

A MODEL OF FIRM BEHAVIOR
ENCOMPASSING PRICE, PRODUCTION,
INVESTMENT, AND EMPLOYMENT DECISIONS

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I. INTRODUCTION

It is usually the case that the price, production, investment, and employment decisions of a firm are analyzed separately rather than within the context of a complete behavioral model of the firm. A few studies have analyzed two of the decisions at a time. Holt, Modigliani, Muth, and Simon [6], for example, have considered the joint determination of production and employment decisions within the context of a quadratic cost-minimizing model. Lucas [7] has recently postulated a general stock-adjustment model in which the stock of one input may influence the demand for another input, and Nadiri and Rosen [11] have used this basic model in an empirical study of employment and investment decisions. Coen and Hickman [2] have worked with a model that takes into account the interrelationship of employment and investment decisions. Mills [9], Hay [5], and Maccini [8] have considered the joint determination of price and production decisions.

The purpose of this paper is to present a model of firm behavior in which all four decisions are determined simultaneously. The objective function of the firm is taken to be the present discounted value of expected future after-tax cash flow. Each period the firm is assumed to solve an optimal control problem by choosing the paths of its price, production, investment, and wage rate¹ that maximize its objective function.

¹In the present model, as will be seen below, the firm's wage-rate decision and its employment decision are not independent. Given one decision, the other is automatically determined. In what follows, reference will sometimes be made to the wage-rate decision and sometimes to the employment decision, depending on the context.

The firm is assumed to convert what would otherwise be a stochastic control problem into a deterministic control problem by setting all of the values of the stochastic variables equal to their expected values before solving.²

Algorithms have been written to solve the maximization problem of the firm, and the behavior of the firm has been analyzed by means of computer simulation.

The underlying technology of the firm is assumed to be of a "putty-clay" type, where at any one time different types of machines with differing worker-machine ratios can be purchased. The worker-machine ratio is assumed to be fixed for each type of machine. Costs of changing employment and capital are postulated. With respect to its price decision, the firm is assumed to have a certain amount of monopoly power in the short run in the sense that raising its price above prices charged by other firms will not result in an immediate loss of all of its customers and lowering its price below prices charged by other firms will not result in an immediate gain of everyone else's customers. There is assumed, however, to be a tendency in the system for a high-price firm to lose customers over time and for a low-price firm to gain customers. This assumption, that a firm's market share is a function of its price relative to the prices of other firms, is common to the studies of Mortensen [10], Phelps [13], Phelps and Winter [14], and Maccini [8].

²This is a common procedure in the control literature. See, for example, Athans [1]. The procedure was followed here because of computational convenience. The resulting solution values must thus be interpreted as being only approximations to the true solution values of the complete stochastic control problem.

Because of the assumption that the firm replaces stochastic variables with their expected values, the model is presented below using expected values directly rather than density functions. A superscript "e" on a variable is used to denote the expected value of that variable.

In the present case the firm is also assumed to expect that the future prices of other firms are in part a function of its own past prices. The firm's market share of labor supplied to it is treated in a similar manner as its market share of goods sold: the firm's market share of labor is assumed to be a function of its wage rate relative to the wage rates of other firms. Likewise, the firm is assumed to expect that the future wage rates of other firms are in part a function of its own past wage rates. The firm must borrow to finance investment expenditures in the model, and both the case in which the firm is constrained in the amount of money it can borrow and the case in which it is not constrained are analyzed.

An advantage of using computer-simulation techniques over standard analytic methods to analyze models is that one can deal with much larger and more complete models. In addition, one need not compromise on specifying the objective of the firm; the objective function can easily be taken to be the present discounted value of expected future after-tax profits or cash flow. It is hoped that this study will stimulate further theoretical work in analyzing more complete behavioral models of the firm, as well as empirical work in testing the empirical implications of this study.

In Table 1 the important symbols used in this paper are presented in alphabetical order. This table should be used as a reference in reading the rest of the paper.

TABLE 1

List of Symbols in Alphabetical Order

Subscript i denotes variable for firm i . Subscript j denotes variable for firm j . Subscript t denotes variable for period t . An e superscript in the text denotes firm i 's expectation of the variable.

CF_{it}	= cash flow before taxes and dividends
\overline{CF}_{it}	= cash flow net of taxes and dividends
d_1	= corporate tax rate
DDF_{it}	= actual demand deposits
DDF_{lit}	= demand deposits set aside for transaction purposes
DDF_{2i}	= demand deposits set aside to be used as a buffer to meet unexpected increases in the cost of investment goods
DEP_{it}	= depreciation
$DIVF_{it}$	= dividends paid
\overline{H}	= maximum number of hours that each machine can be used each period
HP_t	= total hours paid for in the economy
HPF_{it}	= hours paid for (by firm i)
HPF_{jt}	= hours paid for (by firm j)
I_{nit}	= number of machines of type n purchased ($n=1,2$)
INV_{it}	= number of goods purchased for investment purposes
\overline{K}	= minimum number of machines required to be held in each of the last m periods of the decision horizon
K_{nit}	= minimum number of machines of type n required to produce Y_{nit} ($n=1,2$)
K_{nit}^a	= actual number of machines of type n held ($n=1,2$)
KH_{nit}	= number of machine hours worked on machines of type n ($n=1,2$)
LF_{it}	= value of loans taken out

TABLE 1 (continued)

$LFMAX_{it}$	= maximum value of loans that can be taken out
m	= length of life of one machine
MH_{nit}	= number of worker hours worked on machines of type n ($n=1,2$)
MH_{3it}	= number of worker hours required to maintain inventories below β_1 times sales
MH_{4it}	= number of worker hours required to handle fluctuations in sales
MH_{5it}	= number of worker hours required to handle fluctuations in worker hours paid for
MH_{6it}	= number of worker hours required to handle fluctuations in net investment
MH_{it}	= total number of worker hours required
PF_{it}	= price charged (by firm i)
PF_{jt}	= price charged (by firm j)
\overline{PF}_t	= average price level in the economy
PFF_{it}	= price paid for investment goods
RF_{it}	= loan rate paid
$T+1$	= length of decision horizon
$TAXF_{it}$	= taxes paid
V_{it}	= stock of inventories (of firm i)
V_{jt}	= stock of inventories (of firm j)
WF_{it}	= wage rate (of firm i)
WF_{jt}	= wage rate (of firm j)
\overline{WF}_t	= average wage rate in the economy
X_t	= total number of goods sold in the economy
XF_{it}	= number of goods sold (by firm i)

TABLE 1 (continued)

XF_{jt}	= number of goods sold (by firm j)
Y_{nit}	= number of goods produced on machines of type n (n=1,2)
Y_{it}	= total number of goods produced
δ_n	= number of goods it takes to create a machine of type n (n=1,2)
λ_n	= amount of output produced per worker-hour on machines of type n (n=1,2)
μ_n	= amount of output produced per machine-hour on machines of type n (n=1,2)
πF_{it}	= before-tax profits

II. THE EQUATIONS OF THE MODEL

Equations Regarding the Technology and Capital and Labor Requirements

$$(1) \quad MH_{nit} = \frac{Y_{nit}}{\lambda_n}, \quad n=1,2,$$

$$(2) \quad KH_{nit} = \frac{Y_{nit}}{\mu_n}, \quad n=1,2,$$

$$(3) \quad K_{nit} = \frac{KH_{nit}}{H}, \quad n=1,2,$$

$$(4) \quad K_{nit}^a = K_{nit-1}^a + I_{nit} - I_{nit-m}, \quad n=1,2,$$

$$(5) \quad INV_{it} = \sum_{n=1}^2 \delta_n I_{nit},$$

$$(6) \quad Y_{it} = \sum_{n=1}^2 Y_{nit},$$

$$(7) \quad V_{it} = V_{it-1} + Y_{it} - XF_{it},$$

$$(8) \quad MH_{3it} = \beta_2 (V_{it} - \beta_1 XF_{it})^2 \text{ if } V_{it} < \beta_1 XF_{it},$$

$$= 0 \quad \text{otherwise,} \quad \beta_1 > 0, \beta_2 > 0,$$

$$(9) \quad MH_{4it} = \beta_3 (XF_{it} - XF_{it-1})^2, \quad \beta_3 > 0,$$

$$(10) \quad MH_{5it} = \beta_4 (HPF_{it-1} - HPF_{it-2})^2, \quad \beta_4 > 0,$$

$$(11) \quad MH_{6it} = \beta_5 \left[\sum_{n=1}^2 K_{nit}^a - \sum_{n=1}^2 K_{nit-1}^a \right]^2, \quad \beta_5 > 0,$$

$$(12) \quad MH_{it} = MH_{1it} + MH_{2it} + MH_{3it} + MH_{4it} + MH_{5it} + MH_{6it},$$

$$(13) \quad K_{nit}^a \geq K_{nit}, \quad n=1,2,$$

$$(14) \quad HPF_{it} \geq MH_{it}$$

Equation (1) defines the number of worker hours required to produce output Y_{nit} on machines of type n , and equation (2) defines the number of machine hours required. These two equations reflect the putty-clay nature of the technology. Without loss of generality, the number of different types of machines is taken to be 2.³ There is assumed to be no technical progress, so that λ_n and μ_n ($n=1,2$) are not functions of time. Machines are also assumed not to be subject to physical depreciation, so that λ_n and μ_n ($n=1,2$) are not a function of the age of the machines. The machines are assumed to wear out completely after m periods.

Equation (3) defines the minimum number of machines of type n required to produce Y_{nit} . It is assumed that \bar{H} , the maximum number of hours

³It should be obvious in what follows that the number of different types of machines can be generalized to any number. For the simulation work the number was taken to be 2, and so this is the specification presented in the text.

that each machine can be used each period, is constant over time. Equation (4) defines the actual number of machines of each type on hand in period t . Machines purchased in a period are assumed to be able to be used in the production process in that period. In equation (4), I_{nit} is the number of machines of type n purchased in period t , and I_{nit-m} is the number of machines of type n that wear out at the end of period $t-1$ and so cannot be used in the production process in period t . The firm is subject to the restriction (13), which says that the actual number of machines of type n on hand must be greater than or equal to the minimum number required.

There is assumed to be only one good in the system, which can be used either for consumption or investment purposes. δ_n is the number of goods it takes to create a machine of type n . In equation (5) the number of machines purchased in period t is translated into the equivalent number of goods purchased. To rule out the possibility of one type of machine completely dominating the other in efficiency, it was assumed for the simulation work that $\mu_1 = \mu_2$, so that the types differ from each other only in terms of the λ coefficients. Machines of type 1 were assumed to have a lower worker-machine ratio, $\lambda_1 > \lambda_2$, and to require more goods to create one machine, $\delta_1 > \delta_2$. Equation (6) defines total production, and equation (7) defines the stock of inventories. Equations (8) through (11) define various adjustment costs facing the firm, the costs taking the form of increased worker-hour requirements. Equation (8) reflects the assumption that there are costs involved in having inventories be less than a certain proportion of sales, such as workers having to make more trips to the storeroom.

The asymmetrical specification in equation (8) is not a critical assumption of the model, and little would be changed if (8) were made symmetrical. In addition to the costs in (8), any positive stock of inventories is costly to the firm in the sense that the stock must be financed. Equations (9) - (11) reflect the assumptions that there are costs involved in having sales, worker hours paid for, and net investment fluctuate. The use of the lagged change in worker hours paid for in equation (10) is made for computational convenience and is not a critical assumption of the model. Equation (12) defines total worker-hour requirements. The firm is subject to the restriction (14), which says that worker hours paid for must be greater than or equal to worker-hour requirements.

Equations Regarding Financial Variables

$$(15) \quad DEP_{it} = \frac{1}{m}(PFF_{it} INV_{it} + PFF_{it-1} INV_{it-1} + \dots + PFF_{it-m+1} INV_{it-m+1}),$$

$$(16) \quad \pi_{F_{it}} = PF_{it} XF_{it} - WF_{it} HPF_{it} - DEP_{it} - RF_{it} LF_{it},$$

$$(17) \quad TAX_{F_{it}} = d_1 \pi_{F_{it}},$$

$$(18) \quad DIV_{F_{it}} = \pi_{F_{it}} - TAX_{F_{it}},$$

$$(19) \quad CF_{it} = PF_{it} XF_{it} - WF_{it} HPF_{it} - PFF_{it} INV_{it} - RF_{it} LF_{it},$$

$$(20) \quad \overline{CF}_{it} = CF_{it} - TAX_{F_{it}} - DIV_{F_{it}}$$

$$= DEP_{it} - PFF_{it} INV_{it}$$

$$(21) \quad DDF_{it} = DDF_{it-1} + LF_{it} - LF_{it-1} + \overline{CF}_{it}$$

The government is assumed to allow for tax purposes straight-line depreciation, which is reflected in equation (15). Equation (16) defines before-tax profits on an accounting basis, which is the difference between total revenue and the sum of wage costs, depreciation, and interest costs. Taxes are defined in equation (17), where d_1 is the corporate tax rate. The firm is assumed not to retain any earnings, so that the level of dividends, as defined in equation (18), is merely the difference between before-tax profits and taxes. Equation (19) defines cash flow gross of taxes and dividends, and equation (20) defines cash flow net of taxes and dividends. The level of demand deposits, defined in equation (21), is a residual in the model, given the borrowings of the firm and cash flow net of taxes and dividends. The firm's level of borrowings is a decision variable, and its determination is discussed in Section IV. The firm is assumed not to issue any new stock.

Market-Share Equations

$$(22) \quad \frac{XF_{it}}{X_t} = \frac{XF_{it-1}}{X_{t-1}} \left(\frac{PF_{it}}{PF_{jt}} \right)^{\beta_6}, \quad \beta_6 < 0,$$

$$(23) \quad \frac{HPF_{it}}{HP_t} = \frac{HPF_{it-1}}{HP_{t-1}} \left(\frac{WF_{it}}{WF_{jt}} \right)^{\beta_7}, \quad \beta_7 > 0.$$

Equation (22) reflects the assumption that the market share of a firm is a function of its price relative to the prices of other firms. Without loss of generality, there is assumed to be only one other firm, firm j , in existence.⁴ Equation (22) states that the ratio of firm i 's sales to

⁴ Again, it should be obvious how the number of other firms in existence can be generalized to be more than one. There is nothing in the model that is inconsistent with there being a relatively large number of other firms in existence.

total sales is equal to last period's ratio times a function of the ratio of firm i's price to firm j's price. Equation (23) is a similar equation for the firm's market share of labor.

III. THE FORMATION OF EXPECTATIONS

Let $T+1$ be the length of the decision horizon. In order for the firm to solve its control problem at, say, the beginning of period t , it must form expectations of a number of variables for periods t through $t+T$. The assumptions made regarding the formation of these expectations have an important influence on the final properties of the model. Firm i is assumed to form the following expectations:⁵

$$(24) \quad \frac{PF_{jt}^e}{PF_{jt-1}} = \left(\frac{PF_{it-1}}{PF_{jt-1}} \right)^{\beta_8} \left(\frac{V_{jt-1}}{\beta_1 X_{jt-1}} \right)^{\beta_9}, \quad \beta_8 > 0, \beta_9 < 0 \quad \left[\begin{array}{l} \text{expected price} \\ \text{of firm } j \text{ for} \\ \text{period } t \end{array} \right]$$

$$(25) \quad \frac{PF_{jt+k}^e}{PF_{jt+k-1}^e} = \left(\frac{PF_{it+k-1}}{PF_{jt+k-1}^e} \right)^{\beta_8} \quad \left[\begin{array}{l} \text{expected price of firm } j \text{ for period} \\ t+k \text{ (} k=1,2,\dots,T \text{)} \end{array} \right]$$

$$(26) \quad \overline{PF}_{t+k}^e = (PF_{it+k} \cdot PF_{jt+k}^e)^{\frac{1}{2}} \quad \left[\begin{array}{l} \text{expected average price for} \\ \text{period } t+k \text{ (} k=0,1,\dots,T \text{)} \end{array} \right]$$

$$(27) \quad X_t^e = X_{t-1}^e \left(\frac{\overline{PF}_{t-1}}{PF_t^e} \right)^{\beta_{10}}, \quad \beta_{10} > 0 \quad \left[\begin{array}{l} \text{expected aggregate demand} \\ \text{for goods for period } t \end{array} \right]$$

$$(28) \quad X_{t+k}^e = X_{t+k-1}^e \left(\frac{\overline{PF}_{t+k-1}^e}{PF_{t+k}^e} \right)^{\beta_{10}} \quad \left[\begin{array}{l} \text{expected aggregate demand for} \\ \text{goods for period } t+k \text{ (} k=1,2,\dots,T \text{)} \end{array} \right]$$

⁵Since all expectations are made by firm i , no i subscript or superscript has been added to the relevant symbols to denote the fact that it is firm i making the expectation.

$$(29) \frac{WF_{jt}^e}{WF_{jt-1}} = \left(\frac{WF_{it-1}}{WF_{jt-1}} \right)^{\beta_{11}}, \quad \beta_{11} > 0 \quad \left[\text{expected wage rate of firm } j \text{ for period } t \right]$$

$$(30) \frac{WF_{jt+k}^e}{WF_{jt+k-1}^e} = \left(\frac{WF_{it+k-1}}{WF_{jt+k-1}^e} \right)^{\beta_{11}} \quad \left[\text{expected wage rate of firm } j \text{ for period } t+k \text{ (} k=1,2,\dots,T \text{)} \right]$$

$$(31) \overline{WF}_{t+k}^e = (WF_{it+k} \cdot WF_{jt+k}^e)^{\frac{1}{2}} \quad \left[\text{expected average wage rate for period } t+k \text{ (} k=0,1,\dots,T \text{)} \right]$$

$$(32) HP_t^e = HP_{t-1} \left(\frac{WF_t^e}{WF_{t-1}} \right)^{\beta_{12}} \left(\frac{PF_t^e}{PF_{t-1}} \right)^{\beta_{13}}, \quad \beta_{12} > 0, \beta_{13} < 0 \quad \left[\text{expected aggregate supply of labor for period } t \right]$$

$$(33) HP_{t+k}^e = HP_{t+k-1}^e \left(\frac{WF_{t+k}^e}{WF_{t+k-1}^e} \right)^{\beta_{12}} \left(\frac{PF_{t+k}^e}{PF_{t+k-1}^e} \right)^{\beta_{13}} \quad \left[\text{expected aggregate supply of labor for period } t+k \text{ (} k=1,2,\dots,T \text{)} \right]$$

$$(34) PPF_{it+k}^e = \overline{PF}_{t+k}^e \quad \left[\text{expected price of investment goods for period } t+k \text{ (} k=0,1,\dots,T \text{)} \right]$$

$$(35) RF_{it+k}^e = RF_{it} \quad \left[\text{expected loan rate for period } t+k \text{ (} k=1,2,\dots,T \text{)} \right]$$

The first term on the right hand side of equation (24) reflects the fact that firm i expects its price-setting behavior in period $t-1$ to have an effect on firm j 's price-setting behavior in period t . The second term is designed to represent the effect of market conditions on firm i 's expectation of firm j 's rate. If firm j 's stock of inventories at the end of period $t-1$ is greater than a certain proportion of sales, then firm i

is assumed to expect that firm j will respond to this situation by lowering its price in period t in an effort to increase sales and draw down inventories.

Firm i must also form expectations of firm j 's price for periods $t+1$ and beyond. These expectations are specified in equation (25), which is the same as equation (24) without the final term. Equation (25) means that firm i expects that firm j is always adjusting its price toward firm i 's price. If firm i 's price is constant over time, then firm i expects that firm j 's price will gradually approach this value.

In equation (26) firm i 's expectation of the average price level is taken to be the geometric average of its price and its expectation of firm j 's price. The geometric average is used rather than the arithmetic average to make the solution of the model easier. Firm i expects that the aggregate demand for goods is a function of the average price level, as specified in equations (27) and (28).

Firm i 's expectation of firm j 's wage rate is specified in equations (29) and (30). Equation (29) for the wage rate is similar to equation (24) for the price level, without the final term. Firm i is assumed to have no other basis upon which to base its expectation of firm j 's wage rate for period t than its and firm j 's wage rates for period $t-1$. Equation (31), defining firm i 's expectation of the average wage rate, is similar to equation (26). Firm i expects that the aggregate supply of labor is a positive function of the average wage rate and a negative function of the average price level, as specified in equations (32) and (33).⁶

⁶There is an asymmetry in the specification of equations (27) - (28) and (32) - (33). In (27) - (28), firm i 's expectation of the aggregate demand for goods is only a function of prices and not wages, whereas in (32) - (33), its expectation of the aggregate supply of labor is a function of both prices and wages. In general, a firm's expectation of the aggregate demand for goods may also be a function of wages, but for reasons of computational convenience this possibility was not allowed for here.

Equation (34) states that firm i expects that the price that it must pay for investment goods each period is the expected average price level for that period. The firm is assumed not to be able to produce its own investment goods. Equation (35) states that firm i expects that the loan rate for all future periods is going to be the same as the loan rate for period t . The firm is assumed to know the loan rate for period t at the beginning of period t .

IV. BEHAVIORAL ASSUMPTIONS

The objective of the firm is to maximize the present discounted value of expected future after-tax cash flow. The discount rate is assumed to be the loan rate. The objective function of firm i at the beginning of period t is:

$$(36) \text{ OBJF}_{it} = \frac{CF_{it}^e - \text{TAXF}_{it}^e}{(1+RF_{it}^e)} + \frac{CF_{it+1}^e - \text{TAXF}_{it+1}^e}{(1+RF_{it}^e)(1+RF_{it+1}^e)} \\ + \dots + \frac{CF_{it+T}^e - \text{TAXF}_{it+T}^e}{(1+RF_{it}^e)(1+RF_{it+1}^e)\dots(1+RF_{it+T}^e)},$$

where $CF_{it+k}^e - \text{TAXF}_{it+k}^e$ is the expected value of after-tax cash flow for period $t+k$ ($k=0,1,\dots,T$). The decision variables of the firm are its price, PF_{it+k} , its wage rate, WF_{it+k} , the number of each type of machine to buy, I_{lit+k} and I_{2it+k} , the number of goods to produce on each type of machine, Y_{lit+k} and Y_{2it+k} , and the amount of money to borrow, LF_{it+k} ($k=0,1,\dots,T$).

Given a set of paths of the decision variables, the corresponding value of the objective function can be computed as follows:

1. Given firm i 's price path, firm i 's expectation of firm j 's price path can be computed from (24) and (25). The path of the expected average price level can then be computed from (26), followed by the path of expected aggregate demand from (27) and (28). Firm i 's expectation of its own sales path can then be computed from (22).⁷
2. Given firm i 's wage path, firm i 's expectation of firm j 's wage path can be computed from (29) and (30). The path of the expected average wage rate can then be computed from (31), followed by the expected aggregate supply of labor from (32) and (33). Firm i 's expectation of the supply of labor available to it can then be computed from (23).
3. Given paths of the number of each type of machine to buy, the path of investment denominated in goods can be computed from (5). The path of depreciation can then be computed from (15), given the path of the expected price of investment goods from (34).
4. Given the above paths and the path of the expected loan rate from (35), the paths of profits, taxes, and cash flow can be computed from (16), (17), and (19), which then means that the value of the objective function can be computed.

⁷Although equations (1) - (23) are written only for period t , they are also meant to hold for periods $t+1, \dots, t+T$ as well. In addition, an "e" superscript should be added to a variable when firm i only has an expectation of that variable. For example, equation (22) should be written:

$$(22)' \quad \frac{XF_{it}^e}{X_t^e} = \frac{XF_{it-1}}{X_{t-1}} \left(\frac{PF_{it}}{PF_{jt}^e} \right)^{\beta_6}$$

$$(22)'' \quad \frac{XF_{it+k}^e}{X_{t+k}^e} = \frac{XF_{it+k-1}^e}{X_{t+k-1}^e} \left(\frac{PF_{it+k}}{PF_{jt+k}^e} \right)^{\beta_6}, \quad k=1,2,\dots,T.$$

To conserve space, equations (1) - (23) will not be written out in this expanded way, but the expansion in each case is straightforward.

The firm is restricted in each period by (13) and (14) and by various nonnegativity properties, such as the fact that the stock of inventories must be nonnegative. For any set of paths of the decision variables, these restrictions can be checked by solving equations (1) through (12) and then making the appropriate checks. The firm may also be constrained in the amount of money that it can borrow, i.e., in the values of LF_{it+k} ($k=0,1,\dots,T$) that it can choose. The following two end-point constraints were also imposed on the firm:

$$(37) \quad V_{it+T}^e = \beta_1 X_{it+T}^e ,$$

$$(38) \quad K_{1iT-k}^a + K_{2iT-k}^a \geq \bar{K} , \quad k=0,1,\dots,m-1.$$

The level of inventories at the end of the decision horizon was forced to be equal to β_1 times sales of the last period, and the number of machines held in each of the last m periods was required to be greater than or equal to a given number. These conditions were imposed to avoid quirks that would otherwise be likely to show up in the optimal paths near the end of the horizon.

A few general remarks can now be made regarding the control problem of the firm. The firm expects that it will gain customers by lowering its price relative to the expected prices of other firms. The main expected costs to the firm from lowering its price, in addition to the lower price it is charging per good, are the adjustment costs (9), (10), and (11) involved in increasing sales, employment, and investment. The firm also expects that other firms will follow it if it lowers its price, so that it does not expect to be able to capture an ever increasing share of the market without further and future price reductions.

The firm expects that it will lose customers by raising its price relative to the expected prices of other firms. The main costs from doing this, aside from the lost customers, are the adjustment costs. On the plus side, the firm expects that other firms will follow it if it raises its price, so that it does not expect to lose an ever increasing share of the market without further and further price increases.

The firm expects that it will gain workers if it raises its wage rate relative to the expected wage rates of other firms and lose workers if it lowers its wage rate relative to the expected wage rates of other firms. The firm also expects that other firms will follow it if it raises (lowers) its wage rate, so that it does not expect to capture (lose) an ever increasing share of the market without further and further wage rate increases (decreases).

Because of the various adjustment costs, the firm, if it chooses to lower its production, may choose in the current period not to lower its employment and capital stock to the minimum levels required. The firm may thus plan to hold either excess labor or excess capital or both during certain periods.

Before concluding this section, the borrowing behavior of the firm needs to be described. Consider first the values for period t and the demand-deposit needs of the firm. The demand-deposit needs are assumed to be of two kinds: the need for transactions purposes and the need to meet unexpected increases in the cost of investment goods. The need for transactions purposes is assumed to be proportional to the firm's wage bill. Let DDF_{lik} denote the value of demand deposits set aside by firm i for transactions purposes

in period t , and let HPF_{it}^e denote the number of worker hours that it expects to pay for in period t . Then DDF_{lit} is assumed to be:

$$(39) \quad DDF_{lit} = \beta_{14} W_{F_{it}} HPF_{it}^e, \quad \beta_{14} > 0.$$

If it is assumed that the firm never hires more than HPF_{it}^e amount of labor,⁸ then the firm's wage bill cannot exceed $W_{F_{it}} HPF_{it}^e$ and so the firm is assured by setting aside the value of demand deposits in (39) that it will always have enough demand deposits for transactions purposes. With respect to the second need, firm i only has an expectation of the price of investment goods, PPF_{it}^e , and must prepare for the possibility that it underestimates this price and ends up in period t paying more for investment goods than it originally expected. The firm is assumed to prepare for this possibility by planning to hold more demand deposits than are needed for transactions purposes. The firm is assumed from past experience to have a good idea of the largest error it is likely to make in underestimating the cost of investment, and this is the amount that the firm is assumed to plan to hold in demand deposits over and above its requirements for transactions purposes. Denote this amount as DDF_{2i} . DDF_{2i} should be a function of the number of goods chosen to be purchased for investment purposes in the period, but for simplicity DDF_{2i} is assumed to be constant over time.

⁸Since this study is concerned with the firm's decision making process at the beginning of the period and not with the mechanism by which transactions take place during the period, questions of what happens if the firm's expectations turn out to be incorrect can largely be ignored. If the firm overestimates the supply of labor available to it at the wage rate that it sets, then it will be required to cut its production from the level that it had planned at the beginning of the period, unless it has excess labor on hand to meet the shortfall. If the firm underestimates the supply of labor, then presumably it will not hire more labor than it had originally planned, assuming that the firm can make no other adjustments in its decision variables until the beginning of the next period. Questions of this sort are not considered in this paper.

Given its expectations, the firm is assumed to borrow money with the aim of holding amount $DDF_{lit} + DDF_{2i}$ in demand deposits in period t . The aimed-for change in demand deposits is $DDF_{lit} + DDF_{2i} - DDF_{it-1}$, where DDF_{it-1} is the actual value of demand deposits held by firm i in period $t-1$. The firm will need to increase its loans over and above any increase in aimed-for demand deposits if its expected cash flow after taxes and dividends, \overline{CF}_{it}^e , is negative, and conversely if \overline{CF}_{it}^e is positive. From (20), \overline{CF}_{it}^e is equal to $DEP_{it}^e - PFF_{it}^e \cdot INV_{it}$. The change in the value of loans for the firm is thus

$$(40) \quad LF_{it} - LF_{it-1} = (DDF_{lit} + DDF_{2i} - DDF_{it-1}) - \overline{CF}_{it}^e .$$

At the end of the period, after all transactions have taken place, actual demand deposits, DDF_{it} , will be equal to $DDF_{lit} + DDF_{2i}$ only in the case in which the firm's expectation of PFF_{it} is completely accurate. DDF_{it} will be less than $DDF_{lit} + DDF_{2i}$ if the firm underestimates PFF_{it} and has to use some of DDF_{2i} to meet the unexpected investment costs. From the definition of DDF_{2i} , the firm is assured that DDF_{it} will never be less than DDF_{lit} . DDF_{it} will be greater than $DDF_{lit} + DDF_{2i}$ if the firm overestimates PFF_{it} and takes out more loans than it really needed. The actual change in demand deposits of the firm for period t is a residual and is defined by equation (21). The determination of the value of loans for periods $t+1$ and beyond is a straightforward extension of the above analysis.

V. THE SOLUTION OF THE CONTROL PROBLEM OF THE FIRM

It was seen in Section IV that given the paths of the decision variables, the corresponding value of the objective function can be computed.

In order to solve the control problem of the firm, algorithms were written to search over various sets of paths for the optimum. The main algorithm searched over different price paths. The base price path, from which other paths were tried, was taken to be the path in which the price in each period was the same and equal to PF_{jt}^e in (24). PF_{jt}^e is the price that firm i expects firm j to set for period t . From (25) it can be seen that this price path corresponds to firm i expecting that firm j 's price path will be the same as firm i 's price path, which from (22) corresponds to firm i expecting that its market share will remain the same in periods t and beyond as it was in period $t-1$.

For each price path chosen by the algorithm, a sub-maximization problem was solved to determine the optimal production, investment, and employment paths corresponding to the given price path. This sub-maximization problem was solved by scanning over the various possible paths. First, given the expected sales path corresponding to the price path, various production paths were tried. The production paths are constrained, given the sales path, by the fact that inventories cannot be negative and by the terminal condition on inventories. For each production path, various investment paths were tried. The investment paths are constrained by the fact that there must be enough machines on hand to produce the amount of output required from the production path and by the terminal conditions. For each production and investment path, various employment paths were tried. The employment paths are constrained by the fact worker hours paid for each period must be at least as great as worker-hour requirements.

Two extreme production paths that were tried were a path in which production changed as little as possible from period to period and a path in which inventories changed as little as possible from period to period. Other paths were then tried as weighted averages of these two paths. There is a tradeoff between costs of production fluctuations (due to costs of investment and employment fluctuations) and costs of inventory fluctuations, and so trying various weighted averages of the two extreme paths should lead to a computed optimum path that is close to the true optimum path.

Given the level of production for a particular period and given the past history of investment, one can compute the number of machines of type 1 or of type 2 that need to be purchased in the period to produce the output of the period, assuming that all machines are utilized to full capacity (\bar{H} hours per period). Two investment paths that were tried were a path in which only machines of type 1 were purchased and a path in which only machines of type 2 were purchased, both of the paths being characterized by full capacity utilization all of the time, unless full capacity utilization required negative gross investment, which was not allowed. Other paths were tried in which investment fluctuations were lessened by not having the firm be at full capacity utilization all of the time. Paths in which some of type 1 machines and some of type 2 machines were purchased were not tried since it was costly to do so and it did not seem likely that the computed optimum values for period t would be sensitive to this omission.

Given the level of production and the number of the two types of machines on hand for a particular period, given the expected deviation of inventories from β_1 times sales for the period, given the expected change

in sales for the period, given the change in worker hours paid for of the previous period, and given the value of net investment for the period, worker-hour requirements can be computed from equations (1) and (8) - (11). Two extreme employment paths that were tried were a path in which worker hours paid for were always kept equal to worker-hour requirements and a path in which fluctuations in worker hours paid for were kept small. Other paths were then tried as weighted averages of these paths. As was the case for the production paths, trying various weighted averages of the two extreme paths should lead to a computed optimum path that is close to the true optimum path. All paths except the path in which worker hours paid for were equal to worker-hour requirements were characterized by the firm paying for more hours than required during some periods.

Given a path of prices and worker hours paid for and given firm i 's expectation of the path of firm j 's prices and of the path of average prices in the economy, one can compute from equations (29) - (33) and (23) the wage path that firm i expects is necessary to yield the path of worker hours paid for that it has set. In other words, once the firm has chosen its price path and its path of worker-hours paid for, the wage path is automatically determined.

When loan constraints were assumed to be in effect, they were handled by throwing out as infeasible those paths that implied loan values greater than the constraints. It should be stressed that there is no guarantee that the optimal paths found by the algorithms are the true optimal paths. Cost considerations limited the amount of searching that could be

done, both with respect to the scanning involved in the solution of the sub-maximization problem and the searching involved in finding the optimal price path. Particular attention was concentrated, however, on searching and scanning over values of the control variables for the first few periods of the horizon, so that some confidence could be placed on the assumption that the values chosen for the current period are close to the true optimizing values. If a firm reoptimizes each period, after data from the previous period become available, then values of the control variables for periods other than the current period are of importance only insofar as they affect the values for the current period. The solution of the control problem of the firm took about 15 seconds on an IBM 360-91 computer.

VI. SOME EXAMPLES OF SOLVING THE CONTROL PROBLEM OF THE FIRM

Parameter Values and Initial Conditions

The parameter values and initial conditions that were used for the first example are presented in Table 2. The most important parameters are β_8 , the measure of the extent to which firm i expects firm j to respond to firm i 's price-setting behavior; β_6 , the measure of the extent to which firm i loses or gains market share as its price deviates from firm j 's price; β_{11} , the measure of the extent to which firm i expects firm j to respond to firm i 's wage-setting behavior; β_7 , the measure of the extent to which firm i loses or gains its market share of labor as its wage deviates from firm j 's wage; β_9 , the measure of the extent to which firm i expects firm j to raise its price in period t as a result of firm j 's inventory situation in

Table 2

Parameter Values and Initial Conditions

<u>Parameter</u>	<u>Value</u>	<u>Variable</u>	<u>Value</u>
$T+1$	30	K_{lit-1}^a	250.0
d_1	0.5	K_{lit-2}^a	0.0
m	10	$I_{lit-1}, \dots, I_{lit-m}$	25.0, ..., 25.0
λ_1	1.3212	$I_{2it-1}, \dots, I_{2it-m}$	0.0, ..., 0.0
λ_2	1.30	V_{it-1}	52.625
μ_1	1.684	HPF_{it-1}	318.65
μ_2	1.684	HPF_{it-2}	318.65
\bar{H}	1.0	$PFF_{it-1}, \dots, PFF_{it-m+1}$	1.0, ..., 1.0
δ_1	1.0	LF_{it-1}	164.05
δ_2	0.9	DDF_{it-1}	25.15
β_1	0.125	XF_{it-1}	421.0
β_2	0.010	X_{t-1}	842.0
β_3	0.150		
β_4	0.050	PF_{it-1}	1.0
β_5	0.250	PF_{jt-1}	1.0
β_6	-8.0	HP_{t-1}	637.3
β_7	2.0		
β_8	0.5	WF_{it-1}	1.0
β_9	-0.030	WF_{jt-1}	1.0
β_{10}	0.3	XF_{jt-1}	421.0
β_{11}	0.5	V_{jt-1}	52.625
β_{12}	1.0	DDF_{2i}	2.5
β_{13}	-1.0	RF_{it}	0.0750
β_{14}	0.07108	Y_{it-1}	421.0 [= $\lambda_1 HPF_{it-1}$ = $\mu_1 K_{lit-1}^a$]
\bar{K}	250.0		

period $t-1$; and the four parameters reflecting inventory, sales-adjustment, hours-adjustment, and capital-adjustment costs, β_2 , β_3 , β_4 , and β_5 . The parameter values and initial conditions were chosen, after some experimentation, so that the optimum values of each control variable for periods t through $t+T$ would be essentially the same as the initial value for period $t-1$. This was done to make it easier to analyze the effects on the behavior of the firm of changing various initial conditions. As can be seen from Table 2, the initial conditions for the first example correspond to firm i having half of the sales in period $t-1$ and half of the labor employed. The firm holds no excess labor nor excess capital in period $t-1$. The two firms' prices and wage rates in period $t-1$ are the same. All of the machines held by firm i are type 1 machines. The length of the decision horizon is 30 periods, and the length of life, m , of a machine is 10 periods. The values in Table 2 correspond to the firm having profitable investment opportunities in the sense that, ignoring adjustment costs, the present discounted value of the revenue stream generated by an extra unit of investment is greater than the initial cost.

The results of solving the control problem of the firm for the parameter values and initial conditions in Table 2 are presented in the first row of Table 3. Only a small subset of the results are presented in Table 3, as it is not feasible to present all 30 values for each variable. Values of the price variable are given for periods t , $t+1$, and $t+2$, and then values for period t are given for the expected price of firm j , the expected level of sales, the planned level of production, the expected supply of labor, the wage rate, the value of loans, planned excess labor

TABLE 3

Results of Solving the Control Problem of the Firm

Initial Conditions from Table 2 except:	PF_{it}	PF_{it+1}	PF_{it+2}	PF_{jt}^e	XF_{it}^e	Y_{it}	INV_{it}	HPF_{it}^e	WF_{it}	LF_{it}	Planned excess labor for period t	Planned excess capital for period t
1. No exceptions	1.0000	0.9995	1.0000	1.0000	421.0	421.1	25.1	318.75	1.0001	164.12	0.0	0.0
2. (demand increase) ^a	1.0085	1.0090	1.0085	1.0075	427.1	424.1	26.9	326.85	1.0134	166.81	0.0	0.0
3. (demand decrease) ^b	0.9928	0.9928	0.9938	0.9938	414.6	411.3	24.4	313.98	0.9914	162.86	0.0	5.2
4. (excess labor) ^c	0.9995	1.0000	1.0000	1.0000	422.7	422.0	25.6	319.97	0.9917	163.93	0.0	0.0
5. (excess capital) ^d	0.9995	1.0000	1.0000	1.0000	422.7	421.1	25.0	319.60	1.0011	163.02	0.0	11.2
6. RF_{it} 0.0800 (+6.7%)	1.0000	1.0000	1.0005	1.0000	421.0	419.5	24.1	317.72	0.9988	163.13	0.0	0.0
7. RF_{it} 0.0850 (+13.3%)	1.0000	1.0000	1.0005	1.0000	421.0	419.5	21.7 ^e	318.22	0.9995	161.01	0.0	0.0
8. RF_{it} 0.0350 (-53.3%)	0.9995	0.9995	0.9995	1.0000	422.7	422.9	26.1	320.89	1.0027	165.30	0.0	0.0
9. $LFMAX_{it} = 160.84$	1.0000	1.0005	1.0005	1.0000	421.0	418.6	21.2 ^e	317.89	0.9990	160.57	0.0	0.0

^a $X_{t-1} = 863.0$, $XF_{it-1} = 431.5$, $V_{it-1} = 42.1$, $XF_{jt-1} = 431.5$, $V_{jt-1} = 42.1$

^b $X_{t-1} = 821.0$, $XF_{it-1} = 410.5$, $V_{it-1} = 63.1$, $XF_{jt-1} = 410.5$, $V_{jt-1} = 63.1$

^c $HPF_{it-1} = 326.62$, $HPF_{it-2} = 326.62$, $HP_{t-1} = 653.2$, $DDF_{it-1} = 25.71$

^d $K_{lit-1}^a = 262.5$; $I_{lit-1}, \dots, I_{lit-m} = 26.25$

^e Firm switched to machines of type 2.

$(HPF_{it}^e - MH_{it}^e)$, and planned excess capital in units of machines $(K_{lit}^a - K_{lit} + K_{2it}^a - K_{2it})$.⁹ The results in the first row show that the firm set essentially the same values for period t as existed for period $t-1$.

One of the most important reactions of a firm is how the firm responds to an increase or decrease in sales. For the results in row 2 of Table 3, sales in period $t-1$ were increased by 2.5 percent. Production for period $t-1$ was not changed, and so inventories for period $t-1$ were assumed to fall. Both firm i and firm j were assumed to have the same rise in sales and thus the same drop in inventories. The drop in inventories of firm j led firm i to expect firm j 's price for period t to rise to 1.0075. Firm i raised its price a little above this level, which caused its market share to decrease somewhat. Firm i ended up with expected sales of 427.1 for period t , compared to the level of 431.5 that it would have expected had it kept its price equal to the expected price of firm j . Production, investment, employment,¹⁰ and loans were all higher as a result of the sales increase. The wage rate was also higher since firm i needed to attract more workers to meet the increased employment requirements. Also, since firm i expected the average price in the economy to be higher in period t , this had a negative effect on firm i 's expectation of the aggregate supply of labor, which caused the firm to have to raise its wage rate more than it otherwise would have to attract the same amount of labor. Although not shown in the table, the higher expected average price also had a negative effect on firm i 's expectation of the aggregate demand for goods.

⁹The actual values of production, excess labor, and excess capital may, of course, differ from the planned values, depending on the accuracy of the firm's expectations and the mechanism by which transactions take place during the period. As discussed in footnote 8, the concern here is only with the planned values.

¹⁰By employment in this case and in what follows is meant the expected supply of labor.

For the results in row 3 of Table 3, sales in period $t-1$ were decreased by 2.5 percent. The results are essentially the opposite to those in row 2. Firm i lowered its price slightly from what it expected firm j 's price to be, which had the effect of increasing expected sales somewhat from what would have been the case had firm i kept its price the same as the expected price of firm j . Production, investment, employment, loans, and the wage rate were all lower as a result of the sales decrease. The firm also planned to hold excess capital in period t , which means that the firm did not plan to lower gross investment as much as it could have and still produce the planned output.

For the results in row 4 of Table 3, the employment of firm i for period $t-1$ was increased, with no corresponding increase in production. This meant that firm i held excess labor in period $t-1$. This change caused firm i to decrease its employment in period t from the level existing in period $t-1$ and to lower its price slightly in period t and increase its expected sales, planned production, and investment. Excess labor in period $t-1$ thus caused the firm to lower its price and expand slightly in period t . The wage rate was lower in this case, which was caused by the fact that the aggregate supply of labor in period $t-1$ was also increased for this run.

For the results in row 5 of Table 3, the number of machines held by firm i in period $t-1$ was increased, with no corresponding increase in production. This meant that firm i held excess capital in period $t-1$. This change also caused the firm to lower its price slightly in period t and to increase its expected sales. Investment dropped by 1.25 machines--from past gross investments of 26.25 to a gross investment of 25.0 in period t .

The firm chose to hold excess capital in period t of 11.2 machines.

Employment rose because of the investment-adjustment costs.

For the results in row 6 in Table 3, the loan rate was increased to 0.0800 from 0.0750. This caused the firm to raise its price for periods $t+1$ and beyond and to produce, invest, and borrow less in period t . For the results in row 7, the loan rate was increased even more to 0.0850, which caused the firm to switch to type 2 machines. In this case the price and production paths were the same as in row 6, but investment and loans were lower because of the switch to the cheaper machines. Employment was higher than in row 6 because of the greater employment requirements on type 2 machines. For the results in row 8, the loan rate was decreased to 0.0350. This caused the firm to lower its price for period t and for periods $t+2$ and beyond, which caused expected sales to increase and the firm to produce, invest, and borrow more. In this case it is, of course, not possible for the firm to switch to more expensive machines, since only two types of machines were postulated and the firm was already using the more expensive type.

For the results in row 9, the firm was assumed to be restricted in the amount of money that it could borrow. The maximum amount of money that the firm was allowed to borrow, $LFMAX_{it}$, was set to 160.84, compared with the unconstrained choice of the firm of 164.12.¹¹ This constraint caused the firm to raise its price for periods $t+1$ and beyond and to produce

¹¹For this example the firm was assumed not to be constrained for periods $t+1$ and beyond.

and invest less in period t . The firm switched to type 2 machines, which allowed the firm to spend less for investment than it otherwise would have had to, given the level of production, and thus to lower its loan requirements.

For none of the runs in Table 3, given the parameter values used, did the firm plan to hold excess labor in period t . In general, however, one would expect firms to adjust to falling demand situations by holding some excess labor in the current period.

VII. CONCLUSION

The results in Table 3 are not meant to confirm or refute the model in any way, but merely to aid in understanding the properties of the model.

Some of the main properties of the model are the following:

1. When demand increases and inventories decrease, the firm raises its price and increases its production, investment, employment, wage rate, and borrowings. The firm raises its price for two reasons. One is because it expects other firms to raise their prices, and the other is a desire to lower its market share somewhat to avoid having as large an increase in investment and employment as would be required if it kept its market share the same.
2. The opposite effects from 1. take place when demand decreases and inventories increase.
3. The existence of excess labor in a period causes the firm to decrease employment in the next period and to lower its price and expand production slightly.
4. The existence of excess capital in a period causes the firm to decrease investment in the next period and to lower its price slightly. For the results in Table 3, the firm did not choose in this case to increase production in period t , but in general one would expect this to happen given the higher level of expected sales in period t .

5. The firm responds to an interest-rate increase by lowering its production, investment, and borrowings. Employment may respond in either direction depending on whether or not the firm moves into cheaper types of machines with higher employment requirements.
6. The opposite effects from 5. take place for an interest-rate decrease.
7. The firm responds to a constraint on its borrowing behavior in a similar way that it responds to an interest-rate increase, by lowering its production and investment. Lower investment in this case may also take the form of purchasing cheaper machines.

Aside from the results in Table 3, a number of other properties of the model can be considered. One is the relationship of a firm's price decision to its wage decision. When a firm changes its price, this has, other things being equal, two main and opposite effects on the wage rate that it is likely to set. First, if a firm, say, increases its price, expected sales will be less and thus the firm is likely to decrease production. A decrease in production implies lower worker-hour requirements, which means that the firm is likely to want to employ fewer workers. Since in this case the firm has to induce fewer workers to work for it, it expects that it will need to pay a lower wage rate to attract the number of workers that it wants. So on this score a higher price implies a lower wage rate being set. On the other hand, if a firm increases its price, it expects the average price in the economy to be higher, especially a few periods into the future as other firms are expected to respond to the firm's higher price. A higher expected average price has a negative effect on the expected aggregate supply of labor, which implies a tighter aggregate labor market and thus the need to raise wages to attract the same number of workers.

So on this score a higher price implies a higher wage rate being set. The relationship between a firm's price decision and its wage decision is thus ambiguous.

An important property of the model is the asymmetrical behavior that it implies. For example, the firm's reaction to an increase and decrease in demand is asymmetric. First, although this is a fairly minor point, inventory costs are asymmetric in the sense that too high a level of inventories is less costly than too low a level. Second, and much more important, a decrease in demand means that the firm has the opportunity to hold excess labor and excess capital to help smooth out adjustment costs, whereas an increase in demand from a situation in which no excess labor or capital is being held means that the firm must either increase investment and employment immediately or must decrease its inventories. For the results in rows 2 and 3 in Table 3, for example, the firm chose to increase production by only 3.0 units corresponding to a 10.5 increase in sales of the previous period, whereas it chose to decrease its production by 9.8 units corresponding to a 10.5 decrease in sales. In the second case the firm planned to hold excess capital in the current period. Another example of an asymmetrical reaction in the model is reflected in rows 6 and 8 of Table 3. An interest rate increase of only 6.7 percent led to a price increase in periods $t+1$ and beyond, but it took an interest rate decrease of about 50.0 percent to lead to a price decrease. The firm's reaction to the loan constraint is, of course, another asymmetrical reaction in the sense that the firm is forced to respond to the constraint, but is not forced in the opposite direction where there is no constraint.

It was quite evident from examining various runs for the firm, using in many cases different sets of parameter values, that the firm was more inclined to choose to raise its price and lower its expected sales and production than to lower its price and raise its expected sales and production. This asymmetry appears to be due in large part to the possibility of being able to hold excess labor and capital on downswings, but having no corresponding possibility on upswings when already at full capacity.

Some of the important assumptions of the model are the assumptions of a putty-clay technology, the assumption of no technical progress nor other growth features, the adjustment-cost assumptions, the market-share assumptions, and the assumptions regarding the formation a firm's expectations of other firms prices and wage rates.¹² It would be of interest to examine the behavior of the firm under differing sets of assumptions, but such an undertaking is beyond the scope of the present study. Many runs were made in the present case corresponding to different sets of parameter values and initial conditions, but no further insights into the properties of the model were gained from this exercise other than those already reported, and so no further results will be presented here.

A few of the empirical implications of the model are the following. Since the decision of the firm regarding its price, production, investment,

¹²The assumptions of no retained earnings and no new stock issues are not as important as some of the others and are primarily needed only for the loan-constraint case. Even the loan-constraint case would go through in the face of retained earnings and new stock issues as long as the firm had to borrow some money from the outside and was constrained in doing so.

employment, and wage-rate all come out of the same maximization process, one should in empirical work probably consider these decision variables together. One should in particular be wary of including a decision variable on the right hand side of an equation explaining another decision variable. In some cases one may be able to consider the decisions of the firm as being made sequentially and specify, for example, that production is a function of sales and that employment and investment are functions of production. In general, however, one should probably use only nondecision variables as explanatory variables.¹³ In particular, the common practice of specifying a simultaneous-equations model determining prices and wages, in which the current price variable appears in the wage equation and the current wage variable appears in the price equation, is questionable in the present context. If both of these variables are decision variables of firms and thus affected by the same factors, they are likely to be highly correlated, but this does not mean that the variables ought to be explanatory variables of each other. Note also in the present context that inventory investment is a consequence of other decisions and is not a direct decision variable itself. It is thus questionable whether one ought to treat inventory investment as a decision variable, as is done in most macro-econometric models.

¹³Given the expectational assumptions of the present model, some of the important variables affecting the firm's decisions for period t are the prices and wage rates in period $t-1$, inventories in relation to sales in period $t-1$, the supply of labor and demand for goods in period $t-1$, the amount of excess labor and capital on hand at the beginning of period t , and the loan rate and loan constraint for period t .

Two other empirical implications of the model are that excess labor has a negative effect on employment and that excess capital has a negative effect on investment. The negative effect of excess labor on employment is confirmed by the results in [3]. The model also indicates that excess labor and capital have a negative effect on prices, which would be of interest to test. Similarly, the model indicates that the loan rate and the loan constraint may have a positive effect on prices, which would also be of interest to test. It is also of interest to note that the loan rate and other aspects of the cost of capital may have effects on investment that have nothing to do with capital-labor substitution in the sense of the firm purchasing different types of machines. A high loan rate, for example, may cause the firm to smooth production more relative to sales and thus to require less capital to meet peak production needs. A high loan rate, in other words, may cause the firm to hold on average less excess capital and thus to invest on average less. Similarly, a shift in the expected labor-supply schedule of the firm, in which the firm expects to have to pay a higher wage rate for the same amount of labor, may cause the firm to smooth production more relative to sales and thus to hold on average less excess labor. The loan rate and the labor-supply schedule may also, of course, affect the firm's price decision and thus its expected sales, which would further affect employment and investment. One should be aware in empirical work of the different ways in which the costs of labor and capital can affect employment and investment and should not necessarily attribute all of the estimated cost effects to the existence of capital-labor substitution.

It is obvious that expectations play a key role in the model, which emphasize the importance of trying to estimate or account for these expectations in empirical work. The importance of expectations also provides an explanation for why lagged endogenous variables appear to be so important in the explanation of the current endogenous variables in many models. When information is not perfect and decisions are made on the basis of imperfect information, it is quite likely that what has happened in the past will have an important effect on current decisions. One of the most important influences on the firm's price-setting behavior in the model is its expectation of other firms' prices. Consequently, any variable that is likely to influence this expectation is a candidate for inclusion in a price equation. "Cost-push" factors, for example, such as a shift in the labor-supply curve facing the firm, may be important in influencing the firm's price decision by affecting the firm's expectation of other firms' labor-supply curves and thus of their prices. This effect would be, of course, in addition to the direct effect of a shift in the labor-supply curve on the firm's behavior through the maximization process. The fact that expectations of other firms' behavior are so important in influencing the firm's price decision also means that one may observe periods in which prices and current aggregate demand do not move together, since factors other than current demand may at times dominate the determination of the firm's expectations.

Finally, the model indicates that asymmetries of behavior are important. Although asymmetries are difficult to deal with econometrically,