CAPITAL-LABOR SUBSTITUTION AND THE DEMAND FOR CAPACITY

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I. INTRODUCTION

Despite the large amount of research that has been done in the field of investment behavior, there still appears to be no general agreement on how investment expenditures are determined and in particular no general agreement on the degree to which relative factor prices are important in determining investment expenditures.[1] Because of the aggregative nature of much of the empirical work in this field, this lack of agreement is perhaps not too surprising. It may be asking too much of the aggregative data to expect that the data can distinguish, for example, among hypotheses that are based on different assumptions about the degree of substitution between capital and labor. This is especially likely to be true if attempts are made to estimate the degree of substitution between capital and labor without using data on labor. Even much of the data at the level of the firm may not be adequate for testing hypotheses about capital-labor substitution. Capital-stock data at the level of the firm are usually constructed from deflated investment data, and this procedure may lead to large errors in measuring the stock of capital because of possible inaccuracies in the price deflators used and because of the

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[1] This lack of agreement is perhaps best reflected in the extensive debate between Robert Eisner and his collaborators on the one hand and Dale Jorgenson and his collaborators on the other. See Eisner [3] for one of the latest in the series of comments and replies.

restrictive assumptions that usually have to be made about the rate at which the stock of capital depreciates.

For two three-digit United States manufacturing industries -- Cement and Steel -- rather good data on capacity are available, and the purpose of this paper is to use these data to try to determine the importance of capital-labor substitution and to study the determinants of investment in capacity. Many of the problems associated with investment and capital-stock data are avoided by using capacity data, and this study should be able to provide information regarding the importance of capital-labor substitution and the determinants of investment decisions that cannot be provided merely from using investment data. In addition, the availability of capacity data allows one to distinguish between decisions of the firm regarding how much capacity to install and decisions regarding the types of machines to purchase to meet the capacity requirements. It will be seen below that this distinction provides a useful framework for deciding how much investment is due to capital-labor substitution and how much is due to other causes.

Any attempt to determine the degree of capital-labor substitution must take account of short-run fluctuations in capacity utilization. The results in Fair [4] indicate that most firms hold a considerable amount of excess labor during slack periods, and the monthly data on capacity utilization for the Cement and Steel industries indicate that firms operate well below 100 percent capacity much of the time. Part of the present study, therefore, must be concerned with trying to separate the short-run determinants of capacity utilization from the longer-run determinants of capital-labor substitution and the demand for capacity.

The theoretical model upon which this study is based is presented in Section II, and the data and results for the Cement and Steel industries are presented in Sections III and IV respectively. The major conclusions of the study are presented in Section V.

II. THE THEORETICAL FRAMEWORK

Assumptions about the Underlying Technology

The assumptions made here about the underlying technology are similar in many respects to the assumptions made by Johansen [6], Salter [8], and others who have analyzed "putty-clay" models. Some of the notation used here is similar to the notation used by Solow, Tobin, von Weizsacker, and Yaari [9].

Assume that at any point in time there are available k different types of machines that can be purchased. These machines differ in price, in the number of workers that must be used with each machine per unit of time, and in the amount of output that can be produced per machine per unit of time. The worker-machine ratio is assumed to be fixed for each type of machine. Let $\lambda_{\nu i}$ denote the amount of output that can be produced per hour per worker on machines of number i that were purchased in time ν, and let $\mu_{\nu i}$ denote the amount of output that can be produced per hour on one of these machines. Also, let $I_{t\nu i}$ denote the number of machines of number i that were purchased in time ν that are actually operating in time t, let $M_{t\nu i}$ denote the number of workers working in time t on these machines, let $H_{t\nu i}$ denote the number of hours worked per worker and machine in time t on these machines, and let $Y_{t\nu i}$ denote the amount of output produced in time t on these machines. The machines will be assumed not to be subject to physical depreciation, so that $\lambda_{\nu i}$ and $\mu_{\nu i}$ are not a function of t. This assumption and the constancy of the worker-machine ratio imply that

$$(1) \qquad Y_{t\nu i} = \lambda_{\nu i} M_{t\nu i} H_{t\nu i} = \mu_{\nu i} I_{t\nu i} H_{t\nu i} \quad .$$

Let m_t denote the age of the oldest machine that is being used in time t. Then the total amount of output produced in time t, Y_t, is

$$(2) \qquad Y_t = \sum_{\nu=t-m_t}^{t} \sum_{i=1}^{k} Y_{t\nu i} \quad .$$

In other words, the total amount of output produced in time t is the sum of the amount of output produced on each of the machines that are operating in time t. The parameter k in equation (2) may not be constant through time since more types of machines may be available for purchase at one time than at another, but without loss of generality in what follows k can be assumed to be independent of time. What is implied by this assumption is that as a new type of machine becomes available for purchase, one of the old types of machines is phased out.

Technical progress in the model is assumed to be reflected in the new types of machines. It will be convenient in what follows to consider two simple examples of what the underlying technology might be like and of how technical progress might affect the technology. For the first example one new type of machine is assumed to be introduced each period, technical progress is assumed to be of the exponential kind, the oldest type of machine is assumed to be phased out each period, and the types of machines are assumed to be numbered according to age (i equal to 1 being the newest type and i equal to k being the oldest type). These assumptions imply that $\lambda_{\nu i} = \beta(1+\lambda)^{\nu-i-1}$, $i=1,2,\ldots,k$, where λ is the rate of labor-augmenting technical progress, and that $\mu_{\nu i} = \gamma(1+\mu)^{\nu-i-1}$, $i=1,2,\ldots,k$, where μ is the rate of capital-augmenting technical progress. (ν is assumed to be greater than k.) In this example each type of machine changes number each period (machines of number i that are purchased in period ν being the same as machines of number i+1 that are purchased in period ν+1), and any two consecutive types of machines differ from each other in terms of the $\lambda_{\nu i}$ and $\mu_{\nu i}$ coefficients by constant proportions.

For the second example there are assumed to be two lines of machines, the first line being more costly but having a lower worker-machine ratio than the second line. Technical progress is assumed to be the same for both lines of machines and is assumed to be of the exponential kind. One new type of machine for each line is assumed to be introduced each period, and the oldest type of machine for each line is assumed to be phased out each period. The types of machines are assumed to be numbered according to age, where the types of machines of the first line are numbered $1,2,\ldots,k_1$, newest to oldest, and where the types of machines of the second line are numbered $k_1+1,k_1+2,\ldots,k$, newest to oldest. These assumptions imply that $\lambda_{\nu i} = \beta_1(1+\lambda)^{\nu-i-1}$, $i=1,2,\ldots,k_1$, and $\lambda_{\nu i} = \beta_2(1+\lambda)^{\nu-i-1+k_1}$, $i=k_1+1,k_1+2,\ldots,k$, and similarly for $\mu_{\nu i}$. The two lines of machines thus differ from each other in terms of the $\lambda_{\nu i}$ coefficients by the proportion β_1/β_2 for machines of the same age.

In the general case, of course, there can be many different lines of machines, technical progress need be neither exponential nor smooth, and new types of machines need not be introduced every period. The above two examples have been presented merely to aid in the discussion that follows.

Equation (1) refers to the actual amount of output produced on a given type of machine, but it can also be written in terms of capacity output. Let $I_{\nu i}$ denote the total number of machines of number i that were purchased in time ν (regardless of whether they are actually operating in time t or not), let $M_{\nu i}$ denote the number of workers that are required to work on the $I_{\nu i}$ machines, and let $H_{\nu i}$ denote the "capacity" number of hours that can be worked per worker per period on these machines. Then capacity output per period on these machines, $Y_{\nu i}$, is

$$(3) \qquad Y_{\nu i} = \lambda_{\nu i} M_{\nu i} H_{\nu i} = \mu_{\nu i} I_{\nu i} H_{\nu i} \quad .$$

If m_t' denotes the age of the oldest machine in existence in time t,[2] then total capacity output in time t, Y_t^c, is

$$(4) \qquad Y_t^c = \sum_{\nu=t-m_t'}^{t} \sum_{i=1}^{k} Y_{\nu i} \quad .$$

Likewise, the capacity number of workers in time t, M_t^c, is

$$(5) \qquad M_t^c = \sum_{\nu=t-m_t'}^{t} \sum_{i=1}^{k} M_{\nu i} \quad .$$

A Test of Capital-Labor Substitution

For purposes of this subsection, two further simplifying assumptions about the above model will be made. First, the capacity number of hours that can be worked per machine per period will be assumed to be the same for all types and ages of machines: $H_{\nu i} = H$, all ν and i. This assumption is fairly innocuous since it does not seem likely that the maximum number of hours that machines can be worked per period will vary much from machine to machine.

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[2] For simplicity and without loss of generality in what follows, machines of the same age are assumed to go out of existence all at the same time.

Second, the age of the oldest machine in existence in time t will be assumed to be independent of t: $m_t' = m$, all t. This assumption is more restrictive than the assumption about capacity hours, but it will be relaxed below.

If the above two assumptions are made, then equations (3) and (4) imply that

$$(6) \qquad Y_t^c = H \sum_{\nu=t-m}^{t} \sum_{i=1}^{k} \mu_{\nu i} I_{\nu i} ,$$

and equations (3) and (5) imply that

$$(7) \qquad M_t^c = \sum_{\nu=t-m}^{t} \sum_{i=1}^{k} \frac{\mu_{\nu i} I_{\nu i}}{\lambda_{\nu i}} .$$

Lagging equations (6) and (7) one period and subtracting each lagged equation from the respective unlagged equation yields

$$(8) \qquad Y_t^c - Y_{t-1}^c = H \sum_{i=1}^{k} [\mu_{ti} I_{ti} - \mu_{t-m-1,i} I_{t-m-1,i}] ,$$

$$(9) \qquad M_t^c - M_{t-1}^c = \sum_{i=1}^{k} [\frac{\mu_{ti} I_{ti}}{\lambda_{ti}} - \frac{\mu_{t-m-1,i} I_{t-m-1,i}}{\lambda_{t-m-1,i}}] .$$

Equation (8) can then be solved for $\mu_{t1} I_{t1}$ and this expression substituted into equation (9), yielding

$$(10)\qquad M_t^c - M_{t-1}^c = \frac{Y_t^c - Y_{t-1}^c}{H\lambda_{t1}} + \left(\frac{1}{\lambda_{t1}} - \frac{1}{\lambda_{t-m-1,1}} \right) \mu_{t-m-1,1} I_{t-m-1,1}$$

$$- \sum_{i=2}^{k} \left[\left(\frac{1}{\lambda_{t1}} - \frac{1}{\lambda_{ti}} \right) \mu_{ti} I_{ti} - \left(\frac{1}{\lambda_{t1}} - \frac{1}{\lambda_{t-m-1,i}} \right) \mu_{t-m-1,i} I_{t-m-1,i} \right]$$

If k is equal to one in equation (1), so that it is possible to purchase only one type of machine at any given time, and if there is no technical progress, so that λ_{t1} is constant for all t, then equation (10) merely states that the change in the capacity number of workers is directly proportional to the change in capacity output.

Consider next the first example described above, only assume that it is never possible for the firm to purchase any but the newest type of machine at any given time. In this case k is equal to one in equation (10) and λ_{t1} equals $\beta(1+\lambda)^t$, and so equation (10) becomes

$$(11)\qquad M_t^c - M_{t-1}^c = \frac{Y_t^c - Y_{t-1}^c}{H\beta(1+\lambda)^t} + \left[\frac{1}{\beta(1+\lambda)^t} - \frac{1}{\beta(1+\lambda)^{t-m-1}} \right] \mu_{t-m-1,1} I_{t-m-1,1} \, .$$

Equation (11) states that the change in the capacity number of workers is directly proportional to an exponentially decreasing function of the change in capacity output except for a factor that takes into account replacement investment. The change in capacity output reflects net investment, and if, for example, the change in capacity output is zero in time t, there still will be investment taking place to replace the old machines being retired. If technical progress is occuring so that newer machines require fewer workers to work on them than on older machines, then the change in the capacity number of workers will not be directly proportional to an exponentially decreasing function of the change in capacity output, but will differ from this function by a factor that measures how much more efficient (in terms of labor requirements) the new machines just installed are then the old machines just replaced. $I_{t-m-1,1}$ in equation (11) is the number of machines purchased in time t-m-1 and thus the number of machines retired at the end of time t-1.

In the general case, where k is greater than one in equation (10) and technical progress is not necessarily exponential or smooth, the change in the capacity number of workers will definitely not be proportional to any simple function of the change in capacity output. The change in the capacity number of workers will be a function, among other things, of the amount of investment made in each type of machine in time t, of the number of machines retired at the beginning of time t, and of the relative efficiencies of all of these machines.

Let W_t denote the average wage rate in time t and let C_t denote the average cost of new capital in time t. The cost of capital will be different for each type of machine since the cost of capital is a function of the price and length of life of the capital good in question in addition to being a function of the cost of borrowing funds and of tax and depreciation laws. In what follows, however, it will be assumed that the prices of the various types of machines, while differing from each other at any one time, move together over time. If this is true, then an average cost of capital can be defined based on the average price of the various types of machines and on the average length of life of the various types of machines. Since the main concern is with how the cost of capital changes relative to the wage rate, the use of the concept of an average cost of capital should not pose any serious difficulties in the present analysis. The ratio W_t/C_t will be referred to as the wage-rental ratio.

If more than one type of machine can be purchased at any given time, then the wage-rental ratio should have an effect on the types of machine purchased. A high wage-rental ratio should cause firms to purchase high-priced machines with low worker-machine ratios, and a low wage-rental ratio should cause firms to purchase low-priced machines with high worker-machine ratios. This effect of the wage-rental ratio can be considered to be a direct form of capital-labor substitution.

Salter has pointed out two other ways in which capital-labor substitution may take place.[3] First, the direction that technical progress takes may be a product of changing factor prices rather than of new knowledge. To the extent that, say, a rising wage-rental ratio causes technical progress to be biased toward labor-saving techniques (i.e., labor-augmenting technical progress growing faster than capital-augmenting technical progress), this can be considered to be a form of capital-labor substitution. Second, capital-labor substitution may come about by speeding up the rate of replacement investment. If, for example, technical progress is on average biased toward labor-saving techniques, then speeding up the rate of replacement investment will result on average in the purchase of machines with lower worker-machine ratios than the machines being replaced.

There are thus three possible ways in which the wage-rental ratio can affect capital-labor substitution. The ratio can affect the types of machines purchased, the direction of technical progress, and the rate of replacement investment. Since data on Y_t^c are available for the Cement and Steel industries

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[3] Salter [8], pp. 24, 71.

and since, as will be seen below, data on M_t^c can be constructed (at least approximately) for these industries, the following test of the effect of the wage-rental ratio on capital-labor substitution can be made. If labor-augmenting technical progress is exponential and not a function of the wage-rental ratio, if k is equal to one in equation (10), and if replacement investment is not a function of the wage-rental ratio, then estimating the equation,

$$(12) \qquad M_t^c - M_{t-1}^c = \left(\frac{1}{H\beta} \right) \frac{Y_t^c - Y_{t-1}^c}{(1+\lambda)^t} + \gamma_1 \frac{W_t}{C_t} + u_t \quad ,$$

where u_t is an error term, should result in an estimate of γ_1 not significantly different from zero. If technical progress is exponential and k is equal to one, then it can be seen from equation (11) that the only way in which the wage-rental ratio can have a nonzero coefficient in equation (12) is if replacement investment is a function of the wage-rental ratio. Consequently, if replacement investment is not a function of the wage-rental ratio, the coefficient γ_1 in equation (12) should be zero.

If, on the other hand, the wage-rental ratio affects capital-labor substitution in any of the three possible ways mentioned above, then the coefficient γ_1 should be nonzero in equation (12). Consider, for example, the second example described above, where k is greater than one but technical progress is not a function of the wage-rental ratio. Assume also that replacement investment is not a function of the wage-rental ratio. In this case a high wage-rental ratio should cause firms to purchase the more expensive line of machines with the lower worker-machine ratios, and a low wage-rental ratio should cause firms to purchase the less expensive line of machines with the higher worker-machine ratios. A changing wage-rental ratio should thus cause the change in the capacity number of workers in equation (10) not to be directly proportional to any simple function of the change in capacity output, which in turn should cause the wage-rental ratio to be a significant explanatory variable when an equation like (12) is estimated. The purchase of machines with low worker-machine ratios should cause $M_t^c - M_{t-1}^c$ to deviate from $\left(\frac{1}{H\beta}\right) \frac{Y_t^c - Y_{t-1}^c}{(1+\lambda)^t}$ by a negative amount, and the purchase of machines with high worker-machine ratios should have the opposite effect. The wage-rental ratio in equation (12) should thus have a negative effect on $M_t^c - M_{t-1}^c$ if the purchase of different types of machines is influenced by the wage-rental ratio.

Consider next the case in which labor-augmenting technical progress is not exponential, but is a function of the wage-rental ratio. Assume in particular that it is the deviation of the rate of labor-augmenting technical progress from an exponential trend that is a function of the wage-rental ratio. Assume also that k is equal to one in equation (10) and that replacement investment is not a function of the wage-rental ratio. In this case a high (low) wage-rental ratio should cause the deviation of the rate of labor-augmenting technical progress from an exponential trend to be positive (negative), which should in turn cause $M_t^c - M_{t-1}^c$ to deviate from $\left(\frac{1}{H\beta}\right) \frac{Y_t^c - Y_{t-1}^c}{(1+\lambda)^t}$ in equation (12) by a negative (positive) amount. The wage-rental ratio in equation (12) should thus have a negative effect on $M_t^c - M_{t-1}^c$ if the rate of labor-augmenting technical progress is a function of the wage-rental ratio.

Consider finally the case in which replacement investment is a function of the wage-rental ratio. This involves relaxing the assumption made above that the age of the oldest machine in existence, $m_t^!$, is not a function of time, for if $m_t^!$ is constant, then replacement investment is merely a function of the number of machines purchased m periods ago. Assume also that k is equal to one in equation (10) and that the rate of labor-augmenting technical progress is exponential and not a function of the wage-rental ratio. Then from equation (11) it can be seen that replacement investment should have a negative effect on $M_t^c - M_{t-1}^c$, since the term in brackets in the equation is negative. Therefore, if the wage-rental ratio has a positive effect on the rate of replacement investment, as it should have if technical progress is biased toward labor-saving techniques, then the wage-rental ratio should have a negative effect on $M_t^c - M_{t-1}^c$ in equation (12).

In summary, then, if the wage-rental ratio affects capital-labor substitution in any of the three possible ways described above, estimating equation (12) should result in a significantly negative estimate of the coefficient γ_1. Otherwise the coefficient estimate should be insignificant. Even if the rate of labor-augmenting technical progress is not exponential, so that equation (12) is misspecified, it still should not be the case that estimating equation (12) results in a significant estimate of γ_1 if in fact the wage-rental ratio has no effect on any of the three possible ways of capital-labor substitution. It should be noted, of course, that the above test cannot separate out the three possible ways in which the wage-rental ratio may affect capital-labor substitution. In the next two sections equation (12) will be estimated for the Cement and Steel industries. The equation is nonlinear in the parameter λ and so must be estimated by a nonlinear technique.

A Model of the Demand for Capacity

In this study the firm is conceived of as making two basic kinds of investment decisions, the first regarding the desired level of capacity and the second regarding how many machines to replace and what types of machines to purchase to meet the desired level of capacity.[4] It is important to note that in this way of looking at the problem capital-labor substitution relates only to the second kind of decision.

With respect to the first kind of decision, it seems quite likely that the firm's desired level of capacity will be a function, among other things, of the level of expected future sales. The key assumption that is made in this study regarding the demand for capacity is that desired capacity is a function not of the expected level of sales of the whole year but of the expected level of sales of only some peak period of the year.

Assume that decisions on capacity investment are made at the beginning of each year and that capacity is installed at the end of the year. Let SP_t^e denote the expected level of sales of some peak period of year t, the expectations being made at the beginning of year t. Let Y_t^c denote capacity output at the end of year t, and let Y_t^{c*} denote desired capacity output for the end of year t, the decisions on desired capacity being made at the beginning of year t. Then it is assumed that

$$(13) \qquad Y_t^{c*} = \alpha SP_t^e \quad ,$$

where the parameter α is a function of various cost parameters of the firm.

The sales of most firms are subject to seasonal fluctuations, and in many of these firms an attempt is made to smooth production relative to sales by the accumulation and decumulation of inventories. How much production is smoothed relative to sales will depend on such things as the cost of holding inventories relative to the costs of changing the rate of production and the costs of carrying capacity. If production is not smoothed very much relative

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[4]Bischoff [1], p. 6, makes the same kind of distinction between investment decisions.

to sales, then inventory holding costs are low but a lot of capacity is needed to meet production requirements during the peak sales periods, whereas if production is smoothed very much relative to sales, then inventory holding costs are high but less capacity is needed to meet the peak production requirements. Firms presumably weigh these various costs and arrive at an optimal production-smoothing plan. Desired capacity should then be equal to (or slightly greater than if the firm wants to allow for a certain margin of error) the amount of capacity necessary to meet peak production requirements under this plan, given the expected path of sales. The specification of equation (13) is an attempt to approximate this situation. The specification is only approximate since data on inventory costs and costs of changing the rate of production are not available, but it does seem likely that if these costs are not fluctuating very much relative to each other, then desired capacity will be roughly proportional to some expected peak level of sales. The larger is the level of expected peak sales, the more capacity will be needed to meet peak production requirements under the optimal plan. Only if inventory costs, costs of changing the rate of production, and costs of carrying capacity are changing rapidly relative to each other is the specification in equation (13) likely to be a poor one.

It can now be seen why expected peak sales are likely to be more important in determining desired capacity than expected sales for the whole year. Under the optimal production-smoothing plan, desired capacity should be equal to the amount necessary to meet peak production requirements, and it should not matter very much from the point of view of determining desired capacity what the levels of production and sales are during slack periods.

Since data on the cost of capital are available, it will be possible in the next two sections to see if the data can pick up any effect of the cost of capital on the parameter α in (13). The problem with this procedure is that α is also a function of inventory costs and costs of changing the rate of production, and if these costs are changing in a similar manner as the cost of capital, then one is not likely to be able to pick up the effect of the cost of capital on α without also including the other costs as well.

There are likely to be costs involved in changing capacity, and so the following simple lagged adjustment process will be postulated:

$$(14) \qquad Y_t^c - Y_{t-1}^c = \delta(Y_t^{c*} - Y_{t-1}^c) \quad .$$

The parameter δ in equation (14) reflects the timing of expenditures on capacity, and it may be the case that δ is a function of the deviation of the cost of capital from some expected long-run value.[5] If, for example, the cost of capital is particularly high one year and the firm expects that it will be lower next year, then the firm may postpone some of its planned expenditures until the following year. Similarly, the firm may speed up its planned expenditures if the cost of capital is particularly low one year and is expected to be higher next year. It will be possible to test this hypothesis in the next two sections to see if the data can pick up any effect of the cost of capital on δ. Equations (13) and (14) imply that

$$(15) \qquad Y_t^c - Y_{t-1}^c = \delta(\alpha SP_t^e - Y_{t-1}^c) \quad ,$$

and this is the basic equation estimated in the work below.

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[5]See Greenberg [5] and Phelps [7] for the use of this kind of hypothesis.

It is important to note that the cost of capital may affect investment in capacity in ways that have nothing to do with capital-labor substitution. The cost of capital may affect both δ and α in equation (15), and yet in neither case does this effect have anything to do with the choice of the types of machines to purchase to meet capacity requirements. Even if there were no capital-labor substitution in the above three senses, the cost of capital could still affect investment in capacity through its effect on δ and α. The effect on δ is only a matter of timing, but the effect on α is of a more permanent type. The variety of ways in which the cost of capital can affect investment decisions may be one of the reasons why there has been so little agreement on the importance of the cost of capital in determining investment expenditures.[6]

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[6] In Bischoff's work [1], for example, it is not possible to separate the effects of the cost of capital on investment expenditures that are due to capital-labor substitution from those that are due to other causes. Bischoff assumes, among other things, that the desired capacity-output ratio is constant (p. 17) and that replacement investment is a constant proportion of existing capacity (p. 20). Because of these and other assumptions that Bischoff is required to make, it is really not possible to interpret his estimates of the elasticity of substitution as estimates of the degree of capital-labor substitution. At best the estimates reflect some average effect of the cost of capital on investment expenditures. Bischoff's work also suffers, as does most work on aggregate investment functions, from not using data on employment and wage rates in the estimation of the elasticity of substitution. This is not to degrade the importance of Bischoff's work on aggregate investment functions, but only to point out a number of problems that remain with his approach.

III. THE DATA AND RESULTS FOR THE CEMENT INDUSTRY

The Data

The basic period of estimation considered in this study is 1947-1969, although in many cases data limitations prevented the entire period from being used. Yearly data on capacity in the Cement industry are available for the entire period from the Bureau of Mines. In an annual survey of the Bureau of Mines Cement companies are asked the following question: "Assuming no problem of storage, transportation, or plant labor force, report total rated maximum 24 hour kiln capacity to produce clinker at this plant as of December 31." The answers are given in barrels of 376 pounds, and by adding the answers for all plants for each year the Bureau of Mines has been able to construct a yearly series on total Cement capacity. Also available from the Bureau of Mines are monthly data on Cement shipments for the entire period and monthly data on production for the 1947-1964 period. All of these data are in physical units. From the monthly data on production and the data on capacity, a monthly series on capacity utilization can be constructed, and such a series was published by the Bureau of Mines through 1964. Monthly data on employment in the Cement industry are available for the entire period from the Bureau of Labor Statistics. The employment data used in this study refer to the employment of production workers only.

Short-run fluctuations in shipments and production are fairly pronounced in the Cement industry, due in large part to seasonal factors, and the work in [4] indicates that excess labor is held in this industry during slack months. For present purposes, however, what is needed is not a series on excess labor, but rather a series on the number of workers that are required to produce capacity output. For the 1950-1959 period, the Cement industry operated at or near 100 percent capacity during at least one month of the year.

For the 1947-1949 and 1960-1964 periods, the peak operating rate each year was about 92 percent of capacity. For the 1965-1969 period, it also looked like the peak operating rate was about 92 percent of capacity, although this impression had to be gathered from looking at monthly shipments data rather than monthly production data. For some of the years 1950-1959 the peak operating rate was about 110 percent of capacity.

From the monthly data on employment and capacity utilization, a series on the capacity number of workers, M_t^c, was constructed as follows. For each year of the 1950-1959 period, M_t^c was taken to be that level of employment corresponding to a month or months in which capacity utilization was approximately 100 percent. If more than one month in a year had capacity utilization of around 100 percent, then the employment numbers were either averaged to get the number for M_t^c or the most frequently occuring number was taken. There was some subjectivity involved in the choice of the numbers for M_t^c, but in general the choice was fairly straightforward. For each year of the 1947-1949, 1960-1966 period, M_t^c was taken to be equal to a constant, $\bar{M}$, plus that level of employment corresponding to a month or months in which capacity utilization was approximately 92 percent. The choice for the years 1965 and 1966 had to rely on shipments rather than production data. The constant $\bar{M}$ can be determined by the regression equation, as will be seen below. The constant should be negative since fewer than the capacity number of workers are likely to be on hand when capacity utilization is only 92 percent. There may, of course, still be some excess labor held at 92 percent of capacity, but if the amount held at this operating rate does not vary from year to year, then the above procedure is still valid. All that is required is that the capacity number of workers differ from the actual number of workers at 92 percent of capacity by a constant amount.

For the years in which the peak operating rate was 92 percent of capacity, the choice of M_t^c always corresponded to the peak level of employment for the year, but this was not always the case for the other years. For the years in which the peak operating rate was near 110 percent of capacity, the peak level of employment was usually larger than the level corresponding to 100 percent operating rates.

Monthly data on average hourly earnings in the Cement industry are available for the entire period from the Bureau of Labor Statistics. For purposes here, the earnings figure prevailing in February of each year was used as the wage rate for the year. February is a month in which very little overtime is worked in the Cement industry, and so the average hourly earnings figure for this month more closely reflects straight time hourly earnings than, say, the average hourly earnings figure for the whole year.

The measure of the cost of capital used in this study is one constructed by Robert Coen and presented in [2], Table 2. The measure is available yearly for the 1947-1966 period and incorporates various tax incentives for investment that have been passed since 1954. The GNP implicit price deflator for nonresidential fixed investment was used by Coen as the estimate of the price of capital goods. Moody's industrial bond yield was used as the estimate of the interest rate. There are a number of fairly restrictive assumptions involved in the construction of Coen's measure, but it appeared to be the best measure available.

The Results

From the above work, data on M_t^c are available for the 1950-1959 period and data on $M_t^c+\bar{M}$ are available for the 1947-1949 and 1960-1966 periods. Taking first differences means that data on $M_t^c-M_{t-1}^c$ are available for the

1948-1966 period except for the years 1950 and 1960. For 1950 data on $M_t^c - M_{t-1}^c - \bar{M}$ are available, and for 1960 data on $M_t^c - M_{t-1}^c + \bar{M}$ are available. The constant $\bar{M}$ can be estimated and all of the data points used by constructing a dummy variable, say D_t, which is minus one in 1950, one in 1960, and zero everywhere else and including D_t as an explanatory variable in the equation being estimated. For all of the years except 1950 and 1960, D_t will have no effect; for 1950 the coefficient of D_t will be $\bar{M}$ since the dependent variable for 1950 is $M_t^c - M_{t-1}^c - \bar{M}$ rather than the correct $M_t^c - M_{t-1}^c$; and for 1960 the coefficient of D_t will be $\bar{M}$ since the dependent variable for 1960 is $M_t^c - M_{t-1}^c + \bar{M}$. The estimate of the coefficient of D_t in the regression equation will thus be an estimate of $\bar{M}$. The estimate is expected to be negative since $\bar{M}$ is assumed to be negative.

The results of estimating equation (12) for the Cement industry are presented in Table I. The equation is a very simple nonlinear equation and was estimated by a standard gradient method. The estimate of the coefficient of the wage-rental ratio is definitely significant in Table I and of the expected negative sign. The estimate of $\bar{M}$ is of the expected negative sign, but it is quite insignificant. It appears that $\bar{M}$ is either quite small or the sample is not large enough to pick up any errors-of-measurement effects. The estimate of $1+\lambda$ in Table I is above one, as expected, and very significant.

The results in Table I thus indicate that capital-labor substitution has taken place in the Cement industry. If more observations had been available, it would have been desirable to experiment with various distributed lags of the W_t/C_t variable to get a better idea of the timing effects of the wage-rental ratio on capital-labor substitution, but the results in Table I are quite consistent with capital-labor substitution having taken place.

Table I: Results of estimating equation (12) for the Cement industry

$$M_t^c - M_{t-1}^c = \bar{M}D_t + \gamma_0 \frac{Y_t^c - Y_{t-1}^c}{(1+\lambda)^t} + \gamma_1 \frac{W_t}{C_t} + u_t$$

Period of Estimation	$\hat{\bar{M}}$	$(\widehat{1+\lambda})$	$\hat{\gamma}_0$	$\hat{\gamma}_1$	SE	R^2	DW	# of Obs.
1948-1966	-.214 (0.04)	1.157 (18.46)	24.02 (1.96)	-.871 (-3.82)	6.80	.503	1.56	19

(t-statistics are in parentheses.)

The results of estimating equation (15) for the Cement industry are presented in Table II corresponding to different assumptions about expected peak sales, SP_t^e. The best results were obtained by taking SP_t^e to be equal to the level of shipments during the peak month of the year. (Equation (1) in Table II.) Results almost as good were obtained by taking SP_t^e to be equal to the level of shipments during the peak month of the year or to the level of shipments during the previous peak month, whichever is larger. (Equation (2).) Poorer results were obtained by taking SP_t^e to be equal to the average level of shipments during the two largest months of the year, and very poor results were obtained by taking SP_t^e to be equal to the average level of shipments over the whole year. (Equations (3) and (4).)

The implied estimates of α are presented in Table II for each equation. All of the shipments variables are in units per month, and for purposes of estimating equation (15), Y_t^c was also converted into units per month by dividing the annual capacity figures by twelve. The value of α of approximately one for equation (1) in Table II thus means that desired capacity converted to a monthly level is approximately equal to the expected level of shipments during the peak month of the year.

Table II: Results of estimating equation (15) for the Cement industry

$$Y^c_t - Y^c_{t-1} = \delta(\alpha SP^e_t - Y^c_{t-1}) + u_t$$

Equation Number	Period of Estimation	Assumption about SP^e_t	$\hat{\delta}$	$\hat{\delta\alpha}$	Implied $\hat{\alpha}$	SE	R^2	DW	# of Obs.
(1)	1948-1969	(a)	.286 (4.72)	.288 (5.20)	1.01	587.0	.502	0.90	22
(2)	1948-1969	(b)	.293 (4.36)	.293 (4.79)	1.00	614.4	.455	0.80	22
(3)	1948-1969	(c)	.227 (3.14)	.238 (3.54)	1.05	705.5	.281	1.01	22
(4)	1948-1969	(d)	.108 (1.63)	.169 (2.07)	1.57	816.7	.037	0.83	22
(5)	1948-1964	(a)	.233 (3.41)	.244 (3.98)	1.05	588.8	.512	1.02	17
(6)	1948-1964	(e)	.258 (3.44)	.296 (3.96)	1.15	590.6	.509	1.15	17

(a) Peak month of year.

(b) Peak month of year or previous peak month, whichever is larger.

(c) Average of two largest months of the year.

(d) Average of all twelve months of the year.

(e) Peak month of year. Output rather than shipments data used.

(t-statistics are in parentheses.)

There may be a bias in the results in Table II if the level of shipments during a peak month is not exogenous but is restricted by the level of capacity. If the level of shipments is so restricted, then SP_t^e will be a function of Y_t^c as well as Y_t^c being a function of SP_t^e. The level and fluctuation of inventories in the Cement industry are large enough relative to the level of shipments to indicate that this bias is not likely to be very serious, but the bias cannot be completely ruled out. Overall, however, the results definitely do seem to indicate that expected shipments during peak periods are more important in determining desired capacity than are expected shipments during slack periods as well. The results always got worse as more and more slack periods were included in the measure of SP_t^e.

Equation (5) in Table II is the same as equation (1) except estimated for a shorter period, and equation (6) is the same as equation (5) except that output data rather than shipments data were used for the estimate of SP_t^e. The results of equations (5) and (6) are similar in terms of fit and coefficient estimates, which is encouraging since only data on output are available for the Steel industry. Even though peak output is likely to be restricted by the level of capacity, the bias from using output data in equation (15) does not appear to be very great.

Equation (15) was also estimated under the assumption that α is a function of the cost of capital -- $\alpha = \alpha_o + \alpha_1 C_t$ -- but no effect of the cost of capital on α could be found. Depending on the assumption about SP_t^e and the period of estimation, the estimates of α_1 were either insignificant or of the wrong positive sign. Apparently data on inventory costs and costs of changing the rate of production would be needed before any serious attempt could be made to determine the effects of the cost of capital on the desired amount of capacity.

Equation (15) was also estimated under the assumption that δ is a function of the deviation of the cost of capital from its expected long-run value:

$$(16) \qquad \delta = \delta_o + \delta_1(C_t - \bar{C}_t^e) \quad ,$$

where $\bar{C}_t^e$ is the expected long-run value of C_t made at time t. The equation was estimated under various assumptions about $\bar{C}_t^e$, and in no case was the estimate of δ_1 significant.[7] In addition to Coen's measure of the cost of capital, profit and cash flow variables were also tried as measures of C_t in (16). One might expect that higher than expected profits or cash flow would induce firms to speed up their planned investment expenditures, and vice versa for lower than expected profits or cash flow. This did not appear to be the case for the results obtained here, although the profits and cash flow data that were available pertained to all of the Stone, Clay, and Glass industry rather than to just the Cement industry. The Cement industry constitutes only about 13 percent by value added of the Stone, Clay, and Glass industry. A simple interest rate was also tried as a measure of C_t in (16), but again with no success. The results achieved here thus do not indicate that the timing of capacity expenditure is influenced by the deviation of the cost of capital from its expected long-run value, but the sample is small and some of the data are not very good, and perhaps not too much emphasis should be placed on this set of negative results.

Finally, the equations in Table II were estimated under the assumption of first order serial correlation of the error terms since the Durbin-Watson

[7]When equation (15) is estimated under assumption (16), the equation is nonlinear and must be estimated by a nonlinear technique. For purposes here a simple gradient method was used -- the same method used to estimate equation (12).

statistic indicated the existence of positive serial correlation. The estimates of the serial correlation coefficients were between about .5 and .6, but the other coefficient estimates were not changed very much. None of the conclusions reached above appeared to be changed by the serial correlation results, and so these results will not be presented here.

IV. THE DATA AND RESULTS FOR THE STEEL INDUSTRY

The Data

Yearly data on steel capacity -- blast furnace capacity and capacity to produce ingots and steel for castings -- are available for the 1947-1960 period from the American Iron and Steel Institute (AISI). Monthly data on production and capacity utilization are also available from AISI for this period. Blast furnace production accounts for about 11 percent of SIC industry 331, and production of ingots and steel for castings accounts for about 46 percent. The other 43 percent of industry 331 is composed primarily of production of steel mill products. Monthly data on employment for the 1947-1960 period are not available on any more detailed a basis than for industry 331.

For the work in this study, the data on capacity and production of the ingots-and-steel-for-castings part of industry 331 were used for the capacity and production data, and Bureau of Labor Statistics data on the employment of production workers in industry 331 were used for the employment data. Because of the interrelatedness of the various processes in industry 331, there is a very high correlation between the production (of pig iron) from blast furnaces, the production of iron and steel for castings, and the production of steel mill products. It should thus not matter very much for present purposes that the capacity and production data refer to only about

46 percent of industry 331 while the employment data refer to the entire industry. Only if technological developments and trends were quite different in the various parts of industry 331 would the use of the production and employment data pose a serious problem, and this does not appear to have been the case during the period under consideration. Monthly data on shipments of ingots and steel for castings are not available. All of the capacity and production data are in physical units.

Production in the Steel industry is subject to very little seasonal fluctuation, but it is characterized by large cyclical swings and large fluctuations because of strikes. Because of the lack of regular seasonal fluctuations, the construction of M_t^c for the Steel industry was somewhat more difficult than it was for the Cement industry. For the 1947-1960 period, only eight months that were separated by a year or more could be found in which the rate of capacity utilization was around 100 percent. The eight months that were chosen to use for the M_t^c data were August 1947, February 1949, January 1951, August 1952, August 1953, December 1955, September 1956, and February 1960. Data on capacity output for these eight months were constructed by interpolating the yearly capacity figures, the yearly figures being available for the beginning of each year. This procedure is to be contrasted to the procedure used for the Cement industry, where there was no real need to interpolate the yearly capacity figures.

Average hourly earnings figures prevailing in February in industry 331 were used for the wage rate data. The hourly earnings figure of the February preceding the month in question was used as the figure pertaining to the month in question. The same cost of capital variable was used for the Steel industry as was used for the Cement industry.

The Results

The results of estimating equation (12) for the Steel industry are presented in Table III. It should be noted that for purposes of estimation the time trend was not taken to be evenly spaced over the 7 observations, but was taken to be evenly spaced across calendar time. The estimate of the coefficient of the wage-rental ratio is negative, as expected, in Table III, but it is not significantly different from zero at the 95 percent confidence level. The estimate of $1+\lambda$ is above one and is highly significant.

Table III: Results of estimating equation (12) for the Steel industry.

$$M_t^c - M_{t-1}^c = \gamma_o \frac{Y_t^c - Y_{t-1}^c}{(1+\lambda)^t} + \gamma_1 \frac{W_t}{C_t} + u_t$$

Period of Estimation	$(\widehat{1+\lambda})$	$\hat{\gamma}_o$	$\hat{\gamma}_1$	SE	R^2	DW	# of Obs.
See text.	1.044 (54.74)	.278 (1.44)	-6.97 (-1.28)	112.7	.668	1.75	7

(t-statistics are in parentheses.)

The results in Table III for the Steel industry are thus similar to the results in Table I for the Cement industry, although the estimate of the coefficient of the wage-rental ratio is not significant. With only four degrees of freedom, the results in Table III must be interpreted with considerable caution, but at least the results are not unfavorable to the conclusion that some capital-labor substitution has taken place in the Steel industry.

The results of estimating equation (15) for the Steel industry are presented in Table IV corresponding to different assumptions about expected

Table IV: Results of estimating equation (15) for the Steel industry.

$$Y_t^c - Y_{t-1}^c = \delta(\alpha SP_t^e - Y_{t-1}^c) + u_t$$

Equation Number	Period of Estimation	Assumption about SP_t^e	$\hat{\delta}$	$\hat{\delta\alpha}$	Implied $\hat{\alpha}$	SE	R^2	DW	# of Obs.
(1)	1948-1959	(a)	.095 (2.10)	.142 (2.94)	1.50	168.8	.428	1.97	12
(2)	1948-1959	(b)	.334 (2.31)	.382 (2.58)	1.14	178.8	.359	1.56	12
(3)	1948-1959	(c)	.076 (1.96)	.128 (2.94)	1.68	169.0	.427	1.92	12
(4)	1948-1959	(d)	.450 (3.50)	.517 (3.80)	1.15	147.5	.563	1.63	12
(5)	1948-1959	(e)	.047 (1.22)	.104 (2.21)	2.22	188.9	.284	1.92	12
(6)	1948-1959	(f)	.176 (1.66)	.242 (2.02)	1.37	194.4	.242	1.56	12

(a) Peak month of previous year.

(b) Peak month of previous year or previous peak month before that, whichever is larger.

(c) Average of six largest months of previous year.

(d) Average of six largest months of previous year or previous largest average before that, whichever is larger.

(e) Average of all twelve months of previous year.

(f) Average of all twelve months of previous year or previous largest average before that, whichever is larger.

(t-statistics are in parentheses.)

Note: Output data rather than shipments data used for assumptions about SP_t^e.

peak sales. Since data on sales or shipments were not available, data on production had to be used for the different assumptions. The sample period was annual and extended from 1949 through 1959, for a total of 12 observations. The best results for the Steel industry were obtained using for the peak sales variable the average of the six largest months of the previous year or the previous largest average before that, whichever is larger. (Equation (4) in Table IV.) Poorer results were obtained using merely the average of the six largest months of the previous year, the level during the peak month only, or the level during the peak month or the level during the previous peak month, whichever is larger. (Equations (3), (4), and (2).) Considerably poorer results were obtained using yearly averages. (Equations (5) and (6).) The values of the various variables during the previous year gave better results than the values during the current year.

The results in Table IV are thus similar to the results in Table II in that they indicate that expected levels during peak periods are more important in determining desired capacity than are expected levels during slack periods as well. The results in the two tables differ in that the best results for the Steel industry were obtained using the average level of the six largest months of the previous year or the previous largest average before that, whichever is larger, as the peak sales variable rather than merely the level of the peak month of the current year.

As with the Cement industry, equation (15) for the Steel industry was estimated under the assumption that α is a function of the cost of capital and under the assumption that δ is a function of the deviation of the cost of capital from its expected long-run value. The results for the Steel industry were similar to the results for the Cement industry. In neither case could a significant effect be found under any of the assumptions used. For α the negative results are not particularly surprising, but for δ one might have expected to find significant effects. Again, however, the sample is small, and perhaps not too much emphasis should be placed on the negative results for δ.

V. SUMMARY AND CONCLUSION

In this paper an attempt has been made to provide a framework for deciding how much investment is due to capital-labor substitution and how much is due to other causes. Firms are conceived of as making two basic kinds of decisions, the first regarding the desired level of capacity and the second regarding how many machines to replace and what types of machines to purchase to meet the desired level of capacity. Capital-labor substitution relates to the second kind of decision. Capital-labor substitution can be affected by the wage-rental ratio in three possible ways: the wage-rental ratio can affect the types of machines purchased, the direction of technical progress, and the rate of replacement investment. With respect to the first kind of decision, the wage-rental ratio should not affect decisions regarding the desired level of capacity, but the cost of capital by itself may affect these decisions. Decisions on the desired level of capacity are likely to be closely related to production-smoothing decisions, and the desired amount of capacity is likely to be roughly equal to the amount of capacity necessary to meet peak production requirements under the optimal production-smoothing plan, given the expected path of sales. The cost of capital, as well as inventory costs and costs of changing the rate of production, should have an effect on the optimal plan. The cost of capital in the form of deviations from its long-run expected value may also affect the timing of investment expenditures.

The empirical results presented in this paper appear to be consistent with the above framework. The results indicate that capital-labor substitution has taken place and that, as expected, desired capacity is influenced by sales (or production) during peak periods rather than being influenced by sales (or production) during both peak and slack periods.

Although the kinds of data used in this study are not available on an aggregate basis, the framework presented above should be of some use in the specification of aggregate models of investment behavior. Model builders should at least be aware of the three possible ways in which the wage-rental ratio can affect capital-labor substitution and of the variety of ways in which the cost of capital can affect investment decisions. The framework also indicates that capacity decisions should probably be studied within the context of inventory and production-smoothing decisions.

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