A WAGE AND PRICE ENDOGENOUS MODEL FOR PLANNING INCOME DISTRIBUTION POLICY

Irma Adelman

and

Sherman Robinson*

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*The authors are respectively Professor of Economics, University of Maryland and Assistant Professor of Economics, Princeton University. Paper prepared for the World Congress, Econometric Society, Toronto, Canada, August, 1975.

Econometric Research Program
PRINCETON UNIVERSITY
207 Dickinson Hall
Princeton, New Jersey

This paper introduces a new kind of planning model designed for income distribution planning and implemented empirically with data from South Korea. The essential distinguishing features of the model are: (1) it solves for wages and prices endogenously in both factor and product markets; (2) its solution is based on achieving consistency among the results of individual optimizing behavior by a large number of actors (households and firms), (3) it incorporates monetary phenomena, and (4) it is dynamic, with imperfect inter-temperal consistency. In addition, the model incorporates government behavior, the foreign trade sector, and allows for varying principles of market clearing and institutional behavior.

I. Précis of the Model

For each period, the computation of the model is decomposed into three stages. The Stage I model describes the contracts made between firms and the financial markets to spend funds on investment goods. Stage II describes how factor and product markets reach an equilibrium constrained by the investment commitments undertaken in Stage I and by various institutional rigidities imposed by foreign trade and by the operation of product and labor markets. Stage III serves to generate the expectations on which Stage I decisions are based, to set some of the rules of its operation (e.g., the credit regime), and to "age" the model economy. Stage II is the core of the overall model, and represents the basic wage and price endogenous (WPE) general equilibrium model used for comparative-statics experiments. Stages I and III are used only in the dynamic analysis.

Stage I models the loanable funds market. Producers form their demands for loanable funds on the basis of expected sales and expected prices of inputs. Credit is then rationed either by setting an interest rate and allowing the market to clear at that rate (called finance minister 1) or by setting a target rate of expansion of credit and allowing the rate of interest to adjust in order to clear the loanable funds market (finance minister 2). A third possibility (finance minister 3) is the direct rationing of credit to firms by the government -- an option that completely circumvents the market mechanism. The output of Stage I is the allocation of loanable funds among firms and sectors, and an overall injection of credit into the economy. Stage I is diagrammed in Figure 1.

Stage II is a micro-economic market model. On the supply side, there are twenty-nine sectors of production each of which is disaggregated into four firm sizes. These firms are assumed to maximize profits subject to a number of different constraints. On the demand side, the household sector is modelled in some detail. There are fifteen different types of households, each corresponding to a different socio-economic category. Within each category, there are specified distributions of individual households. Sets of demand curves are specified for each category of household. Foreign trade and government spending (and taxes) are also modelled.

The Stage II model is a general equilibrium model in that prices and/or supplies are assumed to adjust so as to clear all markets, subject to various constraints. The Stage II model is itself subdivided into a number of parts representing different computational phases: supply, demand, wage, income and price determination. The output of this is "actual" production, employment, prices, wages and income distribution for the period. Figure 2 portrays the basic model structure.

In each period, the three stages are solved serially. Variables which are assumed fixed in Stage I are allowed to vary in Stage II. Thus the overall

Figure 1
STAGE | MODEL

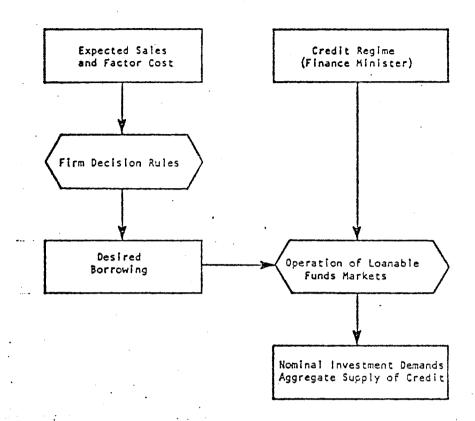
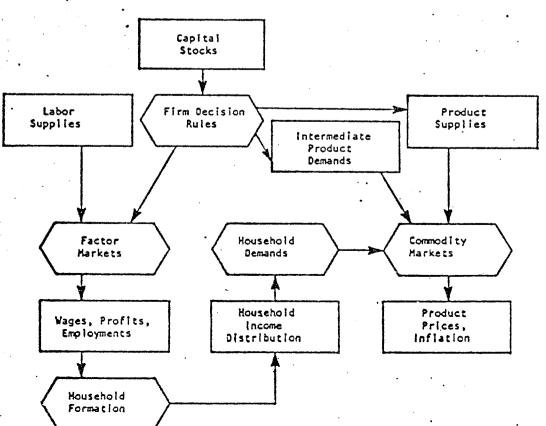


Figure 2

STAGE II HODEL



model distinguishes between expectations and realizations. In Stage III, the differences between expectations and realizations are incorporated into the forecast functions for the expected variables with which calculations are made in the periods subsequent to the first.

The overall dynamic model is an equilibrium model only in a very limited sense. The solution of the overall model is certainly not Pareto-efficient either in the static or dynamic sense. Efficiency in any one period could be improved by solving Stages I and II together, and achieving inter-temporal efficiency would require a much more complex linking of the models over time than is provided by the Stage III phase. In fact, the overall dynamic model represents a kind of "lurching equilibrium" which, it is hoped, represents a more realistic specification of actual growth than would be provided by some intertemporally efficient equilibrium growth model.

II. A Simplified Wage and Price Endogenous (WPE) Model

In this section, we present a very simple WPE model. The model is static and excludes any consideration of government, foreign trade, money, labor migration, or household formation. After discussing this model, we indicate how we have extended it to include the factors listed above. We will not present here the full set of equations for our model. \(\frac{1}{2}\)

THE PRODUCING SECTOR

Production Functions

Output is assumed to be a function of two kinds of inputs:

 $R_{\lambda i}$: resources such as labor and fixed capital which are not produced this period. $R_{\lambda i}$ is use of resource λ in sector i. $\lambda = 1, \ldots, m$ $i = 1, \ldots, n$

 K_{ji} : Intermediate goods (circulating or variable capital). K_{ji} is the amount of good j used by sector i. j = 1,...,n; i = 1,...,n.

In general, production can be seen as a function of both kinds of inputs with substitution possibilities among them. However, we assume that there are substitution possibilities only among the non-produced resources. The requirements for intermediate resources are given by fixed input-output coefficients:

$$K_{ji} = A_{ji}x_{i}$$
.

The production function can be written as:

$$x_i = \min[f_i(R_{1i}, \dots, R_{mi}); K_{ji}/A_{ji}, j=1,\dots,n]$$
.

where $f_i(R_{1i},...,R_{mi})$ is a production function allowing substitution possibilities such as Cobb-Douglas or C.E.S.

It is convenient to assume that requirements for intermediate goods are never a binding constraint and so write the production function as:

(1)
$$x_i = f_i(R_{1i}, ..., R_{mi})$$
 n equations

The demands for intermediate goods will be accounted for in the material balance equations below.

Net Prices

Firms are assumed to maximize profits. Since they are assumed to require intermediate goods in fixed proportions, their demands for such goods are linear functions of output. In terms of the firm's behavior, the cost of intermediate goods is simply proportional to output. The firm's set receipts can be viewed as the sale price minus the fixed charge for intermediate goods. Thus, we can define the net price as:

(2)
$$p_{i}^{*} = p_{i} - \sum_{j} A_{ji} p_{j}$$
 n equations

In terms of hiring decisions for other factors (and assuming perfect competition) the firm views the net price as the marginal revenue from the sale of one more unit of output.

Factor Markets

The first order conditions for profit maximization require that the firm hire factors until wages equal marginal revenue products:

$$\begin{array}{ccc}
* & \partial f_i \\
p_j & \partial R_i & = w_{\lambda} & .
\end{array}$$

We are here assuming that wages for a given factor are equal across all sectors and that there is perfect competition in the product markets.

The first order conditions can be solved to give factor demands as a function of wages, the net price, and the employment of other factors:

(3)
$$R_{\lambda i} = g_{\lambda i}(p_i^*; w_1, ..., w_m; R_{ki}, k=1,...,m \text{ and } k \neq \lambda)$$
 m·n equations

The solution of the first order conditions for the factor demands is straight-forward for Cobb-Douglas functions. For more complicated functions, it can be necessary to solve for the factor demands numerically.

To calculate total demands for factors, simply aggregate the demands by sectors:

(4)
$$\mathbf{r}_{\lambda}^{d} = \sum_{j} \mathbf{R}_{\lambda j} \qquad \qquad \mathbf{m} \text{ equations}$$

The aggregate supplies for factors in this simple model are simply assumed to be fixed.

(5)
$$r_{\lambda}^{s} = \bar{r}_{\lambda}^{s}$$
 m equations

Equilibrium in the factor markets require that factor demands equal factor supplies or that excess demands for factors equal zero.

(6)
$$r_{\lambda}^{d} - r_{\lambda}^{s} = 0 \qquad m \text{ equations.}$$

Wage Determination

Assume for the moment that product prices, p_i , are given. Then the system of equations given by (1) - (6) can be solved simultaneously to give wages, employment, and production. With prices given and net prices from equation (2), the excess demand equations (6) can be seen to be a function of wages, w_{λ} . Simply substitute equations (3), (4), and (5) into (6). The problem then is to find a set of wages that clears the factor markets. Formally, the problem is that of finding the solution of a set of simultaneous non-linear equations. The techniques used to solve such a system are discussed in Adelman and Robinson [1973].

In order to solve the factor market equations, it was assumed that product prices are given. The solution of the factor markets can be seen as one part of the overall solution since solving the overall model will involve clearing product markets as well as factor markets. The solution of the factor markets, as well as yielding wages and employment, gives product supplies from equation (1). Thus, this part of the model can be seen as yielding the supply of goods given prices.

Note that in solving for equilibrium wages, all factors are assumed to be completely mobile among sectors. While this might be a reasonable assumption for some purposes, it is not reasonable to assume a factor like fixed capital is mobile among sectors in the short run. A factor can easily be assumed to be fixed by simply setting its demand exogenously in equation (3). The excess demand in equation (6) is identically zero and its wage -- which will in general now differ by sectors -- is calculated as a side equation from the marginal productivity equation.

THE CONSUMING SECTOR

Product Markets

The demand side of the model involves first calculating total income by factors -- the functional distribution of income. Aggregate income by factors is given by:

(7)
$$y_{\lambda} = \sum_{j} w_{\lambda} R_{\lambda j} \qquad m \text{ equations}$$

In this simple model, each factor type is supposed to consist of a homogeneous group of people whose consumption behavior can be represented by the same expenditure function. Thus the demand for goods by each group can be written as:

(8)
$$C_{i\lambda} = h_{\lambda}(y_{\lambda}; p_{1},...,p_{n})$$
 m·n equations

The demand equations are written quite generally. The only requirement on them is that they satisfy the budget constraint; i.e., that

$$\sum_{j} p_{j} \cdot C_{j\lambda} = y_{\lambda} \qquad \lambda=1,...,m .$$

The literature on systems of demand equations is quite active and the particular choices we made are discussed in Adelman and Robinson [1973]. We have assumed that the demand equations are derived from utility maximizing behavior by consumers.

As described so far, the model apparently does not include any investment. The model can be easily reinterpreted to include investment behavior in a Kaldorian fashion. Assume that one of the factors is capital and that capital income is spent on investment goods -- capitalists save all and invest and everyone else consumes. One of the demand equations is then reinterpreted as determining the demand for investment goods by sector of origin. Somewhat more realistic models could be formulated by assuming different savings behavior by different groups. Our full model is significantly more complicated because firms, workers, and households are all treated separately for various purposes.

A simplification of this model which has been employed in virtually all of the current price endogenous models is to assume that there is only one set of consumer demand equations. For purposes of calculating consumer demand, all consumers are aggregated together. This simplification has some important implications for solution strategies. However, we do not use this approach.

The product demands by groups can be aggregated to determine total demand for goods from each sector.

(9)
$$c_{i}^{d} = \sum_{\lambda} C_{i\lambda} \qquad \qquad n \text{ equations}$$

The aggregate supply of each good, x_i , was determined in the solution of the factor markets. The supply of goods for final demand is determined by subtracting the demand for intermediate goods in the material balance equations:

(10)
$$c_{i}^{s} = x_{i} - \sum_{j} A_{ij} x_{j}$$
 n equations

Equilibrium in the product markets requires that the excess demands for each good be zero:

$$c_{i}^{d} - c_{i}^{s} = 0 n equations$$

Equations (7) - (10) can be substituted into equation (11) to yield n excess demand equations in n prices (assuming that the factor markets are solved for each set of prices). The problem is that of solving a general equilibrium system for market clearing prices and wages. It is well known that such a general equilibrium system can only determine relative prices. Thus we are free to impose some normalization on prices and wages. We choose simply to set the level of the wholesale price index exogenously:

(12)
$$\bar{p} = \sum \omega_i p_i$$
 1 equation

There are a number of different strategies and techniques that can be used to solve the model economy given by equations (1) - (12). We have chosen to iterate on prices following a kind of tatonnement procedure. First, start with an initial guess at prices. Second, solve the factor market equations for wages, employment, and production. Third, solve the product market equations and calculate excess demands. Fourth, raise or lower prices in sectors where there are excess demands or supplies. Fifth, normalize prices according to equation (12) and start another iteration. Stop iterating when all excess demands equal zero. Our particular method is discussed in more detail in Adelman and Robinson [1973]. It is worth noting here, however, that solutions were achieved in five to twenty price iterations in virtually all of our experiments.

This simple model is extended in a number of ways. First, the full model includes a model of household formation which also generates household income. Second, the overall size distribution of income is generated by assuming that each category of households (defined by the job category of the household head) has a within-group size distribution given by a two parameter lognormal distribu-The overall size distribution is obtained by summing the distributions for fifteen different groups. Fourth, the full model includes money supply and demand endogenously. Money is not "neutral" in the full model because the real equations of the full model are not homogeneous of degree zero in wages, prices, and money. Furthermore, the nominal injection of money into the full model is partly determined endogenously. Fifth, the full model includes a specification of foreign trade that includes the possibility of some product prices being fixed by the world price and various specifications of importing and exporting behavior. Sixth, the full model is dynamic and includes a number of sub-models (such as a specification of rural-urban migration) to "age" the economy. In general, in the full model, capital is not assumed to be mobile among sectors and firm sizes. Investment decisions are made in Stage I, ratified in Stage II, and the capital is installed in Stage III. Within Stage II, capital is immobile.

III. Discussion of the Model

There are currently two approaches to specifying endogenous price models: a programming approach and a Walrasian approach. The programming approach is based on the fact that if one maximizes the sum of producers' and consumers' surplus across all markets, the maximum will be reached when prices (and quantities) are those that would be achieved by the operation of competitive markets. Application of the programming approach is computationally very cumbersome. The economic assumptions and theoretical compromises that one must make in order to use this approach as a market simulator are quite restrictive and unrealistic unnecessarily so considering that alternative approaches are operational.

The Walrasian approach represents the general strategy of directly solving for market clearing prices and wages consistent with the specified behavior of the various actors in the economy. Formally, the problem can be expressed as that of finding a fixed point for a set of equations -- the well known economic problem of finding a general equilibrium solution. This direct approach thus permits a great deal of freedom in the specification of how the various actors behave and of how the markets work. One is not restricted to linear equations but the system must satisfy some minimal convexity conditions if one is to be sure a solution exists.

Our model is of a Walrasian type and is quite comprehensive in its degree of "closure," i.e., the number of features of the economy which are endogenous. When compared with other economy-wide models, the model here makes fewer compromises in the names of "simplicity" and "solvability." In this sense, the philosophy of our model is closer to that of simulation models, although it

contrasts sharply with most simulation models in its theoretical and empirical grounding. The costs of this approach are that the model is both large and complex, although neither too large to solve economically nor too complex to be able to trace the major effects through the general equilibrium system.

In our model, the household and income recipient accounts, are determined endogenously in accordance with behavioral specifications. There is strict accounting both of household income and household composition by workers. The model explicitly goes from factor payment and employment to household incomes with savings and expenditure decisions being modelled at the household level rather than at a more aggregate level. Accounting consistency is maintained among: (1) household, firm, government, and trade accounts; (2) national income accounts; (3) input-output accounts; and (4) the national product accounts. The overall size distribution of household incomes is determined by explicit aggregation of household incomes. In addition, consistency is maintained between the labor force and the number of households.

The model also has a number of features in common with macro-economic models. It incorporates a demand for and supply of both loanable funds (Stage I) and new cash holdings (Stage II). The rate of inflation is determined endogenously and money is important in that the real economy is not invariant with respect to changes in the money supply and general inflation. The model solutions indicate that the degree of inflation is an important mechanism by which the model economy achieves internal consistency under shocks imposed by attempts at policy intervention. The inclusion of inflation is important.

The model combines Keynes and Walras in that it can portray open unemployment and in that the demand for cash balances (the Cambridge k) is set for each period (in Stage III) as a function of anticipated inflation and interest rates. The fact that the model is a more-or-less complete microeconomic model with money in it should make it very useful for many other purposes than studying income distribution. It is likely to be useful in inflation control planning and in analyzing the trade-off between inflation and unemployment. The model is in the tradition of economy-wide planning models. While it has as a primary focus the incorporation of the distribution of income, it also includes all the components of more traditional non-optimizing planning models plus monetary elements typical of a macro-model. It could certainly form the basis of a national planning exercise.

IV. Some Empirical Results

The full model has not been developed for forecasting purposes. Rather, it has been developed to provide an empirical laboratory in which one can study the effects of various medium run policy measures and of variations in behavioral parameters on the distribution of income. This focus has strongly influenced our modelling philosophy. It has led to a relative neglect of certain sections of the model, and has affected the manner of disaggregation of productive sectors, factors of production, socio-economic groups and degree of institutional specificity.

Our focus on policy experiments has also led us to construct the model so as to incorporate, from the very beginning, the possibility for alternative specifications of model subsectors. Thus, the model is capable of portraying a large variety of economic and institutional rules of the game. One can vary the degree of monopoly, the principles of operation of credit markets, the clearing principles for labor markets and for commodity markets, and permitted degrees of disequilibrium in individual markets. The model can operate in a Keynesian or in a neoclassical mode, and in an equilibrium or disequilibrium vein. Different functional specifications for production functions, consumption functions and different objective functions for firms and households can be used.

Table 1

DESCRIPTION OF COMPARATIVE STATICS EXPERIMENTS

Adelman/Robinson Model

	·
Experiment	
A-1	Peg agricultural prices at 1.25 of base price.
A-2	Increase agricultural capital stock by 50%.
A-3	Increase agricultural productivity 25% in cereals and 15% is other agriculture. Peg both prices at .90 of base price, and allow free trade in agricultural products.
A-4	Increase productivity in cereals by 25% and give cereals a price subsidy of 25%.
8-1	Double all direct taxes on households and transfer the proceeds to skilled labor, apprentices, and workers.
B-2	Give price subsidy of 20% to labor intensive sectors. Increase their capital stock by 20%. Peg their prices to world prices, allowing unlimited exports.
B-3	Same as B-2 but, in addition, increase agricultural productivity by 25%.
· C-1	Increase entire capital stock by 50%.
C-2	Increase capital stocks by 50% for non-agricultural firms with less than 50 employees (two smallest sizes).
D-1	Increase all exports by 25%.
D-2	Devalue by 30%. Peg prices of traded goods to world prices. Increase capital stocks of trading sectors by 20%.
E-1	Increase supply of engineers, technicians, skilled workers, and clericals by 10%, reducing the number of unskilled workers correspondingly. CES functions.
E-2	Increase elasticity of substitution between capital and labor in manufacturing.
E3	Set minimum wage at 125% of unskilled wage rate.

Table 2. DISTRIBUTION OF HOUSEHOLD INCOME (%) AFTER TAXES AND TRANSFERS

															•	
Exper	iment:	A-1	A-2	A-3	A-4	8-1	8-2	8-3	C-1	C-2	0-1	0-2	E-1	E-2	E-3	Base
Bottom	101	2.41	2.31	2.39	2.40	2.50	2.42	2.06	2.33	2.44	2.45	2.45	2.27	2.10	2.19	2.50
DECILES	20	3.88	3.63	3.75	3.79	3.99	3.89	3.29	3.76	3.88	3.90	3.89	3.70	3.42	3.60	3.90
	30	4.91	4.63	4.75	4.80	5.04	4.93	4.24	4.79	4.90	4.92	4.90	4.75	4.44	4.67	4.90
	40	5.93	5.63	5.75	5.81	6.07	5.95	5.23	5.82	5.91	5.92	5.91	5.78	5.49	5.75	5.90
	50	7.04	6.74	6.85	6.92	7.18	7.10	6.34	6.96	7.01	7.02	7.01	6.93	6.68	6.92	6.97
	60	8.34	8.06	8.15	8.24	8.47	8.38	7.69	8.31	8.32	8.30	8.30	8.28	8.11	8.29	8.24
	70	.10.00	9.77	9.83	9.92	10.08	10.04	9.47	10.03	9.99	9.93	9.95	10.01	9.97	10.03	9.86
	80	12.33	12.20	12.23	12.30	12.32	12.38	12.10	12.45	12.34	12.24	12.28	12.44	12.58	12.45	12.15
	90	16.24	16.40	16.34	16.34	16.07	16.30	16.81	16.51	16.31	16.17	16.24	16.46	15.89	16.46	16.08
	100	28.93	30.62	29.97	29.47	28.27	28.66	32.78	29.03	28.90	29.14	29.07	29.36	30.33	29.63	29.49
· e	ini	.38	.40	.39	.39	.37	.38	.43	.39	.38	.38	.38	.39	.41	.40	.38

Table 3. PERCENT OF HOUSEHOLDS IN THE BOTTOM DECILE Who are the Poor?

	Experiment:	A-1	A-2	A-3	A-4	8-1	8-2	8-3	C-1	C-2	D-1	D-2	E-1	E-2	E-3	Base
1	Engineers	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
2	Technicians	0.1	0.0	0.0	0.0	0.1	0.0	0.0	0.0	0.1	0.0	0.0	1.0	1.0	0.5	0.8
3	Skilled	14.2	3.9	1.5	6.5	3.7	9.5	2.2	17.7	14.4	7.5	6.9	9.0	16.8	3.9	8.8
4	Apprentices	0.4	0.2	7.6	0.3	0.2	0.5	0.2	0.0	0.5	0.3	0.3	0.5	0.6	0.2	0.3
5	Workers	24.3	12.4	7.9	13.8	13.5	17.0	5.4	15.2	16.8	19.1	15.1	5.6	28.5	10.5	19.4
. 6	White Collar	4.5	1.7	0.4	2.2	4.4	2.2	0.2	2.1	2.5	3.4	2.2	6.2	3.5	2.7	3.2
7	Government	8.6	5.4	3.0	9.4	11.1	13.0	8.0	11.0	10.3	9.2	11.8	6.9	4.5	6.5	7.4
8	Self-Employed 1	9.5	5.3	3.3	7.4	10.1	10.6	5.5	13.4	10.4	8.8	ੑ8.8	3.6	5.7	3.0	7.6
9	Self-Employed 2	11.1	7.1	3.1	9.7	12.3	15.9	7.7	18.7	14.2	9.8	12.7	5.4	4.9	5.2	9.8
10	Capitalists	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
11	Ag Labor	5.6	11.2	5.7	9.2	8.9	5.1	13.4	4.4	6.4	8.4	8.5	33.0	22.9	34.2	8.4
12	Agric. 1 (Tiny)	13.4	30.6	36.1	25.6	20.9	17.1	34.4	11.6	14.9	19.7	20.3	18.6	8.0	21.1	20.2
13	Agric. 2 (Small)	7.8	19.6	22.8	14.2	13.8	7.7	21.3	5.8	9.0	12.8	12.6	10.0	3.5	11.8	13.2
14	Agric. 3 (Medium)	0.4	1.4	8.5	0.7	0.9	0.3	1.8	0.2	0.4	0.8	0.7	0.3	0.0	0.3	0.9
15	Agric. 4 (Large)	0.0	0.0	0.2	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
	Average HH Income	45.	47.	31.	51.	48.	52.	52.	52.	49.	47.	.: 48.	45.	43.	42.	47.

Average income is in 1,000's of real (1968) Won.

This model has been implemented for South Korea. Some indication of the validity of the model may be inferred from the fact that the base period solution for the model reproduces the 1968 data to within less than one percent for every one of the approximately 2,000 endogenous variables.

Some of the comparative statics experiments performed to date are listed in Table 1. The results of these experiments are summarized in Tables 2 and 3. Each column of Table 2 indicates the distribution of household income after the model is solved under the conditions of the corresponding experiment, indicated at the top of the column. There appears to be a remarkable insensitivity of the relative income distribution to the experiments performed. Table 3 indicates the composition by socio-economic group of the lowest decile for each of the experiments. The bottom row of the table gives the average real houshold income of the decile. An examination of this table suggests, by contrast, that economic policy, by changing the functional distribution of income, can play a major role in determining who are the poor and, in particular, whether they are primarily urban or rural. There is also somewhat more scope for affecting the absolute level of poverty through economic intervention than for changing the overall distribution of incomes.

The remarkable insensitivity of the relative shares of national income accruing to households by deciles to the wide array of economic policy packages represented in the model suggests that, while one can make small gains in the welfare of the poor through large changes within the system, the goal of equity cannot be achieved without radical reform. This result is consistent with recent work by Adelman-Morris. 6

FOOTNOTES

- 1. A complete discussion of the full model equations is available in Adelman and Robinson [1973].
- 2. See Bronfenbrenner [1971] for a discussion of the use of the lognormal distribution to represent the income distribution.
- 3. See Goreux and Manne [1973] and Takayama and Judge [1971]. Chenery and Raduchel [1971] is a variant of this approach which does not explicitly model market behavior.
- 4. Recent practice indicates that our approach and even the Scarf fixed point method are computationally much more efficient for models of any size.
- 5. There is a growing body of computational experience with such models. The Scarf algorithm -- which treats the problem explicitly as a fixed point problem -- has been extended and improved so that now it can be used on problems of moderate size (say, ten to fifteen "prices"). It is being (or soon will be) applied to somewhat larger problems. See Scarf [1973] and Kuhn [1975]. More classical methods have been or are being applied successfully to a number of models, of which ours is currently the largest. For examples of completed work, see Dervis [1975] and Ahmed [1974]. Johansen [1960] formulated the first empirical price endogenous model which was linear in growth rates and so could be solved as a set of simultaneous linear equations. Taylor and Black [1974] have applied this technique in a model of Chile.

REFERENCES

- Ahmed, F. [1974] "Migration and Employment in a Multisector Model: An Application to Bangladesh." Princeton University, Economics Dept., unpublished PhD dissertation, December.
- Bronfenbrenner, M. [1971] <u>Income Distribution Theory</u>. Chicago: Aldine Atherton.
- Chenery, H. and W. J. Raduchel [1971] "Substitution in Planning Models."

 In H. Chenery, ed. Studies in Development Planning. Cambridge: Harvard University Press.
- Dervis, K. [1975] "Planning Capital-Labor Substitution and Intertemporal Equilibrium with a Non-linear Multi-sector Growth Model." <u>European Economic</u> Review, vol. 6, No.], January.
- Goreux, L. M. and A. S. Manne, eds. [1973] Multi-Level Planning: Case Studies in Mexico. Amsterdam: North Holland.
- Johansen, L. [1960] A Multi-Sectoral Study of Economic Growth. Amsterdam: North Holland.
- Kuhn, H. [1975] "How to Compute Economic Equilibria By Pivotal Methods." Princeton University, Mathematics Dept., January, mimeo.
- Scarf, H. E. and T. Hansen [1973] The Computation of Economic Equilibria.

 New Haven: Yale University Press.
- Takayama, T. and G. Judge [1971] Spatial and Temporal Price and Allocation Models. Amsterdam: North Holland.
- Taylor, L. and S. L. Black [1974] "Practical General Equilibrium Estimation of Resource Pulls Under Trade Liberalization." <u>Journal of International Economics</u>, Vol. 4, No. 1, February.