INTRODUCTION TO SELECTED PAPERS FROM THE THIRD NBER STOCHASTIC CONTROL CONFERENCE

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SUMMARY

We introduce the selected papers from the Third NBER Stochastic Control Conference which are published in the Spring, 1975 issue of the Annals of Economic and Social Measurement. The conference was held in Washington, D.C., from May 29 to May 31, 1974, and was cosponsored by the Board of Governors of the Federal Reserve System. Over seventy persons attended the conference. The papers deal with applications of stochastic control to macroeconomics and microeconomics, and with developments in control theory and methods.

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Following the conferences at Princeton University in 1972 and at the University of Chicago in 1973, the Third NBER Stochastic Control Conference was held at the National Academy of Sciences, Washington, D.C., on May 29-31, 1974, with the Board of Governors of the Federal Reserve System as a cosponsor. Twenty eight papers were presented. The conference program is included in the Appendix to this introduction. About 70 persons participated. In addition, numerous members on the staff of the Federal Reserve Board attended the sessions.

Most of the papers were submitted in response to an announcement and call for papers which I had circulated early in January, 1974. Michael Athans was responsible for a session of survey papers on dynamic game and team problems. David Kendrick helped organize a session on estimation and control. James Pierce coordinated several reports from the FRB staff on the nature of the SMP (SSRC-MIT-Penn) econometric model and its use for optimal control calculations at the Federal Reserve Board. Among the persons who contributed significantly to running this conference and the handling of its local arrangements, Steven M. Roberts and Evelyn Kender of FRB and Anna Trembley of NBER deserve our sincere thanks.

Less than half of the papers presented before the conference are included in this Special Issue of the Annals. Some papers have been submitted to other journals, and others are in the nature of progress reports or expositions already contained in other publications. The included papers do provide a picture of the current research activities in the field of stochastic control in economics. I will try to describe them briefly by way of an introduction. Readers interested in background material on the subject may refer to the introductory essays in the October, 1972, and January, 1974 issues of the Annals which reported on the first and second NBER stochastic control conferences, or to G. C. Chow, Analysis and Control of Dynamic Economic Systems, John Wiley and Sons, Inc., 1975.

The papers in this volume can be divided into three groups. The first is concerned with macroeconomic applications of stochastic control. The second with microeconomic applications and the third with developments in control theory and methods. In the macroeconomics group, the paper by Kenneth Garbade attempts to measure the extent to which discretionary policies could help stabilize the American economy in the 1960's. Discretionary policies are feedback policies. They assign values to the policy or control variables depending on the performance of the controlled system over the planning period so that the instruments are, in part, a function of the random disturbances affecting the system. Thus discretionary policies react to discrepancies between the actual and the desired behavior of the system. In contrast, nondiscretionary policies assign values to the instruments over the future irrespective of what occurs during the planning period. Maintaining constant rates of change for the policy variables exemplifies a nondiscretionary policy. Garbade employs a fairly sophisticated nondiscretionary policy, namely, the solution to the nonstochastic control problem formulated by ignoring the random disturbances in the econometric model. This permits him to measure the gain from feedback control. The model employed is a nonlinear quarterly econometric model consisting of some 43 structural equations. Besides the rates of unemployment and inflation, per capita consumption expenditures, per capita residential housing, period-to-period changes in government purchases of goods and services, in government employment, and in the Treasury bill rate, a federal personal tax scaling factor, Federal Home Loan advances, and government compensation to its employees enter the welfare function, the last six variables being control variables. For the eleven quarters beginning from the second quarter of 1960, Garbade has found that a discretionary policy would yield an expected loss (in weighted sum of squares of deviations of the selected variables from targets) equal to approximately half of the loss from applying a nondiscretionary policy. Besides its substantive conclusions, Garbade's paper has contributed to the methods of obtaining approximately optimal control solutions for nonlinear structural equations with random disturbances, assuming no uncertainty in the estimated parameters.

The paper by Andrew B. Abel attempts to measure the relative effectiveness of monetary and fiscal policies by comparing the optimal expected welfare loss obtained when both sets of instruments are applied optimally and when only one set can be freely used, with the other subject to a constant rate of change. The model used is a very simple one consisting of two equations explaining aggregate consumption and investment expenditures by their lagged values and by government expenditures and money supply. The last two are control variables representing fiscal and monetary policies respectively. It was found that expected welfare loss increases substantially if either instrument is not permitted to perform freely, thus confirming the importance of both instruments, but that the expected loss increases slightly more when government expenditures are restrained to play a passive role. Abel's study employs three different methods of control for linear stochastic systems, one assuming the model parameters to be known for certain, a second allowing for uncertainty in the parameters but ignoring the possibility of future learning about them, and the third incorporating an element of learning in the determination of the control policy for the first period. Although the above major conclusion is supported by calculations obtained by all three methods, his study illustrates the differences which uncertainty in the parameters can make in terms of the optimal feedback control equations and the associated expected welfare losses.

The paper by S. K. Gupta, Lawrence Meyer, Frederick Rains and T. J. Tarn attempts to study the effect of price control on economic stabilization in the context of three versions of a macroeconomic model. The first version is based on the Phillips-Lipsey formulation of the traditional Phillips curve which ignores price expectations. The second is attributed to the Friedman-Phelps-Mortensen formulation which implies the absence of long-run trade-off between inflation and unemployment. The third permits both price expectations and long-run trade-off. The model consists basically of five equations. An aggregate demand equation explains the total of the demand for consumption, investment and government spending. An aggregate output adjustment equation determines the change in output by the difference between aggregate demand and actual output of the preceding period, subject to total output not exceeding potential output. Third, the change in the price level depends on excess demand, expected price change (absent in the first version), and the existence of price control. Four, the expected price change is determined by a weighted sum of lagged expected price change and actual price change and by price control. Fifth, potential output is a function of the difference between actual and expected price change which affects labor supply (absent in the first version) and of the potential decline in supply due to the imposition of price control. The model is deterministic. Welfare loss is quadratic in the difference between actual and maximum output, the inflation rate, the change in government spending and the costs associated with price controls. It was found, for the particular numerical values assigned to the parameters, that the use of price control can reduce welfare loss substantially in the second version of the model but not in the first version.

In the borderline between micro- and macroeconomic applications, the paper by Gordon Rausser and Richard Howitt applies the framework of stochastic control to the regulation of wastes produced by a group of firms. Both firm behavior and the behavior of the government control agency have to be modeled. The three (vector) control variables are the tax rates on waste emissions, the frequencies of measurement of waste concentrations in selected locations, and the legal enforcement efforts. Firms take the costs of producing wastes into account in the maximization of profits, and thus the government tax rates affect the production of wastes and regular outputs. A dynamic system is derived with these two types of production and the waste concentrations in selected locations as the state variables and government tax rates as control variables. Besides, a set of observation equations determine the legally settled amounts of wastes produced by the firms and the measured (rather than the "true") amounts of waste concentrations, with frequencies of measurement and legal enforcement costs as control variables. A quadratic loss function is assumed. The selection of optimal waste taxes is found to be separable from the determination of measurement frequencies and legal efforts. The former is a linear-quadratic stochastic control problem to be solved by applying linear feedback control equations to the estimated states obtained by a Kalman filter. The latter is reduced to a nonlinear but deterministic control problem to be solved for all periods in the finite planning horizon by gradient or other numerical methods. Extensions to the model and directions for empirical implementations are suggested.

In the paper by Charles Tapiero, a random walk model is formulated to explain the effect of advertising on sales. A differential equation specifies the probability of selling x units at t as a function of the rate at which the customers forget and the rate of advertising. A diffusion approximation to this random walk model leads to a stochastic differential equation in sales. The mean of sales turns out to satisfy the same differential equation as the advertising model of Nerlove-Arrow, or of Vidale-Wolfe, depending on whether the effect of advertising is independent of the gap between a preassigned saturation level and existing sales. Thus a justification is provided for each of these models of advertising. Likelihood ratio tests are provided for testing various hypotheses concerning these models, and an application to testing the existence of economy of scale in advertising is given. Although the paper does not deal with the selection of an optimal advertising strategy, the stage is set, by providing the mathematical and statistical tools for the formulation and testing of stochastic models of advertising, for the application of optimal control techniques to the determination of advertising policies.

The paper by Chee-Yee Chong and David Cheng explores the behavioral implications of using adaptive control rules in the context of a monopolist facing a time-invariant linear demand function with unknown parameters. Maximizing expected profit over a finite number of periods, the monopolist behaves as if he were to solve an adaptive control problem of choosing optimal prices by the method of dynamic programming. Analytical solution cannot be obtained if both the slope and the intercept of the demand function are unknown. Several approximate solutions are applied, and results of simulation experiments are reported. It is found that pricing behavior when uncertainty about parameter values is accounted for can be quite different from the certainty-equivalent solution.

The problem posed by Edward Stohr in his paper, "A Model of a Project Activity," is to minimize expected total cost of completing a preassigned amount of work, measured by a scalar, given a production function (with only one input) and a cost function both of which are subject to additive random disturbances. There is no limit on the time required to complete the project. The problem is treated in both continuous and discrete time. For the continuous time problem, it is shown that a policy of applying a constant rate of input per period is optimal. For the discrete time problem, a constant-input policy based on certainty equivalence, an optimal constant-input policy, and the optimal policy are compared. Bounds are obtained for the differences between the first two policies; the second policy is found to be approximately optimal. Applications to some special production and cost functions are given. The implications for the design of control systems for activities of random duration are indicated.

Among the papers on control theory and methods, the one by J. B. Cruz is a survey of Nash and Stackelberg equilibrium strategies in dynamic games. In the classical control problem, there is one control agent whose actions alone, together with random disturbances from nature, determine the state of the system. In a dynamic game, the actions of several players affect the state of the system through a differential or difference equation, and each tries to maximize his own objective function of the state. While each player is assumed to know the differential equation for the state, his own strategy, and his own loss function, he may or may not know the state of the system at present and in the past. Different Nash equilibrium strategies are defined and studied according to the information available to each player. Oligopoly situations with intermediate-run horizons and an armament race between two nations are possible applications of Nash equilibrium strategies. On the other hand, if when there are only two players and one (the follower) can be assumed to take the strategy of the other (the leader) as given, the Stackelberg equilibrium strategies would be relevant. One possible application is to an optimal macroeconomic stabilization problem with the government viewed as the leader and the competitive private sector as the follower, but the government has to take the latter's reaction into consideration in the formation of its stabilization policies.

In a decentralized control problem, the state of the system is affected by the actions of several agents, as in a dynamic game. There is, however, one objective function which measures the overall performance of the entire system. A coordinator is assumed to exist who wishes to achieve the best overall performance of the system by allowing the local agents to operate according to certain rules. The paper by Michael Athans does not contain a well specified mathematical formulation of the decentralized control problem. Rather it reports on several attempts to search for a mathematical formulation which may justify the use of decentralized control. The approach taken is to modify the assumptions of the classical stochastic control problem with a single control agent, especially in regard to the information available to him. For example, the central agent or coordinator is allowed to know only the control actions taken by the local agents, but not their measurements of the state, or to receive local subsystem measurements or decisions only periodically. The information available to the coordinator and to the local agents will affect crucially the nature of the problem and thus the solution for decentralized control.

The importance of the information structure in multi-person optimization problems, whether the individuals are assumed to achieve their individual goals in a gaming situation or to assist in achieving a set of overall objectives for the system, is the subject of a survey lecture given by Y. C. Ho. No paper by Ho is included in this volume, but references to his published works include:

1. Y. C. Ho and K. C. Chu, "Information Structure in Dynamic Multi-Person Control Problems," Automatica, July 1974.

2. Yu-Chi Ho and Fang-Kuo Sun, "Value of Information in Two-Person Zero Sum Problems," Journal of Optimization Theory and Applications, to appear.

3. Tamer Basar and Yu-Chi Ho, "Informational Properties of the Nash Solutions to Two Stochastic Nonzero-Sum Games," Journal of Economic Theory, April 1974.

4. Y. C. Ho, I. Blau, and T. Basar, "A Tale of Four Information Structures," Proceedings of IRIA Symposium on Control Theory, June 1974; Springer-Verlag Notes on Mathematical Systems and Economics, October 1974.

The papers by Athans and Ho emphasize the dynamic aspects of the related problems treated by economists including:

5. Jacob Marschak and Roy Radner, Economic Theory of Teams, Yale University Press, 1972.

6. Theodore Grove and Roy Radner, "The Allocation of Resources in a Team," Journal of Economic Theory, June 1972.

7. Theodore Groves, "Incentives in Teams," Econometrica, July 1973.

Dynamic game and team problems are areas of mutual interests to economists and control scientists. Continuing exchanges are to be expected.

In summary, after editing this Special Issue, I have found that as the methods of optimal control are constantly improved upon to deal with more difficult situations such as nonlinear econometric systems, systems with unknown parameters, and systems with more than one control agent, they are being applied to economic problems of greater complexity. More complicated models are being used than before, as exemplified by the works of Garbade and of Rausser and Howitt. The amount and the variety of research as illustrated in this volume and in the Appendix confirms the fact that the subject is in an established and ongoing stage. At the time of writing this introduction in February 1974, a Fourth NBER Stochastic Control Conference is being planned by David Kendrick and Edison Tse to take place in Cambridge, Mass., sometime in May, 1975. Hopefully, the papers in this volume will serve as a useful progress report in the development of the subject.

APPENDIX

Program of Stochastic Control Conference

Sponsored by

NBER Conference on the Computer in Economic and Social Research

and

Board of Governors of the Federal Reserve System

Washington, D.C., May 29-31, 1974

Wednesday, May 29

Morning — Arrival of Participants
Registration, National Academy of Sciences
2101 Constitution Avenue, N.W.

2:00 - 5:00 — Control in Macroeconomics I

Chairman: Gregory C. Chow (Princeton)

(1) Michael Athans (MIT) "The Interplay Between Modeling Accuracy and the Use of Optimal Feedback Control for Stochastic Linear Econometric Models"

(2) Kenneth D. Garbade (NYU) "Discretion in the Choice of Macroeconomic Policies"

(3) A. L. Norman and James L. Weatherby, Jr. (U. Tex.) "On Selecting Economic Targets"

(4) Roger H. Gordon (MIT) "The Investment Tax Credit as a Supplementary Discretionary Stabilization Tool"

(5) S. K. Gupta, Lawrence H. Meyer, Frederick Q. Rains, and T. J. Tarn (Washington University) "Optimal Coordination of Aggregate Stabilization Policy and Price Control: Some Simulation Results"

(6) Triveni N. Upadhyay and Rex J. Fleming (Texas Instruments), "On the Computational Aspect of Adaptive Control in Econometric Modeling"

6:00 - 7:00 — Reception at Roger Smith Hotel
Petite Ball Room

Thursday, May 30

9:00 - 12:30 — Control in Macroeconomics II

Chairman: James L. Pierce (FRB)

(1) Jared J. Enzler (FRB) "Overview of the SMP (SSRC-MIT-Penn) Model and Its Properties"

(2) Arthur M. Havenner, Jared J. Enzler, and Douglas Battenberg (FRB) "Mini-SMP: Properties and Problems in Estimation"

(3) Peter A. Tinsley, Roger N. Craine, and Arthur M. Havenner (FRB) "Control Solutions to the Mini-SMP"

(4) R. S. Pindyck and Steven M. Roberts (MIT and FRB) "Optimal Monetary Policy - Some Further Results"

(5) Benjamin Friedman and E. P. Howrey (Harvard and U. of Michigan) "Nonlinear Models and Linearly Optimal Policies: An Evaluation"

2:00 - 5:00 — Estimation and Control

Chairman: David Kendrick (U. Texas)

(1) Edison Tse (Systems Control) "Identification Problems in Econometric Models"

(2) Raman Mehra and P. S. Kirshnaprasad (Harvard) "A Unified Approach to the Structural Estimation of Distributed Lag Models and Stochastic Differential Equations"

(3) Gregory C. Chow (Princeton) "A Solution to Optimal Control of Linear Systems with Unknown Parameters"

(4) Andrew Abel (Princeton) "A Comparison of Three Control Algorithms as Applied to the Monetarist-Fiscalist Debate"

(5) Reports of Joint Control Engineer--Economist Projects on Estimation and Control

(a) Kuh, Athans, and Pindyck (MIT)

(b) Kendrick, Tse, Norman, Barshalon (Texas-Systems Control)

7:00 — Dinner, Watergate Terrace Restaurant
Speaker: Governor Andrew Brimmer (FRB)

Friday, May 31

9:00 - 10:30 Dynamic Game and Team Problems

Chairman: Michael Athans (MIT)

(1) J. B. Cruz, Jr. (Univ. of Ill.) "Survey of Dynamic Nash and Stackelberg Strategies"

(2) Y. C. Ho (Harvard) "Information Structures in Many-Person Optimization Problems"

(3) M. Athans (MIT) "Survey of Decentralized Stochastic Control Methods"

10:45 - 12:30 Control in Microeconomics I

Chairman: Gordon C. Rausser (University of Chicago)

(1) Edward A. Stohr (Northwestern U.) "A Model for Project Activities"

(2) Charles S. Tapiero (Columbia U.) "Optimum On-Line Advertising Control and Goodwill Under Uncertainty"

(3) James Thurber and Andrew Whinston (Purdue) "Stochastic Control Problems in Urban Planning"

2:00 - 5:00 Control in Microeconomics II

Chairman: Nils H. Hakansson (University of California, Berkeley)

(1) George Bitros and Harry Kelejian (NYU) "A Stochastic Control Approach to Factor Demand"

(2) David C. Cheng and C. Y. Chong (Georgia) "Multistage Pricing Under Uncertain Demand"

(3) David S. Sibley (Bell Labs.) "Permanent and Transitory Income Effects in a Model of Optimal Consumption with Wage Uncertainty"

(4) Nils H. Hakansson (U. of California, Berkeley) "Convergence to Isoelastic Utility and Policy in Multiperiod Portfolio Choice"

(5) David G. Luenberger (Stanford) "An Optimal Control Problem with a Linear Feedback Solution"

(6) Gordon C. Rausser and Richard Howitt (U. of Chicago and U. of California, Davis) "Optimal Stochastic Control of Environmental Externalities"