

A MODEL OF INHERITED WEALTH

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The many vexing questions in the theory of income distribution can usefully be dichotomized into two groups. An intra-generational model of income distribution takes as given the distributions of inherited wealth and abilities ("human wealth," if you will) and studies the factors which explain how the current distribution of income (or wealth) is derived. An inter-generational model, by contrast, concentrates on the transmission of net worth and human capital across generations. The present paper deals with one important aspect of the latter problem -- i. e. , how is the distribution of inherited wealth determined?¹ This question has received almost no attention in the economic literature. Josiah Stamp's lament that "scientific economic inquiry into the subject inheritance...has thus been very scanty"² is as true in 1971 as it was in 1926. The present model, combined with some model of the inter-generational transmission of human capital, would "close the loop"

*I would like to thank Peter A. Diamond and Robert M. Solow for insightful criticism of earlier drafts of this paper. I also benefited from the comments of several members of the Harvard-M. I. T. Seminar on Income Distribution in 1970, including R. A. Musgrave and L. C. Thurow. None of these people, of course, are implicated in any of the ideas expressed herein.

¹I have elsewhere dealt with the intra-generational aspects of income distribution, given the distribution of inheritances. See my Towards an Economic Theory of Income Distribution, unpublished Ph. D. dissertation, M. I. T., 1971.

²Josiah Stamp, "Inheritance as an Economic Factor," Economic Journal, 1926, pp. 347-348.

between the income distribution of one generation and the income distribution among its heirs.

There is a widely-held belief -- certainly in the mind of the public, and probably also in the minds of economists -- that unequal inheritances are a major source of inequality. Pigou, for example, used this as the basis for explaining why the income distribution is skewed even though the distribution of abilities is (presumably) normal.³ While the popular wisdom probably overstates the quantitative importance of inheritance of non-human wealth,⁴ it is true that the distribution of inheritances is terribly unequal and, as such, is a contributor to the total inequality in incomes.

Over the past decade, several studies in the United States have supplied us with some information on the importance of inheritances. The most comprehensive study was the Survey of Financial Characteristics of Consumers conducted by the Federal Reserve in 1962.⁵ They found that 57% of consumer units in the highest income class (\$100,000 and over) had inherited assets which constituted a "substantial" portion of their total wealth. For other income classes, this percentage never exceeded 14%.⁶ So it would appear that many of the rich are inheritors.

³A. C. Pigou, The Economics of Welfare, Second Edition, London, 1924.

⁴Cf. Blinder, op. cit. Chapter 4.

⁵For a full description of this survey, see D.S. Projector and G.S. Weiss, Survey of Financial Characteristics of Consumers, Washington, 1966.

⁶Ibid., Table A-32, p. 148.

In a regression to explain the logarithm of income, a dummy variable for inheritors achieved a coefficient of .50 (with standard error .06), indicating that being an inheritor, on average, adds some 50% to expected income.⁷ In their notable study of high income individuals, Barlow, Brazer and Morgan⁸ found that about 1/7th of their present assets had been derived from inheritances or gifts, even excluding appreciation on these assets. Still, as compared with lifetime earnings, inheritance does not bulk very large. Based on data collected by the Survey Research Center in 1960, Lansing and Sonquist⁹ guesstimate that the mean inheritance among those who inherited anything was only about \$7,500. If about 40% of the population ever inherits anything, this makes the over-all population mean about \$3,000 -- not a very large number compared with lifetime earnings. Still, inherited wealth may have a disproportionate effect on over-all inequality because it is so unequally distributed. For example, the Gini coefficient calculated by the present author from the data given by Lansing and Sonquist¹⁰ is about .973, not far from the perfectly inegalitarian value of unity.¹¹

⁷ Ibid., Table 1, p. 7.

⁸ Robin Barlow, Harvey Brazer and James Morgan, Economic Behavior of the Affluent, 1966, esp. Ch. VII.

⁹ John B. Lansing and John Sonquist, "A Cohort Analysis of Changes in the Distribution of Wealth," in Lee Soltow (ed.), Six Papers on the Size Distribution of Wealth and Income, New York, NBER, 1969.

¹⁰ Ibid., Table 15, p. 64.

¹¹ The Gini coefficient is twice the area between the Lorenz curve and the hypothetical line of perfect equality.

In this essay I explore ways, under a variety of assumptions, in which we can express the distribution of inheritances as a function of the distribution of terminal wealth of the preceding generation. Combined with an intra-generational model which determines the amount bequeathed (given inheritance), this would constitute a complete model of non-human wealth and income distribution. Section I introduces the basic model, explaining what factors I am going to take into account and what factors I shall assume away. Certainly the actual distribution of inheritances has a great variety of determinants, far too many to be adequately described in a formal model. Section II disposes quickly of the simple case of primogeniture. The analytical convenience of this case is due to the fact that under primogeniture we do not have to worry about who marries whom. Sections III - VI take up three different models in which primogeniture is not the rule; the three models differ by the marriage customs which prevail. In each case the effect on inequality of the passing of a generation is assessed.

I. Elements of a Model of Inheritance

In developing a formal model of inherited wealth, I will consider a highly simplified world characterized by the following:

(a) All people marry and all married couples have two children-- one boy and one girl. This assumption assures us of a constant population, 50 percent male and 50 percent female. While this assumption is obviously inaccurate, it is hoped that it does not stray too far

from the truth for a stationary economy. It would be interesting to extend the results to a growing economy. I have not attempted this here, but would speculate that most of the results hold equally well in an economy undergoing steady state growth.¹²

(b) Each person lives for two periods: childhood and "economic life." In the first period, an individual grows up, acquires some formal education and/or other training, but accumulates no wealth. All economic decisions are made for him by his parents. At the start of the second period--which I shall dub "economic age zero"--he marries, comes into an inheritance (possibly zero), and begins working and/or investing as an homo economicus. At economic age T he dies, passing on his wealth to his heirs.¹³

It should be noted that these two assumptions mean that I am ignoring the effects on the distribution of wealth of such things as population growth, differential fertility among income groups, and

¹²Two earlier efforts have allowed for population growth, but only at the expense of some drastic simplifications elsewhere. For example, A. B. Atkinson's recent paper, "Capital Taxes, the Redistribution of Wealth and Individual Savings," Review of Economic Studies, April 1971, assumes away any differences between males and females, and assumes that all estates are divided equally among the heirs. See also the work on income distribution in the context of an aggregate growth model by J. E. Stiglitz, "Distribution of Income and Wealth Among Individuals," Econometrica, 1969, pp. 382-397.

¹³In a life-cycle model of consumption and labor supply, it is quite possible that a person will retire from the labor force before age T . This is an endogenous decision. See Blinder, op. cit., Chapter 3.

shifts in the age distribution of the population. Regarding the first, there is good reason to believe that the wealth distribution is relatively insensitive to the rate of population growth. Some unpublished work by Atkinson¹⁴ supports this view. Differential fertility, if it exists, can potentially have substantial influence on the wealth distribution.¹⁵

Fortunately, for the U.S. at least, the evidence is overwhelming that fertility differences by economic class are both very minor and narrowing.¹⁶

Finally, shifts in the age distribution of the population are relatively easy to accommodate within the model and present no new conceptual problems.

One other complication which I ignore, gifts inter vivos, is incorporated into the model in a formal sense by treating a gift of G received t periods before one's inheritance as an inheritance of Ge^{rt} ,

¹⁴ I am indebted to Atkinson for sending this interesting paper entitled, "The Distribution of Wealth and the Individual Life-Cycle," to me.

¹⁵ Cf. F.L. Pryor, "The Impact of Social and Economic Institutions on the Size Distribution of Income and Wealth: A Simulation Study," paper read at the December 1969 meetings of the American Economic Association, pp. 8-10.

¹⁶ See, for example, P.M. Blau and O.D. Duncan, The American Occupational Structure, New York, 1967, Chapter 11; C.J. Bajema, "Relation of Fertility to Occupational Status, IQ, Educational Attainment, and Size of Family of Origin," Eugenics Quarterly, 1968, pp. 198-203; and B.K. Eckland, "Theories of Mate Selection," Eugenics Quarterly, 1968, pp. 71-84.

where r is the rate of interest. This device is valid in a certainty model so long as gifts are taxed at the same proportional rate as inheritances. Indeed, it is correct for the same reasons that it is valid to act as if all inheritances are received at economic age zero when, in fact, most people receive them much later.

I have chosen to ignore all these complications in order to concentrate on two institutional factors which appear to be of primary importance in explaining the unequal distribution of inherited wealth, and yet have been virtually ignored in the small economic literature on the subject.¹⁷ Namely, mating habits and customs governing inheritance. Included in the latter are (a) the typical division of the family fortune between the son and the daughter, and (b) the rate of inheritance tax, if any. To get any analytic results, I am forced to assume a flat rate of death taxation.

Regarding marriage customs, there are two extreme cases, with the truth no doubt lying somewhere in between (but closer to which?). In a truly classless society, there might be no relation between the wealth of the husband and the wealth of the wife. We shall refer to such a system as "random mating." At the opposite pole, a man might never marry a woman not of the same wealth or wealth class. I will call this a milieu of "class mating." Intermediate between these two

¹⁷ A notable exception is Frederic Pryor's simulation study, op. cit.

extremes we have a whole variety of cases where, no doubt, virtually every real economy has stood. Under a regime of "assortative mating" the wealth of husband and wife are correlated, but not perfectly. It seems intuitively clear, without any need for formal models, that random mating will tend to equalize the wealth distribution over time, while class mating will not. Wedgwood, in 1928, noted that "the effect of marriage customs on distribution is also an unexplored topic."¹⁸ This paper seeks to rectify this situation.

In considering the laws and mores governing inheritance, we shall assume that, either by custom or statute, a fraction a of every family fortune is inherited by the male heir. Again, a could be allowed to vary by family only at considerable cost to the simplicity of the model.¹⁹ Now if the flat rate of estate tax is τ , the male's share will be $(1-\tau)a$, and the female's share will be $(1-\tau)(1-a)$. Two special cases have been mentioned in the literature.

¹⁸J. Wedgwood, "The Influence of Inheritance on the Distribution of Wealth," Economic Journal, 1928, p. 39.

¹⁹This can be derived from explicit maximizing behavior by decedents if we assume that the utility they receive from leaving a bequest is a homothetic function (of CES type) of the amounts left to the male heir (X) and female heir (Y):

$$U = \frac{X^{1-a}}{1-a} + \beta \frac{Y^{1-a}}{1-a} .$$

It is a trivial matter to show that the fraction received by the male heir (called a in the text) will be: $\frac{1}{1+\beta\frac{1}{a}}$. If the taste parameters a and β are the same for each individual, then a will also be the same.

Case 1: Primogeniture

In this case $\alpha = 1$, i. e., the son's share is whatever the state leaves over. It is clear that primogeniture will tend to perpetuate inequalities from one generation to the next. Primogeniture is a particularly simple case to deal with since it makes mating habits irrelevant. Since all females have zero wealth, the choice of a spouse has no effect on family wealth. It will be seen that mating problems cause all the real difficulties, and that primogeniture is easily analyzed.

Case 2: Equal Division

In this case $\alpha = \frac{1}{2}$, which implies that both son and daughter receive a fraction $\frac{1}{2}(1-\tau)$ of the estate. Other than primogeniture, this is the only case that has been examined in the literature. It is clear that such a custom will tend to break down large family fortunes over time. Equal division is mathematically convenient because it makes the distribution of male and female inheritances identical. It is also of interest since, as might be guessed, the passing of a generation is most potent as an equalizer when legacies are equally divided.²⁰

II. Primogeniture

In an economy in which primogeniture is the rule, we need only consider the distribution of inheritances among males. Since our

²⁰ The paper by Stiglitz, op. cit., contrasts these two polar cases and shows, as is obvious, that there is a continuing tendency towards equality under equal division but not under primogeniture. I am more interested in the intermediate cases.

assumptions guarantee that the population will be uniformly distributed among age groups, for every "family" that dies at a given instant there will be exactly one male heir "born" in the economic sense.

The question is: How does the distribution of inherited wealth depend upon the distribution of terminal wealth of the previous generation? Suppose the economy is in a stationary state so that only age, and not calendar time, matters; and suppose that the density function for wealth in every age cohort is known. Let $f_t [K(t)]$ be the density at age t . That is, the probability that a man of age t has wealth in the range $(K, K+dK)$ is $f_t(K)dK$. Our question now is: How do we go from knowledge of the $f_t [K(t)]$ to the distribution of inheritances, $f_0 [K(0)]$? In the case of primogeniture, this is exceedingly simple. The wealth of a man who dies at the age T is $K(T)$. Since all of this is willed to a specific age zero man (his son), we can easily go from the legacies, $K(T)$, to the inheritances, $K(0)$, since:

$$K(0) = (1 - \tau) K(T)$$

Therefore, by a well-known result of probability theory, the density of $K(0)$ will be

$$(1) \quad f_0(K_0) = \frac{1}{1 - \tau} f_T \left[\frac{K_0}{1 - \tau} \right] .$$

So, under primogeniture, the distributions of wealth at age T and inheritance are related in a very elementary way.

One might wonder how effective death duties are in reducing the inequality of non-human wealth. More concretely, one might ask:

By how much does the estate tax reduce the coefficient of variation of inheritances below the coefficient of variation of bequests?²¹ A routine calculation will show that the answer is: Not at all! It should, of course, be observed that this conclusion that flat-rate death duties are not equalizers is contingent upon the particular inequality measure chosen. The coefficient of variation is one of several measures for which only relative inequality matters. Thus, pulling all quantities in towards zero by the same percentage leaves inequality unchanged. For measures in which absolute inequality matters, of which the variance is the most obvious, such a change would be considered as decreasing inequality. My own taste runs towards relative inequality measures, since they avoid, e. g. , calling a currency reform or devaluation a decrease in inequality. In this paper, when I say a given policy does not effect inequality, I mean only that the policy does not change the Lorenz curve.²²

The remark that flat-rate inheritance taxation has no effect on inequality should be qualified in three further ways. First, it assumes that the government does not use the tax revenues to make redistributive

²¹The coefficient of variation is the standard deviation normalized by dividing by the mean.

²²It is obvious that a proportional decrease in all wealths leaves the Lorenz curve unchanged. The proof is somewhat tiresome, and hence is omitted.

transfers. Second, the statement only refers to the distribution of wealth. If, as is surely the case, income from property is more unequally distributed than income from labor, a proportional inheritance tax will be an egalitarian measure. Finally, it ignores the effect of the inheritance tax on the choice of K_T . This is the subject of an intra-generational model.²³

In order to isolate the effect of inheritance practices, each of the following sections will consider a hypothetical economy in which inheritance is the only way to acquire wealth, i. e. , all income is consumed and there are no death duties. Thus the same total wealth gets re-shuffled with the passing of each generation. Though this mental exercise is trivial in the case of primogeniture, it is rather enlightening for more complex milieus. Of course, under primogeniture not only would the over-all wealth distribution never change, but the same fortune would be attached to the same family name forever.

III. The Mating Function

Now let us consider a more general regime where the male gets a fraction α of the inheritance and the female get $1-\alpha$. (Notethat $\alpha=1$ brings us back to the previous model.) Under such a system, the random variable representing inheritances of males will be:

$$X = c_1 K(T), \quad \text{where } c_1 = \alpha(1-\tau)$$

²³Cf. Blinder, op. cit., Chapter 2.

and the random variable representing female wealth will be)

$$Y = c_2 K(T), \quad \text{where } c_2 = (1-a)(1-\tau).$$

In the previous notation, if $f_T(\cdot)$ is the density function for terminal wealth, then by a well-known result of probability theory the probability that a male individual will have an inheritance in the range $(X, X + dX)$ will be:

$$f_x(X)dX = \frac{1}{c_1} f_T\left(\frac{X}{c_1}\right) dX \quad (\text{as long as } a \neq 0)$$

and the corresponding probability for females will be:

$$f_y(Y)dY = \frac{1}{c_2} f_T\left[\frac{Y}{c_2}\right] dY \quad (\text{as long as } a \neq 1)$$

Since f_T is assumed to be known, f_x and f_y are easily obtained.

However, in order to go from these distributions to the distribution of inheritances, $f_0(K_0)$, we must specify who marries whom, at least in a probabilistic sense.

In the most general and most difficult case, any man might marry any woman, but the likelihood of such a marriage would be a function of the wealth of each. In particular, suppose the wealth of the wife (Y) is a random variable with a distribution conditional upon the wealth of the husband (X). That is, let $P(Y|X)dY$ be the probability that a man of wealth X marries a woman with wealth in the range $(Y, Y+dY)$. Given this conditional probability function, which I shall call the "mating function," we can calculate the probability that a married couple will have wealth in the interval $(K_0, K_0 + dK_0)$:

$$(2') \quad f_0(K_0) dK_0 = \int_0^{K_0} f_X(X) P(K_0 - X|X) dX \quad dK_0$$

so that the density of inheritances will be:

$$(2) \quad f_0(K_0) = \frac{1}{c_1} \int_0^{K_0} f_T\left(\frac{X}{c_1}\right) P(K_0 - X|X) dX .$$

In a formal sense, equation (2) gives us our solution in the most general case. The mating function is a "datum," given by non-economic factors. Knowledge of it, along with knowledge of the distribution of terminal wealth, suffices to determine the distribution of inherited wealth. Unfortunately, this formulation is too general to yield (as yet!) any tractable results. For this reason, the following two sections consider two extreme cases which are analytically simple and which certainly bound the true mating function.

Before leaving this section, however, let us consider what restrictions we might place on permissible mating functions. One obvious one is that, in this model, everyone is assumed to marry. The idea that "every man takes a bride" translates to:

$$(3) \quad \int_0^{\infty} P(Y|X) dY = 1 \text{ for every } X .$$

Second, our intuition suggests that $P(Y|X)$ should have its maximum at $Y=X$ or $Y=\gamma X$ for some constant γ ²⁴. A final

²⁴The rationale for a constant γ different from one is given in Section V below.

restriction, and the one that causes the severe analytical difficulties, is that the probabilities implicit in $P(\cdot)$ must be consistent with the existing distributions of male and female wealth. Given $f_x(X)$, and given $P(Y|X)$, a particular density for Y is implied. In particular, for every Y we must have:

$$(4) \quad \int_0^{\infty} P(Y|X) f_x(X) dX = f_y(Y) .$$

If this were not so our mating function would call for more (or fewer) wives in range $(Y, Y+dY)$ than actually were available. Since both f_x and f_y are derived from the distribution of terminal wealth, f_T , equation (4) restricts the permissible combinations of f_T and $P(Y|X)$:

$$(4') \quad \int_0^{\infty} P(Y|X) f_T\left(\frac{X}{c_1}\right) dX = \frac{c_1}{c_2} f_T\left(\frac{Y}{c_2}\right)$$

In principle, given one of these functions the other can be determined (perhaps several functional forms would do). In practice, this is very hard.

IV. Random Mating

In an extremely egalitarian milieu, the likelihood that girl g will marry boy b will not have anything to do with the wealth of either. I shall call such a regime one of "random mating." What kind of marriage function is implied by a system such as this? The question is easily answered. If mating is random, $P(Y|X)$ really has nothing to do with X .

X and Y are independent random variables. Thus, in equation (4), we may move $P(Y|X) = P(Y)$ out of the integral to get:

$$f_y(Y) = P(Y) \int_0^{\infty} f_x(X) dx$$

But since $f_x(X)$ is a density, this integral is unity and $P(Y) = f_y(Y)$.

In a regime of random mating the mating function, $P(Y|X)$, is identical to the density of female inheritances, $f_y(Y)$.

This result is, of course, "obvious once you think of it." $P(Y)dY$ is the probability that a wife (of any man) will have an inheritance in the interval $(Y, Y+dY)$. But if mating is random this must be the same as the probability of finding a female wealth in this interval, i.e., $f_y(Y)dY$.

With the mating function known, the distribution of inheritances follows immediately from equation (2'). Thus, in the case of random mating the relation between terminal wealth and inherited wealth is:

$$(5') \quad f_0(K_0) = \int_0^{K_0} f_x(X) f_y(K_0 - X) dX$$

$$(5) \quad f_0(K_0) = \frac{1}{c_1 c_2} \int_0^{K_0} f_T\left(\frac{X}{c_1}\right) f_T\left[\frac{K_0 - X}{c_2}\right] dX$$

To readers familiar with probability theory, equation (5') will make it clear that we could have arrived at equation (5) by an alternative route. Simply observe that the assumption of random mating make K_0

the sum of two independent random variables, $X + Y$. The density of K_0 is, therefore, the convolution of the densities $f_x(\cdot)$ and $f_y(\cdot)$. This is precisely the content of equation (5'). This approach yields some fruitful results since it is well-known that for several probability laws if X and Y follow the same law (though, perhaps, with different parameters), and if X and Y are independent, then the sum $X + Y$ (which is K_0) also follows this probability law. This result holds for the normal distribution; but this is of little relevance since wealth distributions are always skewed. Perhaps the two most popular analytical distributions used in economic models of income and wealth are the Pareto and lognormal distributions. Mandelbrot²⁵ has shown that the Pareto-Lévy family of distributions has this "stability" property. The lognormal, on the other hand does not. The sum of two independent lognormal variates is not itself lognormal.²⁶

If mating is random and the distribution of terminal wealth of one generation is Paretian, the distribution of inherited wealth in the next generation will also be Paretian. However, this result is not true of the lognormal distribution.

²⁵ Benoit Mandelbrot, "The Pareto-Lévy Law and the Distribution of Income," International Economic Review, 1960, pp. 79-106.

²⁶ Though J. Aitchison and J. A. C. Brown, "On Criteria for Descriptions of Income Distribution," Metroeconomica, 1954, pp. 88-107, state conditions under which the sum of a large number of lognormal variates is approximately lognormal.

We might note in passing that the Cauchy and Gamma probability laws also possess this stability property.

At this point a "realistic" example might help clarify things.

Suppose terminal wealth follows the exponential probability law:

$$f_T(K) = ae^{-aK} \quad \text{for } K \geq 0.$$

This is a highly skewed and rather unequal²⁷ density.

Then:

$$f_X(X) = \frac{1}{c_1} f_T\left(\frac{X}{c_1}\right) = \frac{a}{c_1} e^{-\frac{a}{c_1} X} \quad X \geq 0$$

$$f_Y(K_0 - X) = \frac{1}{c_2} f_T\left(\frac{K_0 - X}{c_2}\right) = \frac{a}{c_2} e^{-\frac{a}{c_1}(K_0 - X)} = \frac{a}{c_2} e^{-\frac{a}{c_2} K_0} e^{\frac{a}{c_2} X}$$

Thus, applying equation (5):

$$f_0(K_0) = \int_0^{K_0} \frac{a^2}{c_1 c_2} e^{-\frac{a}{c_2} K_0} e^{-\left(\frac{a}{c_1} - \frac{a}{c_2}\right) X} dx =$$

$$= \frac{a^2}{c_1 c_2} e^{-\frac{a}{c_2} K_0} \int_0^{K_0} e^{\frac{a(c_1 - c_2)}{c_1 c_2} X} dx$$

Case 1: $c_1 = c_2$ (equal division)

$$f_0(K_0) = \frac{a^2}{c_1 c_2} e^{-\frac{a}{c_2} K_0} K_0 = \left(\frac{a}{c}\right)^2 K_0 e^{-\frac{a}{c} K_0}, \quad K_0 \geq 0$$

²⁷The Gini coefficient = .50 and coefficient of variation = 1.0.

Case 2: $c_1 > c_2$

$$f_0(K_0) = \frac{a^2}{c_1 c_2} e^{-\frac{a}{c_2} K_0} \cdot \frac{c_1 c_2}{a(c_1 - c_2)} \left[e^{\frac{a(c_1 - c_2)}{c_1 c_2} K_0} - 1 \right]$$

$$f_0(K_0) = \frac{a}{c_1 - c_2} \left[e^{-\frac{a}{c_1} K_0} - e^{-\frac{a}{c_2} K_0} \right], \quad K \geq 0 .$$

In other words, the functional form of $f_0(K_0)$ depends on the division of the estate. But in either case a monotonically declining distribution of terminal wealth is transformed into a distribution of inherited wealth that has the more typically skewed shape of income and wealth distributions.

In Figures I and II below, I sketch $f_0(K_0)$ and $f_T(K_T)$ assuming the following parameter values: $a = .1$, $c = 1/3$ in the case where $c_1 = c_2$; ²⁸ and $a = .1$, $c_1 = .5$, $c_2 = .25$ in the case where division is unequal. ²⁹

It may be instructive to briefly consider how the distributions of K_T and K_0 are related in a very different example. Suppose that K_T were uniformly distributed over the interval $[0, W]$, i. e., there were an equal number of people at every wealth level. This wealth distribution shows moderate inequality (its Gini coefficient is .333 and its coefficient of variation is .578), but is neither peaked nor skewed.

Applying equation (5) with $c_1 = c_2 = c$ to: $f_T(K_T) = \frac{1}{W} \quad 0 \leq K_T \leq W$,

²⁸This corresponds to $a = \frac{1}{2}$ and $\tau = 1/3$.

²⁹This corresponds to $\tau = .25$, $a = 2/3$.

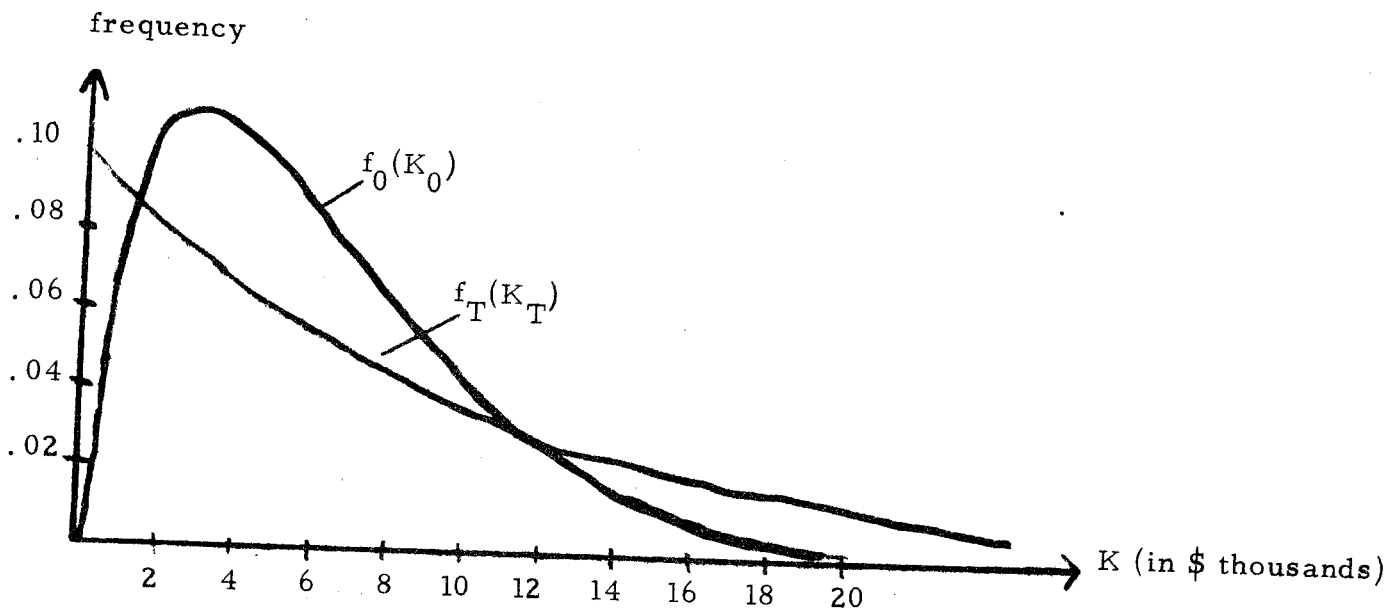


Figure I ($\alpha = \frac{1}{2}$, $\tau = 1/3$)

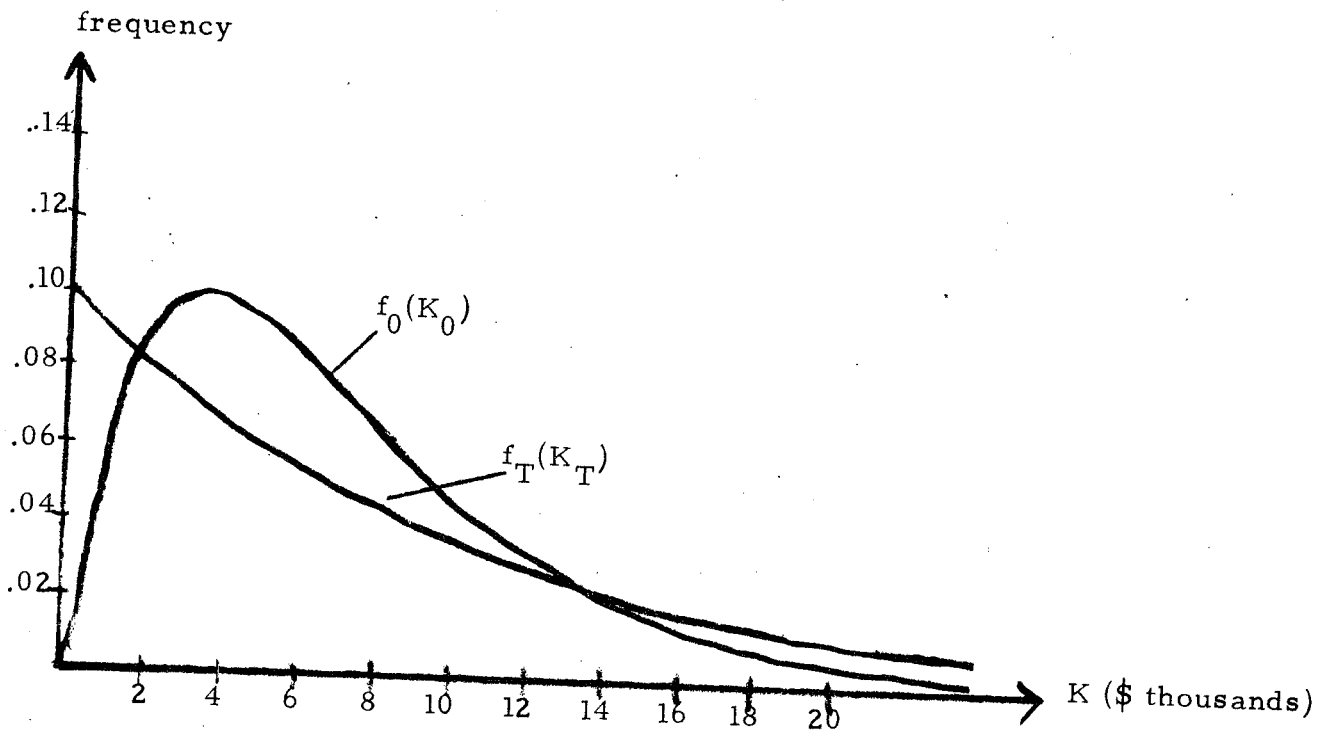


Figure II ($\alpha = 2/3$, $\tau = 1/4$)

as the reader can readily verify, we arrive at:

$$f_0(K_0) = \frac{1}{c^2 W^2} K_0 \quad 0 \leq K_0 \leq cW$$

$$= \frac{2cW - K_0}{c^2 W^2} \quad cW \leq K_0 \leq 2cW$$

as the density of inherited wealth. Symmetry is retained only because $c_1 = c_2$, and considerable "peakedness" is once again introduced by the passing of a generation. (See Figure III below.)

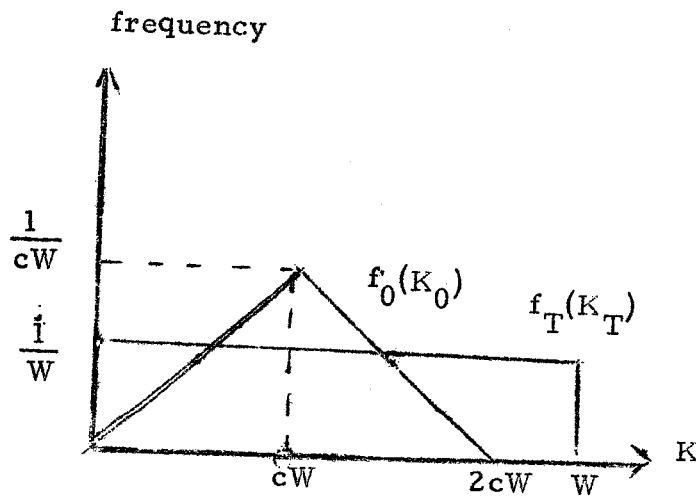


Figure III

If we are only interested in comparing the inequality in the distributions of K_T and K_0 , we need not know the entire density function. Since $K_0 = X + Y$, X and Y are independent, and $X = c_1 K_T$ and $Y = c_2 K_T$, we have by direct computation:

$$E(K_0) = E(X+Y) = c_1 E(K_T) + c_2 E(K_T) = (1-\tau)E(K_T) \quad \text{since}$$

$$c_1 + c_2 = 1-\tau ; \text{ and}$$

$$\begin{aligned}\text{Var}(K_0) &= \text{Var}(X) + \text{Var}(Y) = c_1^2 \text{Var}(K_T) + c_2^2 \text{Var}(K_T) \\ &= (1-\tau)^2 (1 - 2\alpha + 2\alpha^2) \text{Var}(K_T) .\end{aligned}$$

Let us choose to measure the equalization from one generation to the next inherent in the particular inheritance and marriage customs as the ratio of the coefficient of variation of inheritances of new households, $K_0 = X + Y$, to the coefficient for legacies of "dying" households, K_T . If we call this ratio R , then the above calculation shows that R under random mating is:

$$R = \sqrt{1 - 2\alpha(1 - \alpha)}$$

regardless of the value of τ . Once again, estate taxation is powerless to reduce (relative!) inequality if it must be at a flat rate.³⁰ However, customs or laws governing the division of estates do have a powerful effect. Under primogeniture (or its inverse, $\alpha = 0$), this ratio takes on its maximum value of unity, i. e., the passing of a generation leads to no reduction in inequality. As can easily be checked, R is minimized, i. e., the movement towards equality is maximized, when $\alpha = \frac{1}{2}$; and this holds regardless of the tax rate. In other words, no rate of estate taxation (short of 100 percent) can accomplish any equalization. However, anti-primogeniture laws can have potent effects. In the extreme case, if $\alpha = \frac{1}{2}$ (equal division) had force of

³⁰ The reader is reminded of the caveats mentioned with regard to this conclusion in Section II.

law and mating were random, the Lorenz curve would move about 29 percent of the way towards equality in each generation.³¹

Let us now conduct the mental experiment mentioned in Section II. That is, suppose inheritance were the only way to acquire wealth, and every family left a bequest, K_T , exactly equal to its inheritance, K_0 . Suppose the terminal wealth of the "first" generation followed the exponential distribution with parameter a , and (to simplify the calculations) there was equal division. Then, as we have already seen, the inherited wealth of the second generation would be distributed as:

$$f_0^2(K_0) = \left(\frac{a}{c}\right)^2 K_0 e^{-(a/c)K_0} \quad K_0 \geq 0$$

If this is also the distribution of second generation terminal wealth, then the distribution of third generation inheritances will be (by the same procedure used previously):

$$f_0^3(K_0) = \frac{\left(\frac{a}{c}\right)^3}{3!} K_0^3 e^{-\frac{a}{c}K_0} \quad K_0 \geq 0$$

and similarly for the $(n+1)$ th generation:

$$f_0^{n+1}(K_0) = \frac{\left[\frac{a}{c}\right]^{2^n}}{(2^n - 1)!} K_0^{2^n - 1} e^{-\frac{a}{c}K_0}$$

³¹In a hypothetical world where every person died with the same wealth as he inherited, the "half-life" of inequality would be two generations.

The reader can easily see where this process is going. The density functions for wealth in generation $n+1$ form a sequence of Gamma probability laws of the form:

$$f^{n+1}(K_0) = \frac{\lambda_n^{r_n}}{(r_n - 1)!} K_0^{r_n - 1} e^{-\lambda_n K_0} \quad K_0 \geq 0$$

where $\{r_n\} = 2^n$ and $\{\lambda_n\} = \frac{a}{c^n}$.

Observe that if there is no inheritance taxation, $c = \frac{1}{2}$, so $\lambda_n = a2^n$. The first three members of this sequence are sketched in Figure IV below for $a = .1$, $c = \frac{1}{2}$.

It is natural to enquire as to what limit (if any) this distribution tends to after an "infinite" number of generations. The question is easily answered. As can be readily verified, the mean of a gamma distribution is r/λ , so the mean in our sequence of distributions is $(2c)^n/a$. If $c = \frac{1}{2}$, this is $1/a$ for every member of the sequence. If $c < \frac{1}{2}$, it goes to zero as n goes to infinity. The variance of a gamma probability law is r/λ^2 , so the variance in our sequence of distributions is $(2c^2)^n/a^2$, which goes to zero faster than the mean for any value of c . In other words, if $c = \frac{1}{2}$ the limiting distribution is a "spike" at the mean wealth level, $1/a$. If $c < \frac{1}{2}$, the only true limit is a spike at zero, but as it approaches this limit the distribution begins to look

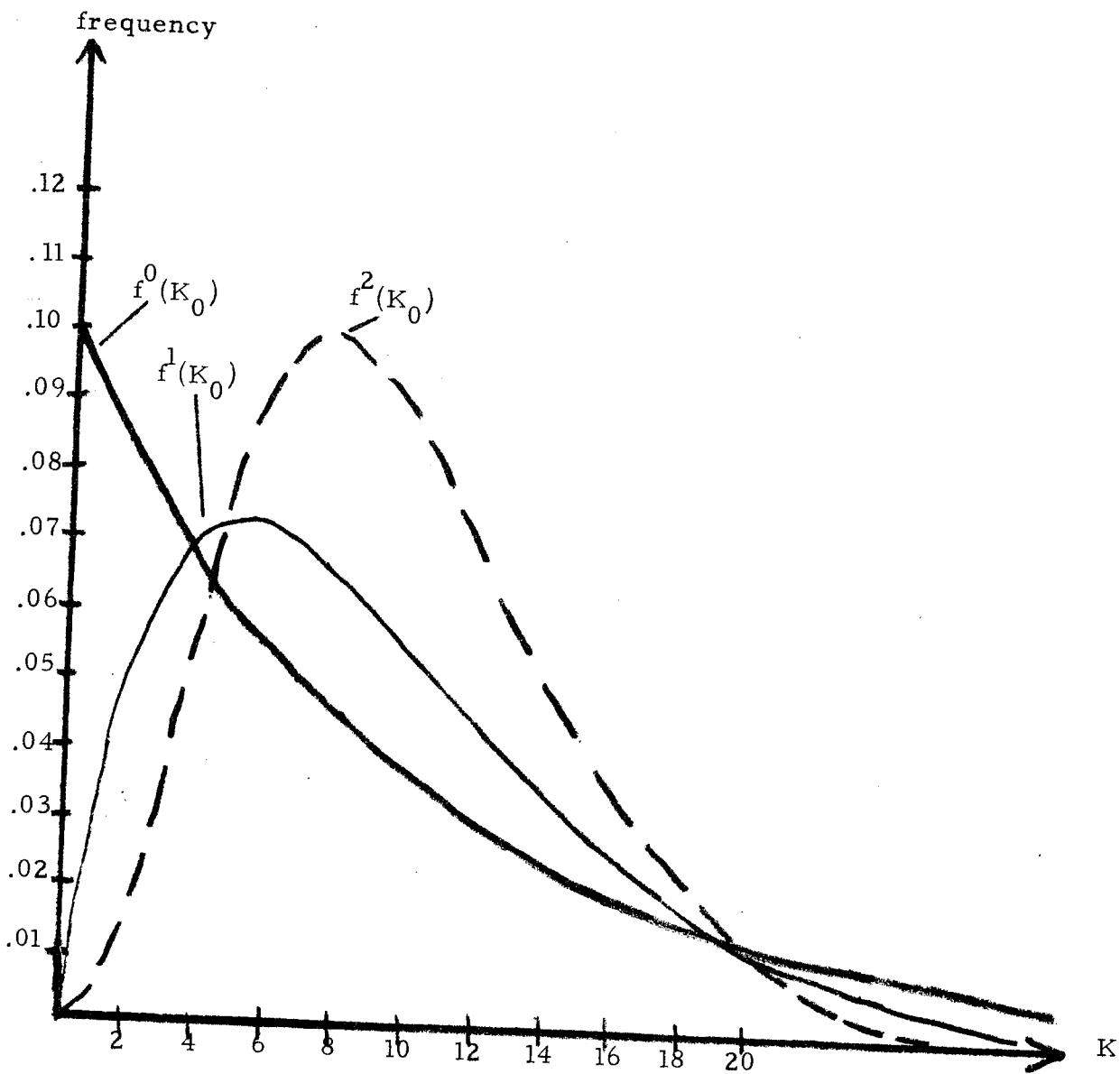


Figure IV ($a = \frac{1}{2}$, $\tau = 0$)

$$f^{n+1}(K) = \frac{(a 2^n)^{2^n}}{(2^n - 1)!} K^{2^n - 1} e^{-(a 2^n) K_0}$$

like a spike at some positive wealth level.³² These results, of course, are what we should expect. If mating were random, and if inheritance were the only way to acquire wealth, the wealth distribution would become perfectly egalitarian in an "infinite" amount of time.

V. Class Mating

Random mating is, to be sure, a rather extreme assumption. As the polar opposite case, we might suppose that knowing the husband's wealth fully determines the wife's wealth, i. e., that $Y = \gamma X$ for some constant γ . In terms of our general model, class mating is a regime where the mating function collapses to a spike for each X with all the probability "piled up" at $Y = \gamma X$. What sort of customs might produce such a system? It turns out, happily, that the natural interpretation of class mating is always consistent, i. e., will always satisfy equation (4), regardless of the wealth distribution. This "natural" interpretation of class mating is a regime in which men always choose a wife from a family of equal wealth. If X is the man's wealth, then X/c_1 was his family's wealth. Similarly, if Y is the wife's wealth, then Y/c_2 was her family's wealth. Thus we are discussing the case where $X/c_1 = Y/c_2$, or $Y = \gamma X$ where $\gamma = c_2/c_1$, a fraction smaller than unity. Note that unless there is equal division of estates ($c_1 = c_2$), this system differs

³² The third central moment is the typical measure of skewness. For any $c \leq \frac{1}{2}$, it can be shown that this moment goes to zero faster than the variance. The usual measure of "peakedness" is "kurtosis," i. e., the fourth central moment divided by the square of the variance. It can be shown that kurtosis approaches a finite limit.

from one in which men always marry women of the same wealth as themselves. Likewise, unless the rate of inheritance taxation is zero, this system differs from one where either men or women choose spouses so as to keep themselves in an equally wealthy family.

It is easy to verify that class mating with $\gamma = c_2/c_1$ is in fact a feasible system, i. e., that there will be the right number of men and women at each wealth level. Since for couples that marry, $Y = \gamma X$, the requirement of equation (4) reduces to:

$$(6) \quad \begin{aligned} f_x(X)dX &= f_y(\gamma X)dY, \text{ or} \\ f_x(X) &= f_y(\gamma X) dY/dX = \gamma f_y(\gamma X). \end{aligned}$$

But we have already seen that:

$$(7) \quad \begin{aligned} f_x(X) &= \frac{1}{c_1} f_T(X/c_1) \quad \text{and} \\ f_y(Y) &= \frac{1}{c_2} f_T(Y/c_2) \quad \text{so that} \end{aligned}$$

$$(8) \quad \gamma f_y(\gamma X) = \frac{\gamma}{c_2} f_T\left(\frac{\gamma}{c_2} X\right).$$

From (7) and (8), it is clear that $\gamma = c_2/c_1$ guarantees that (6) will hold. So class mating with $\gamma = c_2/c_1$ is feasible, and places no restriction on the permissible form of $f_T(K_T)$.³³

³³ There is, however, another way in which (6) could be satisfied. After we have analyzed the present case, I will return to this point.

When $\gamma = \frac{c_2}{c_1} = \frac{1-a}{a}$ it is a simple matter to derive the density function of inherited wealth of new households, $K_0 = X + Y$. Since $Y = \gamma X$, we know $K_0 = (1 + \gamma)X$, so the density of inheritances is:

$$f_0(K_0) = \frac{1}{1+\gamma} f_X\left(\frac{K_0}{1+\gamma}\right)$$

But $f_X(X) = \frac{1}{c_1} f_T\left(\frac{X}{c_1}\right)$ so $f_X\left(\frac{K_0}{1+\gamma}\right) = \frac{1}{c_1} f_T\left[\frac{K_0}{c_1(1+\gamma)}\right]$

Finally since $c_1(1 + \gamma) = c_1 + c_2 = 1 - \tau$, we obtain

$$(9) \quad f_0(K_0) = \frac{1}{1 - \tau} f_T\left(\frac{K_0}{1 - \tau}\right)$$

Referring back to equation (1), which applied in the case of primogeniture, we find that it gives precisely the same relation!

The relation between the distributions of terminal wealth of one generation and inherited wealth of the following generation is the same:

- (a) under primogeniture and arbitrary mating habits
- (b) under class mating with $\gamma = \frac{1-a}{a}$ and arbitrary division of estates.

Once again this result is "obvious once you think of it." If every man marries a woman whose family is as rich as his own, the result is exactly the same (moral implications aside!) as if he had married his sister. But this, in turn, is exactly the same as if the son had received the entire inheritance and married a wife who had no inheritance.

Since class mating with $\gamma = \frac{1-a}{a}$ gives rise to the same basic equation as primogeniture, we can immediately apply the results

obtained there. Namely, that inheritance taxation is powerless to reduce inequality. Also, as we have just observed, the way estates are divided has no bearing on inequality. In a fictitious world where all wealth came from inheritance and there were no inheritance taxes, the wealth distribution would never change.

Let us now turn to the case where equation (6) is satisfied for some $\gamma \neq c_2/c_1 = (1-a)/a$. Substituting (7) and (8) into (6), it is clear that we require:

$$\frac{1}{c_1} f_T(X/c_1) = \frac{\gamma}{c_2} f_T\left(\frac{\gamma X}{c_2}\right) \quad \text{for all } X.$$

But this will be true for an arbitrary γ if and only if $\lambda f_T(\lambda X)$ is independent of λ , i. e., $f_T(\lambda X) = \frac{1}{\lambda} f_T(X)$ for all X , i. e., if f_T is homogeneous of degree -1 . The only possible wealth density function that has this property is:

$$(10) \quad f_T(K_T) = \frac{1}{K_T \cdot \log M} \quad 1 \leq K_T \leq M.$$

This function is graphed in Figure V below.³⁴ It is a simple matter to find the distribution of inheritances implied by class mating and the particular terminal wealth distribution given by (10). As in the previous

³⁴The interval for K_T must have a finite upper bound and a positive lower bound in order for the integral of f_T over its range to converge. The reader should think of the units of measurement of wealth as dollars, so that the lower bound is \$1.

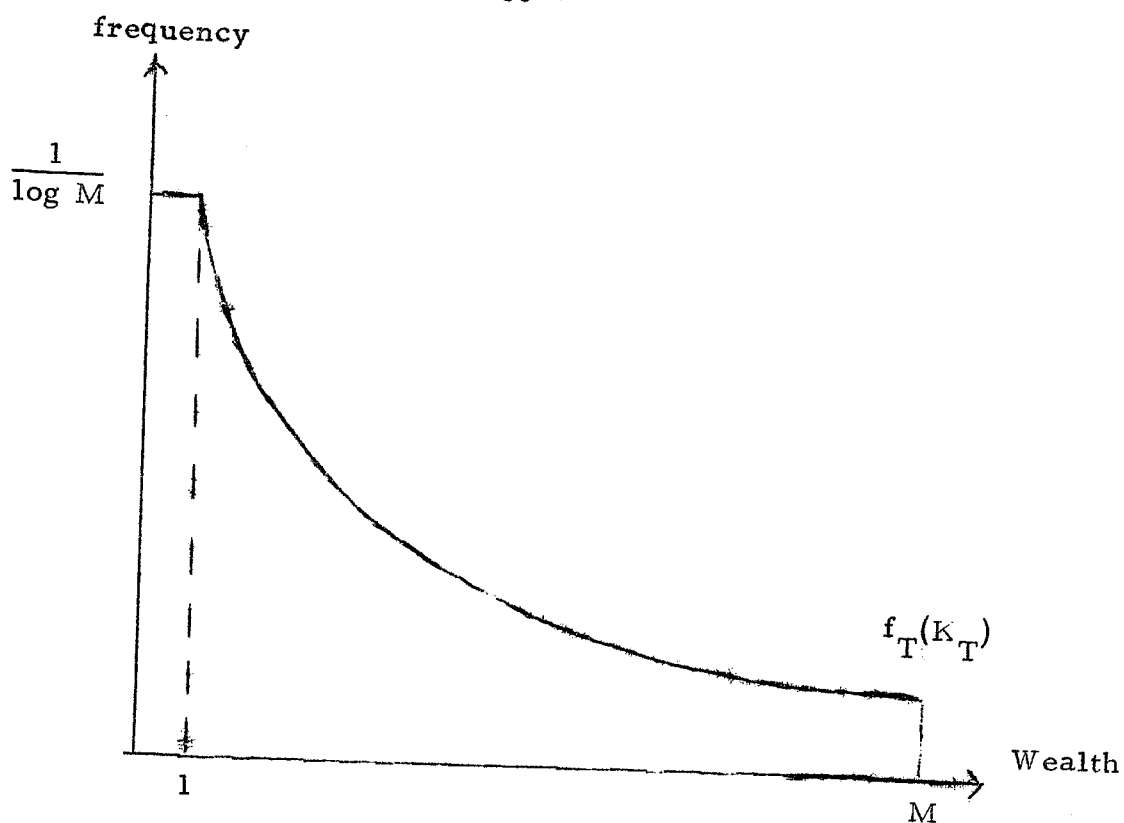


Figure V

case:

$$f_0(K_0) = \frac{1}{c_1 + c_1 \gamma} f_T\left(\frac{K_0}{c_1 + c_1 \gamma}\right) .$$

Therefore, by (10) we obtain ((after noting that $c_1 + c_2 = 1 - \tau$) :

$$(11) \quad f_0(K_0) = \frac{1}{K_0 \cdot \log M} ; 1 - \tau \leq K_0 \leq (1 - \tau) M .$$

i. e. , the distribution of inheritances is the same as the distribution of terminal wealth, except that the flat-rate tax pulls every fortune proportionately towards zero. Once again, the coefficient of variation of K_0 is the same as that for K_T , and in an "inherited wealth only" world the distribution of wealth would never change.

A regime with class mating with arbitrary γ is possible only if the wealth distribution has the hyperbolic shape given by equation (10). In this case, the distribution of inheritances is (except for scale) identical to the distribution of terminal wealth.

VI. Assortative Mating

We now return to the most general social system where there is a correlation between the economic status of husband's families and wife's families, but this correlation is not perfect. As previously stated, not much can be said in general about equation (2) which, in a formal sense, is the solution to our problem. However, if we are content to compare only the coefficients of variation of K_0 and K_T (in order to see by how much the passing of a generation reduces inequality), progress can be made. Clearly,

$$\begin{aligned} E(K_0) &= E(X + Y) = E(X) + E(Y) = (c_1 + c_2) E(K_T) = \\ &= (1 - \tau) E(K_T) \end{aligned}$$

$\text{Var}(K_0) = \text{Var}(X) + \text{Var}(Y) + 2\rho \sigma(X) \sigma(Y)$, where ρ is the correlation between husbands' inheritances and wives' inheritances.

We assume ρ is positive but less than unity. So:

$$\begin{aligned} \text{Var}(K_0) &= c_1^2 \text{Var}(K_T) + c_2^2 \text{Var}(K_T) + 2\rho c_1 c_2 \text{Var}(K_T) \\ &= (1 - \tau)^2 [a^2 + (1 - a)^2 + 2\rho a(1 - a)] \text{Var}(K_T) . \end{aligned}$$

The ratio, R , of the coefficient of variation of K_0 to the coefficient of K_T is, therefore:

$$(12) \quad R = \sqrt{1 - 2a(1 - a)(1 - \rho)} .$$

This last equation shows how two conclusions we have already reached appear as special cases. Namely, if there is either class mating ($\rho = 1$) or primogeniture ($\alpha = 1$), then $R = 1$, regardless of the value of the other parameter. That is, no equalization takes place. Equation (12) points out the obvious facts that the movement towards equality is greater (i) the smaller is ρ , and (ii) the closer α is to $\frac{1}{2}$. Equation (12) has interesting policy implications. Suppose public policy could determine α by statute. Given a value of ρ we might ask: What α will minimize R ? The answer, as the reader can readily verify, is $\alpha = \frac{1}{2}$ regardless of the value of ρ ! Similarly, if ρ was subject to legislation (I am not recommending this!), what would be the optimal ρ , given a fixed α ? The answer is: $\rho = 0$ regardless of α .³⁵ Thus, although equation (12) clearly shows that the two parameters have important interaction effects, the optimal value of either parameter can be determined independently of the value of the other.³⁶ This may be of some interest to a government which can influence α but not ρ , and knows that ρ is bound to vary over time. Values of $1 - R$, the percentage equalization in one generation, corresponding to several (α, ρ) combinations are presented in Table I below.

³⁵ Or $\alpha = -1$ if negative ρ 's were possible.

³⁶ This conclusion, of course, is based on the notion that α and ρ are independent social parameters.

In our hypothetical world where all people die with the same wealth they received as an inheritance, equation (12) enables us to calculate the half-life of inequality, and see how it depends on social customs (as represented by the parameters α and ρ). The half-life is the solution, h , of the equation $R^h = \frac{1}{2}$. Table II below tabulates this half life for various values of α and ρ .

TABLE I
Percentage Equalization in One Generation (1-R)

| $\rho \backslash \alpha$ | 0 | .2 | .4 | .5 | .6 | .8 | 1.0 |
|--------------------------|---|-----|-----|-----|-----|-----|-----|
| 0 | 0 | .18 | .28 | .29 | .28 | .18 | 0 |
| .25 | 0 | .13 | .20 | .21 | .20 | .13 | 0 |
| .50 | 0 | .08 | .13 | .13 | .13 | .08 | 0 |
| .75 | 0 | .04 | .06 | .07 | .06 | .04 | 0 |
| 1.00 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

TABLE II
Half-Life of Inequality (in generations)

| $\rho \backslash \alpha$ | .2 | .4 | .5 | .6 | .8 |
|--------------------------|-------|-------|-------|-------|-------|
| 0 | 3.59 | 2.05 | 2 | 2.05 | 3.59 |
| .25 | 5.03 | 3.11 | 2.95 | 3.11 | 5.03 |
| .50 | 7.95 | 5.03 | 4.82 | 5.03 | 7.95 |
| .75 | 16.63 | 10.84 | 10.38 | 10.84 | 16.63 |

The most striking result that emerges from these tables is that the speed of equalization is very much more sensitive to ρ , which is a measure of the degree of social stratification, than it is to α , which is a measure of the "degree of primogeniture." This suggests that laws prohibiting primogeniture or encouraging equal division will be rather less egalitarian in their effect than policies that tend to break down economic class barriers in marriage. An example of the former might be progressive taxation of inheritances received rather than estates bequeathed. Since the empirical evidence on mate selection documents the fact that the educational levels of husbands and wives are more highly correlated than any other variable,³⁷ perhaps policies which lead to equality of opportunity in education might be an example of the latter.

What are some reasonable estimates of the values of the two crucial parameters for the U.S. today? The only studies of α , the division of estates among heirs, known to me were conducted by J. Wedgwood and C. D. Harbury³⁸ on English estate records. Wedgwood found that estates were typically not divided equally among the heirs.

³⁷ See, for example, B. L. Warren, "A Multiple Variable Approach to the Assortative Mating Phenomenon," Eugenics Quarterly, 1965-66, pp. 285-290; or R. J. Garrison, V. E. Anderson and S. C. Reed, "Assortative Marriage," Eugenics Quarterly, 1968, pp. 113-127.

³⁸ J. Wedgwood, op. cit.; and C. D. Harbury, "Inheritance and the Distribution of Personal Wealth in Britain," Economic Journal, 1962, pp. 845-868.

"It appeared to be usual, among the wealthier... for the sons to receive a larger share than the daughters," while leavers of smaller estates tended to divide them more equally.³⁹ Although there was great variation, a typical value of α , according to Wedgwood, might be .55. Suppose we accept this for the U.S. today. Estimates of the correlation between the wealth of husbands and the wealth of wives is even scarcer. Table 3 below tabulates the husband-wife correlations of certain variables related to wealth which I have been able to cull from the sociological literature. Based on these data, I might hazard a guess that the correlation of wealth between men and women who marry is between .3 and .5. Using the "optimistic" values that $\alpha = .50$ and $\rho = .25$, the half-life of inequality is 2.95 generations, or about 74 years. Using more pessimistic values, $\alpha = .60$ and $\rho = .50$, the half-life is 5.03 generations, or about 126 years. These crude calculations suggest that,

TABLE 3
Husband-Wife Correlations of Some Variables

| <u>Variable</u> | <u>Source</u> | <u>Correlation</u> |
|---------------------------------|---|--------------------|
| Occupation of Father | Blau and Duncan (<u>op. cit.</u> , p. 354) | .3 |
| Intelligence | Garrison, Anderson and Reed (<u>op. cit.</u> , p. 115) | .31 - .33 |
| Education | Warren (<u>op. cit.</u> , p. 287) | .39 - .70 |
| Socio-economic Status of Father | Warren (<u>op. cit.</u> , p. 287) | .11 - .37 |

³⁹Wedgwood, op. cit., p. 48.

with existing institutions, the passing of generations can be expected to break down the inequality in wealth only very slowly. A heroic guess might be that inequality would be reduced 50 percent in a century!

Thus, it would appear that De Tocqueville was excessively optimistic when he wrote:

When the legislator has once regulated the law of inheritance, he may rest from his labor. The machine once put in motion will go on for ages, and advance, as if self-guided, towards a point indicated beforehand.⁴⁰

⁴⁰ Alexis De Tocqueville, Democracy in America, the Henry Reeve text as revised by Francis Bowen, Alfred A. Knopf, New York, 1945, Vol. I, Chapter III, p. 48.