecasting purposes. In our current investigations, we have already shown that, in a first-order auto-regressive model, the Bayesian estimator of the auto-regressive coefficient for making forecasts one period ahead is different from the estimator for making forecasts two periods ahead, where the loss is the square of the forecast error. The result can be extended to systems of auto-regressive equations, and we propose to study the general problem of optimal forecasting both analytically and by Monte Carlo methods.

c. Analyses of Empirical Models

The analysis of the dynamic characteristics of macro-econometric models is not only closely related to the above estimation problems; it is an area in which we have initiated a substantial amount of theoretical work, and it is also an area in which we have substantial empirical experience, with Chow, Fair, and Goldfeld having built models of their own and Branson, de Menil, and Jaffee having made major contributions to the FRB-MIT model. Research in this area will proceed along the following lines.

First, we hope to devise formal methods to study specification errors as they occur in a system of econometric equations, to detect them and to measure their importance in the system. Second, we will devote serious attention to the comparison of large and small models and to a systematic study of possible gains from disaggregation. (See next paragraph.) Some results have already been recorded by (Fair, 1971a) concerning relative forecasting abilities. It appears that both the problems of specification errors and of optimal aggregation can be studied in the framework of optimal control. Using a welfare function, one can hope to measure the cost of errors due to misspecification and that due to over or under aggregation.

For the purpose of obtaining good control policies for the stability and growth of the american economy, one needs to have solid econometric models. Specifically, we plan to converge on two small to medium sized econometric models for policy purposes - one based partly on the ingredients of the Chow-Moore model (Chow and Moore, 1971) and the Fair model (Fair, 1971a), and the other to be an aggregation and simplification of the FRB-MIT model. We have sufficient strength for making significant progress on fundamental empirical issues in macro-economics not only from our collective experience but also because of our strategy of concentrating on two models which are small and easy to comprehend and analyze. First, we believe that at the present time much knowledge can be gained by careful study and re-specification of a few key equations in the framework of small macro-models. Second, our focus on two such models will permit us - through comparison - to determine the relative importance of the different features in these models. Up until now, detailed comparison of existing macro-econometric models has been hindered by their complexities and by the considerable differences in size. We think our strategy will permit us to break new ground in this comparative study. Third, condensation and aggregation of the FRB-MIT model will provide us with a means for studying the costs and benefits of dissaggregation, as mentioned above.

Among the key questions to be studied in the framework of small models, that of the determination of the price level ranks high in importance. We have made contributions to this area (Fair, 1970b), but further investigations are necessary, partly because the rate of inflation is an important ingredient of welfare and the mechanism for its determination is essential for the derivation of optimal policies. The problem can be studied both on the aggregative level or on the more disaggregative level. The latter approach can provide a foundation for, and help reformulate, the price behavior in the aggregate. In the latter approach, the influence of the composition of demand and supply in both the product and labor markets on the inflation-unemployment tradeoff will be examined, including the influence of the characteristics of the rates of growth of components of GNP on product prices and the influence of the geographical and occupational distribution of unemployment on wage rates. It is also planned to study the influence of unions on wages. Based on the progress made on a game-theoretic model of union-nonunion differentials (de Menil, 1970a), we plan to extend the effort to the development and testing of a model of nonunion wage determination, to the estimation of a simultaneous-equation model of the union-nonunion differential by integrating the above model with the game-theoretic model, and to the derivation of Phillips-type disequilibrium wage adjustment equations for both the union and non-union sectors which are consistent with the above simultaneous equation model.

We also intend to continue studies in the area of the housing and mortgage markets. Our previous contributions (Jaffee and Gramlich, 1971) have provided aggregate models of these markets which are used as sectors of the FRB-MIT model. The main emphasis in future work will be to adapt the current models in order to answer specific policy questions of how to stimulate residential construction efficiently. An empirical study testing the theoretical model developed in (Goldfeld and Jaffee, 1970) on Savings and Loan Association behavior is also anticipated.

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