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A NEW LOOK AT
ECONOMIC TIME SERIES ANALYSIS

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More than 25 years have gone by since A. Wald [19] formulated with decisive clarity the problem of the "decomposition" of economic time series as understood by economists at that time. His additional work on seasonal variations has had surprisingly little influence, although it could have become the point of departure for other work by virtue of the fact that the logic and the limitations of the conventional approach of distinguishing the usual four components were nowhere shown as clearly. It fell to Burns and Mitchell [3] to bring the underlying, essentially qualitative approach to economic time series to its present state, which is unquestionably a state of perfection but also one of impasse since their procedure does not seem to be capable of significant extension. This does not mean, of course, that this method may not yield insights into the behavior of time series which may be useful in economic analysis. Recent work by G. H. Moore on statistical indicators of business cycles is an example of an interesting application. But by and large, the Burns and Mitchell method, which is essentially one of forming simple averages on the basis of points marked off on the given series by means of purely subjective judgment, has gained more in the way of respectful comment by researchers not directly connected with the National Bureau of Economic Research, than in forming the basis for their own independent research. Moreover, writers like Schumpeter [15] have made rather far-reaching statements about the (alleged) existence of several types of cycles which are not easily supported by the evidence revealed by the National Bureau method. More recent writers on business cycles seem to pay little, if any, attention to the problems of decomposition of time series. So we gain the impression that the entire field either has been arrested in its development or that nothing more need be said.

This is in vivid contrast to the renewed interest in time series, other than economic, and to the steady development of powerful methods for their analysis. The interest stems from the needs of communication and information theory, and is therefore closely tied up with electrical engineering, although seismography has had an increasing share in this development. The new methods, notably spectral analysis, have roots that go back directly to Fourier analysis. It is safe to say that modern communication theory, with its enormous and growing impact on our daily life, could not have reached its present position without our improved ability to comprehend the complicated relationships between signal and noise.

We can conceive of the economy as being in this position: Producers and consumers, being subject to very regular, in many cases strictly periodic, influences, send signals of these activities into the (linear) time invariant "black box" called "economy." Their intermingling and the impinging of chance influences produces the aggregative data which confronts the economist. He wishes to analyze the data and understand what periodic influences are responsible for the behavior of the aggregative phenomena.

There can be no doubt about the existence of great regularities, beginning from the daily repetition of consumption, production, and transportation, via the more recondite regularities encompassing weeks, months, and sometimes years, to regularities that may escape our direct observation. We might indeed contemplate the great regularity of our economic activities: Trains and planes have regular schedules; factories open and close at fixed hours. Millions of people go to and from work at regular times. Theater performances start and end at fixed times. We rise in the morning and go to bed at night in general in a regular pattern. Schools have their calendars; banks and stock exchanges keep regular hours and days; all agricultural activity is highly regular in a narrow sense (feeding and tending of cattle), and in a wider sense, planting, fertilizing, harvesting, etc. Holidays follow a set pattern, as do fashions and temperature changes. Energy output is determined by season, length of day, climate, etc. In the face of the predominant regularities and periodicities pervading virtually all of our economic activities in all countries and at all times—though subject to the state of the technological art—it is difficult to see how anyone could expect the aggregative economic activity to be devoid of great regularities. Or, rather, why it should be impossible to look through the aggregates and to discover the underlying, constituent rhythms. Once these periodicities and rhythms have been found, the task of the economist is to eliminate them in order properly to analyze the remainder which contains the true puzzle of the behavior of the system as we see it now—the remainders presumably being a random series.

The disassembly of the joint product of all these activities is similar to the task the human ear performs when it identifies the various musical instruments that participate in an orchestral performance: the ear Fourier-analyzes successfully. If it were not for the ear's ability to apply Fourier analysis with a high degree of accuracy, though not with mathematical perfection—that is clearly not needed—there would be no music; the many instruments of an orchestra would merely produce a ghastly, aggregative noise, devoid of any structure. The physiology of the ear makes it an analogue computer admirably designed for this purpose.

Economists have been much impressed by Slutzky's demonstration [16] that smooth time series with intuitively distinct periodicities can be obtained from the summation and averaging of mere random inputs. They were quick to conclude (correctly) that the utmost care is needed to avoid the assumption that there must be underlying periodic inputs even when the resulting series appears to show periods. On the other hand there is no question whatsoever that economic time series are not random series, as even the simplest test will show. We can therefore also reject the hypothesis that the countless periodic inputs mentioned above produce nothing but random fluctuations in the various aggregates. This "converse" of Slutzky's theorem is certainly not true in general. Consequently it ought to be possible to discover the regularities which are demonstrably present in many basic input series.

It would be surprising if economic data were not accessible to methods specifically designed for finding periodicities and determining their energy. Economic analysis clearly cannot stop at the level of intuitive, qualitative, and subjective interpretation of time series in an age when it is possible to discover in the United States the existence of oceanic waves of 1 mm. amplitude, a length of one kilometer and originating at a distance of 10,000 miles, in the Indian Ocean; or to find the echo of a signal sent from Earth to Venus amongst the radio noise emanating from the region of that planet. (Incidentally, the latter achievement originally took one year to accomplish; the repetition of the experiment is now a matter of a few hours at most.)

Several attempts were made, of course, over the last few decades to test the applicability of Fourier and periodogram analysis for economic time series [1,4,5,6,10,11,21]. The results were generally deemed unsatisfactory by the authors themselves and viewed with suspicion by other economists. The efforts involved in the main only short time series and as a rule only a few coefficients were computed. On that basis it was, indeed, difficult to see the fruitfulness of the more advanced methods. Yet it is difficult to understand, as, e.g., Hafstad has pointed out [8], why Fourier analysis, Schuster's periodogram analysis and others derived from it (such as Bartel's technique), were so quickly discarded. Much of this attitude apparently stems from the belief that the situation in the social sciences in general, and in economics in particular, is entirely different from that of the physical sciences. This is quite at variance—in a most paradoxical way—with the fact that until the advent of game theory, though of course still persisting, economic theory had been modeled after classical mechanics! Yet when it comes to analyzing the dynamic behavior of the economy, the mathematical methods used by physicists to describe most complicated regular and also highly irregular oscillations and fluctuations of mechanical, electrical, and atomic phenomena, often beset by severe and frequent random disturbances, are supposedly not useful at all! This is an anomaly of the first order. One reason, perhaps the principal one, is that an entirely unwarranted weight has been placed on the fact that periodicity in aggregate economic affairs is not "obvious" to the common man, nor to those who are untrained in the use of the advanced methods (though they claim to have the ability to recognize visually fairly regular "cycles" of varying types). But science has long since passed the point where it can rely principally on the mere, direct and immediate evidence of the senses and use them as the chief instruments with which to describe adequately the phenomena to be investigated. (In all scientific activity there remains, of course, the unresolved personal judgment and commitment of the investigator.) Clearly economics will also go far beyond the point where the data given by immediate personal experience are not the principal elements on which to base economic analysis. In fact, this point has been reached; therefore it is necessary to draw the consequences.

The limitation of the work done in economics was furthermore partly conditioned by computational difficulties. Obviously the physical scientists had been in no better position. But with the experience of many centuries, if not millenia, they were conditioned to undertaking lengthy, tedious computations, especially in astronomy. Furthermore, they were quick to develop analogue computers suitable for special purposes, such as the determination of the tides. As long as computational difficulties are great, the computation has to be justified by very strong support from theory [12]. But in economics the usefulness of the mathematical theory itself was doubted! Now the situation is, of course, completely different: computation has become easy to a degree never imagined by earlier researchers, even in their wildest dreams. Consequently almost any theory can be used in an experimental manner. Electronic computers are likely to transform the entire field profoundly, provided they are used as scientific instruments and not merely for the purpose of mass application of existing, but analytically weak methods. Routine processing of economic time series is at any rate a rather dubious procedure, no matter what method may be fashionable at the moment.

Straight Fourier analysis and the more flexible periodogram analysis are far more powerful than the few economic applications would suggest. One should not forget that ships enter and leave harbors on the basis of tables of the tides, obtained for decades from applying Fourier analysis to information hardly more revealing than that available in economics.

Of course, the theory has not stood still, so that at present we command tools of a considerably more sophisticated nature. Present spectral analysis, firmly anchored in Fourier analysis, is still in a state of vigorous development.

Spectral analysis determines a function $f(\omega)$ which measures the true contribution to the total variance of the given series that is attributable to a specific frequency ω (the inverse of the more commonly used "period"), as described, for example, by Hannan [9]. $f(\omega)$ is called the "power spectral density function." If we were able to estimate $f(\omega)$, we would be led to the discovery of important economic mechanisms that underly the particular, given time series. Important results on spectral estimators, without which the field would remain barren, are due to J. Tukey [2,17]. They are now being further investigated, and are being applied to a large set of time series of the American economy.

An important aspect of spectral analysis lies in its restriction to stationary processes, i.e., to those that are "temporally homogeneous," or, in more conventional terms, show no trends of various kinds. Since most economists would say that trends, in the mean especially, not only exist, but are in fact one of the more significant characteristics of the economic system, it appears that this would make spectral analysis of little value for economics, restricting its use to the comparatively few series without such trends—assuming that all other objections, such as the alleged lack of economic periodicities, have been overcome. But the matter does not rest here.

First, it is possible to proceed in the ordinary way and to eliminate the trend in the mean by applying the standard devices currently in use and then to apply spectral theory to the remaining series.

Second, we can take a rigorous position based on statistical theory. Then the situation is this: even though trends in the mean or the variance appear to exist, we cannot necessarily draw the conclusion that the economic system is non-stationary. It may very well be non-stationary; but it may also be that the system is stationary and that the (apparent) trend in the mean is nothing but the high power residing in the low frequencies. The trend in the variance is much more difficult to deal with and to interpret since it does not appear clearly in the "spectrum" estimated without paying any attention to the nonstationarity. These and other nonstationarities can be looked upon as changing spectra and are at present being actively investigated. Though the trend in the mean presents difficulties, other forms of non-stationarity are even more complex and accordingly more difficult to establish. It is hoped, however, that one can show that for the practical purposes of the economist, the spectrum does not depend on stationarity.

Which of these two possibilities is true, is difficult to decide, largely because the universe from which the sample under consideration is drawn yields only one sample for each economic variable. This makes a clearcut decision on the basis of strictly statistical considerations impossible. It is then necessary to introduce other evidence, not described by the time series under consideration. Such evidence exists in favor of assuming non-stationarity, i.e., existence of trends in at least the mean of the series. The reasons adduced are essentially qualitative and heuristic, and stem largely from a basic knowledge about technological interdependencies of different trends.

In view of these uncertainties the proper approach is to proceed irrespective of the present, preliminary statements about the stationarity or non-stationarity of the population from which economic time series are drawn and to find out whether the propositions that it is possible to make when spectral analysis is successfully used become valid or not. It should be understood that the problem of stationarity is a very deep one. It cannot be properly formulated without a thorough analysis of stochastic processes and it bears on the foundation of the theory of probability.

The opinion that spectral analysis is inapplicable to economic and social time series because of their apparent non-stationarity [7] is thus certainly unwarranted: it is clearly more difficult to devise spectral analysis in the first place and to apply it to stationary series than in the conventional way to identify and eliminate the trends in the mean where they exist. As might be expected, the view that the spectral method applies only to strictly stationary processes lends support to the skepticism regarding the usefulness of powerful mathematical methods in economics. Yet how could a non-mathematical approach possibly be more powerful? We therefore conclude that the above observation is justified, according to which trends in the mean can be identified and computed either with or without spectral theory, and that analysis of the series is warranted in the spirit of the underlying theory.

The idea that either the present state of affairs as represented by the Burns-Mitchell method is satisfactory or that truly mathematical methods are inapplicable, is curiously defeatist. It means either that we can do no better than Burns-Mitchell, or that it is impossible to invent new mathematical techniques designed to analyze the (alleged) peculiarities of economic and social time series. For example, the analysis and application of "almost periodic" functions is far from concluded and may offer pleasant surprises. Whatever one may think of the future of mathematics—and who could justify pessimism?—the idea that one could stay forever satisfied with the non-mathematical techniques certainly has to be rejected. But it will be much more difficult—I think, in fact, hopeless in the long run—to suggest measures capable of producing decisive progress in that direction than to apply existing mathematical procedures which have shown their power in a startling manner in so many diverse fields.

The fact that the subjective-qualitative approach cannot carry much further is illuminated by the limitation of distinguishing only four components in an economic time series. On the basis of laying down initially any arbitrary number of frequencies as the only possible ones, the wave components that make up complicated physical phenomena would never have been discovered. Even qualitative, descriptive work devoted to some specific economic time series has yielded the conclusion that other cycles exist intermingled in complicated patterns with those generally accepted (e.g., "short cycles," which are neither seasonal nor of business cycle length [13]). There may be many more. Yet the conventional inspection of time series is too uncertain, too primitive, for the investigator to read off visually, but with confidence, a complex pattern of cycles or periods, let alone to find new ones.

This difficulty does not arise from an attempt to describe <u>exact</u> periods or cycles. Economists have, quite correctly, never tried to think in terms of "exactness" in connection with business cycles. But this does not exclude the use of modern statistical methods, in particular since these are not, at any rate, designed to measure "exactly" where cycles lie. Rather, their purpose is to measure the energy at various frequencies. Even a radio receiver does not identify a <u>precise</u> frequency but rather a bandwidth, and there is no doubt that electrical waves are more stable and more easily identified than economic cycles.

One of the chief aims of time series analysis is to <u>discover</u> whether periodic fluctuations occur, where they lie, and to measure their energy. To assert from the outset that such and such periods do or do not exist is at best preliminary to the investigation. It is at best a heuristic procedure but it certainly cannot determine once and for all whether a particular method for the discovery of frequencies and amplitudes is suitable. That task arises only after the analysis has been made and the very difficult problem of the economic interpretation of the results has to be faced.

When frequencies are found with which we are now not familiar, it may be unwise to discard them as economically meaningless. We may, instead, have to enlarge our concepts, make new models of the economy or of the particular sector being investigated, and seek to interpret the evidence thus revealed. It should be observed also that all time series analysis carries with it the idea of prediction. This is latent in all methods even though some who developed them may not have said so expressly. However, great care has to be used in judging the value of some methods solely on the basis of successful prediction: even false theories can sometimes give "alarmingly accurate descriptions" and excellent predictions. The history of science is full of important examples [20].

The undisputed fact that economic time series are smooth—in other words, definitely not strings of random numbers—plays havoc with the idea of using the theory of correlation in comparing several series. This is true whether or not lags are introduced between two or more series. Correlation measures postulate randomness, i.e., irrelevance of the time order in which the sample is drawn; but in economic time series a given observation is not independent of the preceding one and this is the case for each series of any pair that are to be correlated. Thus if one wishes to assume a rigorous, logically unassailable position—as, indeed, one can hardly avoid doing—serial correlation as commonly understood and practiced, is of little avail. Yet the desire to measure the interdependence of time series is legitimate. It is, in fact, nothing different from expressing the belief that an interdependence of economic variables exists and has a certain structure. The attempt to correlate time series is merely the extension of these notions into time. In the same spirit, lead and lag relationships are of interest.

Thus the question arises whether spectral analysis offers procedures that are not subject to the above limitations and objections. This is, indeed, the case. It would lead too far to describe the technical procedure [2], especially since we have only given the barest idea of spectral analysis itself. Suffice it to say that it is possible to compute the "in phase" cospectral density function and the "out of phase" quadrature spectral density function. From both we can determine the "coherence" of the two series at a given frequency. Other measures derived from these two density functions allow us to determine whether the lag is constant or varies with different frequencies.

It is one of the great attractions of these measurements that it is now possible to study the interrelations of economic time series from a new viewpoint; instead of looking at the overall behavior of entire series (allegedly "correlated"), we can for the first time determine the lead-lag relations for the different frequencies and explore the possibility that these relationships might depend on, and change with, the short-term—long-term relations prevailing among economic activities. In other words, the lag may or may not depend on the particular frequency observed. For the same set of series there may prevail another relation when seasonal fluctuations are considered than when their trends are investigated. Trends are more closely related to trends than trends to seasonals, each set of these variations having a structure of its own which needs to be explored, but for which purpose the usual techniques appear to be inadequate. The exploration of these areas by means of investigating the coherencies of time series promises to be exciting and capable of revealing important new insights. One can easily imagine that if these more complicated structures are established, our picture of the economy could undergo considerable changes [14]. This will be the consequence of our ability to make finer and finer distinctions in our observations and measurements—a goal all scientific activity strives to reach, although thereby our life becomes much more complicated.

It is interesting to note that the development and subsequent application of spectral analysis will also become of great significance to business. When mass produced commodities are to be scheduled so that the production runs can be matched as closely as possible to the incoming orders so as to minimize costs, it is important to identify, and perhaps predict, the fluctuations in sales. Present methods do not go beyond the elimination of seasonal variations where they are easily discernable and the elimination of trends, but the more recondite other fluctuations escape observation and do not enter into production planning as fully as is desirable. A further practical interest is, of course, in the area of stock markets for which a bewildering number of methods has been proposed.

The information obtained from time series analysis would be of little value unless it is put into an econometric model of the economy. The study of individual series is only a first step. We cannot stop there. Entire sets of series have to be viewed together and that requires the formulation of suitable models. By now this should be a trivial remark. Yet when we see important institutions still trying to avoid commitment to any theoretical model, still hoping that more and more description by means of a time series analysis based only on rudimentary manipulations and not rooted in the fertile developments of modern mathematical statistics which would bring about a deeper knowledge of the economy, we cannot but wonder.

Building models is at once easy and difficult: it is easy if a rather abstract, general model of little applicability is envisaged; it is difficult when it is more realistic. The proper medium position is dictated by the availability of data in specific form and by the complication of the model that can possibly be handled mathematically and computationally (i.e., the more "realistic" it is the more unwieldy it becomes analytically). In this particular instance the further complication is introduced that a discrete step forward in the amount of description of the time series is being taken. The new results may then be difficult to interpret intuitively, depending, of course, on the expectations of the investigator. This might apply, for example, to cases (which have actually been found) where two series are little lagged for their "short-term" but have a greater lag for their "long-term" components.[1]

The usual dilemma arises that the longest series are the most desirable from the point of view of analysis, but together they may not yield a reasonable model sufficiently similar to reality. Furthermore, the longer the time span considered, the greater the structural changes in the economy, due to advancing technology, new products, new institutions—in fact, one of the aspects of the stationarity problem. Yet these difficulties affect all theorizing in economics and are not restricted to the problem of time series analysis. But it is in the use of long time series where they unfold completely.

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1. This has actually been found to be the case for the New York call money rate and the New York commercial paper rate, 1876-1914. Cf. C.W.J. Granger, First Report on the Time Series Project (Econometric Research Program, Res. Mem. No. 12, 3 Feb. 1960). This memorandum describes the initial thinking and the first steps of a Project being actively pursued at Princeton University.

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