CONTROL METHODS FOR MACROECONOMIC POLICY ANALYSIS

Gregory C. Chow

ECONOMETRIC RESEARCH PROGRAM
Research Memorandum No. 189
November 1975

Paper to be presented before the annual meetings of the American Economic Association in Dallas, Texas, on December 28, 1975. Financial support from the National Science Foundation is gratefully acknowledged. This paper was typed by Alice Furth.

Econometric Research Program
PRINCETON UNIVERSITY
207 Dickinson Hall
Princeton, New Jersey

CONTROL METHODS FOR MACROECONOMIC POLICY ANALYSIS

Gregory C. Chow

I. Why Optimal Stochastic Control?

The use of econometric models for the quantitative analysis of macroeconomic policy is now generally accepted. A great deal of research in the last several years has been devoted to the development and application of optimal stochastic control techniques in policy analysis. This development appears to be natural and necessary. Using an econometric model, one can make projections of the key economic variables in future periods given a set of proposed future values for the policy variables or instruments. One may then examine the nature of these projections in order to evaluate a policy proposal. This approach to policy analysis is deficient for two reasons. First, the dynamic response of the economic variables to a particular course assigned to the policy variables is complicated and unpredictable. This makes the selection of policy by trial and error extremely inefficient. There is a need to specify a loss function of the key economic variables and to minimize its value with respect to the policy instruments. The specification of an objective or loss function and the derivation of a policy solution by optimization is an essential feature of optimal control. It is much more efficient than the method of trial and error. The objective function is also useful for the evaluation of other policy proposals than the optimum policies.

The second, and perhaps more important, reason is due to the uncertainty of economic projections. Given the proposed time paths for the policy variables, one cannot rely on an econometric model to make perfect predictions of the important economic variables. This uncertainty not only makes it difficult to evaluate a given policy path, but it makes the evaluation of such a path unrealistic and irrelevant. The former difficulty can be resolved by stochastic simulations which incorporate the random elements in the econometric model in making projections; the means and variances of the future paths can then be calculated. Because of uncertainty in the economy, a policy maker often will not adhere to a fixed plan irrespective of future developments. Future decisions will be made on the basis of future observations of the economy. Hence, it is usually unrealistic and irrelevant to evaluate the consequences of a preassigned sequence of policy actions. A more realistic policy takes the form of a reaction function, or a feedback control equation; it is a rule to determine the future values of the policy variables according to future economic observations. By the technique of optimal stochastic control, the solution to a multiperiod planning problem under uncertainty is given by a set of feedback control equations. The solution in feedback form is important from both the descriptive and the normative points of view.

We can therefore conclude that in policy analysis it is necessary to optimize, and optimization has to be performed in a stochastic setting to yield optimal feedback control equations. Deterministic control theory ignores uncertainty in the econometric model and yields a preassigned future path for the policy variables as a solution. This deterministic solution path, nevertheless, has one interesting characteristic. If the econometric model is linear with an additive random disturbance, if its parameters are known for certain, and if the objective is to minimize the expected value of a quadratic loss function for T periods, then according to a theorem due to Simon and Thiel, the optimum solution for the first period is identical with that of the above deterministic path. This result is known as the first-period certainty equivalence principle. Imagine how this principle can be applied to evaluate the outcome of an optimal sequential plan for a stochastic control problem with twenty periods as the planning horizon. A twenty-period deterministic control problem is solved to obtain x_1, the optimum setting of the vector of policy variables for the first period. Given x_1, we generate y_1, the vector of endogenous variables in period 1, using the econometric model and a random drawing for the vector u_1 of random disturbances. Having generated y_1 stochastically, we have the initial condition to solve a nineteen-period deterministic control problem to obtain the optimal x_2 for the second period. We then generate y_2 from x_2 and a random drawing of the vector u_2 of random disturbances for the second period, using the econometric model. An eighteen-period deterministic control problem is then solved to obtain x_3, and so forth until x_{20} and y_{20} are obtained. All this will give one observation of a sequence of the x's and the y's for twenty periods. The process can be repeated n times. Each observation provides a value for the loss function, and the expected loss can be estimated by averaging over the n observations. This would be a laborious and costly procedure. By contrast, using the technique of stochastic control to be described in the next section, we derive a set of feedback control equations which determine the optimal values for the future policy variables in terms of the values of future economic observations. The minimum expected loss associated with the optimal policy can be calculated by a simple formula. Furthermore, the means, variances and covariances of the future paths of all economic variables can be obtained analytically without resort to stochastic simulations. They contain much useful information concerning the dynamic performance of the economy under the rules of optimal stochastic control.

II. Techniques of Stochastic Control

Before presenting some important applications of stochastic control to the analysis of macroeconomic policy, I will survey briefly the available techniques. Of most importance is the derivation of the optimal feedback control equations and the associated expected welfare loss. Solutions are now available for linear as well as nonlinear econometric models, as described in Chow (1975; 1976a; 1976b). The techniques can also be classified according to the treatment of uncertainty. Beginning with the case of complete certainty where all model parameters are assumed to be known and all random disturbances are set equal to their mean value zero, one can first introduce random disturbances and secondly allow for uncertainty in the model parameters. We will postpone till the end of this paper the treatment of a third kind of uncertainty, namely in the specification of the model itself. For the control of a linear model with a vector of random disturbances but known parameters, assuming a quadratic loss function, we can find the optimal solution as a set of linear feedback control equations. When uncertainty is introduced into the parameters of a linear model, and when either the parameters or the disturbances in a nonlinear model are treated as uncertain, the solutions available are only approximately optimal. Stochastic control theory is still useful for the calculation of the expected loss associated with the approximately optimal policies and the derivation of the important dynamic characteristics of the economy under control.

For expository purpose in this section, I will illustrate the solution to the optimal control of a linear system with unknown parameters using a quadratic loss function. The system is written as

$$y_t = A_t y_{t-1} + C_t x_t + b_t + u_t \tag{1}$$

where y_t is a vector of endogenous variables at time t, x_t is a vector of policy variables at time t, b_t is a vector combining the effects of all exogenous variables not subject to control, the matrices A_t, C_t, and b_t consist of unknown parameters whose probability distribution is assumed to be given, and u_t is a vector of random disturbances having mean 0, covariance matrix V, and being serially uncorrelated. Endogenous variables and policy variables with higher-order lags can be eliminated by defining new endogenous variables so as to retain the form (1) of a system of first order linear stochastic difference equations in which only the current control variables x_t appear. We can include the policy variables in the vector y_t so that x_t need not be an argument of the loss function. After suitable transformations, the model (1) can also be used to deal with serially correlated random disturbances. These technical details are explained in Chow (1975). The loss function for a T period control problem is

$$W = \sum_{t=1}^{T} (y_t - a_t)' K_t (y_t - a_t) \tag{2}$$

where a_t is a vector of targets for the variables y_t and K_t is a diagonal matrix giving the relative penalties for the squared derivations of various variables from their targets.

To ease the exposition without effecting the essential argument, let me assume that $a_t = 0$ and $b_t = 0$ for all t. The problem is to minimize the expected value of the loss function for T periods by choosing a strategy for $x_1, x_2, \ldots, x_T$. The control variables will be selected sequentially, the vector x_t for each period being determined only after the up-to-date information is available. This information consists mainly of y_{t-1} which includes the observations of all past endogenous variables and policy variables affecting the current endogenous variables at time t. Using the method of dynamic programming, we first solve the problem for the last period T by minimizing

(3) $$V_T = E_{T-1}(y_T'K_Ty_T) = E_{T-1}(y_T'H_Ty_T)$$
$$= y_{T-1}'(E_{T-1}A_T'H_TA_T)y_{T-1} + x_T'(E_{T-1}C_T'H_TC_T)x_T$$
$$+ 2x_T'(E_{T-1}C_T'H_TA_T)y_{T-1} + E_{T-1}u_T'H_Tu_T$$

where we have used (1) to substitute for y_T, taken mathematical expectation E_{T-1} conditioned on the data available at the end of period $T-1$, assumed $a_t = 0$ and $b_t = 0$, and defined $K_T = H_T$ for future convenience. Minimizing (3) with respect to the vector x_T, one obtains the optimal feedback control equation for the last period,

(4) $$\hat{x}_T = -(E_{T-1}C_T'H_TC_T)^{-1}(E_{T-1}C_T'H_TA_T)y_{T-1} = G_Ty_{T-1} .$$

(Without the assumptions $a_t = 0$ and $b_t = 0$, the solution (4) would contain an intercept g_T .) The minimum expected loss for the last period associated with the optimum policy is obtained by substituting (4) for x_T into (3):

(5) $$\hat{V}_T = y'_{T-1}(EA'_T H_T A_T)y_{T-1} - y'_{T-1}(EA'_T H_T C_T)(EC'_T H_T C_T)^{-1}(EC'_T H_T A_T)y_{T-1} + Eu'_T H_T u_T \ .$$

To solve the problem for the last two periods, we apply the principle of optimality in dynamic programming to minimize with respect to x_{T-1} the expression

(6) $$V_{T-1} = E_{T-2}(y'_{T-1}K_{T-1}y_{T-1} + \hat{V}_T) \ .$$

The rationale for (6) is that whatever x_{T-1} is chosen, which will affect the outcome y_{T-1} , we shall choose the optimum policy $\hat{x}_T$ according to the feedback control equation (4) to yield the minimum expected loss $\hat{V}_T$. The optimal policy for both periods can thus be obtained by minimizing (6) with respect to x_{T-1} , given that x_T will be optimally chosen. Substituting (5) into (6) and letting

(7) $$H_{T-1} = K_{T-1} + (EA'_T H_T A_T) - (EA'_T H_T C_T)(EC'_T H_T C_T)^{-1}(EC'_T H_T A_T) \ ,$$

we rewrite (6) as

(8) $$V_{T-1} = E_{T-2}(y'_{T-1}H_{T-1}y_{T-1}) + Eu'_T H_T u_T \ .$$

Note that, except for a constant, (8) has the same form as (3) with the subscript $T-1$ replacing T. Therefore, by the same argument, the optimum $\hat{x}_{T-1}$ will be given by equation (4) with $T-1$ replacing T; the minimum total expected loss $\hat{V}_{T-1}$ for periods $T-1$ and T will then be found as in (5). The process continues until $\hat{x}_1$ and $\hat{V}_1$ are obtained. $\hat{V}_1$ is the minimum total expected loss for all T periods. It can be computed by equation (5) with the subscript 1 replacing T, where H_1 is obtained by solving the difference equation (7) for H_t backward in time. In these calculations, the approximation has been made that all expectations involving the unknown parameters A_t and C_t in the future are based only on data available at the beginning of period 1. Otherwise, the derivation will be more complicated because the expectations in equation (4) are functions of the data y_{T-1}, y_{T-2}, etc. We have just demonstrated how the optimal feedback control policies and the associated expected loss for all periods can be obtained analytically. The above derivations can be extended to deal with nonlinear econometric models with unknown parameters, as described in Chow (1976a).

III. Applications to Macroeconomic Policy Analysis

Having elaborated on the two major concepts in optimal stochastic control, namely, the minimum expected loss and the feedback control equations, we will illustrate their applications to the analysis and formulation of macroeconomic policies.

Using the minimum expected loss, one can compare various policy proposals as described by different feedback control equations or reaction functions. The candidates for comparison include such control equations as the ones precribing the policy variables to grow at constant percentage rates, reaction functions estimated from time series data as descriptive of the historical decision making process, and policies which rely exclusively on a subset of active instruments while keeping the remaining passive instruments to grow at some constant rates in order to study the relative effectiveness of fiscal and monetary policies. The minimization of expected loss can also be used to measure the trade-off possibilities for inflation and unemployment implicit in an econometric model. Without using the techniques of optimal control, one can experiment with different policies using an econometric model and observe the resulting rates of inflation and unemployment, hoping to trace out the trade-off possibilities. However, this approach is defective because the inflation-unemployment combinations generated by the experiments may not be the best possible. By varying systematically the parameters of the loss function in an optimal control formulation, one can trace out the best combinations of inflation and unemployment that can be achieved according to a given econometric model. Finally, the minimum expected loss can be used to measure the value of information and the costs of delays. The amount of information can be described by the variances of the estimated parameters and of observation errors; the variance-covariance structure, when incorporated into the analysis, will affect the minimum expected loss. Delays in information and in carrying out the decisions can also be modeled in the framework of stochastic control and the resulting minimum expected losses can be compared, Chow (1975, pp. 182ff, 202).

Secondly, the feedback control equations are extremely useful for policy analysis. When combined with the other econometric equations, they become a part of a stochastic model whose dynamic properties can be studied by analytical techniques. For the analysis of any policy, optimal or not, one would like to ascertain the means, variances and covariances of the important time series generated by an econometric model given the policy. Spectral properties of these time series can also be deduced analytically, as described in Chow (1975). Since automatic stabilizers can be specified by feedback equations, one can study the dynamic properties of an economy subject to different stabilization schemes. One can even optimize with respect to the parameters in the automatic stabilization schemes by the techniques of stochastic control for nonlinear systems in order to find a good scheme. Various dynamic policy multipliers, including the impact, delayed, interim, and long-run multipliers, can be calculated from a nonlinear econometric model after appropriate linearizations; they are by-products of the optimal control calculations for nonlinear models which I have proposed. The tools of stochastic control can also be used to compare econometric models, in terms of the optimal feedback control equations which they imply, of the degrees of stability under the optimal stabilization policies, and of the sensitivities of the control solution to changes in the parameters of the models or the loss functions. In short, the techniques of stochastic control can be used to deduce the policy implications of econometric models systematically and efficiently.

Let me conclude this paper by a proposal to deal with the third kind of uncertainty in econometric models, the uncertainty associated with the specification of the model itself. The decision maker, when faced with a serious choice between two or three competing models, is advised to calculate the optimal policies based on these models, and to examine how these and other proposed policies perform under the assumptions of the different models. A payoff matrix should be used, with different columns corresponding to the different models and different rows corresponding to the different policies to be examined, each element in the matrix being the expected loss associated with a combination of model and policy. Such an analysis might uncover policies that are superior to the inactive policies for all models. This finding would be extremely important. If no such active policies could be uncovered which dominate the inactive policies for all the seriously competing models, much useful information would have been obtained on the degrees of differences in these econometric models, the needed areas of econometric research for macroeconomic decision making, and the empirical basis for current policy recommendations or decisions. A Bayesian would assign probabilities to the competing econometric models and choose that strategy which minimizes the expected value of the entry in the above payoff matrix. To any rational economic decision maker, Bayesian or not, the techniques of optimal stochastic control are important not because existing econometric models are nearly perfect but because how imperfect the models are, what imperfections matter for policy analysis, and what areas require most research can be ascertained efficiently only by the use of these techniques.

References

G. C. Chow, Analysis and Control of Dynamic Economic Systems, New York 1975.

———, "On the Control of Nonlinear Econometric Systems with Unknown Parameters," Econometrica, May, 1976, 44. (a)

———, "An Approach to the Feedback Control of Nonlinear Econometric Systems," Annals of Economic and Social Measurement, Summer, 1976, 5. (b)

H. A. Simon, "Dynamic Programming Under Uncertainty with a Quadratic Criterion Function," Econometrica, Jan. 1956, 24, 74-81.

H. Theil, "A Note on Certainty Equivalence in Dynamic Planning," Econometrica, April, 1957, 25, 346-349.