ARE INCOME TAXES INFLATIONARY OR DEFLATIONARY?: AN EXPOSITORY NOTE

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In recent discussions of the experience with the 1968 income tax surcharge[1] it has been suggested that increases in income tax rates may not have the depressing effect on the price level conventionally attributed to them. Certainly, they lower aggregate demand, and thus exert downward pressure on prices. This effect, which we may call the "income effect" is the one generally stressed in the textbooks and in discussions of the multiplier. But, simultaneously, they raise costs, and this will put upward pressure on prices. For example, when the income tax rate rises, workers may demand a higher <u>before-tax</u> wage to compensate them for the disutility of the marginal hour of work. Such effects, which we may dub "substitution effects," are typically de-emphasized -- without apparent theoretical or empirical reasons.[2] The purpose of this note is to examine, in the context of some highly simplified aggregative models, the conditions under which one effect might dominate the other. For concreteness, I shall direct my inquiry at the following question: Are increases in the rate of income taxation inflationary or deflationary?[3] For simplicity, I restrict myself to comparative statics, ignoring short-run disequilibrium behavior.

I. A Model with No Money Illusion

The lengthy "Keynes and the classics" debate has by now made it clear that, on the level of pure static theory at least, the distinguishing feature of the Keynesian model is money illusion on the part of workers, or, what is in fact a special case of that, rigid money wages. Others have pointed out that this is a slender reed upon which to support a macro theory. So let us begin our investigation with a model which is "Keynesian" except in its (illusion-free) treatment of the labor market. We shall see that, for the question now under consideration, money illusion does not much matter.

In the usual Keynesian treatment of the goods market, consumption (C) is assumed to be a function of disposable income (Y^d), investment (I) is a function of the rate of interest (r), and government spending (G) is exogenous.[4] The equilibrium condition is that these three components of aggregate demand sum up to the supply of output (Y):

$$(1) \qquad Y = f(N) = C(Y^d) + I(r) + G, \quad 1 > C > 0, \quad I' < 0,$$

where $f(N)$ is a neoclassical production function and N is employment. Disposable income is defined as income after taxes:

$$(2) \qquad Y^d = Y - T(Y, t)$$

where $T(\cdot)$ is a tax function and t is a set of tax parameters.

For simplicity, I shall employ a linear tax function:

$$T(Y, t) = T_o + t\,Y . \qquad 0 < t < 1 .$$

In the usual Keynesian treatment of the money market, desired real balances are taken to be a function of the interest rate and income. Thus the condition for money market equilibrium is:

(3) $$\frac{M}{P} = L(r, Y) \qquad L_r < 0 \; L_y > 0 .$$

Finally, in the labor market we assume perfect competition on the demand side so that the aggregate marginal revenue product is equated to the real wage:

(4) $$f'(N) = \frac{W}{P} \equiv \omega$$

where $f'(N) > 0$, $f''(N) < 0$, and W is the money wage <u>before tax</u>. On the supply side we depart, for the moment, from Keynes and assume that labor supplied is a function of the <u>after-tax real</u> wage. This inverse supply function can be written:

(5) $$\omega(1-t) = \omega(N) , \qquad \omega' > 0 .$$

As is well-known, such a model "segments," which makes the analysis of the tax hike simple in this case. It follows from the labor market alone (i.e., from equations (4) and (5)) that:

(6) $$(1-t)\,f'(N) = \omega(N) ,$$

which is one equation in the variable N alone.

From (6) it is immediately clear that an increase in t will lead to a decline in employment and in the after-tax real wage, $\omega(1-t)$, but to an increase in the pre-tax real wage, ω.[5,6]

But what are the possibilities for the price level? This is where the other markets enter. From the money market (equation (3)) it would appear that some combination of price level increases and interest rate decreases are needed in view of the reduced real output. But how much? Can r fall enough, so that P actually falls? To answer this question, we must analyze the other two market-clearing equations:

$$(8) \qquad f(N) = C[(1-t)f(N) - T_o] + I(r) + G$$

$$(9) \qquad M = PL[r, f(N)] \quad .$$

In the Appendix, it is proven that the effect on the price level is ambiguous. A necessary and sufficient condition for the tax increase to be deflationary, i.e. for $\frac{1dP}{Pdt} < 0$, is:

$$(10) \qquad \frac{m'}{\alpha} > -\frac{1}{N}\frac{dN}{dt}$$

where m' is the ordinary multiplier in the IS-LM model for exogenous shifts in the level of taxes (dY/dT_o) and α is labor's share in national income. It can further be shown that:

$$(11) \qquad \frac{1}{N}\frac{dN}{dt} = \frac{1/(1-t)}{\frac{1}{e_d} - \frac{1-t}{e_s}}$$

where e_d and e_s are respectively the elasticities of demand

and supply for labor with respect to the real wage. And from (10) and (11), we have:

$$(12) \qquad \frac{m'}{\alpha} > \frac{\frac{1}{1-t}}{\frac{1-t}{e_s} - \frac{1}{e_d}} \; .$$

Thus, the conditions which favor the tax increase having its desired deflationary effect are:

(A) a high income-tax multiplier (which makes the desirable "income effects" large);

(B) a low share for labor in total output (which makes the cost-push "substitution effect" less important);

(C) a low tax rate

(D) a small (positive) elasticity of supply of labor and small (negative) elasticity of demand (which make the wage increase less severe).

All of these are in accord with common sense.[7]

II. A Model with Rigid Money Wages

In the strictly Keynesian case of absolutely rigid money wages, the analysis is not so simple. Replacing equation (5) by:

$$(5') \qquad (1-t)W = \bar{W}$$

the equilibrium condition (6) becomes:

$$(6') \qquad (1-t)Pf'(N) = \bar{W} \, , \qquad \text{a constant.}$$

It now would appear possible that employment might actually rise. For a rise in N induces a decline in the marginal productivity of labor, $f'(N)$, which is still consistent with the constancy of $(1-t)f'(N)P$ in equation (6') as long as there is sufficient inflation. However, it is proven in the Appendix that this perverse case cannot arise. This follows from the money equation -- equation (3). If N (and hence Y) rises, the demand for real balances increases. However, if at the same time the supply of real balances declines due to inflation, a large increase in r will be required to restore equilibrium in the money market. The IS curve (equation (8)), however, makes it clear that if N rises, only a decrease in r is consistent with equilibrium in the goods market. So the perverse case is ruled out.

While income and employment must rise, the actual behavior of the price level remains ambiguous. In the Appendix, it is proven that a necessary and sufficient condition for the tax rate increase to be deflationary is:

$$(13) \qquad \frac{m'}{\alpha} > \frac{-e_d}{1-t} ,$$

which is an obvious special case of (12) when $e_s = \infty$. The same conditions as in Section I are favorable to the tax hike.

III. A Model with Money Illusion

The more general labor supply function with money illusion, but without rigid wages, is:

(5") $\quad (1-t)W = W(N) \quad , \qquad W'(N) > 0 \quad ,$

where $W'(N) = 0$ returns us to the rigid-wages case. Here again consideration of the labor market alone cannot rule out the possibility that the tax increase causes both inflation and increases in employment. However, by the same reasoning used in Section II above, it is possible to show that the money and goods markets rule out the possibility that N might rise. This is proven in the Appendix, where the condition for the tax increase to be deflationary is also derived:

$$(14) \qquad \frac{m'}{\alpha} > \frac{1/(1-t)}{\frac{1-t}{e_s} - \frac{1}{e_d}} \quad .$$

This is identical to condition (12) except that here the supply elasticity is with respect to the money wage, W, whereas there it was with respect to the real wage, $\omega \equiv W/P$. The conclusions reached in Section I, then, remain valid.

IV. A Quantity Theory Model

Let us alter our model a little bit in the direction in which the monetarists would have us go. Specifically, suppose the interest elasticity of the demand for money is negligible, so to a very good degree of approximation equation (3) becomes:

$$(3') \qquad M = PL(Y) \quad .$$

The evaluation of the tax hike as an anti-inflation policy is trivially simple in such a world -- as the monetarists have known for years! Since N and Y (and hence L(Y)) have to fall, the price level must rise, i.e., it must cause inflation to "absorb" the existing stock of nominal money balances. I hasten to point out that the symmetrical case implies that a tax cut is deflationary.

V. Some Plausible Values

Ignoring the simplification implied by the quantity theory, is it at all likely that an increase in the rate of income taxation will prove to be deflationary? That is, for plausible parameter values, are conditions (12) or (14) likely to be satisfied? There are five parameters in (14): the marginal tax rate, t ; the tax multiplier, m' ; the share of labor in income, α ; and the supply and demand elasticities of labor, e_s and e_d . To avoid being too taxonomic, suppose we settle on $\alpha = .75$, $t = .25$ as reasonable approximations. This reduces the condition for deflation to:

$$(15) \qquad m' > \frac{1}{\frac{3}{4e_s} - \frac{1}{e_d}} .$$

There is general agreement that the supply of labor is rather inelastic. Certainly, no one would argue that e_s exceeds unity. The demand elasticity is a little harder. For concreteness, suppose the production function is Cobb-Douglas: $Y = AN^{\alpha}$.

Then, $e_d = \frac{f'(N)}{Nf''(N)} = \frac{1}{\alpha - 1} = -4$ in our example. Using these as upper bounds, the condition (15) becomes that the tax multiplier (m') exceeds unity. This seems a conservative estimate. For any $e_s < 1$ and $e_d > -4$, the required minimum multiplier will be even less than one. We conclude that for plausible values, at least in this simple model, income tax rate increases are very likely to have their desired deflationary effects.[8,9]

APPENDIX

This appendix proves the results cited in the text. First consider the two equations which hold in all versions of the model (save the quantity theory case) -- the money market and goods market equations:

$$\text{(i)} \quad f(N) - C[(1-t)f(N) - T_o] = I(r) + G$$

$$\text{(ii)} \quad M = PL(r, f(N)) \quad .$$

Differentiating this system totally with respect to t gives:

$$\text{(iii)} \quad f'[1 - C'(1-t)] \frac{dN}{dt} + C'f = I' \frac{dr}{dt}$$

$$\text{(iv)} \quad - \frac{M}{P} \frac{1}{P} \frac{dP}{dt} = L_r \frac{dr}{dt} + L_y f' \frac{dN}{dt} \quad .$$

Using (iii) to find dr/dt and substituting the result into (iv), we can solve for the percentage increment in the price level as a function of dN/dt:

$$\text{(v)} \quad \frac{1}{P} \frac{dP}{dt} = - \frac{P}{M} f'(N) \frac{L_r}{I'} \left\{ [1-C'(1-t) + \frac{L_y I'}{L_r}] \frac{dN}{dt} + \frac{C'f(N)}{f'(N)} \right\} .$$

Now the term in square brackets is just the reciprocal of the ordinary multiplier in the IS-LM model; call it $1/m$. Since, under the usual sign restrictions, the term outside the curly brackets is negative, the condition for $\frac{1}{P} \frac{dP}{dt}$ to be negative is that:

$$\text{(vi)} \quad \frac{1}{m} \frac{dN}{dt} + \frac{C'f(N)N}{Nf'(N)} > 0 \quad .$$

Now since $f'(N) = W/P$, $Nf'(N)/f(N) = WN/PY$ = labor's share in output, which we call α, the second term in (vi) is $\frac{C'N}{\alpha}$. Finally, multiplying (vi) through by m/N, we arrive at:

$$\frac{1}{N}\frac{dN}{dt} + \frac{C'm}{\alpha} > 0 ,$$

which, since $C'm$ is the tax multiplier, m', can be written:

$$\frac{m'}{\alpha} > -\frac{1}{N}\frac{dN}{dt} ,$$

which is inequality (10) in the text.

Now, in the first model, with no money illusion, it was found in the text (See footnote 5.) that:

$$\frac{1}{N}\frac{dN}{dt} = \frac{f'(N)}{(1-t)f''(N)N - \omega'(N)N} .$$

So, dividing the numerator and denominator by $f'(N) = \omega(N)/(1-t)$, we have:

$$\frac{1}{N}\frac{dN}{dt} = \frac{1}{(1-t)\frac{Nf''(N)}{f'(N)} - (1-t)\frac{N\omega'(N)}{\omega(N)}} .$$

But since $\frac{Nf''(N)}{f'(N)}$ is the inverse of the elasticity of labor supply with respect to the real wage, ω, and $\frac{1}{1-t}\left[\frac{N\omega'(N)}{\omega(N)}\right]$ is the inverse of the elasticity of labor demand with respect to ω, this becomes:

$$\frac{1}{N}\frac{dN}{dt} = \frac{\frac{1}{1-t}}{\frac{1}{e_d} - \frac{1-t}{e_s}} < 0$$

which is equation (11) in the text.

Next, consider the model with rigid wages. Equation (v) above remains valid, as it only depends on the money and goods markets. From the labor market equation:

$$(1-t)\,P\,f'(N) = \bar{W} ,$$

it follows that:

$$(1-t)Pf''(N)\,\frac{dN}{dt} + (1-t)\,\frac{dP}{dt}\,f'(N) - Pf'(N) = 0 .$$

Using the facts that $f' = W/P$, $(1-t)\,W = \bar{W}$, and $(1-t)P = \bar{W}/f'$, this can be re-written as:

$$\text{(vii)} \qquad \frac{1}{N}\frac{dN}{dt} = \frac{\frac{1}{1-t} - \frac{1}{P}\frac{dP}{dt}}{\frac{1}{e_d}} ,$$

where inflation becomes relevant due to money illusion.

Solving (vii) and (v) simultaneously leads to:

$$\text{(viii)} \qquad \frac{1}{N}\frac{dN}{dt} = \frac{\frac{1}{1-t} + \frac{B}{\alpha}}{\frac{1}{e_d} - \frac{B}{m'}} < 0$$

where $B \equiv \dfrac{f'(N)\,N\,L_r\,C'}{(M/P)\,I'} > 0$,

and

$$\frac{1}{P}\frac{dP}{dt} = -B\left\{\frac{\frac{1}{1-t} + \frac{B}{\alpha}}{\frac{m'}{e_d} - B} + \frac{1}{\alpha}\right\} ,$$

which is ambiguous in sign. The condition for $\frac{1}{P}\frac{dP}{dt} < 0$ is:

$$\frac{1}{\alpha} > \frac{\frac{1}{1-t} + \frac{B}{\alpha}}{B - \frac{m'}{e_d}} .$$

which reduces to:

$$\frac{m'}{\alpha} > -\frac{e_d}{1-t} ,$$

which is inequality (13) of the text.

Finally, consider the general money illusion model, where the labor market equation is:

$$(1-t)Pf'(N) = W(N) .$$

Differentiating this totally with respect to t gives:

$$(1-t)Pf'' \frac{dN}{dt} + (1-t)\frac{dP}{dt} f' - Pf' = W'(N)\frac{dN}{dt} .$$

This can be simplified by dividing the left-hand side by $Pf'(N)$ and the right-hand side by $W(N)/(1-t)$, to obtain:

$$\frac{1}{N}\frac{dN}{dt} = \frac{\frac{1}{1-t} - \frac{1}{P}\frac{dP}{dt}}{\frac{1}{e_d} - \frac{1-t}{e_s}} .$$

Equation (v) remains true, and solving them simultaneously for the percentage increments in employment and the price level yields:

$$\text{(ix)} \qquad \frac{1}{N}\frac{dN}{dt} = \frac{\frac{1}{1-t} + \frac{B}{\alpha}}{\frac{1}{e_d} - \frac{1-t}{e_s} - \frac{B}{m'}}$$

$$\text{(x)} \qquad \frac{1}{P}\frac{dP}{dt} = -\frac{B}{m'}\left\{\frac{\frac{1}{1-t} + \frac{B}{\alpha}}{\frac{1}{e_d} - \frac{1-t}{e_s} - \frac{B}{m'}} + \frac{m'}{\alpha}\right\} .$$

Equation (ix), which shows that employment must decline, is a generalization of equation (viii), and shows that the employment response is smaller (in absolute value) when the supply curve has positive slope. Equation (x) once again tells us that the price level effect is ambiguous. The condition for a deflationary impact is:

$$\frac{\frac{1}{1-t} + \frac{B}{\alpha}}{\frac{1}{e_d} - \frac{1-t}{e_s} - \frac{B}{m'}} + \frac{m'}{\alpha} > 0$$

$$- \frac{m'}{\alpha}\left(\frac{1}{e_d} - \frac{1-t}{e_s}\right) + \frac{B}{\alpha} > \frac{1}{1-t} + \frac{B}{\alpha}$$

or

$$\frac{m'}{\alpha} > \frac{\frac{1}{1-t}}{\frac{1-t}{e_s} - \frac{1}{e_d}}$$

which is condition (14) in the text. Q.E.D.

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FOOTNOTES

1. See J. H. Hotson (1971), B. Hansen, and R. Eisner (1971a, 1971b).

2. For an interesting discussion of this, see J.H. Hotson (1967). Similar points have been made in a different context by G. Brennan and D.A.L. Auld.

3. I define "inflation" as a rise in the price level. It would appear that Keynesians have long been guilty of treating "lowers aggregate demand" as synonomous with "deflationary"

while (correctly, but hypcritically) accusing the monetarists of treating "raising the money supply" and "inflationary" as synonyms.

4. As is well-known, it costs nothing but notational complication so allow C to depend on r and I to depend on Y.

5. Proof: Differentiate (6) with respect to t to obtain:

$$(1-t)f''(N)\frac{dN}{dt} - f'(N) = \omega'(N)\frac{dN}{dt}, \quad \text{or}$$

$$(7) \qquad \frac{dN}{dt} = \frac{f'(N)}{(1-t)f''(N) - \omega'(N)} < 0 .$$

6. With no money illusion the analysis is formally equivalent to that of an ad valorem excise tax.

7. Nor will simple-minded application of Samuelson's correspondence principle help remove the ambiguity. Suppose we hypothesize the most elementary type of disequilibrium price-adjustment mechanism where each price responds (linearly) only to excess demand in its own market:

$$\dot{p} = k_1[C((1-t)Y) + I(r) + G - Y]$$

$$\dot{r} = k_2[PL(r, Y) - M]$$

$$\dot{\omega} = k_3[g(\omega) - h(\omega)], \qquad k_1, k_2, k_3 > 0$$

where $g(\omega)$ and $h(\omega)$ are respectively the demand and supply functions for labor in the case of no money illusion. We can derive the stability conditions for such a system by (a) substituting $Y = f(N) = f[g(\omega)]$ or $f[h(\omega)]$, (b) linearizing the model by a Taylor approximation, and (c) finding the eigenvalues of the resulting determinant. The conditions that all three eigenvalues be negative turn out to be:

(i) $f''(1-t) - \omega' < 0$

(ii) $L_r < 0$

(iii) $I_r < 0$,

all of which have already been assumed.

8. Of course, such reasoning is by no means conclusive. What is true of this very small macro model might not be true of the larger macro-econometric models. However, I should be very much surprised if these models reversed the conclusions reached herein.

9. Much of the criticism of increases in tax rates as deflationary devices in the references cited in footnote 1 was directed at increases in excise taxes rather than income taxes. My argument, of course, in no way undercuts this criticism. In fact, if one constructs a similar model for an excise tax, where the consumers' price level, $P(1+t)$, exceeds the producers' price level, P , one finds that excise tax increases are unambiguously inflationary and employment-reducing.