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ECONOMETRIC ANALYSIS OF THE UNITED STATES MANGANESE PROBLEM

Final Report
Part II

Econometric Research Program
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TABLE OF CONTENTS

LIST OF TABLES

PREFACE

This is the second part of the Final Report of the Manganese Project conducted by the Econometric Research Program of Princeton University. As in the first part (Research Memorandum No. 3, issued 15 October, 1958), it describes the results of applying mathematical programming techniques to the determination of the most economical way of providing the United States with the amounts of manganese that it will need in a future period under certain assumed and carefully specified conditions.

In the first part of the report a description was given of a non-stochastic model which led to a quadratic programming problem. This problem was solved using a modified version of the simplex technique. In the following pages of this report, a stochastic model is described which is more comprehensive than the preliminary one given on pages 40-51 of the previous report. It has essentially the same structure as the non-stochastic model although some simplifications were made in order to keep the computations within manageable proportions. The present work is, thus, a continuation of the previous study and should be examined together with Part I.

The Introduction to Part I (pp. i-iv) set forth the underlying philosophy which guided the entire project. The basic objective of this study is the same as before: the problem is to find the optimal solution that guarantees the supply of manganese at minimum-cost under certain postulated conditions. Compared to the reality of future possibilities, these conditions have been severely simplified as is necessary in a novel undertaking of this kind. But we believe, nevertheless, that they contain many of the essential features encountered by those who, at present, have to make decisions without the benefit of prior scientifically justifiable and properly advanced investigations and subsequent computations.

The study as a whole reveals, in our opinion, our inability to "guess" at results merely on the basis of intuition or business and administrative experience. This is, of course, not surprising and merely expresses the need to distinguish clearly the bases on which decisions can be made.

Here, as in the previous report, we have used the best data that we could obtain. If better information should become available, it can be used by recomputing. Different information will produce different results; they cannot be guessed at even with the results before us used as possible guide posts. This is in keeping with the fact that we are analyzing a complicated situation by a sophisticated procedure that is naturally sensitive to changes in inputs.

The reader who wishes to evaluate this investigation should bear in mind the complexity of the problem. Apart from the question of data input — among which we should list technology as well as the business data such as cost, prices, freight rates, etc. — there are possible modifications of the model. These are restricted by the possibility (or impossibility) of handling conceptually more complicated — which may, but need not, mean more realistic — situations, by the power of mathematical methods, and by the capacity of even the most modern electronic computing equipment. Only when all these components are fully understood can a valid judgment be pronounced.

Looking back on the entire project, we think that it has been demonstrated successfully that modern techniques of mathematical programming can be applied with confidence to difficult and important problems of national policy. In view of the rapid progress in the basic techniques we may also expect novel turns in the applied field. The results obtained in this first investigation will then be left far behind. One way of inducing progress would be to tackle other strategic materials (e.g., copper or zinc) which are, perhaps, more important for the economy and have a great diversity of final uses.

The manganese model presented in the following pages was solved using the dynamic programming technique developed by Richard Bellman of the RAND Corporation. The results obtained were further explored along lines indicated by Harlan D. Mills of Mathematica, Princeton, N. J.; this led to a second computation which revealed an interesting ergodic property of the system.

The first computation for the stochastic model was made on the Atomic Energy Commission's IBM 704 at the Institute of Mathematical Sciences, New York University. The second computation was made on the Commission's IBM 650 at Project Matterhorn, in Princeton. The flow chart for the first computation was drawn by Stuart Dreyfus of the RAND Corporation; the procedure for the second computation was given by Harlan Mills. The Fortran code for the IBM 704 and the basic code for the IBM 650 were both written by Herman F. Karreman who, in writing these codes, greatly benefitted from the computer experience of Irving Rabinowitz of Project Matterhorn.

The two parts of this Final Report were written by Herman F. Karreman with the exception of the last section of Research Memorandum No.3; this section, describing the preliminary stochastic model, was written by Harlan D. Mills. The editing of this second report was in the able hands of Dorothy Green.

I want to thank all the members associated now or formerly with the Manganese Project for their contributions, and, once again, the Atomic Energy Commission for making its computing facilities so readily available.

My particular thanks and appreciation go once more to Herman F. Karreman who has been the mainstay during the entire period of this project. Without his contributions as originator and executor, the project would not have accomplished what we hope it did achieve.

Oskar Morgenstern, Director
Econometric Research Program

15 March, 1960
Princeton University

I. INTRODUCTION

This study is directed towards the determination of the lowest-cost policy which would provide the United States' economy with the required quantities of a certain strategic material — manganese. This policy should enable the requirements to be met over a given period of time and in view of various possible political situations.[1] The factors to be taken into account in determining such a policy are: the various types of alloys and the quantities of them that will be required by the steel industry, the availability of foreign ores and the circumstances under which they can be brought to this country, the availability of (lower grade) domestic ores and the various techniques of upgrading them, the conditions governing the policy of stockpiling ores, and the various political situations that might arise during the ten-year period covered by this study.

In the first part of this final report two models were presented which also dealt with the manganese problem: a non-stochastic model which covered a six-year period of "limited war," and a preliminary stochastic model which took into account possible changes in the political situation. These models are described briefly in the following pages of this Introduction.

The remainder of this paper is devoted to a description of a more comprehensive stochastic model and the results obtained from it.

* * * * *

The non-stochastic model had a rather detailed structure.[2] A

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1 The interested reader is referred to the General Introduction in the first part of this report, Econometric Analysis of the United States Manganese Problem, Econometric Research Program, Research Memorandum No. 3, 15 October 1958, for a more general description of the purpose of this study. A discussion of what is meant here by a policy can be found on pages 40-41 of that paper.

2 This model is described fully in Final Report, Part I, op. cit., pp. 1-39.

distinction was made between the two forms of manganese alloys used in the production of steel, viz., ferro-manganese and silico-manganese. The required manganese could be supplied from six different sources: it could be imported from southern Asia, South and West Africa, and Latin America; and it could be supplied domestically from the Cuyuna deposit in Minnesota, the open-hearth slags of the steel mills, and the Aroostook deposit in Maine. Each of the ores obtained from domestic sources has a special character and, therefore, has to be upgraded in a particular way. For the purposes of this model, four upgrading processes were selected, namely, the Dean process for upgrading Cuyuna ore and the Udy process for Aroostook ore, the Sylvester-Dean and the Wright processes for open-hearth slags. The variables which entered into this econometric model included the quantities of ore in the stockpile and in domestic deposits at the beginning of the six-year period to be analyzed, the quantities of manganese required by the steel industry, the quantities of ore to be imported, the quantities of domestic ores to be mined and upgraded and the capacities of the domestic plants. The interactions among these variables were expressed by relations which were in the form of inequalities.[3]

The costs of running the manganese system over a six-year period included the cost of importing ore (which depend on quantities imported), upgrading domestic manganese ores, expanding domestic facilities, and the costs of inventory and stockpile maintenance. The total cost of operating the manganese system for this period was written as an expression (objective function) in these variables; the costs were minimized subject to the constraints imposed by the relations.

This turned out to be a quadratic programming problem, i.e., the objective function was non-linear, containing quadratic as well as linear terms. The problem was solved using a modified version of the powerful simplex technique. This technique works as follows: once at a given point of the objective function, all other possible points of that function

3 Ibid., Mathematical Appendix, pp. 52-63.

are examined in order to find the shortest path to the extremum of that function — here the minimum of total cost. In other words, it is a continuous extremization technique. One of its consequences is that the shadow prices (changes in the total cost of the program resulting from small changes in the requirements and/or constraints) really represent marginal changes in total cost.

The solution of this non-stochastic model produced three kinds of information:

1) the extent to which an activity (e.g., the production of alloys from stockpiled ores or upgraded domestic ores, the amount to be imported, etc.) should be used in order to meet the requirements at the lowest cost;

2) the total cost of the program broken down by import cost, operating cost of domestic plants, cost of expanding domestic plants, etc.; and

3) the amounts which have to be added to (subtracted from) the total cost if the requirements are slightly increased (decreased).

However the simplex technique has certain disadvantages. This technique, in minimizing the objective function, does not allow for negative quadratic terms in that function. In other words, the cost function must be convex. Therefore, any decreases in domestic production costs that could be expected from producing larger quantities had to be neglected in order to render the problem computable.*

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* Since this computation was made (two years ago), new techniques have been developed to solve quadratic programming problems. One of them is the gradient-projection method developed by Dr. J. B. Rosen of the Shell Development Company, Emeryville, California. This method can be used for objective functions with positive as well as negative quadratic terms. When it was applied to the same problem that was solved at RAND two years ago (the results of which were reported on pages 25-35 of the first part of this report), it came up with a solution in which the costs of the six-year program were about $928.04 million instead of the $968.8 million found before. The method will be described in the forthcoming March issue of the Journal of the Society for Industrial and Applied Mathematics.

Another, and more serious drawback to this non-stochastic model, was its deterministic nature, i.e., its dependence on a particular political forecast. It could not take into account the uncertainty due to possible changes in the political situation, hence the assumption of a limited war lasting for six years was made.

Because of the restrictive assumptions underlying the non-stochastic model — a convex cost curve, and a static political situation — a stochastic model has been developed.[4]

In the preliminary version of the stochastic model, emphasis was no longer placed only on the internal complexities of the manganese system. The new model was dynamic in that new information, based on changing political circumstances which affect the variables, was used to find the best policy. Five political situations were considered in this model — peace, cold war with no blockade, cold war with minor blockade, hot war with minor blockade and hot war with major blockade.[5]

This stochastic model was restricted to the procurement of ores only, rather than to alloys. Moreover, only one method for upgrading domestic ores was considered, although there exist at the moment at least two and perhaps even more methods by which these low-grade domestic ores can be economically processed into alloys. The cost constants used in the equations were largely aggregates of the costs of the non-stochastic model.

Since the stochastic model has no restrictions on the objective cost function, it may contain negative as well as positive high-order terms. In other words decreases in domestic production costs due to increased production can now be retained in the objective function. On the other hand, there is presently no computational technique which can be used to search in a continuous way for the extremum of the objective function. Costs, therefore, were computed only at

4 Ibid., pp. 40-51.

5 The political situations were defined in terms of the amounts of manganese which would be required in each situation and the availability of foreign supplies.

discrete points of the objective function and the analysis has a lesser degree of exactness than has the non-stochastic model. In this preliminary stochastic model, the points for which the costs were actually computed lay 400,000 Net Tons apart. The analysis was, therefore, not very exact, and, the conclusions derived from this model were tentative.

This stochastic model has now been further developed and its latest, more comprehensive version is described in the following pages. In this new version, the procurement of alloys as well as that of ores is considered. Furthermore, this model distinguishes between two types of alloys — as does the non-stochastic model — and between two different methods for the economical production of alloys from domestic ores. Therefore, it not only retains the advantage of the preliminary stochastic model for dealing with political uncertainty, but it also has much more "structure" than that early model, and, in fact, has most, although not all, of the details of the non-stochastic model.

The objective function which is to be minimized has a quite general character containing, besides linear terms, positive as well as negative quadratic terms. The computations have been carried out for many more (discrete) points of the objective function than before, all lying within a smaller region than before. Since the solution becomes more precise as the grid of points which is chosen becomes finer, the analysis is much sharper and the conclusions have, therefore, a much more definite character.

The solution of this problem yields three kinds of information, namely:

1) the minimum-cost at which the requirements of a ten-year period can be met, taking into account the various ways in which the political situation might develop;

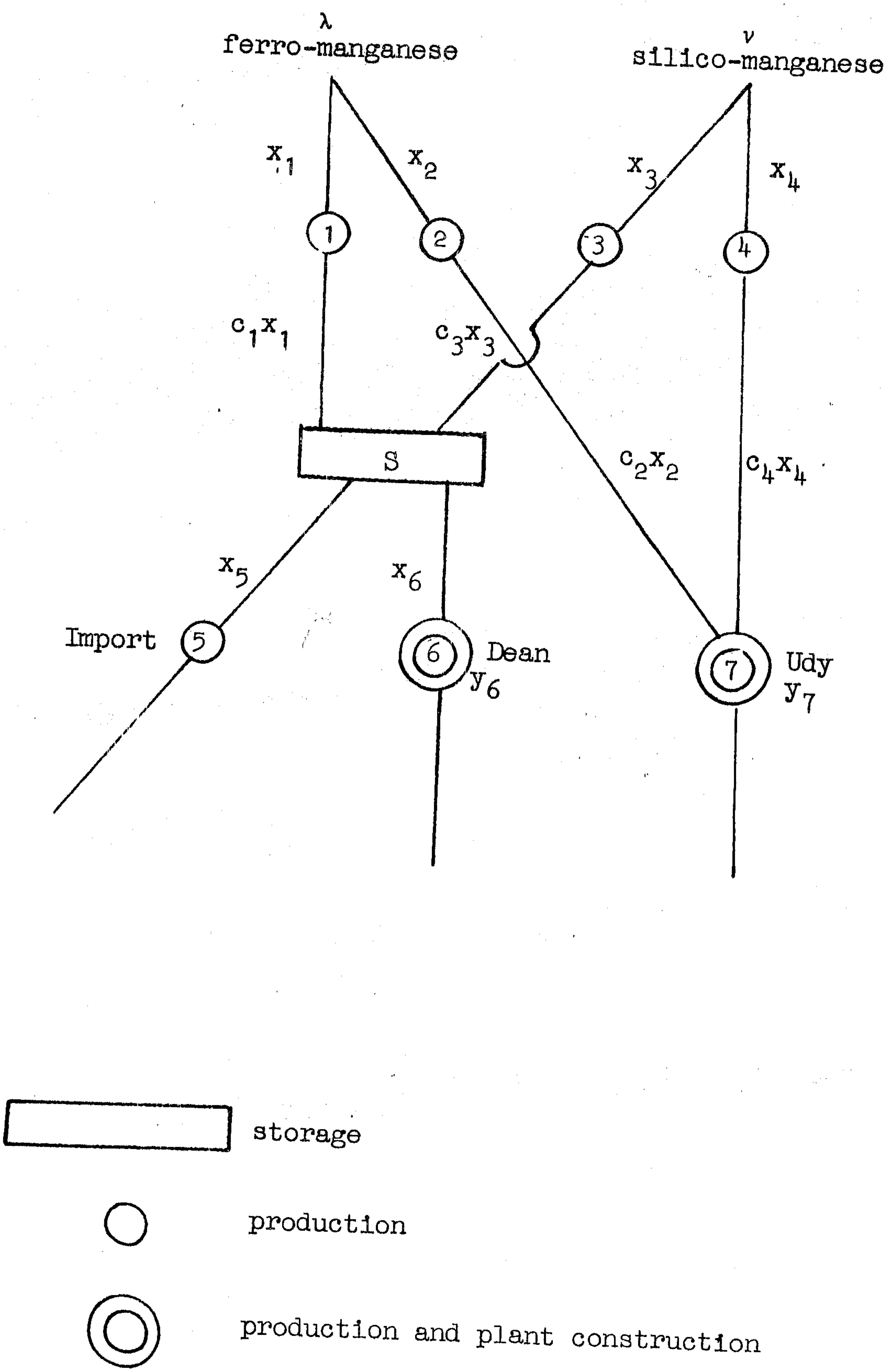
2) the state of the manganese system at the beginning of each year, i.e., the amount of ore in the stockpile and the capacities of the domestic beneficiation plants; and

3) the final state of the manganese system which would result from the adoption of a ten-year program at the end of each year.

II. THE MODEL

The structure of this stochastic model is illustrated by the diagram below. The diagram shows the distinction between the two alloys, ferro- and silicomanganese, which are required by the steel industry. Moreover, it indicates the two ways of processing domestic ores and slags which have been incorporated into this model. One of them (the Dean process, y_6) upgrades the non-silicate Cuyuna ores and produces a high-grade ore from which the alloys can be obtained using conventional methods. The other process (Udy, y_7) carries the silicate Aroostook ores over into either ferro- or silicomanganese, and is therefore an integrated process. These two processes seemed to be the most promising of the four considered in the non-stochastic model and have therefore been selected for this model. The three import areas could not be treated as separate factors lest the model become too complicated. However, this model, unlike the first stochastic model, has no upper limit placed on the quantities of ore that can be imported, since neither major nor minor blockades have been included in the possible political situations considered in this model. Also, there is no restriction on the quantities of ores and slags which can be extracted from domestic sources for the simple reason that in the foreseeable future these quantities will not exceed the amounts contained in these resources (more will be said about this later). Furthermore, there is no provision in this model for shortages of alloys, i.e., it has been made mandatory that the amounts of ferro- and silicomanganese that will be needed in the future will also be produced. Finally, there was in this case no need for constraints on the electric energy required for processing ore since the possibility of total war has been left out of consideration.

The model covers a period of 10 years. The situation at the beginning of each year (i.e., at the end of the preceding year) is characterized by four so-called state variables, namely:


λ
ferro-manganese
ν
silico-manganese
x_1
x_2
x_3
x_4
1
2
3
4
c_1x_1
c_3x_3
c_2x_2
c_4x_4
S
x_5
x_6
Import
5
6
Dean
y_6
7
Udy
y_7
storage
production
production and plant construction

1) the political situation
2) the capacity of the Udy beneficiation plants.[6]
3) the capacity of the Dean plants
4) the quantity of manganese contained in the stockpile

The first state variable represents the general "climate" under which the manganese system has to operate during the coming years; it is outside the programmer's control (this was called an "external" state in the preliminary stochastic model). The assumption is made here that the "climate" at the beginning of a year will remain the same throughout the year and may change only at the end of that year. This is, of course, a crude approximation to reality but it is necessary to the solution of the problem. The three other state variables describe the manganese system itself at the beginning of a year (these are called "internal" states in the previous report). They represent the starting conditions from which the manganese system will further develop.

In addition to the state variables there are the <u>activities</u>. They perform the double function of meeting the year's requirements and, at the same time, transforming the state of the system at the beginning of the year into that state at the end of the year which will most favorably meet the requirements in future years. There are, in general, two groups of activities. The first group (x_1 through x_6 in the diagram) has to do with the operational part of the system; they include:

1) the production of ferro-manganese from stockpiled ores, i.e., imported high-grade ore as well as domestic ore upgraded by the Dean process.

2) the production of ferro-manganese from Aroostook ores by the Udy process

3) the production of silico-manganese from stockpiled ores

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[6] It has been assumed that the Udy alloy plants already have sufficient capacity to process the product of the beneficiation plants.

4) the production of silico-manganese from Aroostook ores by the Udy process

5) the importation of high-grade ores from foreign countries

6) the upgrading of Cuyuna ores by the Dean process.

The second group of activities provides for the expansion of the capacities of the domestic beneficiation plants; they are:

7) increasing the capacity of the Dean plants

8) increasing the capacity of the Udy beneficiation plants.[7]

It was necessary to set a limit on the maximum capacity of both types of plants in order to keep the amount of computational work within manageable proportions. The limit set for each type of plant was 200,000 Net Tons of manganese per year, so that the maximum amount of domestic ore that could possible be treated in any one year by all the plants does not exceed 400,000 N.T. of manganese. This restriction seems to be rather severe in the case of limited war when requirements and import prices will both be high. However, since this model also includes the possibilities of cold war and no war, a limit had to be chosen for the domestic capacities which would be reasonable for all three situations. A domestic production capacity of 400,000 N.T. of manganese per year seems to serve this purpose. It will probably be adequate in times of cold war. In peacetime, when the price of imported manganese will be low, this domestic capacity would cover about 50 per cent of the total requirements.

Since the productive capacity of each type of plant is limited to 200,000 N.T. of manganese per year, there will always be sufficient ore of domestic origin available for beneficiation. Consequently, it was unnecessary to incorporate into the model restrictions which would rule out solutions requiring more domestic ore to be beneficiated than is contained in the resources. The amount of manganese contained in the Cuyuna deposit would enable the Dean plants to produce 200,000

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[7] As above, it has been assumed that the Udy alloy plants already have sufficient capacity so that only the capacity of the beneficiation plants has to be increased if necessary.

N.T. of manganese per year for a period of 20 to 25 years and the Aroostook deposits contain enough ore to sustain a production of 200,000 N.T. of manganese per year during a period of about 40 years.[8]

Furthermore, it has been assumed that there will always be enough electrical energy available to make possible the production of the desired quantities of manganese even in the case of limited war (the electricity constrains turned out to be rather ineffective in the non-stochastic model). In other words, there are no electricity-constraints in this stochastic model.

Altogether there are 5 relationships connecting the 8 activity-variables of each year; the total number of relationships for the ten-year period covered by this model is, accordingly, 50 and the total number of variables, 80.

The objective is to find that combination of the 80 variables which will meet the requirements for manganese over these 10 years at minimum cost. The costs are a function of these 80 variables (plus a few other parameters of which more will be said later). The function is, as before, a second degree polynomial with positive and negative quadratic terms as well as linear terms in these variables. The computational technique applied in the solution of this stochastic model, although permitting computation of costs only at discrete points, does not impose any restrictions on the form of the objective function — the function can be <u>any</u> type of polynomial of arbitrary degree! (More will be said about this in the section that deals with the computational aspects.)

The precise mathematical formulation of this model is given in Appendix I below.

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[8] See Final Report, Part I, *op. cit.*, pp. 6-8.

III. THE DATA

The quantities of manganese which are required by the steel industry varies with changes in the political situation. Therefore, we consider in this model three possible political situations: "no war," "cold war," and "limited war."[9] This is a closer and more useful approximation to reality than is the case of "limited war within the next six years," the only one considered in the non-stochastic model.

The assumption is made that the political situation at the beginning of each year will be one of these three possible states. Furthermore, the political situation at the beginning of the year is assumed to remain stable during that year (this assumption is necessary to the solution of the problem). In other words, it may remain the same or may change into one of the other two situations only at the beginning of each year. The numerical values which were assigned to the probabilities of transfer from one state to another were chosen on the basis of the definitions of the political situations and in consideration of the present state of world affairs. These transfer probabilities, are assumed to remain the same throughout the ten-year period for which the program has been designed. Thus, given the political situation in year n, we assume that the probable situation in year n + 1 will be as follows:

Situation in year n	Probable occurrence of situation in year n + 1: no war	cold war	limited war
no war	.70	.20	.10
cold war	.05	.85	.10
limited war	.10	.40	.50

[9] Limited war is taken to mean a Korean-type war, i.e., a local war fought with conventional weapons. In that situation, the requirements will be high and, at the same time, the importation of foreign ores will be restricted because of high transportation costs. On the other hand, in the no war situation, the requirements will be minimal and the cost of transporting foreign ores will be low. The cold war situation lies somewhere between these two extremes.

If these probabilities were to remain the same for an indefinitely long period of time, then each of these political situations would turn up with the following frequencies:

no war	.166		1
cold war	.666	or	4
limited war	.166		1

This chance distribution more or less reflects current opinion with regard to the probable course of the political situation; therefore it seems to us that the assigned probabilities are reasonable.

The manganese requirements in these three political states have been assumed to be:

Political State	ferro-manganese	silico-manganese	all manganese
no war	700,000 NT	100,000 NT	800,000 NT
cold war	800,000 NT	200,000 NT	1,000,000 NT
limited war	900,000 NT	300,000 NT	1,200,000 NT

The requirements are assumed to be the same for every year for each political situation. (In the non-stochastic model, the limited-war requirements were assumed to lie in a range from 850 to 925×10^3 N.T.)[10] In this stochastic model, the requirements had to be multiples of 100,000 N.T. of Mn, which is the cut-off point used in the calculations. (The solution technique, it will be remembered, is not continuous.) This had two consequences. First, it could hardly be assumed that the difference between the lowest and the highest annual requirement for each of the three political situations would be much more than 100,000 N.T. of Mn in a period of only ten years. It is for this reason that the requirements in each political situation could be taken to be the same for every year. Second, in order to make a distinction between the requirements of the three political situations, it was necessary to set the requirements for silico-manganese

[10] These requirements were estimated on the basis of projected steel production. See Final Report, Part I, p. 6 and Robert E. Kuenne, Estimates of National and Regional Steel Production in the United States for 1970, Econometric Research Program, Research Memorandum No. 6, 2 February, 1959.

at 100,000, 200,000, and 300,000 N.T. respectively, and for ferro-manganese at 700,000, 800,000, and 900,000 N.T. respectively. Consequently, the total requirements are higher (and less realistic than those in the non-stochastic model) than would actually be necessary in each of the political situations considered in this model.[11]

These requirements are the same as those assumed in the preliminary stochastic model for the situation of peace, cold war (no blockade) and hot war (minor blockade). But since there is no place in the present model for a blockade situation — this would almost certainly mean total war — there is no restriction on the quantities of ore which can be imported from foreign countries.

The cost of importing these foreign ores in any one year will certainly depend on the political situation during that year, due to potential fluctuations in the costs of transport and insurance premiums and in the costs of mining these ores in the foreign countries. These costs have sharply reflected political changes in the past. As for mining costs, it might be expected that decreases due to improved technology will be offset by increases due to lower accessibility of the ores in the existing deposits. (There is, of course, the possibility of discovering new and rich deposits in the future, but that is of too random a character to be taken into account.) There are three cost functions for the importation of foreign ores, one for each political situation, namely:

1) no war $u_{5,n} = 8.0\ x_{5,n} + 20.-$
2) cold war $u_{5,n} = 8.0\ x_{5,n} + 40.-$
3) limited war $u_{5,n} = 8.0\ x_{5,n} + 75.-$

$u_{5,n}$, the import price in year n, is expressed in \$/NT of manganese per year and $x_{5,n}$, the quantity imported by the United States in year n, in 100,000 N.T. The function for the case of limited war is a weighted average of the three cost functions of the non-stochastic model after elimination of the negative terms.[12] The terms $8.0\ x_{5,n}$ stand for the rise in the import price due to increases in the annual quantities bought by the United States. The following example

[11] A finer grid of points could have partially overcome this defect, but only at a greatly increased computational cost.

[12] Final Report, Part I, pp. 9-10.

might serve as an illustration:

Annual quantity bought by the USA (Net Tons)	Import price (in $/NT of Mn)		
	no war	cold war	limited war
800,000	$84.-	$104.-	$139.-
900,000	$92.-	$112.-	$147.-
1,000,000	$100.-	$120.-	$155.-
1,100,000	$108.-	$128.-	$163.-
1,200,000	$116.-	$136.-	$171.-

The cost functions used in this model for the upgrading of domestic ores and the production of the alloys in year n, are as follows:

1) Ferro-mang. from stockpiled ores $u_{1,n} = 105.4 - 0.3x_{1,n}$
2) Ferro-mang. by the Udy process $u_{2,n} = 237.1 - 1.0x_{2,n}$
3) Silico-mang. from stockpiled ores $u_{3,n} = 168.1 - 0.5x_{3,n}$
4) Silcio-mang. by the Udy process $u_{4,n} = 272.6 - 1.5x_{4,n}$
5) High-grade ore by the Dean process $u_{6,n} = 115.8 - 0.6x_{6,n}$

As above, the costs are expressed in $/NT of Mn. per year and the quantities in 100,000 NT of Mn. These cost functions include the costs of production and mining. The constants in these functions are, by and large, the same as those used in the second computation of the non-stochastic model.[13]

In estimating the constants in the cost function of the beneficiation processes (Udy, Dean), certain technical coefficients, the c's in the diagram on page 7, were taken into account. The technical coefficients are a measure of the amount of manganese in the form of ore which is needed to produce one net ton of manganese alloy. The technical coefficients used in this model, which are the same as those used in the non-stochastic model,[14] are:

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13 Ibid., p. 13.

14 Ibid., pp. 14-16.

1) Ferro-mang. from stockpiled ores $c_1 = 1.111$

2) Ferro-mang. by the Udy process $c_2 = 1.176 \times 1.053 = 1.238$

3) Silico-mang. from stockpiled ores $c_3 = 1.111$

4) Silico-mang. by the Udy process $c_4 = 1.176 \times 1.053 = 1.238$

As in the non-stochastic case, it has been assumed here that these coefficients will remain the same during the period under consideration.

The second, i.e., x-terms of the cost functions represent the decreases in cost due to increases in the scale of production. It can be seen from those equations that these decreases are larger for the new processes (Dean and particularly Udy) than for the conventional ones since the former are ~~more~~ apt to be improved than are the latter.

Besides these operating costs, there are also construction costs of the new plants, for which the following cost functions have been used:

1) Dean plants $v_{6,n} = 203.0 - 0.2y_{6n}$

2) Udy benef. plants $v_{7,n} = 188.5 - 0.8y_{7,n}$

Again, the costs are expressed in \$/NT and the quantities in 100,000 N.T. per year. The depreciation rule is the same as that in the non-stochastic case: the entire cost is written off in 10 equal stallments over the period starting with the year following that in which the plants are built or their existing capacities increased.

IV. THE PROGRAMS

a) Computational Aspects

There exists as yet no technique that provides an analytical solution for the type of problem that is discussed here (the non-stochastic problem, it will be remembered, could be solved by the simplex technique). Instead, one is forced to compute the value of the objective function for certain combinations of (usually integer-valued) activities and then to select from them that combination of activities that meets the requirements at minimum cost. In general it can be said that the more combinations of (integer-valued) activities one examines, the more one can hope to approximate the absolute minimum-cost combination.

The computations made for the first stochastic model were based on only a few combinations of activities (the results were described on page 46 of the first part of this report). The capacity of all domestic plants together was permitted to take the values 0, 400,000 and 800,000 N.T. only (being respectively 0, 50 and 100 per cent of the peace-time requirements) and the stockpile the values 0, 800,000, 1,600,000 and 2,400,000 N.T. Consequently, only a few points of the objective functions could be examined and the resulting solution was inexact.

The main objective of the present computation was to increase the degree of accuracy of the solution. Thus, there is much less difference between the integer values that the state variables are permitted to take. In all cases this difference is not more than 100,000 N.T., as opposed to the 400,000 or 800,000 N.T. of the previous computation; so, the number of possible combinations of activities has been greatly increased. The first stochastic model considered only 3 activities, importation of foreign ores, beneficiation of domestic ores and increments to the capacity of the beneficiation plants, whereas the present model considers 8 activities. Clearly these two factors — the greater number of activities as well as the greater number of values that each activity can take — vastly increased the number of combinations of activities that had to be examined.

As a matter of fact, the number of possible combinations to be examined became so large, that it was necessary to restrict the range of the state variables in order to keep the total number of combinations that had to be examined within reasonable proportions (the relationship between activities, state variables, and number of combinations will be made clear by an example later on).

The first step was to reduce the number of external states, i.e., political situations that had to be considered from five to three, namely, "no war," "cold war" and "limited war." It was felt that this selection covered sufficiently the various possible political situations. The second step was to set upper limits to the stockpile and to the two domestic capacities. The following table compares the upper limits of the internal state variables in the two computations of the stochastic models.

	Upper limits	
	in present computation	in previous computation
1) stockpile	1,900,000 NT	2,400,000 NT
2) domestic capactities		
a) Dean plants	200,000 NT	800,000 NT
b) Udy plants	200,000 NT	

The upper limit of the stockpile in the present computation is the same as that of the first computation of the non-stochastic model. The reasons for restricting the domestic capacities to a total of 400,000 NT have been discussed on pages 9 and 10 above.

Setting limits to the stockpile as well as to the domestic capacities means, of course, restricting the region that contains the points of the objective function for which the costs have to be calculated. On the other hand, the distance between these points is much smaller than before, since the integral values of the activities lie much closer together. The result is an examination in some detail of that part of the objective function that is the most interesting from a practical point of view.

The amount of computational work that has to be done to find the lowest cost combination of activities depends on the number of initial states, activities and final states that have to be examined. The following example together with the diagram on page 7 above should make this clear.[15]

Suppose that we find the internal state of the system at the beginning of a year to be:

a)	capacity of Udy beneficiation plants	2×10^5NT/yr
b)	capacity of Dean plants	1×10^5NT/yr
c)	quantity of manganese stockpiled	13×10^5NT

Suppose also that we want the internal state of the system at the end of that year to be:

a)	capacity of Udy beneficiation plants	2×10^5NT/yr
b)	capacity of Dean plants	2×10^5NT/yr
c)	quantity of manganese stockpiled	10×10^5NT

Furthermore, suppose that the political situation at the beginning of that year is that of no war, and that this situation will last during that whole year. The amounts of manganese required in that year are then 7×10^5 N.T. ferro-manganese and 1×10^5 N.T. silico-manganese (see table on page 12 above).

The possible combinations of activities (in units of 100,000 NT), transforming the initial state to the final state and at the same time meeting the specified requirements are shown with the costs (in units of $100,000) associated with them at the top third of Table 1.

[15] For the purposes of this example, the technical coefficients, the c's in the diagram (see p. 15 above), have been set equal to one.

TABLE 1

Combinations of activities, with their associated costs, that meet the requirements and carry state (2, 1, 13) over into state (2, 2, 10).

1. No War (requirements: 7 ferro- and 1 silico-manganese)

	1	2	3	4	5	6	1	2	3	4	5	6
x_2	0	0	0	0	0	0	—	—	—	—	—	—
x_4	0	0	1	1	*)	*)	—	—	271.1	271.1		
x_6	0	1	0	1			—	115.2	—	115.2		
x_1	7	7	7	7			723.1	723.1	723.1	723.1		
x_3	1	1	0	0			167.6	167.6	—	—		
x_5	5	4	4	3			300.0	208.0	208.0	132.0		
y_6	1	1	1	1			**)	**)	**)	**)		
y_7	0	0	0	0			—	—	—	—		
							1190.7	1213.9	1202.2	1241.4	—	—

2. Cold War (requirements: 8 ferro- and 2 silico-manganese)

	1	2	3	4	5	6	1	2	3	4	5	6
x_2	0	0	0	0	0	0	—	—	—	—	—	—
x_4	0	0	1	1	2	2	—	—	271.1	271.1	539.2	539.2
x_6	0	1	0	1	0	1	—	115.2	—	115.2	—	115.2
x_1	8	8	8	8	8	8	824.0	824.0	824.0	824.0	824.0	824.0
x_3	2	2	1	1	0	0	334.1	334.1	167.6	167.6	—	—
x_5	7	6	6	5	5	4	672.0	528.0	528.0	400.0	400.0	288.0
y_6	1	1	1	1	1	1	**)	**)	**)	**)	**)	**)
y_7	0	0	0	0	0	0	—	—	—	—	—	—
							1830.1	1801.3	1790.7	1777.9	1763.2	1766.4

3. Limited War (requirements: 9 ferro- and 3 silico-manganese)

	1	2	3	4	5	6	1	2	3	4	5	6
x_2	0	0	0	0	0	0	—	—	—	—	—	—
x_4	0	0	1	1	2	2	—	—	271.1	271.1	539.2	539.2
x_6	0	1	0	1	0	1	—	115.2	—	115.2	—	115.2
x_1	9	9	9	9	9	9	924.3	924.3	924.3	924.3	924.3	924.3
x_3	3	3	2	2	1	1	499.7	499.7	334.1	334.1	167.6	167.6
x_5	9	8	8	7	7	6	1323.0	1112.0	1112.0	917.0	917.0	738.0
y_6	1	1	1	1	1	1	**)	**)	**)	**)	**)	**)
y_7	0	0	0	0	0	0	—	—	—	—	—	—
							2747.0	2651.2	2641.5	2561.7	2548.1	2484.3

*) When $x_2 = 0$, then x_4 can take the value of 2, but that would exceed the silico-manganese requirements in this case.

**) See page 20.

TABLE 1 (contd.)

Combinations of activities, with their associated costs, that meet the requirements and carry state (2, 1, 13) over into state (2, 2, 10)

1. No War (requirements: 7 ferro- and 1 silico-manganese)

	7	8	9	10	11	12	7	8	9	10	11	12
x_2	1	1	1	1	2	2	236.1	236.1	236.1	236.1	470.2	470.2
x_4	0	0	1	1	0	0	—	—	271.1	271.1	—	—
x_6	0	1	0	1	0	1	—	115.2	—	115.2	—	115.2
x_1	6	6	6	6	5	5	621.6	621.6	621.6	621.6	519.5	519.5
x_3	1	1	0	0	1	1	167.6	167.6	—	—	167.6	167.6
x_5	4	3	3	2	3	2	208.0	132.0	132.0	72.0	132.0	72.0
y_6	1	1	1	1	1	1	**)	**)	**)	**)	**)	**)
y_7	0	0	0	0	0	0	—	—	—	—	—	—
							1233.3	1272.5	1260.8	1316.0	1289.3	1344.5

2. Cold War (requirements: 8 ferro- and 2 silico-manganese)

	7	8	9	10	11	12	7	8	9	10	11	12
x_2	1	1	1	1	2	2	236.1	236.1	236.1	236.1	470.2	470.2
x_4	0	0	1	1	0	0	—	—	271.1	271.1	—	—
x_6	0	1	0	1	0	1	—	115.2	—	115.2	—	115.2
x_1	7	7	7	7	6	6	723.1	723.1	723.1	723.1	621.6	621.6
x_3	2	2	1	1	2	2	334.1	334.1	167.6	167.6	334.1	334.1
x_5	6	5	5	4	5	4	528.0	400.0	400.0	288.0	400.0	288.0
y_6	1	1	1	1	1	1	**)	**)	**)	**)	**)	**)
y_7	0	0	0	0	0	0	—	—	—	—	—	—
							1821.3	1808.5	1797.9	1801.1	1825.9	1829.1

3. Limited War (requirements: 9 ferro- and 3 silico-manganese)

	7	8	9	10	11	12	7	8	9	10	11	12
x_2	1	1	1	1	2	2	236.1	236.1	236.1	236.1	470.2	470.2
x_4	0	0	1	1	0	0	—	—	271.1	271.1	—	—
x_6	0	1	0	1	0	1	—	115.2	—	115.2	—	115.2
x_1	8	8	8	8	7	7	824.0	824.0	824.0	824.0	723.1	723.1
x_3	3	3	2	2	3	3	499.7	499.7	334.1	334.1	499.7	499.7
x_5	8	7	7	6	7	6	1112.0	917.0	917.0	738.0	917.0	738.0
y_6	1	1	1	1	1	1	**)	**)	**)	**)	**)	**)
y_7	0	0	0	0	0	0	—	—	—	—	—	—
							2671.8	2592.0	2582.3	2518.5	2610.0	2546.2

**) The costs of increasing the plant capacities are equally distributed over the <u>next</u> 10 years; hence no depreciation is charged to the year in which the plant is contructed or its capacity increased.

The lowest cost combination occurs at the point where $x_1 = 7$, $x_3 = 1$ and $x_5 = 5 \times 10^5$ NT; the cost associated with it is $\$1190.7 \times 10^5$. In other words the lowest cost solution in this example is obtained by producing all the required ferro- and silico-manganese by conventional methods from stockpiled ores, x_1 and x_3, and replacing them as far as is necessary by importing foreign ores, x_5. In addition, there is the required increase of 1×10^5 NT in the capacity of the Dean plants, y_6, at the end of the year; the costs of this increase will be written off in the next 10 years.

Keeping the internal states of the system at the beginning and the end of the year the same as before, let us now suppose that the political situation at the beginning of the year is that of cold war. The requirements to be met in that year are then 8×10^5 NT of ferro- and 2×10^5 NT silico-manganese.

The possible combinations of activities and their associated costs are now given in the middle third of Table 1. In this case the lowest cost combination turns out to be where $x_4 = 2$, $x_1 = 8$, and $x_5 = 5 \times 10^5$ NT, with an associated cost of $\$1763.2 \times 10^5$. (In this case the Udy plants is used to its full capacity, producing 2×10^5 NT more than is required in the no war situation.) This turns out to cost $\$1830.1 - 1763.2 = \$66.9 \times 10^5$ less than it would if the silico-manganese was produced from stockpiled ore, x_3, with imports providing the additional 2×10^5 NT. The Dean plant is not used in this lowest-cost combination despite the higher cost of importing foreign ore.

Finally, suppose that the political situation at the beginning of the year is that of limited war. The requirements in that year are 9×10^5 NT ferro- and 3×10^5 NT silico-manganese. The minimum-cost combination then turns out to be (see lower third of Table 1), $x_4 = 2$, $x_6 = 1$, $x_1 = 9$, $x_3 = 1$ and $x_5 = 6 \times 10^5$ NT with a total associated cost of $\$2484.3 \times 10^5$. Again, the Udy plant is used at its full capacity for the production of silico-manganese; and, since the Dean plant is also used to produce high-grade ore, a smaller quantity of ore has to be imported.

So far, only one year has been considered, starting with one and finishing with another internal state. However, in order to demonstrate the technique that has been used to solve this problem, it is necessary to go back one year in time; this is the procedure that is followed in the actual computations. That is, first the costs that are associated with the various possible combinations of activities in the last year are computed, then the costs of the year preceding the last, and so on all the way back.[16]

Continuing the examples on pages 18-20, above, suppose that the internal state at the beginning of the previous year was:

a)	capacity of Udy beneficiation plants	1×10^5 NT/yr
b)	capacity of Dean plants	1×10^5 NT/yr
c)	quantity of manganese stockpiled	19×10^5 NT

The internal state at the end of this year (which is the same as the internal state at the beginning of the first year in this example) is:

a)	capacity of Udy beneficiation plants	2×10^5 NT/yr
b)	capacity of Dean plants	1×10^5 NT/yr
c)	quantity of manganese stockpiled	13×10^5 NT

Starting again with the political situation of no war (requirements 7×10^5 NT ferro- and 1×10^5 NT silico-manganese), we find that there are six possible combinations of activities (see top third of Table 2). The lowest-cost combination turns out to be where: $x_1 = 7$, $x_3 = 1$ and $x_5 = 2 \times 10^5$ NT with a total associated cost of $\$962.7 \times 10^5$. In addition there has now to be an increase in the capacity of the Udy plants of 1×10^5 NT/yr; as before, the costs involved will be written off in the next 10 years.

In order to compute the cost for both years, it is necessary to bring in the transition probabilities for no war → no war (.70), no war → cold war (.20) and no war → limited war (.10). Moreover, the first 10 per cent of the cost of increasing the capacity of the

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[16] This procedure greatly reduces the computational load, since fewer combinations of activities have to be examined in order to find the least-cost path. See pp. 31-32 below.

TABLE 2

Combinations of activities, with their associated costs, that meet the requirements and carry state (1, 1, 19) over into state (2, 1, 13).

1. No War (requirements: 7 ferro- and 1 silico-manganese).

	1	2	3	4	5	6	1	2	3	4	5	6
x_2	0	0	0	0	1	1	——	——	——	——	236.1	236.1
x_4	0	0	1	1	0	0	——	——	271.1	271.1	——	——
x_6	0	1	0	1	0	1	——	115.2	——	115.2	——	115.2
x_1	7	7	7	7	6	6	723.1	723.1	723.1	723.1	621.6	621.6
x_3	1	1	0	0	1	1	167.6	167.6	——	——	167.6	167.6
x_5	2	1	1	0	1	0	72.0	28.0	28.0	——	28.0	——
y_6	0	0	0	0	0	0	——	——	——	——	——	——
y_7	1	1	1	1	1	1	**)	**)	**)	**)	**)	**)
							962.7	1033.9	1022.2	1109.4	1053.3	1140.5

2. Cold War (requirements: 8 ferro- and 2 silico-manganese).

	1	2	3	4	5	6	1	2	3	4	5	6
x_2	0	0	0	0	1	1	——	——	——	——	236.1	236.1
x_4	0	0	1	1	0	0	——	——	271.1	271.1	——	——
x_6	0	1	0	1	0	1	——	115.2	——	115.2	——	115.2
x_1	8	8	8	8	7	7	824.0	824.0	824.0	824.0	723.1	723.1
x_3	2	2	1	1	2	2	334.1	334.1	167.6	167.6	334.1	334.1
x_5	4	3	3	2	3	2	288.0	192.0	192.0	112.0	192.0	112.0
y_6	0	0	0	0	0	0	——	——	——	——	——	——
y_7	1	1	1	1	1	1	**)	**)	**)	**)	**)	**)
							1446.1	1465.3	1454.7	1489.9	1485.3	1520.5

3. Limited War (requirements: 9 ferro- and 3 silico-manganese)

	1	2	3	4	5	6	1	2	3	4	5	6
x_2	0	0	0	0	1	1	——	——	——	——	236.1	236.1
x_4	0	0	1	1	0	0	——	——	271.1	271.1	——	——
x_6	0	1	0	1	0	1	——	115.2	——	115.2	——	115.2
x_1	9	9	9	9	8	8	924.3	924.3	924.3	924.3	824.0	824.0
x_3	3	3	2	2	3	3	499.7	499.7	334.1	334.1	499.7	499.7
x_5	6	5	5	4	5	4	738.0	575.0	575.0	428.0	575.0	428.0
y_6	0	0	0	0	0	0	——	——	——	——	——	——
y_7	1	1	1	1	1	1	**)	**)	**)	**)	**)	**)
							2162.0	2114.2	2104.5	2072.7	2134.8	2103.0

**) The costs of increasing plant capacity are distributed in equal installments. See note **) to Table 1.

Udy beneficiation plants ($\$18.77 \times 10^5$) has to be taken into account. Finally, the cost in the last year (last in sequence of time, first in computational order) has to be discounted in order to put the cost in both years on the same basis. The discount factor used is .95, corresponding to an interest rate of 5 per cent.

Accordingly, the cost of the least expensive program covering both years will then amount to: $\$962.7 + .95\ (.70 \times 1190.7 + .20 \times 1763.2 + .10 \times 2484.3 + 20.28) = \$2344.8 \times 10^5$.

Next, the political situation of cold war at the beginning of the first year (first in sequence of time, last in computational order) has to be considered. The requirements in that year will then be 8×10^5 N.T. ferro- and 2×10^5 N.T. silico-manganese. Again there are six possible combinations of activities (see middle third of Table 2). The lowest cost combination occurs when $x_1 = 8$, $x_3 = 2$, $x_5 = 4$ and $y_7 = 1 \times 10^5$ N.T. with an associated total cost of $\$1446.1 \times 10^5$. Hence, the cost of the least expensive program covering both years will now amount to: $\$1446.1 + .95\ (.05 \times 1190.7 + .85 \times 1763.2 + .10 \times 2484.3 + 20.28) = \$3181.7 \times 10^5$.

Finally, the political situation of limited war at the beginning of the first year (first in sequence of time, last in computational order) has to be considered. The requirements to be met in that year are 9×10^5 ferro- and 3×10^5 silico-manganese. The lowest-cost combination of activities (see lower third of Table 2) is where $x_4 = 1$, $x_6 = 1$, $x_1 = 9$, $x_3 = 2$, $x_5 = 4$ and $y_7 = 1 \times 10^5$ N.T. with an associated cost of $\$2072.7 \times 10^5$. The cost of the least expensive program covering both years will now amount to $\$2072.7 + .95\ (.10 \times 1190.7 + .40 \times 1763.2 + .50 \times 2484.3 + 20.28) = \$4055.1 \times 10^5$.

It should be noted that the phrase "least expensive" program is a conditional one. It is least expensive only as long as the (2,1, 13) intermediate state is maintained. But if another intermediate state is selected, another and perhaps even better "least expensive" program will be found. In other words, if the "path" connecting the two end points is changed, another "least expensive" program will probably be found. To make this clear, another "path" has been selected which starts with the same initial state as the example above (see page 21),

a) capacity of Udy beneficiation plants 1×10^5 NT/yr

b) capacity of Dean plants 1×10^5 NT/yr

c) quantity of manganese stockpiled 19×10^5 NT

but this state leads to another intermediate state,

a) capacity of Udy beneficiation plants 1×10^5 NT/yr

b) capacity of Dean plants 2×10^5 NT/yr

c) quantity of manganese stockpiled 15×10^5 NT

and ends in the same final state, (see page 18),

a) capacity of Udy beneficiation plants 2×10^5 NT/yr

b) capacity of Dean plants 2×10^5 NT/yr

c) quantity of manganese stockpiled 10×10^5 NT

Starting again with the beginning of the last year (last in sequence of time) and taking first the political situation of no war, we find that in this case that nine different combinations of activities are possible. These activities can be found in the top third of Table 3. The lowest-cost combination turns out to be where $x_1 = 7$, $x_3 = 1$ and $x_5 = 5 \times 10^5$ N.T. with an associated cost of $\$1022.7 \times 10^5$.

The lowest cost combination in the case of cold war turns out to be where $x_4 = 1$, $x_1 = 8$, $x_3 = 1$ and $x_5 = 5 \times 10$ N.T. with an associated cost of $\$1550.7 \times 10^5$ (see middle third of Table 3). In the limited war situation the lowest cost combination is apparently that of $x_4 = 1$, $x_6 = 2$, $x_1 = 9$, $x_3 = 2$ and $x_5 = 4 \times 10^5$ N.T. with an associated cost of $\$2186.7 \times 10^5$.

Stepping back one year in time, i.e., from initial state (1, 1, 19) to intermediate state (1, 2, 15), we find the following lowest cost combinations (see Table 4):

TABLE 3

Combinations of activities, with their associated costs that meet the requirements and carry state (1,2,15) over into state (2,2,10).

1. No War (requirements: 7 ferro- and 1 silico-manganese).

	1 2 3 4 5 6 7 8 9	1	2	3	4	5	6	7	8	9
x_2	0 0 0 0 0 0 1 1 1	—	—	—	—	—	—	236.1	236.1	236.1
x_4	0 0 0 1 1 1 0 0 0	—	—	—	271.1	271.1	271.1	—	—	—
x_6	0 1 2 0 1 2 0 1 2	—	115.2	229.2	—	115.2	229.2	—	115.2	229.2
x_1	7 7 7 7 7 7 6 6 6	723.1	723.1	723.1	723.1	723.1	723.1	621.6	621.6	621.6
x_3	1 1 1 0 0 0 1 1 1	167.6	167.6	167.6	—	—	—	167.6	167.6	167.6
x_5	3 2 1 2 1 0 2 1 0	132.0	72.0	28.0	72.0	28.0	—	72.0	28.0	—
y_6	0 0 0 0 0 0 0 0 0	—	—	—	—	—	—	—	—	—
y_7	1 1 1 1 1 1 1 1 1	**)	**)	**)	**)	**)	**)	**)	**)	**)
		1022.7	1077.9	1147.9	1066.2	1137.4	1223.4	1097.3	1168.5	1254.5

2. Cold War (requirements: 8 ferro- and 2 silico-manganese).

	1 2 3 4 5 6 7 8 9	1	2	3	4	5	6	7	8	9
x_2	0 0 0 0 0 0 1 1 1	—	—	—	—	—	—	236.1	236.1	236.1
x_4	0 0 0 1 1 1 0 0 0	—	—	—	271.1	271.1	271.1	—	—	—
x_6	0 1 2 0 1 2 0 1 2	—	115.2	229.2	—	115.2	229.2	—	115.2	229.2
x_1	8 8 8 8 8 8 7 7 7	824.0	824.0	824.0	824.0	824.0	824.0	723.1	723.1	723.1
x_3	2 2 2 1 1 1 2 2 2	334.1	334.1	334.1	167.6	167.6	167.6	334.1	334.1	334.1
x_5	5 4 3 4 3 2 4 3 2	400.0	288.0	192.0	288.0	192.0	112.0	288.0	192.0	112.0
y_6	0 0 0 0 0 0 0 0 0	—	—	—	—	—	—	—	—	—
y_7	1 1 1 1 1 1 1 1 1	**)	**)	**)	**)	**)	**)	**)	**)	**)
		1558.1	1561.3	1579.3	1550.7	1569.9	1603.9	1581.3	1600.5	1634.5

3. Limited War (requirements: 9 ferro- and 3 silico-manganese).

	1 2 3 4 5 6 7 8 9	1	2	3	4	5	6	7	8	9
x_2	0 0 0 0 0 0 1 1 1	—	—	—	—	—	—	236.1	236.1	236.1
x_4	0 0 0 1 1 1 0 0 0	—	—	—	271.1	271.1	271.1	—	—	—
x_6	0 1 2 0 1 2 0 1 2	—	115.2	229.2	—	115.2	229.2	—	115.2	229.2
x_1	9 9 9 9 9 9 8 8 8	924.3	924.3	924.3	924.3	924.3	924.3	824.0	824.0	824.0
x_3	3 3 3 2 2 2 3 3 3	499.7	499.7	499.7	334.1	334.1	334.1	499.7	499.7	499.7
x_5	7 6 5 6 5 4 6 5 4	917.0	738.0	575.0	738.0	575.0	428.0	738.0	575.0	428.0
y_6	0 0 0 0 0 0 0 0 0	—	—	—	—	—	—	—	—	—
y_7	1 1 1 1 1 1 1 1 1	**)	**)	**)	**)	**)	**)	**)	**)	**)
		2341.0	2277.2	2228.2	2267.5	2219.7	2186.7	2297.8	2250.0	2217.0

**) See Table 1, note **).

TABLE 4

Combinations of activities, with their associated costs, that meet the requirements and carry state (1,1,19) over into state (1,2,15).

	1	2	3	4	5	6	1	2	3	4	5	6
1. No War (requirements: 7 ferro- and 1 silico-manganese).												
x_2	0	0	0	0	1	1	——	——	——	——	236.1	236.1
x_4	0	0	1	1	0	0	——	——	271.1	271.1	——	——
x_6	0	1	0	1	0	1	——	115.2	——	115.2	——	115.2
x_1	7	7	7	7	6	6	723.1	723.1	723.1	723.1	621.6	621.6
x_3	1	1	0	0	1	1	167.6	167.6	——	——	167.6	167.6
x_5	4	3	3	2	3	2	208.0	132.0	132.0	72.0	132.0	72.0
y_6	1	1	1	1	1	1	**)	**)	**)	**)	**)	**)
y_7	0	0	0	0	0	0	——	——	——	——	——	——
							1098.7	1137.9	1126.2	1181.4	1157.3	1212.5
2. Cold War (requirements: 8 ferro- and 2 silico-manganese).												
x_2	0	0	0	0	1	1	——	——	——	——	236.1	236.1
x_4	0	0	1	1	0	0	——	——	271.1	271.1	——	——
x_6	0	1	0	1	0	1	——	115.2	——	115.2	——	115.2
x_1	8	8	8	8	7	7	824.0	824.0	824.0	824.0	723.1	723.1
x_3	2	2	1	1	2	2	334.1	334.1	167.6	167.6	334.1	334.1
x_5	6	5	5	4	5	4	528.0	400.0	400.0	288.0	400.0	288.0
y_6	1	1	1	1	1	1	**)	**)	**)	**)	**)	**)
y_7	0	0	0	0	0	0	——	——	——	——	——	——
							1686.1	1673.3	1662.7	1665.9	1693.3	1696.5
3. Limited War (requirements: 9 ferro- and 3 silico-manganese).												
x_2	0	0	0	0	1	1	——	——	——	——	236.1	236.1
x_4	0	0	1	1	0	0	——	——	271.1	271.1	——	——
x_6	0	1	0	1	0	1	——	115.2	——	115.2	——	115.2
x_1	9	9	9	9	8	8	924.3	924.3	924.3	924.3	824.0	824.0
x_3	3	3	2	2	3	3	499.7	499.7	334.1	334.1	499.7	499.7
x_5	8	7	7	6	7	6	1112.0	917.0	917.0	738.0	917.0	738.0
y_6	1	1	1	1	1	1	**)	**)	**)	**)	**)	**)
y_7	0	0	0	0	0	0	——	——	——	——	——	——
							2536.0	2456.2	2446.5	2382.7	2476.8	2413.0

**) See Table 1, note **).

a) no war: $x_1 = 7$, $x_3 = 1$ and $x_5 = 4 \times 10^5$ NT
associated cost $\$1098.7 \times 10^5$

b) cold war: $x_4 = 1$, $x_1 = 8$, $x_3 = 1$ and $x_5 = 5 \times 10^5$ NT
associated cost $\$1662.7 \times 10^5$

c) limited war: $x_4 = 1$, $x_6 = 1$, $x_1 = 9$, $x_3 = 2$ and $x_5 = 7 \times 10^5$ NT
associated cost $\$2382.7 \times 10^5$.

The cost of the "least expensive" program covering both years and following this "path" will then amount to:

a) no war:
$1098.7 + .95\ (.70 \times 1022.7 + .20 \times 1550.7 + .10 \times 2186.7 + 18.77) = \$2299.0 \times 10^5$

b) cold war:
$1662.7 + .95\ (.05 \times 1022.7 + .85 \times 1550.7 + .10 \times 2186.7 + 18.77) = \$3189.0 \times 10^5$

c) limited war:
$2382.7 + .95\ (.10 \times 1022.7 + .40 \times 1550.7 + .50 \times 2186.7 + 18.77) = \$4125.6 \times 10^5$.

The costs of the "least expensive" programs for each "path" are compared in the table below:

Cost
(in $100,000)

	Path No. 1	Path No. 2
no war	2344.8	2299.0
cold war	3181.7	3189.0
limited war	4055.1	4125.6

The results of this computation show that at the beginning of the two-year period, Path No. 1 is more costly in the case of no war, slightly less costly in the case of cold war and considerably less costly in the case of limited war than Path No. 2. These results are what might reasonably be expected without making the actual computations, with the exception of the cold war situation. In this case, the results are so close as to make a choice between the paths impossible without the benefit of the computations.

So far only two paths have been examined in this two-year example. However, in order to find the minimum-cost solution, it is necessary to examine all the possible paths between the two end points. The number of possible paths depends in part on the political situation. This is shown in the following example:

a) no war

The requirements in this case are 7×10^5 N.T. of ferro-manganese and 1×10^5 NT of silico-manganese, or a total of 8×10^5 NT of manganese. To end up the second year with a stockpile of 10×10^5 NT of Mn it is possible, in principle, to start that year with a stockpile ranging from 18×10^5 NT to 0.[17] If the stockpile at the beginning of the second year is 0, it would be necessary to import and/or to produce domestically all the 8×10^5 NT of Mn that is required in that second year plus the 10×10^5 NT of Mn that is supposed to be in the stockpile by the end of that year. Going one year back in time and, again assuming a no-war situation at the beginning of that year, we see that, starting that year with a stockpile of 19×10^5 NT of Mn, it is possible to meet the requirement and to end that year with a stockpile ranging from 19×10^5 NT to 11×10^5 NT of Mn. In the former case, all the requirements of the first year are met by importation or domestic production. In the latter case, all the requirements are met by ore that comes out of the stockpile. Thus, the amount of ore in the stockpile at the end of the first year (or at the beginning of the second year) can range in this case from 18×10^5 to 11×10^5 of Mn.

b) cold war

The requirements in this case amount to a total of $8 + 2 = 10 \times 10^5$ NT of Mn. Accordingly, the quantity of ore in the stockpile

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17

A stockpile of 19×10^5 NT of Mn at the beginning of the second year is ruled out since this could never lead to a stockpile of 10×10^5 NT of Mn at the end of that year with only 8×10^5 NT of Mn required.

at the end of the first year (or at the beginning of the second year) has a lower limit of 9×10^5 of Mn and the same upper limit of 19×10^5 NT of Mn as before.

c) limited war

The total requirements are now $9 + 3 = 12 \times 10^5$ NT of Mn and the amount of ore in the stockpile at the end of the first year will now range between 7×10^5 NT and 19×10^5 NT of Mn.

Besides the stockpile, there are also the capacities of the plants to be considered in the search for all possible paths. These capacities have an upper limit of 2×10^5 NT per year; the lower limit, on the other hand, is not fixed. This is due to the fact that once the capacity has been increased it either remains that way, or is further increased, until it becomes obsolete. It is assumed for the purposes of this example that it will not become obsolete within the next ten years. The only capacities to be considered in the given example are, therefore, 1×10^5 NT/yr and 2×10^5 NT/yr; this applies both to the Udy plants and to the Dean plants.

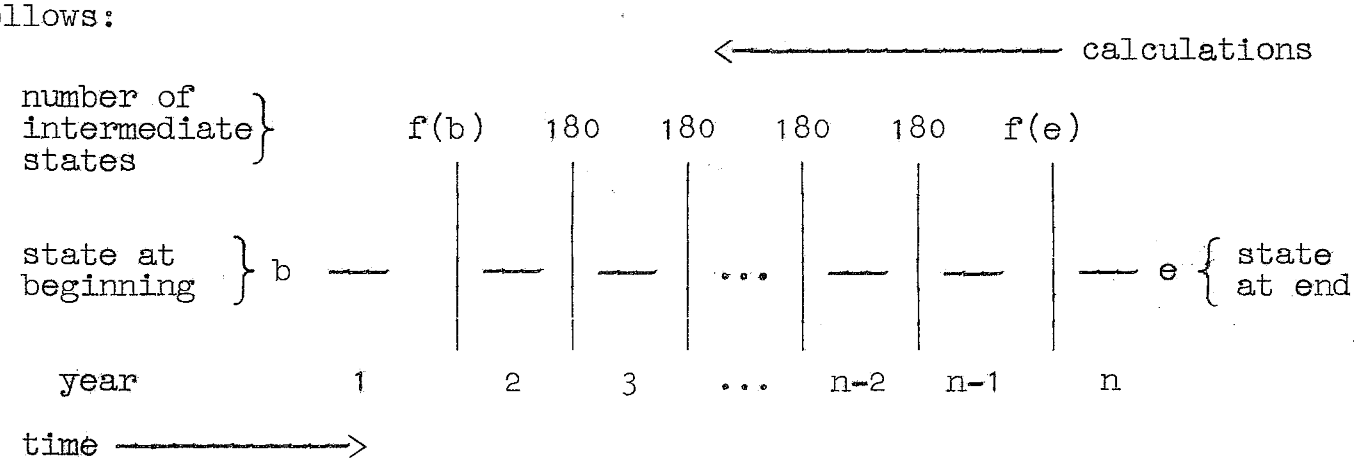
It follows from the foregoing that the number of possible intermediate states connecting these two end points is at least $2 \times 2 \times 8 = 32$ (no war in both years) and at most $2 \times 2 \times 13 = 52$ (limited war in both years). The intermediate state which actually occurs will usually lie between these two extremes.

Accordingly, different combinations of activities can be associated with each possible path. For each of these combinations, the costs have to be computed taking into account the probabilities that the political state of the first year will go over into one of the three possible states of the second year. Then the costs of both years for each possible path have to be compared in order to find the lowest cost path connecting the two end points. The result will be three minimum-cost paths, one for each political situation, at the beginning of the first year.

Clearly, another set of intermediate states would have been possible if another internal state at the beginning of the two-year period had been chosen. For example, if the internal state at the beginning of the two-year period had been (0, 0, 8) and that at the end of the period the same as before (2, 2, 10), then the stockpile at the end of the first year could lie anywhere between the limits 0 and 19×10^5 NT and the capacities of the beneficiation plants between 0 and 2×10^5 NT per year. Consequently, $3 \times 3 \times 20 = 180$ intermediate states would have to be examined in this case, and similarly if other internal states were chosen at the beginning of the two-year period.

The number of intermediate states that have to be calculated for various internal states at the beginning of the two-year period becomes apparent if we step back one more year in time. Starting at that point, it is necessary to examine all possible intermediate states at the end of that year as well as at the end of the following year. From the foregoing analysis it follows that the number of possible intermediate states at the end of the first year now under consideration will lie somewhere between $1 \times 1 \times 9$ and $3 \times 3 \times 20$ depending on the internal state and the political state at the beginning of that year; the number of intermediate states at the end of the second year will remain 52 as long as the internal state at the end of the period is kept the same.

Again stepping back one year in time, the number of possible intermediate states at the end of the second year will be 180, while the number at the end of the first year will again depend on the internal and political states at the beginning of that year and the number at the beginning of the last year will depend on the state at the end of that year, and so on. The situation can be pictured as follows:



Starting with year n, there will be a number of different possible paths for each of the three political states. For each path there are several possible combinations of activities. The costs for each of the possible combinations of activities have to be computed and compared in order to find the lowest cost combination for each political state in that year. Then, one of the many possible internal states at the beginning of year n - 1 has to be connected with the one at the end of that period. In principle several paths are now possible, each one leading over one of the possible intermediate states. All combinations of activities that are compatible with each path have to be examined in order to find the lowest-cost combination for that path. Then another path between the two end points has to be tried and so on, until all possible paths have been examined. The result will again be three lowest-cost paths, one for each political situation in year n - 1, connecting that particular initial state with the final state. The same procedure is followed for all the other possible internal states at the beginning of year n - 1.

The next step is to extend the period from two to three years and to repeat this process over and over again. Clearly, a great many cost calculations have to be made and their results compared. It would be very time-consuming to examine all the combinations of activities that are possible between the states at the beginning and the states at the end of a period of some length if it were not possible to take advantage of a feature which all computations of this sort have in common: once the lowest-cost path has been found between the internal state at the beginning and the internal state at the end of a period, it remains the lowest-cost path in all ensuing computations running over that initial internal state.

To illustrate this, let us assume that in our previous example the lowest-cost paths between the internal state (2, 2, 10) at the end and (1, 1, 19) at the beginning of the two-year period are the paths that run over the (1, 1, 15) intermediate state in case of no war and over the (2, 1, 13) intermediate state in the other 2 cases. Going back one year in time, let us assume that (0, 1, 15) is the internal state at the beginning of the three-year period and that we would like to find the lowest-cost path between the end points (2, 2, 10) and (0, 1, 15) in the case of no war at the beginning of that period. We will then start to compute the costs of all sets of activities in the 3rd year which start out from the (0, 1, 15) internal state, meet the requirements and end up with one of the possible intermediate states at the end of that year. One of these states will be the (1, 1, 19) internal state. Of the two sets of activities which are then possible, the one with the lowest cost is the set $x_5 = 11$ and $x_6 = 1 \times 10^5$ NT plus $y_7 = 1 \times 10^5$ NT/yr with an associated cost of $\$1303.2 \times 10^5$. In finding the lowest-cost path between the two end points, leading over the (1, 1, 19) intermediate state, we can now make use of our previous findings since this path will also lead over the (1, 2, 15) intermediate state if the no-war situation continues for another year, and over the (2, 1, 13) intermediate state in case the political situation changes. The lowest-cost path leading over the (1, 1, 19) intermediate state will have associated costs of $\$1303.2 + .95$ $(.70 \times 2299.0 + .20 \times 3181.7 + .10 \times 4055.1 + 37.38) = \$3857.3 \times 10^5$. It should be noted that there are other possible paths connecting the (0, 1, 15) and (2, 2, 10) endpoints which do not run along the (1, 1, 19) intermediate state, and which, therefore, may have costs below the one found here. The path with the lowest-cost between these two end points will be determined as a result of the computations.

This numerical example shows two things. First, it shows how information on the external state, i.e., the prevailing political situation, is used in the program. To be more specific, the lowest-cost path will run over the (1, 2, 15) intermediate state if there is still no-war at the beginning of the second year, but it will run over the (2, 1, 13) intermediate state if the political situation has changed by that time. Second, the example shows how the results of previous computations are used in the ensuing computations thereby eliminating the need for examining a host of combinations of internal states.

However, it will be clear from this example that there still remains a large number of combinations of states that actually have to be examined. In fact, it took the IBM 704 about 8 1/2 hours to compute all the relevant costs and to come up with a program that covers the requirements of a ten-year period at lowest-cost.

b) Costs of a ten-year program.

The costs of a program will, in general, depend on the external as well as on the internal state of the system at the beginning of the period that is covered by that program. As for the external states, one expects the costs to be lower in the no-war case than in the cold-war case and lower in the cold-war case than in the limited-war case at the beginning of the period. As for the internal states, one expects that if more ore has been stockpiled or the capacities of the domestic plants are larger at the beginning of the period, the lower the costs of the program will be.

The results of the computations, shown in Table 5, clearly fulfills both expectations: in the case of no war at the beginning of the first year, the costs lie between \$1652 million and \$1378 million, depending on the internal state of the system at that time, in the case of cold war between \$1834 million and \$1525 million and in the case of limited war between \$1986 million and \$1611 million.

In comparing these results with those obtained from the first stochastic model, it should be kept in mind that these costs refer to the production of alloys whereas those given in the first report refer only to the procurement of high-grade ores. Roughly speaking, it can be said that the latter amount to about 50 to 60 per cent of the costs of the alloys. Keeping this in mind, we can make the following comparison (figures in brackets refer to the previous computation):

TABLE 5

Lowest Cost (in $ million) at which the Requirements of a Ten-Year Program can be covered.*)

Polit. Situation	Udy Capac. 10^5 NT	Dean Capac. 10^5 NT	Quantity of Mn (in 10^5 NT) stockpiled at the beginning of the period. 0	1	2	3	4	5	6	7	8	9
no war	0	0	1652	1638	1623	1609	1595	1582	1569	1557	1544	1532
	0	1	1631	1617	1604	1591	1579	1566	1554	1541	1529	1518
	0	2	1614	1601	1588	1576	1563	1552	1540	1529	1517	1506
	1	0	1632	1617	1603	1589	1576	1563	1551	1538	1526	1514
	1	1	1611	1598	1585	1572	1560	1548	1537	1525	1514	1502
	1	2	1595	1583	1571	1560	1548	1537	1525	1514	1503	1492
	2	0	1615	1601	1588	1575	1562	1549	1537	1525	1513	1501
	2	1	1597	1584	1571	1559	1547	1535	1524	1512	1501	1490
	2	2	1582	1570	1558	1547	1535	1524	1513	1502	1491	1480
cold war	0	0	1834	1814	1797	1781	1766	1753	1739	1725	1712	1699
	0	1	1808	1790	1774	1760	1746	1732	1719	1706	1694	1681
	0	2	1783	1767	1753	1740	1727	1715	1702	1690	1677	1665
	1	0	1808	1791	1775	1760	1746	1732	1719	1705	1692	1680
	1	1	1784	1768	1753	1740	1726	1713	1700	1687	1675	1663
	1	2	1761	1747	1734	1721	1709	1696	1684	1672	1660	1649
	2	0	1784	1768	1753	1739	1725	1712	1698	1685	1673	1660
	2	1	1761	1747	1733	1719	1706	1693	1680	1668	1656	1644
	2	2	1739	1727	1714	1702	1689	1678	1666	1655	1643	1632
limited war	0	0	1986	1960	1935	1913	1892	1872	1854	1838	1823	1808
	0	1	1953	1929	1906	1885	1865	1848	1831	1816	1801	1787
	0	2	1922	1899	1878	1859	1841	1824	1809	1794	1780	1766
	1	0	1954	1929	1907	1886	1866	1848	1832	1817	1802	1787
	1	1	1923	1900	1879	1859	1841	1825	1810	1795	1781	1766
	1	2	1893	1872	1852	1835	1818	1803	1788	1774	1760	1747
	2	0	1922	1900	1879	1859	1841	1825	1810	1795	1780	1766
	2	1	1893	1872	1852	1835	1818	1803	1788	1774	1759	1745
	2	2	1865	1846	1828	1811	1796	1781	1767	1753	1740	1727

*) Figures have been rounded off to the nearest $ million.

TABLE 5 (contd.)

Lowest Cost (in $ million) at which the Requirements of a Ten-Year Program can be covered.*)

Quantity of Mn (in 10^5 NT) stockpiled at the beginning of the period.										Dean Capac.	Udy Capc.	Polit. Situation
10	11	12	13	14	15	16	17	18	19	10^5 NT	10^5 NT	
1520	1508	1496	1484	1472	1461	1450	1438	1427	1416	0	0	no war
1506	1495	1483	1472	1461	1450	1439	1428	1417	1407	1	0	
1495	1483	1472	1462	1451	1440	1430	1419	1409	1398	2	0	
1502	1491	1479	1468	1457	1447	1436	1425	1415	1404	0	1	
1491	1480	1469	1458	1448	1437	1427	1416	1406	1396	1	1	
1481	1470	1460	1449	1439	1429	1418	1408	1398	1388	2	1	
1490	1478	1467	1456	1445	1435	1424	1414	1403	1393	0	2	
1479	1468	1457	1447	1436	1426	1415	1405	1395	1385	1	2	
1470	1459	1449	1438	1428	1418	1408	1398	1388	1378	2	2	
1686	1673	1661	1648	1636	1623	1611	1599	1587	1575	0	0	cold war
1669	1656	1644	1632	1620	1608	1597	1585	1574	1563	1	0	
1653	1642	1630	1619	1607	1596	1585	1573	1562	1551	2	0	
1667	1654	1642	1630	1617	1606	1594	1582	1571	1559	0	1	
1651	1639	1627	1615	1604	1592	1581	1570	1559	1548	1	1	
1637	1626	1615	1603	1592	1581	1570	1560	1549	1538	2	1	
1647	1635	1623	1611	1599	1588	1576	1565	1554	1543	0	2	
1632	1621	1610	1598	1587	1576	1565	1554	1544	1533	1	2	
1621	1610	1599	1588	1577	1566	1556	1546	1535	1525	2	2	
1793	1779	1764	1750	1737	1724	1711	1698	1685	1673	0	0	limited war
1772	1758	1745	1732	1719	1706	1693	1681	1668	1656	1	0	
1753	1740	1728	1715	1702	1689	1677	1665	1654	1642	2	0	
1773	1758	1744	1731	1718	1705	1692	1679	1666	1654	0	1	
1752	1739	1726	1713	1700	1688	1675	1663	1651	1639	1	1	
1734	1722	1709	1696	1684	1673	1661	1650	1638	1627	2	1	
1752	1737	1724	1711	1698	1685	1672	1660	1647	1635	0	2	
1732	1719	1706	1693	1681	1669	1657	1645	1634	1622	1	2	
1715	1702	1691	1679	1668	1656	1645	1633	1622	1611	2	2	

*) Figures have been rounded off to the nearest $ million.

Comparison of Costs of the
Two Stochastic Models

Political Situation	Udy Cap.	Dean Cap.	Quantity stockpiled 0		8		16	
no war	0	0	$1652	(900)	$1544	(805)	$1450	(715)
	2	2	1582	(780)	1491	(700)	1408	(625)
cold war	0	0	1834	(1000)	1712	(895)	1611	(795)
	2	2	1739	(870)	1643	(775)	1556	(690)
limited war	0	0	1986	(1070)	1823	(925)	1711	(810)
	2	2	1865	(910)	1740	(790)	1645	(680)

The figures between brackets on the last two lines are those of the previous computation for the case of "cold war, minor blockade" which comes closest to the case of "limited war" that is considered in this model.[18] There is a rather good correspondence between the figures of both computations, since the costs in the present computation are approximately twice as high as those in the previous computation.

In comparing the results obtained from this computation with those of the non-stochastic model, two things should be kept in mind. First, the non-stochastic model covered a period of only six years so that its results have to be compared with those of the stochastic model covering the same number of years (the results of the stochastic programs covering 1, 2, ..., 10 years have been printed out by the computer). Second, the non-stochastic model only provided for the eventuality of limited war in which case the manganese requirements were approximately 900×10^3 NT a year. In this stochastic model the annual requirements are assumed to be 1200×10^3 NT in the limited war situation. A straight comparison gives the following picture:

Political situation	Udy cap.	Dean cap.	Quantity stockpiled 9.5	19.5
limited war	0	0	$1179.9 (915.6)	$1048.5 (968.8)

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[18] See Final Report, Part I, op. cit., p. 46.

The cost-figures outside the brackets for the stochastic model have been obtained by linear interpolation; those inside the brackets refer to the non-stochastic model and can be found on pages 22 and 32 of the first part of this report. In order to allow for the difference in the requirements, the costs of the non-stochastic model have to be multiplied by at least 4/3 (actually more since the costs of importing foreign ores go up more than proportionally). However, it can be seen from the table above that the costs of the stochastic model lie well below 4/3 times the costs of the non-stochastic one. This is what one would expect since the stochastic model also accounts for the possibilities that the limited war situation will turn into one of cold war or no war; these possibilities are neglected in the non-stochastic model.

Going back to the lowest-cost table (Table 5) and looking at the costs in the same row, it can be seen that the differences between these costs tend to become smaller and smaller going from left to right. These differences indicate by how much the total cost would drop if there were one more unit of ore in the initial stockpile; this is quite in line with the expectation that these successive differences will become smaller and smaller the more ore there is already in the stockpile. The nature of these differences is essentially the same as that of the shadow prices in the non-stochastic model except for the fact that we are dealing here with discrete rather than infinitely small additions to the initial stockpile.[19] Going down the columns two lines at a time one can see the same tendency. Here the differences indicate by how much the total cost of the program would drop if the capacity of the Udy or Dean plants were one unit larger at the start. Again going from left to right, one notices that the differences between two rows become smaller and smaller, which again is in agreement with what one would normally expect.

c) <u>Internal States</u>

Of particular interest in this type of problem are the internal states in which the system will be at the turn of the year if the

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[19] Ibid., pp. 23-25.

minimum-cost program is carried out. These internal states have been printed out and are shown in Appendix II for the case of no war (pages 1 to 9), cold war (pages 10 to 18), and limited war (pages 19 to 27). In reading these tables, it should be kept in mind that the years are numbered in reverse order of time. In other words, a ten-year program starts at the beginning of the year number 10 in these tables and ends at the end of the year numbered 1. Furthermore, the first column contains a (shorthand) description of the internal states at the beginning of each of these 10 years. The following example is given in order to show how these tables can be used to trace the sequence of internal states selected by the program, given the external state at the beginning of each year, for programs covering periods of various lengths.

First, let us consider a program that covers the needs of only 4 years. Assume that the external state at the beginning of the four-year program is that of cold war and the internal state of the system at that time is characterized by (0, 0, 19). That is to say, the capacities of the Udy plants and the Dean plants at that moment are zero and there is a stockpile of high-grade ore of 19×10^5 NT of Mn. At the end of the first year (that is year No. 4 in the tables) the system will then be in the (0, 0, 13) internal state (see Appendix II, page 10, line 20, column 5). Now suppose that the political situation has changed during that first year so that it has become one of no war at the beginning of the second year (year No. 3 in the tables). Since the internal state at the beginning of that year is (0, 0, 13), the system will be in the (0, 0, 9) state at the end of the second year (see Appendix II, page 1, line 14, column 4). Next, suppose that the political situation has again changed and become one of limited war at the beginning of the third year (year No. 2 in the tables). With an internal state of (0, 0, 9) at the beginning of that year, the system will be in the (2, 2, 1) internal state at the end of that year (see Appendix II, page 19, line 10, column 3). Finally, suppose that the situation of limited war extends itself into the 4th year (year No. 1 in the tables). Starting that last year with an internal state of (2, 2, 1) the system will end up with a (2, 2, 0) internal state at the end of the 4 years here considered (see Appendix II, page 27, line 2, column 2).

Next, let us consider the first four years of a ten-year program. Assuming the same sequence of political states during these 4 years and the same (0, 0, 19) internal state at the beginning, the system will go through the following sequence of internal states:

Internal state beginning of year	Political situation beginning of year	Internal state end of year	Appendix II page	line	column
(0, 0, 19)	cold war	(0, 0, 14)	10	20	11
(0, 0, 14)	no war	(0, 0, 12)	1	15	10
(0, 0, 12)	limited war	(2, 0, 4)	19	13	9
(2, 0, 4)	limited war	(2, 2, 0)	25	5	8

It turns out in both cases that the system will be in the same internal state at the end of the 4-year period although the paths are not the same! This immediately raises the question, which of these two paths, connecting the same end points, operates at lowest cost? This question is answered by the following comparison of the costs (in $ million) associated with each of these paths:

	4-year program		10-year program	
Year	operating costs	depreciation	operating costs	depreciation
1	144.610	———	155.810	———
2	109.870	———	129.870	———
3	185.200	7.790	185.200	3.738
4	277.730	———	236.910	———
	717.410	7.790	707.790	3.738
Total Cost	725.200		711.528	

It should be noted that in the 10-year program the depreciation charged to the 3rd year actually amounts to 7 × $3.738 = $26.166 million and that charged to the 4th year 6 × \$4.052 = \$24.312 million. But, since the comparison is made for the first 4 years only, the depreciation charges were reduced accordingly.

It turns out then that the costs associated with the first 4 years of the 10-year program are \$725.200 - 711.528 = \$13.672 million lower that those associated with the 4-year program. But we cannot conclude from this calculation alone that the path selected by the 4-year program is not a minimum-cost path. It is true, that for this particular sequence of political states the 10-year program turns out to be less costly. However, 26 other sequences of political states would have been possible in these 4 years, each having a certain probability of occurrence. For some of these other sequences the 4-year program would definitely turn out to be cheaper than the first 4 years of the 10-year program, even if we take into account the fact that both programs will not end up with the same internal states at the end of these 4 years. This is illustrated in the following example:

4-year Program

Internal state beginning of year	Political situation beginning of year	Internal state end of year	Appendix II page	line	col.
(0, 0, 19)	cold war	(0, 0, 13)	10	20	5
(0, 0, 13)	limited war	(2, 0, 5)	19	14	4
(2, 0, 5)	no war	(2, 0, 3)	7	6	3
(2, 0, 3)	no war	(2, 0, 0)	7	4	2

10-year Program

Internal state beginning of year	Political situation beginning of year	Internal state end of year	Appendix II page	line	col.
(0, 0, 19)	cold war	(0, 0, 14)	10	20	11
(0, 0, 14)	limited war	(2, 0, 6)	19	15	10
(2, 0, 6)	no war	(2, 0, 5)	7	7	9
(2, 0, 5)	no war	(2, 0, 4)	7	6	8

The costs associated with these two programs are as follows:

Year	4-year program operating costs	4-year program depreciation	10-year program operating costs	10-year program depreciation
1	144.610	——	155.810	——
2	185.200	7.476	185.200	7.476
3	129.520	——	140.320	——
4	102.270	——	140.320	——
	561.600	7.476	621.650	7.476
Credit for ore in stockpile at end of 4th year	——	——	56.315	——
	561.600	7.476	565.335	7.476
Total Cost	569.076		572.811	

The credit for the 4 units of ore that will still be in the stockpile at the end of the 4th year in the 10-year program has been computed as follows:

		Costs of a 6-year program no war	cold war	limited war
initial state	(2, 0, 0)	1018.202	1176.710	1316.633
initial state	(2, 0, 4)	965.707	1119.468	1235.433
		52.495	57.242	81.200
transition probability of no war		.70	.20	.10
		36.74650	11.44840	8.12000
		11.44840		
		8.12000		
Total credit		56.31490		

In this case the 4-year program turns out to cost \$572.811 - \$569.076 = \$3.735 million less than the 10-year program.

If we were to make similar calculations for the other 25 possible sequences of external states, we would find that at times, depending on the sequence, the 4-year program would be less expensive and at other times the 10-year program would be the least expensive. If the costs of both programs for each possible sequence of external states were weighted by the probable occurrence of that sequence, then the over-all costs of both programs would be the same. For example, the two sequences of external states that we took above have the following probabilities of occurrence (see page 11):

$$P(cw \rightarrow nw \rightarrow lw \rightarrow lw) = .05 \times .10 \times .50 = .0025$$

$$P(cw \rightarrow lw \rightarrow nw \rightarrow nw) = .10 \times .10 \times .70 = .0070 .$$

The weighted cost of the 4-year program is: \$725.200 × .0025 + \$569.076 × .0070 = \$5.796532 million; that of the 10-year program is: \$711.528 × .0025 + \$572.811 × .0070 = \$5.788497 million. Thus the costs are approximately the same.

V. THE FINAL STATE OF THE SYSTEM

In the previous section we have outlined the problem-solving procedure. The first step in the computation is always to choose one of the internal states in combination with one of the external states at the beginning of a year. Changes in the external state <u>during</u> the year are neglected, but the two possible conditions of that state at the end of the year are taken into account: either a) the same state will still be in existence, or b) it will have transformed intself into one of the other two external states. The best combination of activities is then computed. These activities will meet the requirements of this year and set up an internal state at the end of the year which is most favorable for meeting the requirements of future years, given the probabilities with which each of the external states might appear in those years.

The situation at the beginning of the next year will then be characterized by the new (most favorable) internal state and the external state actually present at that moment. This situation provides the starting point for the next computation which then makes use of the information on the external state that is available at that point in time. This procedure is repeated for all the years covered by the program.

Suppose that we are now at the beginning of a 10-year program and that the system is in a particular internal state combined with one of the three external states. The program then tells us what the internal state will be at the end of the first year and the probability matrix (page 11 above) gives us the probability that each of the three external states will show up at the end of the first year. Thus, at the beginning of the first year we know the three possible situations, all with the same internal state but with different external states, with which we will end up that first year, as well as the chances that each of these three possibilities will show up.

With each of these three possibilities as starting points, we place ourselves at the beginning of a new 10-year program. Each of these three possible starting-points will again lead to 3 different situations at the end of the second year. Assuming now for a moment that different initial situations will not lead to the same final situation, we would end up the second year with 9 different situations, each with a certain probability of occurrence. Repeating the process for the third time and making the same assumption as before, we would end up the third year with 27 different situations.

Another repetition would lead to 81, the next one to 243 and the following one to 729 different situations — provided the last number of situations is actually possible. However, there are only 3 different political situations, 3 different capacities of Udy plants, 3 different capacities of Dean plants and 20 different levels of stockpile permitted, so the total number of possible situations is limited to a maximum of $3 \times 3 \times 3 \times 20 = 540$. Clearly our assumption was incorrect: different initial situations will lead to the same final situation if the process is repeated long enough.

Now an interesting question arises: if we adopt a 10-year program at the beginning of every year, what results will we find at the end? Will this procedure lead to an ever-changing internal state of the system, or will the system end up in just one final internal state no matter how the sequence of external states manifests itself in the future? To put the question in technical terms, will there be convergence in policy space? If there is convergence, then we would like to know what the final state of the system would be if this 10-year program were repeated over and over again. It turns out that there is such a convergence in this system; this may be illustrated by the following example.

Suppose that we were faced with a situation in which neither the Udy nor the Dean plants have any capacity, and in which there is no ore in the stockpile; moreover, there is a no war situation at that moment. A 10-year program then leads to a (1, 1, 0) internal state at the end of the first year (see Appendix II, Table I, page 1, line 1, column 11); the probability matrix on page 11 tells us that the chance that there will be no war at the end of that first year is .7, cold war .2 and limited war .1.

Taking first the case of no war and a (1, 1, 0) state of the system at the beginning of the second year, a 10-year program leads to a (1, 1, 1) internal state (see Appendix II, Table I, page 5 , line 1, column 11) at the end of the second year. And again there is a probability of .7 that there will be no war, .2 for cold war and .1 for limited war. These probabilities have to be multiplied by .7, the probability that the first year will end with no war. In this case, therefore, we will end up the second year with a (1, 1, 1) internal state combined with a probability of .7 × .7 = .49 for no war, of .7 × .2 = .14 for cold war and of .7 × .1 = .07 for limited war.

In the case of cold war at the beginning of the second year and a (1, 1, 0) internal state, a 10-year program results in a (2, 2, 0) internal state (see Appendix II, Table II, page 14, line 1, column 11) at the end of the second year. The probability that the second year will end up in no war will now be .05, cold war .85, and limited war .10. In this case, therefore, we will end up the second year with a (2, 2, 0) internal state combined with a chance of no war of .2 × .05 = .01, cold war .2 × .85 = .17 and limited war .2 × .1 = .02.

Finally taking the case of limited war at the beginning of the second year and a (1, 1, 0) internal state, a 10-year program also results in a (2, 2, 0) internal state (see Appendix II, Table III, page 23, line 1, column 11) at the end of the second year. That state will then be combined in this case with a probability of .1 × .1 = .01 for no war, .1 × .4 = .04 for cold war and .1 × .5 = .05 for limited war.

The expected results at the end of the second year will then be:

1) a (1, 1, 1) internal state of the system with
   - .49 probability of no war,
   - .14 probability of cold war and
   - .07 probability of limited war.

2) a (2, 2, 0) internal state of the system with

.01 + .01 = .02 probability of no war,

.17 + .04 = .21 probability of cold war and

.02 + .05 = .07 probability of limited war.

The total probability is again 1.00 as it should be.

Continuing this argument along the same lines, we can expect the following situations to show up at the end of the third and at the end of the fourth years:

a) Third year

internal state	external state		probability
(1, 1, 2)	nw		.3430
(1, 1, 2)	cw		.0980
(1, 1, 2)	lw		.0490
(2, 2, 0)	nw	.0070 + .0105 + .0070 + .0070 =	.0315
(2, 2, 0)	cw	.1190 + .1785 + .0280 + .0280 =	.3535
(2, 2, 0)	lw	.0140 + .0210 + .0350 + .0350 =	.1050
(2, 2, 1)	nw		.0140
(2, 2, 1)	cw		.0040
(2, 2, 1)	lw		.0020
			1.0000

b) Fourth year

(1, 1, 3)	nw		.2401
(1, 1, 3)	cw		.0686
(1, 1, 3)	lw		.0343
(2, 2, 0)	nw	.0049 + .0177 + .0002 + + .0049 + .0105 + .0002 =	.0384
(2, 2, 0)	cw	.0833 + .3005 + .0034 + + .0196 + .0420 + .0008 =	.4496
(2, 2, 0)	lw	.0098 + .0353 + .0004 + + .0245 + .0525 + .0010 =	.1235
(2, 2, 1)	nw		.0221
(2, 2, 1)	cw		.0063
(2, 2, 1)	lw		.0031
(2, 2, 2)	nw		.0098
(2, 2, 2)	cw		.0028
(2, 2, 2)	lw		.0014
			1.0000

It can be seen that the probable occurrence of the (2, 2, 0) internal state becomes greater the more years the 10-year program is repeated. This is illustrated in the following table which gives the likelihood of occurrence of the (2, 2, 0) internal state over a six-year period:

state	years 1	2	3	4	5	6
no war	——	.02	.0315	.038375	.03903375	.04253694
cold war	——	.21	.3535	.449575	.45245375	.50867994
limited war	——	.07	.1050	.123550	.12678750	.13456187
	——	.30	.4900	.611500	.61827500	.68577875

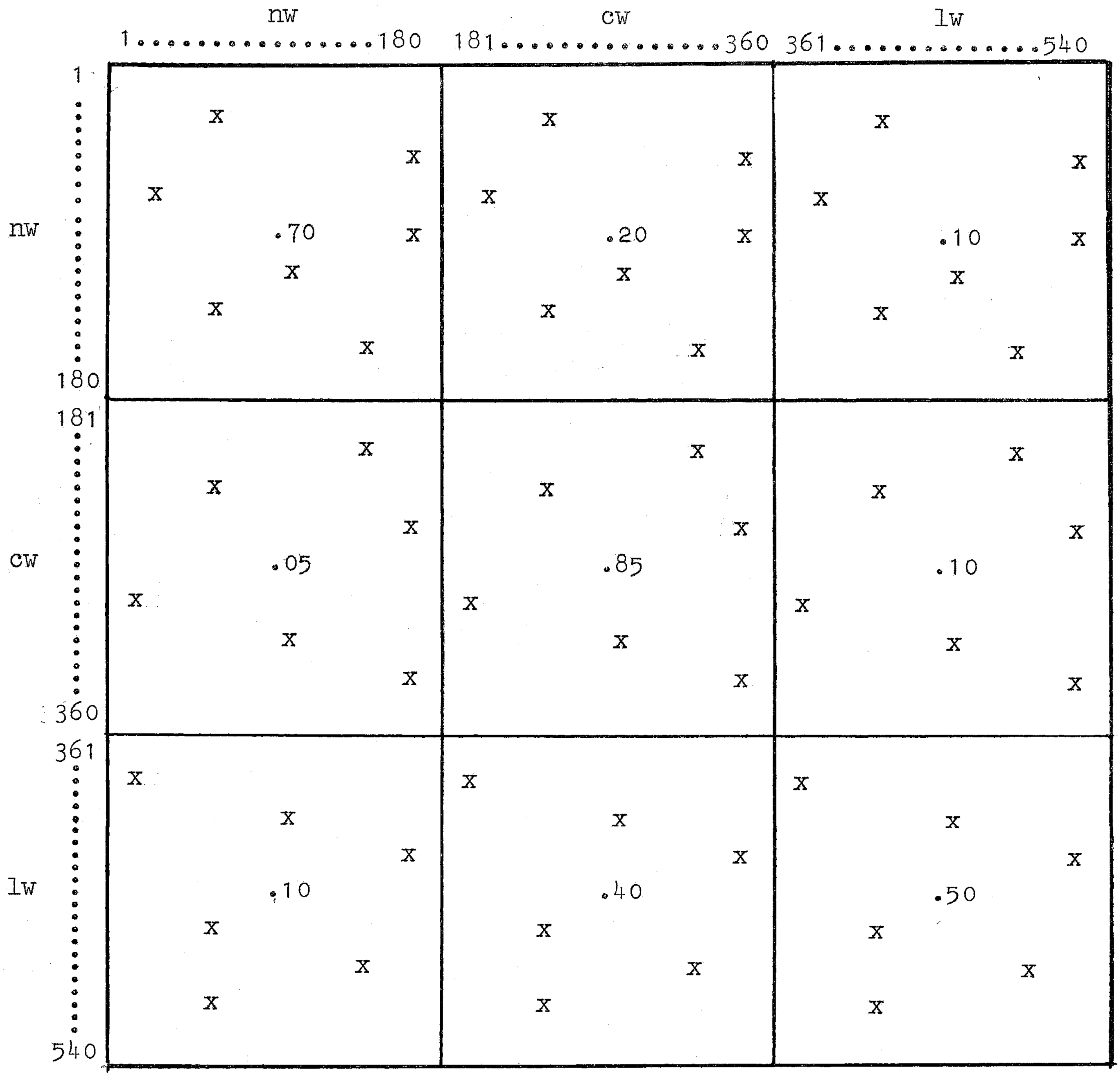
The total probability that we will end up with the (2, 2, 0) internal state will be approximately 90 per cent after 10 years and 95 per cent after 15 years. So we may conclude that starting from a situation of no war and a (0, 0, 0) internal state we would almost certainly end up with a (2, 2, 0) internal state by repeating a 10-year program over and over again.

Even more interesting is the fact that this will always be the result, no matter what the situation is at the beginning! That is to say, if we were to start out from a cold war and a (0, 1, 19) situation, by repeating this particular program we would again end up with a (2, 2, 0) internal state 95 per cent of the time at the end of 15 years. And if the initial situation were one of limited war and a (2, 2, 19) internal state, we would again end up with a (2, 2, 0) situation 95 per cent of the time at the end of 15 years.

The technique of dynamic programming, as it is applied here, has the interesting ergodic property that it will always lead to the same internal state in the end no matter what the situation is at the beginning. This is, of course, subject to the condition that the transfer probabilities and the cost functions, given in Section III above, will remain the same during the period covered by the program.

The computations that led to this interesting result involve the multiplication of a vector of 540 elements by a 540 × 540 matrix. Each element of the vector represents one of all the possible situations at the beginning of the program. The total number of situations is obtained by combining each one of the 3 political possibilities with each of the $3 \times 3 \times 20 = 180$ internal states. Each element of the matrix represents the probable occurrence of one particular combination of external and internal states. Since all three political situations can appear at the end of every year, there are always three elements in each row and their sum is always 1. Thus, the initial situation of no war and a (0, 0, 0) internal state could be expected to be transformed by the end of the first year into a no war and (1, 1, 0) state with a probability of .7, into a cold war and (1, 1, 0) state with a probability of .2 and into a limited war and (1, 1, 0) state with a probability of .1. The matrix can be partitioned into 3 strips of 540 rows and 180 columns, each strip having in each row the non-zero element at exactly the same place as the other two strips. Moreover, each strip can again be partitioned into 3 square blocks of 180 rows and columns with the elements of each block having the same magnitude, i.e., one of the transfer probabilities (see p. 11 above). The situation can be pictured as on the following page.

Because of the special feature of this problem, it was possible to perform the computations on an IBM 650 equipped with floating point arithmetic and 3 index-registers. By making extensive use of the table look-up facility, it took this machine about 3 minutes to carry out one multiplication.



x's represent non-zero elements.

VI. SUMMARY

The manganese problem has an element of uncertainty in it as do all other problems that deal with the future. In this case, the uncertainty is due to the various ways in which the political situation might develop.

The dynamic programming technique has been applied in this problem in order to take full account of these uncertainties. The power of this technique is demonstrated by the fact that it enables one to figure out exactly which activities have to be taken in each successive year to arrive at a minimum-cost solution for every possible sequence of political states during the period under consideration. That is to say, it is possible to determine beforehand the proper course of action for every possible development of the political situation. In other words, it is possible to make full use of the information on the political situation as it becomes available in the course of time.

The method gives the various sequences of activity combinations, one sequence for every possible development of the political situation, that meet the given requirements at minimum cost. The flexibility of the method is demonstrated by the fact that it enables one to include as many years in the program as one desires, the only limitation being the number of available computer-hours. Moreover, the method is not confined to just one initial situation but can take any possible situation as its starting point. Also, there are no limitations on the shape of the cost functions or, what amounts to the same thing, all cost-quantity relationships are permissable.

It is necessary, however, to limit the various activity levels to a preconceived set of (integer) values in order to restrict the number of possible combinations of activities that have to be examined. This limitation will keep the computational burden within manageable proportions. Moreover, the political situation is taken into account only at the beginning of each year and changes in this situation during a year are neglected. Consequently, the technique is handicapped in two ways: the activities can take only integer values and the political situations can be evaluated only at certain points in time.

The minimum-costs at which the requirements of a 10-year period can be met, taking into account the various ways in which the political situation might develop, are given above in Table 5, Section IV. Each figure in this table refers to a particular combination of external (political) and internal states at the beginning of the 10-year period. A comparison with the results obtained from the non-stochastic model, which was assumed to cover a 6-year period of limited war, showed that there is a rather good correspondence between the figures of both models.

Another interesting result obtained from this stochastic model are the tables of internal states, given in Appendix II. These tables allow one to trace the minimum-cost path for each particular situation that might occur, the sequence of internal states over which it leads and the costs associated with it. Comparisons have been made between the costs of a 4-year program and the first four years of a 10-year program and the way in which the figure can be brought into agreement has been indicated.

Finally we have shown what the results would be if one were to adopt a 10-year program at the beginning of every year. In that case there is a very high probability that one would end up with one particular internal state rather than with an ever-changing sequence of states. This particular state is the one in which both the Udy and the Dean plants each would have the capacity to produce 200,000 N.T. of manganese annually and in which there would be no ore in the stockpile.

APPENDIX I

Mathematical Formulation* for the case of a probabalistic political forecast

In this model there is a distinction between the part λ of the manganese requirements for which only ferro-manganese has to be supplied and the part ν for which only silico-manganese has to be supplied.

Denoting the total quantity of manganese required in year i by Q_i we have

$$Q_i = \lambda_i + \nu_i \qquad \text{for} \qquad i = 1, 2, \ldots, n$$

where n is the number of years covered by the program.

The first relationship of the model is an equality expressing the two ways in which the required quantity of ferro-manganese can be obtained:

$$(1) \qquad x_{1,i} + x_{2,i} = \lambda_i \qquad \text{for} \qquad i = 1, 2, \ldots, n$$

where $x_{1,i}$ stands for the quantity of ferro-manganese to be produced from stockpiled ore and $x_{2,i}$ for that from Aroostook ores.

The second relationship is similar to the first one, showing the two ways in which silico-manganese can be obtained:

$$(2) \qquad x_{3,i} + x_{4,i} = \nu_i \qquad \text{for} \qquad i = 1, 2, \ldots, n .$$

The activities indicated by (1) and (3) in the diagram stand for the conventional ways of producing ferro- and silico-manganese and those indicated by (2) and (4) for the Udy integrated process. The

* The reader should refer to the diagram on page 7 above which illustrates the relationships formulated here.

basic assumption that is made is that the required quantities of ferro- and silico-manganese always have to be produced. In other words, there is no place in this model for a shortage of alloys. On the other hand there is also no allowance made for stockpiling alloys (there was also no such allowance in the non-stochastic model).

The third relationship merely states that the quantity of ore that is in the stockpile at the beginning of any year plus what will be added to it during the year must equal the quantity that is taken out of it, plus what remains in it at the end of that year:

(3) $S_i + x_{5,i} + x_{6,i} = c_1 x_{1,i} + c_3 x_{3,i} + S_{i+1}$ for $i = 1, 2, \ldots, n.$

The letters c_1 and c_3 represent the quantity of manganese in the form of ore that is needed to produce 1 N.T. of manganese in the form of ferro- and silico-manganese alloys. They are the technical coefficients (described on pages 14 and 15 of the previous report and page 15 above); again they are assumed to remain the same during the years covered by the program.

The fourth and fifth relationships assure that the quantity of ore that can be processed in any year by the Udy and Dean plants cannot exceed the capacity of these plants at the beginning of that year. The capacity at the beginning of the year is then equal to the capacity at the beginning of the previous year, plus what has been added to it in that year:

(4) $x_{6,i} \leq K_{6,i} = K_{6,i-1} + y_{6,i-1}$ for $i = 1, 2, \ldots, n$

(5) $c_2 x_{2,i} + c_4 x_{4,i} \leq K_{7,i} = K_{7,i-1} + y_{7,i-1}$ for $i = 1, 2, \ldots, n.$

The left-hand side of these expressions contain inequalities as well as equalities since the quantities of ore to be processed by these plants can be smaller, or at most equal to their capacities K_6 and K_7. The variables $y_{6,i-1}$ and $y_{7,i-1}$ stand for the increases in the capacities of these plants in the previous year. It has been assumed that the Udy alloy plants will always have enough capacity to process the quantities of upgraded ores that will be delivered to them; the same assumption is made for the other alloy plants. Hence there was no need for another inequality in the model.

The electricity-constraints proved to be rather ineffective in the non-stochastic model and therefore have not been included in this model.

Furthermore, there are no constraints in this model with respect to the availability of sufficient quantities of ore in the deposits since the capacities of the plants have been restricted to a minimum of 200,000 N.T. of manganese per year. There is sufficient ore known to be present in the Cuyuna-deposit to keep the Dean plants producing at maximum capacity for a period of 20-25 years, and the Aroostook-deposits seem to contain an almost inexhaustible quantity of ore.

Finally, since we are concerned with economic data, there is the condition that all x and y variables have to be positive:

$$(6)\quad \begin{aligned} & x_{j,i} \geq 0 \quad \text{for} \quad j = 1, 2, \ldots, 6 \quad \text{and} \quad i = 1, 2, \ldots, n \\ & y_{j,i} \geq 0 \quad \text{for} \quad j = 6 \text{ and } 7 \quad \text{and} \quad i = 1, 2, \ldots, n \ . \end{aligned}$$

In summary, it can be said that the model consists of 3 equalities and 2 inequalities (besides the one just mentioned) for each year involving a total of 8 activities, of which 6 are related to the actual production of manganese and the other 2 to the construction and expansion of the plants.

The objective is then to select that combination of the x's and y's, one combination for every year, that meets the requirements for all successive years at minimum-cost. This minimum-cost will depend on the situation at the beginning of the program. Hence, there will be as many cost-minima as there are different initial situations.

The initial situations in this model are characterized by a political climate (k), which is an external state since it is outside the programmer's control, and an internal state which is the state of the manganese system itself. As for the former, three possible political states have been distinguished: no war, cold war and limited war; the latter is characterized by the capacity of the Udy plants (K_7), the Dean plants (K_6) and the quantity of ore in the stockpile. The activities of a certain year not only meet the requirements of a year but, in general, they also transform the internal state of the system at the beginning of that year into a different one at the end of it. The latter state then is the one that is most favorable from an economic point of view for meeting the requirements of future years in light of what can be expected to happen politically.

Let $M_{k,r,n}$ be the minimum-cost of an n-year program starting from an initial situation characterized by the k^{th} external and the r^{th} internal state. Then $M_{k,r,n}$ can be defined as follows:

$$M_{k,r,n} = \min_{s} [C_{k,r,s,n} + \alpha(p_{kk}M_{k,s,n-1} + p_{k\ell}M_{\ell,s,n-1} + p_{km}M_{m,s,n-1})]$$

$C_{k,r,s,n}$ is the cost of meeting the requirements of the n^{th} year (being a function of the external state k) and transforming the r^{th} internal state at the beginning of the n^{th} year into the s^{th} internal state at the end of it. Or, what amounts to the same thing, $C_{k,r,s,n}$ depends on the corresponding 8 activities of the n^{th} year that perform this dual function:

$$C_{k,r,s,n} = x_{1,n}u_{1,n} + x_{2,n}u_{2,n} + x_{3,n}u_{3,n} + x_{4,n}u_{4,n} +$$

$$+ x_{5,n}u_{5,n} + x_{6,n}u_{6,n} + y_6v_{6,n} + y_7v_{7,n} \quad .$$

The u and v cost functions that have actually been used are given in Section III above. They are linear functions of the corresponding variables ($u_{5,n}$ also depends on the political situation k and $v_{6,n}$ and $v_{7,n}$ also depend on n). Hence the total cost function

$C_{k,r,s,n}$ is in this case a quadratic function of the x's and y's, although it could be a polynomial of any degree.

α is a discount factor (.95 in the actual computations). p_{kk}, $p_{k\ell}$ and p_{km} stand for the probabilities that the k^{th} external state at the beginning of the n^{th} year will be transformed into the k^{th}, ℓ^{th} and m^{th} external state respectively at the beginning of the $n-1^{st}$ year* (the years are numbered in reverse order of time).

$M_{k,s,n-1}$, $M_{\ell,s,n-1}$ and $M_{m,s,n-1}$ represent the minimum-cost of an $(n - 1)$ year program starting from the k^{th}, the ℓ^{th} and the m^{th} external state respectively combined with the s^{th} internal state at the beginning of the $n-1^{st}$ year.

As has been mentioned in Section IV above, the computations have the uncommon feature of starting with the <u>last</u> year, going <u>back</u> in time and finishing with the first year of the program. This greatly reduces the number of combinations of states which have to be examined.

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* See page 11 for the probabilities actually used in the computations.

APPENDIX II

TABLES OF INTERNAL STATES
OF
MINIMUM-COST PROGRAMS

TABLE I

No War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
0 0 0	0 0 0	2 1 0	2 1 0	2 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 1 0
0 0 1	0 0 0	2 1 0	2 1 0	2 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 0 1
0 0 2	0 0 0	2 0 1	2 0 1	2 0 1	1 1 1	1 1 1	1 0 2	1 0 2	1 0 2	1 0 2
0 0 3	0 0 0	2 0 1	1 0 2	1 0 2	1 0 2	1 0 2	1 0 2	1 0 2	1 0 3	1 0 3
0 0 4	0 0 0	2 0 2	1 0 3	1 0 3	1 0 3	1 0 3	1 0 3	1 0 3	1 0 3	1 0 3
0 0 5	0 0 0	0 0 3	1 0 3	1 0 4	1 0 4	1 0 4	1 0 4	1 0 4	1 0 4	1 0 4
0 0 6	0 0 0	0 0 4	1 0 4	1 0 4	1 0 5	1 0 5	1 0 5	1 0 5	1 0 5	1 0 5
0 0 7	0 0 0	0 0 4	0 0 5	1 0 5	1 0 5	1 0 5	1 0 5	0 0 6	0 0 6	0 0 6
0 0 8	0 0 0	0 0 5	0 0 6	0 0 6	0 0 6	1 0 6	0 0 6	0 0 6	0 0 6	0 0 7
0 0 9	0 0 1	0 0 5	0 0 7	0 0 7	0 0 7	0 0 7	0 0 7	0 0 7	0 0 7	0 0 7
0 0 10	0 0 2	0 0 6	0 0 7	0 0 8	0 0 8	0 0 8	0 0 8	0 0 8	0 0 8	0 0 8
0 0 11	0 0 3	0 0 6	0 0 8	0 0 9	0 0 9	0 0 9	0 0 9	0 0 9	0 0 9	0 0 9
0 0 12	0 0 4	0 0 7	0 0 9	0 0 10	0 0 10	0 0 10	0 0 10	0 0 10	0 0 10	0 0 10
0 0 13	0 0 5	0 0 7	0 0 9	0 0 10	0 0 11	0 0 11	0 0 11	0 0 11	0 0 11	0 0 11
0 0 14	0 0 6	0 0 8	0 0 10	0 0 11	0 0 12	0 0 12	0 0 12	0 0 12	0 0 12	0 0 12
0 0 15	0 0 7	0 0 8	0 0 11	0 0 12	0 0 12	0 0 13	0 0 13	0 0 13	0 0 13	0 0 13
0 0 16	0 0 8	0 0 8	0 0 11	0 0 13	0 0 13	0 0 13	0 0 14	0 0 14	0 0 14	0 0 14
0 0 17	0 0 9	0 0 9	0 0 12	0 0 13	0 0 14	0 0 14	0 0 15	0 0 15	0 0 15	0 0 15
0 0 18	0 0 10	0 0 10	0 0 13	0 0 14	0 0 15	0 0 15	0 0 15	0 0 16	0 0 16	0 0 16
0 0 19	0 0 11	0 0 11	0 0 14	0 0 15	0 0 16	0 0 16	0 0 16	0 0 16	0 0 17	0 0 17

TABLE I (Continued)

No War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
0 1 0	0 1 0	2 1 0	2 1 0	2 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 1 0
0 1 1	0 1 0	2 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1
0 1 2	0 1 0	2 1 1	1 1 2	1 1 2	1 1 2	1 1 2	1 1 2	1 1 2	1 1 2	1 1 2
0 1 3	0 1 0	1 1 2	1 1 2	1 1 2	1 1 3	1 1 3	1 1 3	1 1 3	1 1 3	1 1 3
0 1 4	0 1 0	0 1 3	1 1 3	1 1 3	1 1 3	1 1 3	1 1 3	0 1 4	0 1 4	0 1 4
0 1 5	0 1 0	0 1 3	0 1 4	0 1 4	0 1 4	0 1 4	0 1 4	0 1 5	0 1 5	0 1 5
0 1 6	0 1 0	0 1 3	0 1 5	0 1 5	0 1 5	0 1 5	0 1 5	0 1 5	0 1 5	0 1 5
0 1 7	0 1 0	0 1 4	0 1 5	0 1 6	0 1 6	0 1 6	0 1 6	0 1 6	0 1 6	0 1 6
0 1 8	0 1 0	0 1 4	0 1 6	0 1 6	0 1 7	0 1 7	0 1 7	0 1 7	0 1 7	0 1 7
0 1 9	0 1 1	0 1 5	0 1 6	0 1 7	0 1 7	0 1 8	0 1 8	0 1 8	0 1 8	0 1 8
0 1 10	0 1 2	0 1 6	0 1 7	0 1 8	0 1 8	0 1 8	0 1 8	0 1 8	0 1 8	0 1 8
0 1 11	0 1 3	0 1 6	0 1 8	0 1 8	0 1 9	0 1 9	0 1 9	0 1 9	0 1 9	0 1 9
0 1 12	0 1 4	0 1 7	0 1 9	0 1 9	0 1 10	0 1 10	0 1 10	0 1 10	0 1 10	0 1 10
0 1 13	0 1 5	0 1 7	0 1 9	0 1 10	0 1 10	0 1 11	0 1 11	0 1 11	0 1 11	0 1 11
0 1 14	0 1 6	0 1 8	0 1 10	0 1 11	0 1 11	0 1 12	0 1 12	0 1 12	0 1 12	0 1 12
0 1 15	0 1 7	0 1 8	0 1 11	0 1 12	0 1 12	0 1 12	0 1 13	0 1 13	0 1 13	0 1 13
0 1 16	0 1 8	0 1 8	0 1 11	0 1 13	0 1 13	0 1 13	0 1 13	0 1 13	0 1 13	0 1 14
0 1 17	0 1 9	0 1 9	0 1 12	0 1 13	0 1 14	0 1 14	0 1 14	0 1 14	0 1 14	0 1 14
0 1 18	0 1 10	0 1 10	0 1 13	0 1 14	0 1 15	0 1 15	0 1 15	0 1 15	0 1 15	0 1 15
0 1 19	0 1 11	0 1 11	0 1 13	0 1 15	0 1 16	0 1 16	0 1 16	0 1 16	0 1 16	0 1 16

TABLE I (Continued)

No War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
0 2 0	0 2 0	2 2 0	1 2 0	1 2 1	1 2 1	1 2 1	1 2 1	1 2 1	0 2 1	0 2 1
0 2 1	0 2 0	1 2 1	1 2 1	1 2 1	1 2 1	1 2 1	0 2 2	0 2 2	0 2 2	0 2 2
0 2 2	0 2 0	0 2 2	0 2 2	0 2 2	0 2 3	0 2 3	0 2 3	0 2 3	0 2 3	0 2 3
0 2 3	0 2 0	0 2 3	0 2 3	0 2 3	0 2 3	0 2 3	0 2 3	0 2 3	0 2 3	0 2 3
0 2 4	0 2 0	0 2 2	0 2 4	0 2 4	0 2 4	0 2 4	0 2 4	0 2 4	0 2 4	0 2 4
0 2 5	0 2 0	0 2 3	0 2 4	0 2 5	0 2 5	0 2 5	0 2 5	0 2 5	0 2 5	0 2 5
0 2 6	0 2 0	0 2 3	0 2 4	0 2 4	0 2 5	0 2 5	0 2 5	0 2 5	0 2 5	0 2 5
0 2 7	0 2 0	0 2 4	0 2 5	0 2 5	0 2 5	0 2 5	0 2 5	0 2 5	0 2 5	0 2 5
0 2 8	0 2 0	0 2 4	0 2 6	0 2 6	0 2 6	0 2 6	0 2 6	0 2 6	0 2 6	0 2 6
0 2 9	0 2 1	0 2 5	0 2 6	0 2 7	0 2 7	0 2 7	0 2 7	0 2 7	0 2 7	0 2 7
0 2 10	0 2 2	0 2 5	0 2 7	0 2 7	0 2 8	0 2 8	0 2 8	0 2 8	0 2 8	0 2 8
0 2 11	0 2 3	0 2 6	0 2 8	0 2 8	0 2 9	0 2 9	0 2 9	0 2 9	0 2 9	0 2 9
0 2 12	0 2 4	0 2 6	0 2 9	0 2 9	0 2 9	0 2 10	0 2 10	0 2 10	0 2 10	0 2 10
0 2 13	0 2 5	0 2 7	0 2 9	0 2 10	0 2 10	0 2 10	0 2 10	0 2 11	0 2 11	0 2 11
0 2 14	0 2 6	0 2 8	0 2 10	0 2 11	0 2 11	0 2 11	0 2 11	0 2 11	0 2 11	0 2 11
0 2 15	0 2 7	0 2 8	0 2 11	0 2 12	0 2 12	0 2 12	0 2 12	0 2 12	0 2 12	0 2 12
0 2 16	0 2 8	0 2 8	0 2 11	0 2 13	0 2 13	0 2 13	0 2 13	0 2 13	0 2 13	0 2 13
0 2 17	0 2 9	0 2 9	0 2 12	0 2 13	0 2 14	0 2 14	0 2 14	0 2 14	0 2 14	0 2 14
0 2 18	0 2 10	0 2 10	0 2 13	0 2 14	0 2 15	0 2 15	0 2 15	0 2 15	0 2 15	0 2 15
0 2 19	0 2 11	0 2 11	0 2 13	0 2 15	0 2 16	0 2 16	0 2 16	0 2 16	0 2 16	0 2 16

TABLE I (Continued)

No War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
1 0 0	1 0 0	2 1 0	2 1 0	2 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 1 0	1 0 1
1 0 1	1 0 0	2 0 1	2 0 1	2 0 1	1 1 1	1 1 1	1 0 2	1 0 2	1 0 2	1 0 2
1 0 2	1 0 0	2 0 1	1 0 2	1 0 2	1 0 2	1 0 2	1 0 2	1 0 2	1 0 3	1 0 3
1 0 3	1 0 0	2 0 2	1 0 3	1 0 3	1 0 3	1 0 3	1 0 3	1 0 3	1 0 3	1 0 3
1 0 4	1 0 0	1 0 3	1 0 3	1 0 4	1 0 4	1 0 4	1 0 4	1 0 4	1 0 4	1 0 4
1 0 5	1 0 0	1 0 3	1 0 4	1 0 4	1 0 5	1 0 5	1 0 5	1 0 5	1 0 5	1 0 5
1 0 6	1 0 0	1 0 3	1 0 5	1 0 5	1 0 5	1 0 5	1 0 5	1 0 5	1 0 5	1 0 5
1 0 7	1 0 0	1 0 4	1 0 6	1 0 6	1 0 6	1 0 6	1 0 6	1 0 6	1 0 6	1 0 6
1 0 8	1 0 0	1 0 4	1 0 6	1 0 7	1 0 7	1 0 7	1 0 7	1 0 7	1 0 7	1 0 7
1 0 9	1 0 1	1 0 5	1 0 7	1 0 8	1 0 8	1 0 8	1 0 8	1 0 8	1 0 8	1 0 8
1 0 10	1 0 2	1 0 5	1 0 7	1 0 9	1 0 9	1 0 9	1 0 9	1 0 9	1 0 9	1 0 9
1 0 11	1 0 3	1 0 6	1 0 8	1 0 9	1 0 10	1 0 10	1 0 10	1 0 10	1 0 10	1 0 10
1 0 12	1 0 4	1 0 7	1 0 9	1 0 10	1 0 10	1 0 11	1 0 11	1 0 11	1 0 11	1 0 11
1 0 13	1 0 5	1 0 7	1 0 9	1 0 10	1 0 11	1 0 11	1 0 12	1 0 12	1 0 12	1 0 12
1 0 14	1 0 6	1 0 8	1 0 10	1 0 11	1 0 12	1 0 12	1 0 12	1 0 12	1 0 13	1 0 13
1 0 15	1 0 7	1 0 8	1 0 11	1 0 12	1 0 12	1 0 13	1 0 13	1 0 13	1 0 13	1 0 13
1 0 16	1 0 8	1 0 8	1 0 11	1 0 13	1 0 13	1 0 14	1 0 14	1 0 14	1 0 14	1 0 14
1 0 17	1 0 9	1 0 9	1 0 12	1 0 13	1 0 14	1 0 14	1 0 15	1 0 15	1 0 15	1 0 15
1 0 18	1 0 10	1 0 10	1 0 13	1 0 14	1 0 15	1 0 15	1 0 15	1 0 16	1 0 16	1 0 16
1 0 19	1 0 11	1 0 11	1 0 13	1 0 15	1 0 16	1 0 16	1 0 16	1 0 16	1 0 16	1 0 17

TABLE I (Continued)

No War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
1 1 0	1 1 0	2 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1	1 1 1
1 1 1	1 1 0	2 1 1	1 1 2	1 1 2	1 1 2	1 1 2	1 1 2	1 1 2	1 1 2	1 1 2
1 1 2	1 1 0	1 1 2	1 1 2	1 1 2	1 1 3	1 1 3	1 1 3	1 1 3	1 1 3	1 1 3
1 1 3	1 1 0	1 1 2	1 1 3	1 1 3	1 1 3	1 1 3	1 1 3	1 1 3	1 1 4	1 1 4
1 1 4	1 1 0	1 1 3	1 1 4	1 1 4	1 1 4	1 1 4	1 1 4	1 1 4	1 1 4	1 1 4
1 1 5	1 1 0	1 1 3	1 1 4	1 1 5	1 1 5	1 1 5	1 1 5	1 1 5	1 1 5	1 1 5
1 1 6	1 1 0	1 1 3	1 1 5	1 1 5	1 1 5	1 1 6	1 1 6	1 1 6	1 1 6	1 1 6
1 1 7	1 1 0	1 1 4	1 1 5	1 1 6	1 1 6	1 1 6	1 1 6	1 1 6	1 1 6	1 1 6
1 1 8	1 1 0	1 1 4	1 1 6	1 1 7	1 1 7	1 1 7	1 1 7	1 1 7	1 1 7	1 1 7
1 1 9	1 1 1	1 1 5	1 1 6	1 1 7	1 1 8	1 1 8	1 1 8	1 1 8	1 1 8	1 1 8
1 1 10	1 1 2	1 1 5	1 1 7	1 1 8	1 1 8	1 1 9	1 1 9	1 1 9	1 1 9	1 1 9
1 1 11	1 1 3	1 1 6	1 1 8	1 1 8	1 1 9	1 1 9	1 1 9	1 1 10	1 1 10	1 1 10
1 1 12	1 1 4	1 1 6	1 1 8	1 1 9	1 1 10	1 1 10	1 1 10	1 1 10	1 1 10	1 1 11
1 1 13	1 1 5	1 1 7	1 1 9	1 1 10	1 1 10	1 1 11	1 1 11	1 1 11	1 1 11	1 1 11
1 1 14	1 1 6	1 1 8	1 1 10	1 1 11	1 1 11	1 1 11	1 1 12	1 1 12	1 1 12	1 1 12
1 1 15	1 1 7	1 1 8	1 1 11	1 1 12	1 1 12	1 1 12	1 1 12	1 1 13	1 1 13	1 1 13
1 1 16	1 1 8	1 1 8	1 1 11	1 1 13	1 1 13	1 1 13	1 1 13	1 1 13	1 1 13	1 1 14
1 1 17	1 1 9	1 1 9	1 1 12	1 1 13	1 1 14	1 1 14	1 1 14	1 1 14	1 1 14	1 1 14
1 1 18	1 1 10	1 1 10	1 1 13	1 1 14	1 1 15	1 1 15	1 1 15	1 1 15	1 1 15	1 1 15
1 1 19	1 1 11	1 1 11	1 1 13	1 1 15	1 1 16	1 1 16	1 1 16	1 1 16	1 1 16	1 1 16

TABLE I (Continued)

No War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
1 2 0	1 2 0	1 2 1	1 2 1	1 2 1	1 2 1	1 2 1	1 2 1	1 2 1	1 2 1	1 2 1
1 2 1	1 2 0	1 2 2	1 2 2	1 2 2	1 2 2	1 2 2	1 2 2	1 2 2	1 2 2	1 2 2
1 2 2	1 2 0	1 2 1	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3	1 2 3
1 2 3	1 2 0	1 2 2	1 2 3	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4
1 2 4	1 2 0	1 2 3	1 2 3	1 2 4	1 2 5	1 2 5	1 2 5	1 2 5	1 2 5	1 2 5
1 2 5	1 2 0	1 2 3	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4
1 2 6	1 2 0	1 2 3	1 2 5	1 2 5	1 2 5	1 2 5	1 2 5	1 2 5	1 2 5	1 2 5
1 2 7	1 2 0	1 2 4	1 2 5	1 2 5	1 2 6	1 2 6	1 2 6	1 2 6	1 2 6	1 2 6
1 2 8	1 2 0	1 2 4	1 2 6	1 2 6	1 2 6	1 2 7	1 2 7	1 2 7	1 2 7	1 2 7
1 2 9	1 2 1	1 2 5	1 2 6	1 2 7	1 2 7	1 2 7	1 2 7	1 2 3	1 2 8	1 2 8
1 2 10	1 2 2	1 2 5	1 2 7	1 2 7	1 2 8	1 2 8	1 2 8	1 2 8	1 2 8	1 2 8
1 2 11	1 2 3	1 2 6	1 2 8	1 2 8	1 2 9	1 2 9	1 2 9	1 2 9	1 2 9	1 2 9
1 2 12	1 2 4	1 2 6	1 2 8	1 2 9	1 2 9	1 2 10	1 2 10	1 2 10	1 2 10	1 2 10
1 2 13	1 2 5	1 2 7	1 2 9	1 2 10	1 2 10	1 2 10	1 2 10	1 2 10	1 2 10	1 2 11
1 2 14	1 2 6	1 2 8	1 2 10	1 2 11	1 2 11	1 2 11	1 2 11	1 2 11	1 2 11	1 2 11
1 2 15	1 2 7	1 2 8	1 2 11	1 2 12	1 2 12	1 2 12	1 2 12	1 2 12	1 2 12	1 2 12
1 2 16	1 2 8	1 2 8	1 2 11	1 2 12	1 2 13	1 2 13	1 2 13	1 2 13	1 2 13	1 2 13
1 2 17	1 2 9	1 2 9	1 2 12	1 2 13	1 2 14	1 2 14	1 2 14	1 2 14	1 2 14	1 2 14
1 2 18	1 2 10	1 2 10	1 2 13	1 2 14	1 2 15	1 2 15	1 2 15	1 2 15	1 2 15	1 2 15
1 2 19	1 2 11	1 2 11	1 2 13	1 2 15	1 2 16	1 2 16	1 2 16	1 2 16	1 2 16	1 2 16

TABLE I (Continued)

No War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
2 0 0	2 0 0	2 1 0	2 1 0	2 0 1	2 0 1	2 0 1	2 0 1	2 0 1	2 0 1	2 0 1
2 0 1	2 0 0	2 0 1	2 0 1	2 0 1	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2
2 0 2	2 0 0	2 0 1	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2
2 0 3	2 0 0	2 0 2	2 0 2	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3
2 0 4	2 0 0	2 0 3	2 0 3	2 0 3	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4
2 0 5	2 0 0	2 0 3	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4
2 0 6	2 0 0	2 0 3	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5
2 0 7	2 0 0	2 0 4	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6
2 0 8	2 0 0	2 0 4	2 0 6	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7
2 0 9	2 0 1	2 0 5	2 0 6	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8
2 0 10	2 0 2	2 0 5	2 0 7	2 0 8	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9
2 0 11	2 0 3	2 0 6	2 0 8	2 0 9	2 0 9	2 0 10	2 0 10	2 0 10	2 0 10	2 0 10
2 0 12	2 0 4	2 0 6	2 0 8	2 0 9	2 0 10	2 0 10	2 0 11	2 0 11	2 0 11	2 0 11
2 0 13	2 0 5	2 0 7	2 0 9	2 0 10	2 0 11	2 0 11	2 0 11	2 0 11	2 0 12	2 0 12
2 0 14	2 0 6	2 0 8	2 0 10	2 0 11	2 0 11	2 0 12	2 0 12	2 0 12	2 0 12	2 0 12
2 0 15	2 0 7	2 0 8	2 0 11	2 0 12	2 0 12	2 0 13	2 0 13	2 0 13	2 0 13	2 0 13
2 0 16	2 0 8	2 0 8	2 0 11	2 0 12	2 0 13	2 0 13	2 0 14	2 0 14	2 0 14	2 0 14
2 0 17	2 0 9	2 0 9	2 0 12	2 0 13	2 0 14	2 0 14	2 0 14	2 0 14	2 0 15	2 0 15
2 0 18	2 0 10	2 0 10	2 0 13	2 0 14	2 0 15	2 0 15	2 0 15	2 0 15	2 0 15	2 0 15
2 0 19	2 0 11	2 0 11	2 0 13	2 0 15	2 0 16	2 0 16	2 0 16	2 0 16	2 0 16	2 0 16

TABLE I (Continued)

No War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
2 1 0	2 1 0	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1
2 1 1	2 1 0	2 1 1	2 1 1	2 1 1	2 1 2	2 1 2	2 1 2	2 1 2	2 1 2	2 1 2
2 1 2	2 1 0	2 1 1	2 1 2	2 1 2	2 1 2	2 1 2	2 1 2	2 1 3	2 1 3	2 1 3
2 1 3	2 1 0	2 1 2	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
2 1 4	2 1 0	2 1 3	2 1 3	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4
2 1 5	2 1 0	2 1 3	2 1 4	2 1 4	2 1 4	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5
2 1 6	2 1 0	2 1 3	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5
2 1 7	2 1 0	2 1 4	2 1 5	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6
2 1 8	2 1 0	2 1 4	2 1 6	2 1 6	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7
2 1 9	2 1 1	2 1 5	2 1 6	2 1 7	2 1 7	2 1 8	2 1 8	2 1 8	2 1 8	2 1 8
2 1 10	2 1 2	2 1 5	2 1 7	2 1 7	2 1 8	2 1 8	2 1 8	2 1 9	2 1 9	2 1 9
2 1 11	2 1 3	2 1 6	2 1 8	2 1 8	2 1 9	2 1 9	2 1 9	2 1 9	2 1 9	2 1 9
2 1 12	2 1 4	2 1 6	2 1 8	2 1 9	2 1 9	2 1 10	2 1 10	2 1 10	2 1 10	2 1 10
2 1 13	2 1 5	2 1 7	2 1 9	2 1 10	2 1 10	2 1 11	2 1 11	2 1 11	2 1 11	2 1 11
2 1 14	2 1 6	2 1 8	2 1 10	2 1 11	2 1 11	2 1 11	2 1 11	2 1 11	2 1 11	2 1 12
2 1 15	2 1 7	2 1 8	2 1 11	2 1 12	2 1 12	2 1 12	2 1 12	2 1 12	2 1 12	2 1 12
2 1 16	2 1 8	2 1 8	2 1 11	2 1 12	2 1 13	2 1 13	2 1 13	2 1 13	2 1 13	2 1 13
2 1 17	2 1 9	2 1 9	2 1 12	2 1 13	2 1 14	2 1 14	2 1 14	2 1 14	2 1 14	2 1 14
2 1 18	2 1 10	2 1 10	2 1 13	2 1 14	2 1 15	2 1 15	2 1 15	2 1 15	2 1 15	2 1 15
2 1 19	2 1 11	2 1 11	2 1 13	2 1 15	2 1 15	2 1 16	2 1 16	2 1 16	2 1 16	2 1 16

TABLE I (Continued)

No War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
2 2 0	2 2 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
2 2 1	2 2 0	2 2 1	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2
2 2 2	2 2 0	2 2 1	2 2 2	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3
2 2 3	2 2 0	2 2 2	2 2 2	2 2 3	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4
2 2 4	2 2 0	2 2 2	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3
2 2 5	2 2 0	2 2 3	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4
2 2 6	2 2 0	2 2 3	2 2 4	2 2 4	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5
2 2 7	2 2 0	2 2 4	2 2 5	2 2 5	2 2 5	2 2 5	2 2 6	2 2 6	2 2 6	2 2 6
2 2 8	2 2 0	2 2 4	2 2 5	2 2 6	2 2 6	2 2 6	2 2 6	2 2 7	2 2 6	2 2 7
2 2 9	2 2 1	2 2 5	2 2 6	2 2 7	2 2 7	2 2 7	2 2 7	2 2 7	2 2 7	2 2 7
2 2 10	2 2 2	2 2 5	2 2 7	2 2 7	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8
2 2 11	2 2 3	2 2 6	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8	2 2 9	2 2 9
2 2 12	2 2 4	2 2 6	2 2 8	2 2 9	2 2 9	2 2 9	2 2 9	2 2 9	2 2 9	2 2 9
2 2 13	2 2 5	2 2 7	2 2 9	2 2 10	2 2 10	2 2 10	2 2 10	2 2 10	2 2 10	2 2 10
2 2 14	2 2 6	2 2 8	2 2 10	2 2 11	2 2 11	2 2 11	2 2 11	2 2 11	2 2 11	2 2 11
2 2 15	2 2 7	2 2 8	2 2 11	2 2 11	2 2 12	2 2 12	2 2 12	2 2 12	2 2 12	2 2 12
2 2 16	2 2 8	2 2 8	2 2 11	2 2 12	2 2 13	2 2 13	2 2 13	2 2 13	2 2 13	2 2 13
2 2 17	2 2 9	2 2 9	2 2 12	2 2 13	2 2 14	2 2 14	2 2 14	2 2 14	2 2 14	2 2 14
2 2 18	2 2 10	2 2 10	2 2 13	2 2 14	2 2 14	2 2 15	2 2 15	2 2 15	2 2 15	2 2 15
2 2 19	2 2 11	2 2 11	2 2 13	2 2 15	2 2 15	2 2 16	2 2 16	2 2 16	2 2 16	2 2 16

TABLE II

Cold War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
0 0 0	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 1	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 2	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 3	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 4	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 5	0 0 0	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1
0 0 6	0 0 0	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2
0 0 7	0 0 0	2 0 2	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3
0 0 8	0 0 0	2 0 3	2 0 3	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4
0 0 9	0 0 0	1 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 5	2 0 5	2 0 5
0 0 10	0 0 0	0 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 6
0 0 11	0 0 1	0 0 5	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6
0 0 12	0 0 2	0 0 6	1 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7
0 0 13	0 0 3	0 0 6	0 0 8	2 0 8	2 0 8	2 0 8	2 0 8	0 0 8	0 0 8	0 0 8
0 0 14	0 0 4	0 0 7	0 0 8	0 0 9	0 0 9	0 0 9	0 0 9	0 0 9	0 0 9	0 0 9
0 0 15	0 0 5	0 0 7	0 0 9	0 0 10	0 0 10	0 0 10	0 0 10	0 0 10	0 0 10	0 0 10
0 0 16	0 0 6	0 0 8	0 0 10	0 0 11	0 0 11	0 0 11	0 0 11	0 0 11	0 0 11	0 0 11
0 0 17	0 0 7	0 0 9	0 0 11	0 0 11	0 0 12	0 0 12	0 0 12	0 0 12	0 0 12	0 0 12
0 0 18	0 0 8	0 0 9	0 0 12	0 0 13	0 0 13	0 0 13	0 0 13	0 0 13	0 0 13	0 0 13
0 0 19	0 0 9	0 0 10	0 0 12	0 0 13	0 0 14	0 0 14	0 0 14	0 0 14	0 0 14	0 0 14

TABLE II (Continued)

Cold War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
0 1 0	0 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 1 1	0 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 1 2	0 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 1 3	0 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 1 4	0 1 0	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1
0 1 5	0 1 0	2 1 1	2 1 1	2 1 2	2 1 2	2 1 2	2 1 2	2 1 2	2 1 2	2 1 2
0 1 6	0 1 0	2 1 2	2 1 2	2 1 2	2 1 2	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
0 1 7	0 1 0	2 1 2	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
0 1 8	0 1 0	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4
0 1 9	0 1 0	0 1 4	2 1 4	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5
0 1 10	0 1 0	0 1 5	0 1 6	2 1 6	2 1 6	2 1 6	0 1 6	0 1 6	0 1 6	0 1 6
0 1 11	0 1 1	0 1 5	0 1 6	0 1 7	0 1 7	0 1 7	0 1 7	0 1 7	0 1 7	0 1 7
0 1 12	0 1 2	0 1 6	0 1 7	0 1 8	0 1 8	0 1 8	0 1 8	0 1 8	0 1 8	0 1 8
0 1 13	0 1 3	0 1 6	0 1 8	0 1 8	0 1 9	0 1 9	0 1 9	0 1 9	0 1 9	0 1 9
0 1 14	0 1 4	0 1 7	0 1 8	0 1 8	0 1 9	0 1 9	0 1 10	0 1 10	0 1 10	0 1 10
0 1 15	0 1 5	0 1 7	0 1 9	0 1 10	0 1 10	0 1 11	0 1 11	0 1 11	0 1 11	0 1 11
0 1 16	0 1 6	0 1 8	0 1 10	0 1 10	0 1 11	0 1 11	0 1 11	0 1 11	0 1 11	0 1 12
0 1 17	0 1 7	0 1 8	0 1 11	0 1 11	0 1 12	0 1 12	0 1 12	0 1 12	0 1 12	0 1 12
0 1 18	0 1 8	0 1 9	0 1 12	0 1 12	0 1 12	0 1 13	0 1 13	0 1 13	0 1 13	0 1 13
0 1 19	0 1 9	0 1 10	0 1 12	0 1 13	0 1 13	0 1 13	0 1 13	0 1 13	0 1 13	0 1 14

TABLE II (Continued)

Cold War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
0 2 0	0 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 2 1	0 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 2 2	0 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 2 3	0 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 2 4	0 2 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
0 2 5	0 2 0	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2
0 2 6	0 2 0	0 2 3	1 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	0 2 3	0 2 3
0 2 7	0 2 0	0 2 3	0 2 4	1 2 4	2 2 4	0 2 4	0 2 4	0 2 4	0 2 4	0 2 4
0 2 8	0 2 0	0 2 3	0 2 4	0 2 5	0 2 5	0 2 5	0 2 5	0 2 5	0 2 5	0 2 5
0 2 9	0 2 0	0 2 4	0 2 4	0 2 5	0 2 6	0 2 6	0 2 6	0 2 6	0 2 6	0 2 6
0 2 10	0 2 0	0 2 5	0 2 6	0 2 7	0 2 6	0 2 7	0 2 7	0 2 7	0 2 7	0 2 7
0 2 11	0 2 1	0 2 5	0 2 6	0 2 7	0 2 8	0 2 8	0 2 8	0 2 8	0 2 8	0 2 8
0 2 12	0 2 2	0 2 6	0 2 6	0 2 7	0 2 8	0 2 8	0 2 8	0 2 9	0 2 9	0 2 9
0 2 13	0 2 3	0 2 6	0 2 7	0 2 7	0 2 8	0 2 8	0 2 8	0 2 8	0 2 8	0 2 8
0 2 14	0 2 4	0 2 7	0 2 8	0 2 8	0 2 8	0 2 9	0 2 9	0 2 9	0 2 9	0 2 9
0 2 15	0 2 5	0 2 7	0 2 9	0 2 9	0 2 9	0 2 9	0 2 9	0 2 9	0 2 10	0 2 10
0 2 16	0 2 6	0 2 8	0 2 10	0 2 10	0 2 10	0 2 10	0 2 11	0 2 11	0 2 11	0 2 11
0 2 17	0 2 7	0 2 8	0 2 11	0 2 11	0 2 11	0 2 11	0 2 11	0 2 11	0 2 11	0 2 11
0 2 18	0 2 8	0 2 9	0 2 12	0 2 12	0 2 12	0 2 12	0 2 12	0 2 12	0 2 12	0 2 12
0 2 19	0 2 9	0 2 10	0 2 12	0 2 13	0 2 13	0 2 13	0 2 13	0 2 13	0 2 13	0 2 13

TABLE II (Continued)

Cold War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
1 0 0	1 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 0 1	1 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 0 2	1 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 0 3	1 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 0 4	1 0 0	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1
1 0 5	1 0 0	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2
1 0 6	1 0 0	2 0 2	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3
1 0 7	1 0 0	2 0 3	2 0 3	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4
1 0 8	1 0 0	1 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 5	2 0 5	2 0 5
1 0 9	1 0 0	1 0 4	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 6
1 0 10	1 0 0	1 0 5	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6
1 0 11	1 0 1	1 0 5	1 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7
1 0 12	1 0 2	1 0 6	1 0 7	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	1 0 8
1 0 13	1 0 3	1 0 6	1 0 8	1 0 9	1 0 9	1 0 9	1 0 9	1 0 9	1 0 9	1 0 9
1 0 14	1 0 4	1 0 7	1 0 9	1 0 10	1 0 10	1 0 10	1 0 10	1 0 10	1 0 10	1 0 10
1 0 15	1 0 5	1 0 7	1 0 10	1 0 10	1 0 11	1 0 11	1 0 11	1 0 11	1 0 11	1 0 11
1 0 16	1 0 6	1 0 8	1 0 10	1 0 11	1 0 12	1 0 12	1 0 12	1 0 12	1 0 12	1 0 12
1 0 17	1 0 7	1 0 8	1 0 11	1 0 12	1 0 12	1 0 13	1 0 13	1 0 13	1 0 13	1 0 13
1 0 18	1 0 8	1 0 9	1 0 12	1 0 12	1 0 13	1 0 14	1 0 14	1 0 14	1 0 14	1 0 14
1 0 19	1 0 9	1 0 10	1 0 12	1 0 13	1 0 14	1 0 14	1 0 15	1 0 15	1 0 15	1 0 15

TABLE II (Continued)

Cold War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
1 1 0	1 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 1 1	1 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 1 2	1 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 1 3	1 1 0	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1
1 1 4	1 1 0	2 1 1	2 1 1	2 1 2	2 1 2	2 1 2	2 1 2	2 1 2	2 1 2	2 1 2
1 1 5	1 1 0	2 1 2	2 1 2	2 1 2	2 1 2	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
1 1 6	1 1 0	2 1 2	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
1 1 7	1 1 0	2 1 3	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4
1 1 8	1 1 0	1 1 4	2 1 4	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	1 1 5
1 1 9	1 1 0	1 1 4	1 1 5	2 1 6	2 1 6	2 1 6	1 1 6	1 1 6	1 1 6	1 1 6
1 1 10	1 1 0	1 1 5	1 1 6	1 1 7	1 1 7	1 1 7	1 1 7	1 1 7	1 1 7	1 1 7
1 1 11	1 1 1	1 1 5	1 1 6	1 1 7	1 1 8	1 1 8	1 1 8	1 1 8	1 1 8	1 1 8
1 1 12	1 1 2	1 1 6	1 1 7	1 1 7	1 1 8	1 1 8	1 1 9	1 1 9	1 1 9	1 1 9
1 1 13	1 1 3	1 1 6	1 1 8	1 1 8	1 1 9	1 1 9	1 1 9	1 1 9	1 1 10	1 1 10
1 1 14	1 1 4	1 1 7	1 1 9	1 1 9	1 1 9	1 1 10	1 1 10	1 1 10	1 1 10	1 1 10
1 1 15	1 1 5	1 1 7	1 1 9	1 1 10	1 1 10	1 1 10	1 1 11	1 1 11	1 1 11	1 1 11
1 1 16	1 1 6	1 1 8	1 1 10	1 1 11	1 1 11	1 1 11	1 1 11	1 1 11	1 1 11	1 1 12
1 1 17	1 1 7	1 1 8	1 1 11	1 1 12	1 1 12	1 1 12	1 1 12	1 1 12	1 1 12	1 1 12
1 1 18	1 1 8	1 1 9	1 1 12	1 1 12	1 1 13	1 1 13	1 1 13	1 1 13	1 1 13	1 1 13
1 1 19	1 1 9	1 1 10	1 1 12	1 1 13	1 1 13	1 1 14	1 1 14	1 1 14	1 1 14	1 1 14

TABLE II (Continued)

Cold War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
1 2 0	1 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 2 1	1 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 2 2	1 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 2 3	1 2 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
1 2 4	1 2 0	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2
1 2 5	1 2 0	2 2 1	1 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	1 2 3	1 2 3
1 2 6	1 2 0	2 2 2	1 2 3	1 2 4	2 2 4	1 2 4	1 2 4	1 2 4	1 2 4	1 2 4
1 2 7	1 2 0	1 2 3	1 2 3	1 2 4	1 2 5	1 2 5	1 2 5	1 2 5	1 2 5	1 2 5
1 2 8	1 2 0	1 2 4	1 2 4	1 2 4	1 2 5	1 2 6	1 2 6	1 2 6	1 2 6	1 2 6
1 2 9	1 2 0	1 2 4	1 2 5	1 2 5	1 2 5	1 2 5	1 2 7	1 2 7	1 2 7	1 2 7
1 2 10	1 2 0	1 2 5	1 2 5	1 2 5	1 2 6	1 2 6	1 2 6	1 2 6	1 2 6	1 2 6
1 2 11	1 2 1	1 2 5	1 2 6	1 2 6	1 2 6	1 2 7	1 2 7	1 2 7	1 2 7	1 2 7
1 2 12	1 2 2	1 2 6	1 2 7	1 2 7	1 2 7	1 2 7	1 2 7	1 2 7	1 2 8	1 2 8
1 2 13	1 2 3	1 2 6	1 2 8	1 2 8	1 2 8	1 2 8	1 2 8	1 2 8	1 2 8	1 2 8
1 2 14	1 2 4	1 2 7	1 2 9	1 2 9	1 2 9	1 2 9	1 2 9	1 2 9	1 2 9	1 2 9
1 2 15	1 2 5	1 2 7	1 2 9	1 2 10	1 2 10	1 2 10	1 2 10	1 2 10	1 2 10	1 2 10
1 2 16	1 2 6	1 2 8	1 2 10	1 2 11	1 2 11	1 2 11	1 2 11	1 2 11	1 2 11	1 2 11
1 2 17	1 2 7	1 2 8	1 2 11	1 2 11	1 2 12	1 2 12	1 2 12	1 2 12	1 2 12	1 2 12
1 2 18	1 2 8	1 2 9	1 2 11	1 2 12	1 2 12	1 2 13	1 2 12	1 2 13	1 2 13	1 2 13
1 2 19	1 2 9	1 2 10	1 2 12	1 2 13	1 2 13	1 2 13	1 2 13	1 2 13	1 2 13	1 2 13

TABLE II (Continued)

Cold War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
2 0 0	2 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 0 1	2 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 0 2	2 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 0 3	2 0 0	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1
2 0 4	2 0 0	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2	2 0 2
2 0 5	2 0 0	2 0 2	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3	2 0 3
2 0 6	2 0 0	2 0 3	2 0 3	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4
2 0 7	2 0 0	2 0 3	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 5	2 0 5	2 0 5
2 0 8	2 0 0	2 0 4	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 6
2 0 9	2 0 0	2 0 4	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6
2 0 10	2 0 0	2 0 4	2 0 6	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7
2 0 11	2 0 1	2 0 5	2 0 7	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8
2 0 12	2 0 2	2 0 6	2 0 8	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9
2 0 13	2 0 3	2 0 6	2 0 8	2 0 9	2 0 10	2 0 10	2 0 10	2 0 10	2 0 10	2 0 10
2 0 14	2 0 4	2 0 7	2 0 8	2 0 10	2 0 11	2 0 11	2 0 11	2 0 11	2 0 11	2 0 11
2 0 15	2 0 5	2 0 7	2 0 9	2 0 11	2 0 11	2 0 12	2 0 12	2 0 12	2 0 12	2 0 12
2 0 16	2 0 6	2 0 8	2 0 10	2 0 12	2 0 12	2 0 12	2 0 13	2 0 13	2 0 13	2 0 13
2 0 17	2 0 7	2 0 8	2 0 10	2 0 11	2 0 13	2 0 13	2 0 13	2 0 13	2 0 13	2 0 14
2 0 18	2 0 8	2 0 9	2 0 12	2 0 12	2 0 14	2 0 14	2 0 14	2 0 14	2 0 14	2 0 14
2 0 19	2 0 9	2 0 10	2 0 12	2 0 13	2 0 15	2 0 15	2 0 15	2 0 15	2 0 15	2 0 15

TABLE II (Continued)

Cold War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
2 1 0	2 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 1 1	2 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 1 2	2 1 0	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1	2 1 1
2 1 3	2 1 0	2 1 1	2 1 1	2 1 2	2 1 2	2 1 2	2 1 2	2 1 2	2 1 2	2 1 2
2 1 4	2 1 0	2 1 2	2 1 2	2 1 2	2 1 2	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
2 1 5	2 1 0	2 1 2	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
2 1 6	2 1 0	2 1 3	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4
2 1 7	2 1 0	2 1 3	2 1 4	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5
2 1 8	2 1 0	2 1 4	2 1 5	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6
2 1 9	2 1 0	2 1 4	2 1 5	2 1 6	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7
2 1 10	2 1 0	2 1 4	2 1 6	2 1 6	2 1 7	2 1 7	2 1 8	2 1 8	2 1 8	2 1 8
2 1 11	2 1 1	2 1 5	2 1 7	2 1 7	2 1 8	2 1 8	2 1 8	2 1 8	2 1 8	2 1 8
2 1 12	2 1 2	2 1 6	2 1 8	2 1 8	2 1 8	2 1 9	2 1 9	2 1 9	2 1 9	2 1 9
2 1 13	2 1 3	2 1 6	2 1 8	2 1 9	2 1 9	2 1 9	2 1 9	2 1 9	2 1 9	2 1 9
2 1 14	2 1 4	2 1 7	2 1 8	2 1 10	2 1 10	2 1 10	2 1 10	2 1 10	2 1 10	2 1 10
2 1 15	2 1 5	2 1 7	2 1 9	2 1 11	2 1 11	2 1 11	2 1 11	2 1 11	2 1 11	2 1 11
2 1 16	2 1 6	2 1 8	2 1 10	2 1 12	2 1 12	2 1 12	2 1 12	2 1 12	2 1 12	2 1 12
2 1 17	2 1 7	2 1 8	2 1 10	2 1 11	2 1 13	2 1 13	2 1 13	2 1 13	2 1 13	2 1 13
2 1 18	2 1 8	2 1 9	2 1 11	2 1 12	2 1 14	2 1 14	2 1 14	2 1 14	2 1 14	2 1 14
2 1 19	2 1 9	2 1 10	2 1 12	2 1 12	2 1 13	2 1 15	2 1 15	2 1 15	2 1 15	2 1 15

TABLE II (Continued)

Cold War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 2 1	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 2 2	2 2 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
2 2 3	2 2 0	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2
2 2 4	2 2 0	2 2 1	2 2 2	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3
2 2 5	2 2 0	2 2 2	2 2 2	2 2 3	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4
2 2 6	2 2 0	2 2 3	2 2 3	2 2 3	2 2 4	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5
2 2 7	2 2 0	2 2 3	2 2 4	2 2 3	2 2 4	2 2 4	2 2 4	2 2 5	2 2 4	2 2 4
2 2 8	2 2 0	2 2 4	2 2 4	2 2 5	2 2 4	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5
2 2 9	2 2 0	2 2 4	2 2 5	2 2 5	2 2 5	2 2 5	2 2 6	2 2 5	2 2 6	2 2 6
2 2 10	2 2 0	2 2 4	2 2 6	2 2 6	2 2 6	2 2 6	2 2 6	2 2 6	2 2 6	2 2 6
2 2 11	2 2 1	2 2 5	2 2 7	2 2 7	2 2 7	2 2 7	2 2 7	2 2 7	2 2 7	2 2 7
2 2 12	2 2 2	2 2 6	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8
2 2 13	2 2 3	2 2 6	2 2 7	2 2 9	2 2 9	2 2 9	2 2 9	2 2 9	2 2 9	2 2 9
2 2 14	2 2 4	2 2 7	2 2 8	2 2 10	2 2 10	2 2 10	2 2 10	2 2 10	2 2 10	2 2 10
2 2 15	2 2 5	2 2 7	2 2 8	2 2 11	2 2 11	2 2 11	2 2 11	2 2 11	2 2 11	2 2 11
2 2 16	2 2 6	2 2 8	2 2 10	2 2 10	2 2 12	2 2 12	2 2 12	2 2 12	2 2 12	2 2 12
2 2 17	2 2 7	2 2 8	2 2 10	2 2 11	2 2 11	2 2 13	2 2 13	2 2 13	2 2 13	2 2 13
2 2 18	2 2 8	2 2 9	2 2 11	2 2 12	2 2 12	2 2 12	2 2 12	2 2 12	2 2 14	2 2 14
2 2 19	2 2 9	2 2 10	2 2 12	2 2 12	2 2 13	2 2 13	2 2 13	2 2 13	2 2 13	2 2 13

TABLE III

Limited War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
0 0 0	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 1	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 2	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 3	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 4	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 5	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 6	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 7	0 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 0 8	0 0 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
0 0 9	0 0 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
0 0 10	0 0 0	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 1 3
0 0 11	0 0 0	2 2 3	2 2 3	2 2 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
0 0 12	0 0 0	2 2 3	2 1 4	2 1 4	2 1 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4
0 0 13	0 0 1	2 1 4	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5
0 0 14	0 0 2	2 0 5	2 0 5	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6
0 0 15	0 0 3	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6
0 0 16	0 0 4	2 0 6	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7
0 0 17	0 0 5	1 0 7	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8
0 0 18	0 0 6	0 0 8	0 0 9	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9
0 0 19	0 0 7	0 0 8	0 0 10	0 0 10	0 0 10	0 0 10	0 0 10	0 0 10	2 0 10	1 0 10

TABLE III (Continued)

Limited War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
0 1 0	0 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 1 1	0 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 1 2	0 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 1 3	0 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 1 4	0 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 1 5	0 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 1 6	0 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 1 7	0 1 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
0 1 8	0 1 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
0 1 9	0 1 0	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 1 3
0 1 10	0 1 0	2 2 3	2 2 3	2 2 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
0 1 11	0 1 0	2 2 3	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4
0 1 12	0 1 0	2 1 4	2 1 4	2 1 4	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5
0 1 13	0 1 1	2 1 4	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5
0 1 14	0 1 2	2 1 4	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6
0 1 15	0 1 3	2 1 5	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7
0 1 16	0 1 4	1 1 6	0 1 8	0 1 8	2 1 8	2 1 8	2 1 8	0 1 8	0 1 8	0 1 8
0 1 17	0 1 5	0 1 7	0 1 9	0 1 9	0 1 9	0 1 9	0 1 9	0 1 9	0 1 9	0 1 9
0 1 18	0 1 6	0 1 7	0 1 9	0 1 10	0 1 10	0 1 10	0 1 10	0 1 10	0 1 10	0 1 10
0 1 19	0 1 7	0 1 8	0 1 9	0 1 11	0 1 11	0 1 11	0 1 11	0 1 11	0 1 11	0 1 11

TABLE III (Continued)

Limited War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
0 2 0	0 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 2 1	0 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 2 2	0 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 2 3	0 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 2 4	0 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 2 5	0 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
0 2 6	0 2 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
0 2 7	0 2 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
0 2 8	0 2 0	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2
0 2 9	0 2 0	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3
0 2 10	0 2 0	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3
0 2 11	0 2 0	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4
0 2 12	0 2 0	2 2 4	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5	0 2 5
0 2 13	0 2 1	2 2 4	2 2 6	2 2 6	2 2 6	2 2 6	2 2 6	0 2 6	0 2 6	0 2 6
0 2 14	0 2 2	2 2 4	0 2 7	0 2 7	0 2 7	0 2 7	0 2 7	0 2 7	0 2 7	0 2 7
0 2 15	0 2 3	0 2 6	0 2 8	0 2 9	0 2 8	0 2 8	0 2 8	0 2 8	0 2 8	0 2 8
0 2 16	0 2 4	0 2 6	0 2 8	0 2 9	0 2 9	0 2 9	0 2 9	0 2 9	0 2 9	0 2 9
0 2 17	0 2 5	0 2 7	0 2 8	0 2 9	0 2 10	0 2 10	0 2 10	0 2 10	0 2 10	0 2 10
0 2 18	0 2 6	0 2 7	0 2 8	0 2 10	0 2 10	0 2 11	0 2 11	0 2 11	0 2 11	0 2 11
0 2 19	0 2 7	0 2 8	0 2 9	0 2 9	0 2 10	0 2 11	0 2 11	0 2 11	0 2 11	0 2 11

TABLE III (Continued)

Limited War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
1 0 0	1 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 0 1	1 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 0 2	1 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 0 3	1 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 0 4	1 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 0 5	1 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 0 6	1 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 0 7	1 0 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
1 0 8	1 0 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
1 0 9	1 0 0	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 1 3
1 0 10	1 0 0	2 2 3	2 2 3	2 2 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
1 0 11	1 0 0	2 2 3	2 1 4	2 1 4	2 1 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4
1 0 12	1 0 0	2 1 4	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5
1 0 13	1 0 1	2 0 5	2 0 5	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6
1 0 14	1 0 2	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6
1 0 15	1 0 3	2 0 6	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7
1 0 16	1 0 4	1 0 7	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8
1 0 17	1 0 5	1 0 7	1 0 9	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9
1 0 18	1 0 6	1 0 7	1 0 9	1 0 10	2 0 10	2 0 10	1 0 10	1 0 10	1 0 10	1 0 10
1 0 19	1 0 7	1 0 8	1 0 10	1 0 11	1 0 11	1 0 11	1 0 11	1 0 11	1 0 11	1 0 11

TABLE III (Continued)

Limited War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
1 1 0	1 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 1 1	1 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 1 2	1 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 1 3	1 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 1 4	1 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 1 5	1 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 1 6	1 1 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
1 1 7	1 1 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
1 1 8	1 1 0	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 1 3
1 1 9	1 1 0	2 2 3	2 2 3	2 2 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
1 1 10	1 1 0	2 2 3	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4
1 1 11	1 1 0	2 1 4	2 1 4	2 1 4	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5
1 1 12	1 1 0	2 1 4	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5
1 1 13	1 1 1	2 1 4	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6
1 1 14	1 1 2	2 1 5	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7	1 1 7
1 1 15	1 1 3	1 1 6	1 1 8	1 1 8	2 1 8	1 1 8	1 1 8	1 1 8	1 1 8	1 1 8
1 1 16	1 1 4	1 1 6	1 1 8	1 1 9	1 1 9	1 1 9	1 1 9	1 1 9	1 1 9	1 1 9
1 1 17	1 1 5	1 1 7	1 1 8	1 1 10	1 1 10	1 1 10	1 1 10	1 1 10	1 1 10	1 1 10
1 1 18	1 1 6	1 1 7	1 1 9	1 1 10	1 1 11	1 1 11	1 1 11	1 1 11	1 1 11	1 1 11
1 1 19	1 1 7	1 1 8	1 1 10	1 1 10	1 1 11	1 1 12	1 1 12	1 1 12	1 1 12	1 1 12

TABLE III (Continued)

Limited War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
1 2 0	1 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 2 1	1 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 2 2	1 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 2 3	1 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 2 4	1 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
1 2 5	1 2 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
1 2 6	1 2 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
1 2 7	1 2 0	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2
1 2 8	1 2 0	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3
1 2 9	1 2 0	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3
1 2 10	1 2 0	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4
1 2 11	1 2 0	2 2 4	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5	1 2 5
1 2 12	1 2 0	2 2 4	2 2 6	2 2 6	2 2 6	2 2 6	2 2 6	1 2 6	1 2 6	1 2 6
1 2 13	1 2 1	2 2 4	1 2 7	1 2 7	1 2 7	1 2 7	1 2 7	1 2 7	1 2 7	1 2 7
1 2 14	1 2 2	2 2 5	1 2 7	1 2 8	1 2 8	1 2 8	1 2 8	1 2 8	1 2 8	1 2 8
1 2 15	1 2 3	1 2 6	1 2 7	1 2 8	1 2 9	1 2 9	1 2 9	1 2 9	1 2 9	1 2 9
1 2 16	1 2 4	1 2 6	1 2 7	1 2 9	1 2 9	1 2 9	1 2 10	1 2 10	1 2 10	1 2 10
1 2 17	1 2 5	1 2 6	1 2 8	1 2 8	1 2 9	1 2 10	1 2 10	1 2 10	1 2 10	1 2 10
1 2 18	1 2 6	1 2 7	1 2 9	1 2 9	1 2 9	1 2 9	1 2 10	1 2 10	1 2 10	1 2 10
1 2 19	1 2 7	1 2 8	1 2 9	1 2 10	1 2 10	1 2 10	1 2 10	1 2 10	1 2 10	1 2 10

TABLE III (Continued)

Limited War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
2 0 0	2 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 0 1	2 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 0 2	2 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 0 3	2 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 0 4	2 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 0 5	2 0 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 0 6	2 0 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
2 0 7	2 0 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
2 0 8	2 0 0	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 1 3
2 0 9	2 0 0	2 2 3	2 2 3	2 2 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
2 0 10	2 0 0	2 2 3	2 1 4	2 1 4	2 1 4	2 0 4	2 0 4	2 0 4	2 0 4	2 0 4
2 0 11	2 0 0	2 1 4	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5	2 0 5
2 0 12	2 0 0	2 0 5	2 0 5	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6
2 0 13	2 0 1	2 0 6	2 0 6	2 0 6	2 0 6	2 0 6	2 0 7	2 0 6	2 0 6	2 0 6
2 0 14	2 0 2	2 0 6	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7	2 0 7
2 0 15	2 0 3	2 0 7	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8	2 0 8
2 0 16	2 0 4	2 0 6	2 0 8	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9	2 0 9
2 0 17	2 0 5	2 0 7	2 0 9	2 0 10	2 0 10	2 0 10	2 0 10	2 0 10	2 0 10	2 0 10
2 0 18	2 0 6	2 0 7	2 0 10	2 0 10	2 0 11	2 0 11	2 0 11	2 0 11	2 0 11	2 0 11
2 0 19	2 0 7	2 0 8	2 0 11	2 0 11	2 0 12	2 0 12	2 0 12	2 0 12	2 0 12	2 0 12

TABLE III (Continued)

Limited War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
2 1 0	2 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 1 1	2 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 1 2	2 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 1 3	2 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 1 4	2 1 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 1 5	2 1 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
2 1 6	2 1 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
2 1 7	2 1 0	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 1 3
2 1 8	2 1 0	2 2 3	2 2 3	2 2 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3	2 1 3
2 1 9	2 1 0	2 2 3	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4	2 1 4
2 1 10	2 1 0	2 1 4	2 1 4	2 1 4	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5
2 1 11	2 1 0	2 1 4	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5	2 1 5
2 1 12	2 1 0	2 1 4	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6	2 1 6
2 1 13	2 1 1	2 1 5	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7	2 1 7
2 1 14	2 1 2	2 1 6	2 1 7	2 1 7	2 1 8	2 1 8	2 1 8	2 1 8	2 1 8	2 1 8
2 1 15	2 1 3	2 1 6	2 1 7	2 1 9	2 1 9	2 1 9	2 1 9	2 1 9	2 1 9	2 1 9
2 1 16	2 1 4	2 1 6	2 1 8	2 1 9	2 1 9	2 1 10	2 1 10	2 1 10	2 1 10	2 1 10
2 1 17	2 1 5	2 1 6	2 1 9	2 1 9	2 1 9	2 1 11	2 1 11	2 1 11	2 1 11	2 1 11
2 1 18	2 1 6	2 1 7	2 1 10	2 1 10	2 1 10	2 1 11	2 1 11	2 1 11	2 1 11	2 1 11
2 1 19	2 1 7	2 1 8	2 1 10	2 1 11	2 1 11	2 1 11	2 1 11	2 1 11	2 1 11	2 1 11

TABLE III (Continued)

Limited War

Minimum-Cost Programs

Internal States (Udy Capacity, Dean Capacity, and Stockpile)

Begin. of Year 1...10	End of Year 1	End of Year 2	End of Year 3	End of Year 4	End of Year 5	End of Year 6	End of Year 7	End of Year 8	End of Year 9	End of Year 10
2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 2 1	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 2 2	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 2 3	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0	2 2 0
2 2 4	2 2 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
2 2 5	2 2 0	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1	2 2 1
2 2 6	2 2 0	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2	2 2 2
2 2 7	2 2 0	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3
2 2 8	2 2 0	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3	2 2 3
2 2 9	2 2 0	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4	2 2 4
2 2 10	2 2 0	2 2 4	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5	2 2 5
2 2 11	2 2 0	2 2 4	2 2 6	2 2 6	2 2 6	2 2 6	2 2 6	2 2 6	2 2 6	2 2 6
2 2 12	2 2 0	2 2 4	2 2 6	2 2 7	2 2 7	2 2 7	2 2 7	2 2 7	2 2 7	2 2 7
2 2 13	2 2 1	2 2 5	2 2 6	2 2 7	2 2 7	2 2 7	2 2 8	2 2 8	2 2 8	2 2 8
2 2 14	2 2 2	2 2 5	2 2 6	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8
2 2 15	2 2 3	2 2 5	2 2 7	2 2 7	2 2 7	2 2 7	2 2 9	2 2 9	2 2 9	2 2 9
2 2 16	2 2 4	2 2 6	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8	2 2 8
2 2 17	2 2 5	2 2 6	2 2 9	2 2 9	2 2 9	2 2 9	2 2 9	2 2 9	2 2 9	2 2 9
2 2 18	2 2 6	2 2 7	2 2 8	2 2 10	2 2 10	2 2 10	2 2 10	2 2 10	2 2 10	2 2 10
2 2 19	2 2 7	2 2 8	2 2 9	2 2 11	2 2 11	2 2 11	2 2 11	2 2 11	2 2 11	2 2 11