OBSERVABLE PUBLIC GOOD PREFERENCES

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1. Introduction

In his early papers on public expenditure theory, Paul A. Samuelson [1954, 1955, 1958] laid great stress on the problem of preference revelation for goods that are collectively consumed. Noting that for private goods "each person is motivated to do the signalling of his tastes needed to define and reach the attainable-bliss point," he observed that "no decentralized pricing system can serve to determine optimally [these] levels of collective consumption," since "it is in the selfish interest of each person to give false signals, to pretend to have less interest in a given collective consumption activity than he really has, etc." [1954, pp. 388-389, italics in original]. Later Samuelson said, "If there were only private goods we could rely on each man to calculate and present his demand function once we gave him his budget income and market prices at which he could trade freely. With public goods he has every reason not to provide us with the revelatory demand functions." [1969, p. 103]

A careful reading of these pathbreaking articles leaves one in some doubt as to the precise meaning of these assertions, which were directed at settling issues left open by less formal methods in the existing theory of benefit taxation and, in particular, at the "voluntary exchange" theory of public expenditure and taxation. The simple Lindahl model was specifically discussed by Samuelson, although the full analogy of Lindahl equilibrium, in a world with public goods, with competitive

equilibrium in a world with private goods only, remained to be spelled out (e.g., Duncan Foley [1970], T. C. Bergstrom [1971]). The later analyses showed how efficient outcomes with collective goods could be identified and sustained through prices under certain restrictions as to convexity of preferences and production technology. In these models the details of the "auction" process by which preferences are successively revealed are omitted (as they typically are also in all-private good models) and it is evidently to the question of individual incentive to respond honestly during this process of search for equilibrium that Samuelson was giving his discouraging answer.

Two important strands of recent work have sought to overcome the problem of preference revelation for public goods. Both of these have done this by looking for methods of obtaining preference information in the course of solving the problem of finding an efficient allocation for the economy.[1] Jacques Dreze and D. de la Vallee Poussin [1971] propose a continuous adjustment process whereby consumers bid the maximum they are willing to pay for small increments of the public goods, these bids being used to determine both whether to increase or decrease the level of provision and the financing of any change. With honest participants the process is shown to converge to a Lindahl static equilibrium under certain technical conditions. The incentive for true preference revelation is, however, rather weak: the authors show that telling the truth is a minimax strategy. Theodore Groves and John Ledyard [1974] also propose a system for obtaining preferences in the process of solving the efficiency problem. Their method is highly ingenious and has the incentive feature that the policy of revealing true demand is a Nash

equilibrium (the best thing to do given that the other consumers are also
telling the truth). Each consumer conveys a description of his demand
for public goods to a central agency which calculates the efficient
allocation of resources and distributes transfers among the set of all
consumers. The desirable incentive properties are achieved by making the
payoff to a consumer a positive function of the value generated for the
whole group and otherwise independent of the information conveyed by the
individual (thus ruling out taxation according to individual benefit).

Both of these methods for circumventing the problem seen by
Samuelson would rely on institutions of considerable sophistication to
collect preference information in the course of solving the allocation
problem. The Dreze-de la Vallee Poussin procedure is conceptually the
simpler, but it appears quite vulnerable to manipulation, and thus likely
to generate inefficient solutions. The Groves-Ledyard procedure has
greater appeal for its incentive features but the way in which the demand
information is related to individual payoffs makes it somewhat unattrac-
tive as a practical political tool.

In this paper we suggest a method for observing preferences for
certain classes of public goods in market data. The procedure is a pure
information-gathering one, which is uncoupled from any particular process
of optimizing resource allocation by the key assumption that consumers
regard levels of public good provision and of taxes as parameters. This
assumption, it is argued, is the natural analog of the assumption in a
pure market system that agents regard prices as parameters.

The proposed method exploits the selfish interest of each consumer
to communicate in the market true signals about preferences for private

goods, thereby simultaneously providing the information needed about public goods as well. The basic approach is to take advantage of the fact that the levels of public goods enter as arguments of demand functions for private goods. By observing the effect on demand schedules for private goods we can infer the value consumers place on changes on the levels of public goods.

An example suggests the basic approach and the nature of the conditions under which it will yield the desired information. A consumer's demand schedule for telephone calls will be a function of the size of the telephone network, which can be regarded as a public good. If the telephone network may be assumed to be of no interest except as it affects the value of the opportunity to initiate a call, we can measure the value to the consumer of a unit increase in the public good "network size" by the increase in his consumer's surplus. The latter is observed from data on his demand for the private good, telephone calls.

While there have been attempts to evaluate such public expenditure activities as education and recreation facilities through the study of demand for such private goods as housing and transportation service (Oates [1972], Knetsch and Davis [1966]), the generality of the underlying theory involved seems not to have been recognized. Indeed, if one regards the value of all public goods as contingent upon the availability of some private goods (if there is no butter, there is no demand for guns), the result says that all the information required for efficient public good provision is embedded in private good demand functions.[2]

As an aid to understanding we shall consider successive

complications, starting in Section 2 where we analyze a one-private

good, one-public good model, under the "no income effect" assumption,

that preferences are additive in the level of the numeraire good. This

results in identity between Hicks compensated and ordinary or Marshallian

demand functions and makes optimal non-numeraire allocations independent

of distribution. Furthermore, together with a demand interdependency

assumption, variations of which play a key role throughout the entire

analysis, this assumption permits the results to be stated in terms of

aggregate or market demand functions. In Section 3 the analysis is

extended to the many-private good, many-public good worlds. The results

of Section 2 relating to the use of aggregate demand functions are

generalized, and we show that a sufficient condition

allowing their use is a demand interdependency assumption plus constancy

of the marginal utility of income. We then show in Section 4 that when

the constancy assumption is relaxed, the results relating to public good

preferences being observable continue to hold, but individual rather than

market demand functions may be required. In Section 5 we discuss the

incentive problem as it arises in the context of this analysis. There

is a brief concluding section.

2. One Private and One Public Good Jointly Consumed

We start our analysis with the conventional Samuelson derivation

of the optimality conditions which apply when there is public good con-

sumption. A Bergson social welfare function of the utility levels of I

consumers, $W(U^1, \ldots, U^I)$, will be used to remind us that Pareto optimality

is a necessary condition for social optimality. $U^i(m_i, x_i, q)$ is a

preference indicator for the ith consumer, where m_i is the amount of a numeraire good consumed, x_i is the amount of another ("the") private good, and q is the level of provision of the single public good. The conditions of production are summarized by

$$m^o = m + C(x,q) ,$$

where m^o equals the initial level of the numeraire good which is available to the economy, and $m = \sum_i m_i$, and $x = \sum_i x_i$ equal the aggregate amount of the numeraire good and the private good consumed. $C(x,q)$ is the minimum numeraire good cost of producing a total of x units of the private good together with level q of the public good.

We assume that we know in advance that m_i, x_i, and q will be consumed in positive amounts. To characterize the interior optimum, we first write the Lagrangian associated with this social optimization problem,

$$L = W(U^1(m_1,x_1,q),\ldots,U^I(m_I,x_I,q)) + \gamma[m^o - \sum_i m_i - C(\sum_i x_i,q)].$$

The first order or stationary conditions can be derived in the standard fashion, and one obtains

$$U_2^i/U_1^i = C_1 \qquad i=1,\ldots,I \tag{1}$$

$$\sum_i (U_q^i/U_1^i) = C_q \tag{2}$$

$$W_i U_1^i = W_j U_1^j \qquad i,j=1,\ldots,I \ .^3$$

For the purposes of this analysis, (2) is not a convenient behavioral condition since we cannot directly observe the marginal rates of substitution, U_q^i/U_1^i. However, we can find these marginal rates implicit in the demand schedule for the _private_ good. In order to transform (2)

into a condition in terms of data observable in a market system, let us
first compute

$$\frac{\partial U_q^i/U_1^i}{\partial x_i} = \frac{U_1^i U_{q2}^i - U_q^i U_{12}^i}{(U_1^i)^2} \; .$$

Thus, we can write

$$U_q^i/U_1^i = \int_0^{x_i} \frac{\partial U_q^i/U_1^i}{\partial \xi_i} \, d\xi_i + \frac{U_q^i(m_i,0,q)}{U_1^i(m_i,0,q)}$$

$$= \int_0^{x_i} [\frac{U_{q2}^i}{U_1^i} - (\frac{U_q^i}{U_1^i})(\frac{U_{12}^i}{U_1^i})] \, d\xi_i + \frac{U_q^i(m_i,0,q)}{U_1^i(m_i,0,q)} \; , \qquad (3)$$

where the functions inside the integral are evaluated at the point
$(m_i, \, \xi_i, \, q)$.

Roughly speaking, condition (3), which is simply a calculus
identity, expresses the marginal rate of substitution of numeraire good
for public good as the derivative (with respect to the quantity consumed
of the private good) of the integral of the inverse demand function for
the public good. The latter is not observable. However, under a "no
income effect" assumption, the right hand side of condition (3) will be
identically equal to the derivative with respect to public good provision
of the integral of the (observable) inverse demand function for x.

The No-Income Effect Assumption

Under the no-income effect assumption (the demand for x and the
marginal evaluation of q are independent of budget level) the ith
consumer's preferences can be represented by a function additive in the
level of the numeraire good (Katzner [1970] refers to such functions as
"quasi-linear with respect to m_i"):

$$U^i = m_i + b^i(x_i,q) \; .$$

Under this assumption (3) becomes

$$U_q^i = \int_0^{x_i} U_{qx_i}^i (m_i, \xi_i, q) d\xi_i + U_q^i (m_i, 0, q) , \qquad (4)$$

where $U_q^i (m_i, 0, q) = b_q^i(0, q)$.

The left hand side of (1) can now be written

$$U_2^i(m_i, x_i, q) = P^i(x_i, q) ,$$

where $P^i(x_i, q)$ is the ith individual's inverse demand function for x.

Since we therefore have

$$U_{x_i q}^i = P_q^i(x_i, q) ,$$

and $U_{x_i q}^i = U_{q x_i}^i$, one can write (4) as

$$U_q^i = \int_0^{x_i} P_q^i (\xi_i, q) d\xi_i + b_q^i (0, q) .$$

The first term on the right is the derivative (with respect to the level of the public good) of the area under the inverse demand function for x. The second term on the right is the individual's demand price for the public good when the quantity of "the" private good consumed is zero; if we knew this value we would have the solution to the problem of observing the marginal evaluation of the public good.

The Demand Interdependence Assumption

Some public goods are so complementary to certain private goods that the value of the former would be zero without the latter. Examples of public and private goods in this complementary relation to one another include:

Public Good	Private Good
public highways	transportation vehicles
air safety	air travel
public recreational areas	transportation to areas
local public goods	residences
public television	television sets

The formal expression of this sort of demand interdependence in our case is the assumption $b^i(0,q) = $ constant, and hence $b^i_q(0,q) = 0$. With this assumption the ith individual's marginal rate of substitution of the numeraire good for public good becomes

$$U^i_q = \int_0^{x_i} P^i_q (\xi_i, q) d\xi_i \tag{5}$$

and our efficiency condition (2) becomes

$$\sum_{i=1}^{I} \int_0^{x_i} P^i_q(\xi_i, q) d\xi_i = C_q . \tag{6}$$

From empirical data on individual demand functions for a private good one can then determine whether this condition is satisfied.

From Individual to Aggregate Demand Functions

For practical purposes, the fact that data on _individual_ inverse demand functions are required to test for efficiency is a disadvantage. Fortunately, however, condition (5), which is formulated in terms of individual inverse demand functions, holds as well for the aggregate or market inverse demand function, $P(x,q)$.

Let the individual and aggregate direct demand functions be written as

$$x_i = D^i (p,q) , \quad i=1,\ldots, I \tag{7}$$

$$x = D (p,q) = \sum_{i=1}^{I} D^i(p,q) .$$

(Under our no-income effect assumption the individual and aggregate demands are independent of the distribution of the numeraire good.) We shall show that

$$\sum_{i=1}^{I} \int_{o}^{x_i} P_q^i(\xi_i,q)d\xi_i = \int_{o}^{x} P_q(\xi,q)d\xi \ , \tag{8}$$

where the upper limits are the individual and aggregate quantities demanded at a given price according to (7).

The expression on the right hand side of (5) can be written as

$$\int_{o}^{x_i} P_q^i(\xi_i,q)d\xi_i = \partial[\int_{o}^{x_i} P^i(\xi_i,q)d\xi_i]/\partial q \ . \tag{9}$$

Integration by parts of the expression inside the brackets on the right hand side of (9) then yields

$$\int_{o}^{x_i} P^i(\xi_i,q)d\xi_i = \int_{P^i(x_i,q)}^{\infty} D^i(\tau_i,q)d\tau_i + x_i P^i(x_i,q) \ , \tag{10}$$

where use is made of the fact that by definition

$$x_i \equiv D^i(P^i(x_i,q),q) \ .$$

Using this fact again and differentiating (10) with respect to q, one obtains

$$\int_{o}^{x_i} P_q^i(\xi_i,q)d\xi_i = \int_{P^i(x_i,q)}^{\infty} D_q^i(\tau_i,q)d\tau_i - D^i(P^i,q)P_q^i(x_i,q) + x_i P_q^i(x_i,q)$$

$$= \int_{P^i(x_i,q)}^{\infty} D_q^i(\tau_i,q)d\tau_i \ .$$

By definition of the upper limit quantities

$$P^i(x_i,q) = P^j(x_j,q) = P \ ,$$

so that

$$\sum_i \int_0^{x_i} P_q^i(\xi_i,q)d\xi_i = \int_p^{\infty} \sum_i D_q^i(\tau_i,q)d\tau_i = \int_p^{\infty} D_q(\tau,q)d\tau = \partial[\int_p^{\infty} D(\tau,q)\ d\tau]/\partial_q \ .$$

Integration by parts now yields

$$\int_p^{\infty} D(\tau,q)d\tau = \int_o^{x=D(p,q)} P(\xi,q)d\xi - pD(p,q) \ .$$

Then by differentiating with respect to q one obtains

$$\int_p^{\infty} D_q(\tau,q)d\tau = \int_o^{x} P_q(\xi,q)d\xi \tag{11}$$

thereby satisfying (8).

Thus, a necessary condition for efficient production of private

good x and public good q is that

$$\int_o^{x} P_q(\xi,q)d\xi = C_q(x,q) \tag{12}$$

be satisfied simultaneously with

$$P^i(x_i,q) = P(x,q) = C_1(x,q) \ , \quad i=1,\ldots,I \ .$$

Both of these conditions can be verified using observable relationships

which consumers will reveal in the course of private good purchases.[4]

Thus, for this special case, the preference revelation problem is solved.

3. Many Private and Public Goods with Demand Interdependency

For the case in which the demands for S private goods depend on the

level of provision of R public goods, we find it convenient to use an

alternative approach.[5] The technique of proof followed thus far starts

from the Samuelson efficiency conditions, which can be naturally viewed

as statements about inverse demand functions. It is apparent from (11)

that the efficiency condition (12) could also be written in terms of the aggregate direct demand function. Reformulating the problem in terms of the indirect utility function, we shall show that an analogous result applies to the many good case provided the marginal utility of income is constant and a demand interdependency assumption is satisfied. Furthermore, this reformulation permits an easier transition to the case where consumer preferences do not have the constancy property.

Let the preferences of the ith of a total of I consumers be represented by the indirect utility function, $V^i(p,q,m_i)$, where p is a vector of prices for S private goods, q is a vector of R public goods, and m_i is the budget level spendable on private goods, or money income. The numeraire or S+1'st private good will be assumed to have an unchanging price.

Let F(x,q)=0 be the implicitly defined production possibility function of the economy, $D^i(p,q,m_i)$ be the ith consumer's (vector) demand function, and D(p,q,m) the aggregate ordinary (vector) demand function, where m now denotes the vector $(m_1, \ldots m_I)$ of individual money budget levels. Then the Lagrangian expression associated with the maximization problem defining efficiency can be written

$$L(p,q,m_1,\ldots,m_I) = V^1(p,q,m_1) + \sum_{i=2}^{I} \gamma^i V^i(p,q,m_i) +$$

$$\theta F(D(p,q,m),q) . \tag{13}$$

Making use of the well-known property of indirect utility functions (Katzner [1970, p. 60]) that (in vector notation)

$$-\frac{V^i_p}{V^i_{m_i}} = D^i(p,q,m_i) , \tag{14}$$

the first order conditions associated with efficiency can be written

$$\gamma^i = V^i_{m_1} / V^i_{m_i}, \quad i=2,\ldots,I \tag{15}$$

$$p = F_x / F_{x_{S+1}} \equiv C_x \tag{16}$$

$$\sum_{i=1}^{I} (V^i_q / V^i_{m_i}) = F_q / F_{x_{S+1}} \equiv C_q. \tag{17}$$

Conditions (15) can be thought of as assuring that the distribution of income is appropriate while conditions (16) require marginal cost pricing of private goods. Conditions (17) require that the sum over individuals of the marginal numeraire evaluation of each public good equal the marginal numeraire good cost of that good. Our objective is to show that under certain restrictions on preferences the left hand side of (17) can be empirically observed from data on demand functions for the private goods.

Public Good Indifference at Some Price Vector

The first restriction is an analog of the assumption made in the previous section that the public good is valueless in the absence of any private good. We could maintain this restriction and add to it the condition that for given q and m_i there is a "sufficiently high" price vector for all consumers such that no non-numeraire consumption is chosen. However it suffices to make the related assumption that, for given q and m_i, there is some price vector $\bar{p}$ such that the marginal value of q is zero for each consumer. Thus we are assuming that for every q and m_i there is a price vector $\bar{p}$ such that,

$$V_q^i(\overline{p},q,m_i) = 0 \quad, \quad i=1,\dots,I. \tag{18}$$

This assumption (and the fact that $V_{qp}^i = V_{pq}^i$) allows us to write[6]

$$V_q^i = \int_{\overline{p}}^{p} V_{pq}^i \ (\xi,q,m_i)d\xi \ . \tag{19}$$

Constancy of the Marginal Utility of Income

It is well known that the no income effect assumption of Section 2 is a special case of constancy of the marginal utility of income (Samuelson [1942]). We shall now explicitly assume that the marginal utility of income is independent of p and q ($V_{m_i p}^i = V_{m_i q}^i = 0$). Using (14) and this assumption, we can write (19) as

$$V_q^i = - V_{m_i}^i \int_{\overline{p}}^{p} D_q^i(\xi,q,m_i)d\xi \tag{20}$$

and (17) as

$$\int_{p}^{\overline{p}} \sum_{i=1}^{I} D_q^i(\xi,q,m_i)d\xi = \int_{p}^{\overline{p}} D_q(\xi,q,m)d\xi = C_q \ . \tag{21}$$

This is the desired empirically implementable condition.[7]

The constancy of the marginal utility of income has been analyzed

by Samuelson [1942]. His well-known results have been generalized by Robert Willig [1972] who has proved that the marginal utility of income is constant with respect to the prices of a set of goods if, and only if, the income elasticities of demand for the goods are identical and independent of own prices. We shall assume that this result holds for any q, and select as the relevant set, the set of S private goods.[8] Thus, constancy with respect to prices occurs if, and only if (for clarity, dropping the individual consumer index, the index now representing a particular good)

$$\varepsilon^k(p,q,m) = \varepsilon^j(p,q,m) \qquad\qquad k,j \leq S$$

$$\partial\varepsilon^k(p,q,m)/\partial p_k = 0 \qquad\qquad k \leq S$$

where

$$\varepsilon^k(p,q,m) = (\partial D^k(p,q,m)/\partial m)(m/D^k(p,q,m)) \ .$$

In view of (14), it is clear that constancy of the marginal utility of income with respect to variations in prices and the level of provision of the public goods implies that the individual demand functions must be of the form

$$D^k(p,q,m) = y(m) \, h^k(p,q,) \qquad\qquad k \leq S \ .$$

When the income elasticity is equal to a constant, η, then it is easy to verify that

$$D^k(p,q,m) = m^\eta h^k(p,q) \qquad\qquad k \leq S \ .$$

Special cases of constant income elasticity include the situation where preferences are homothetic ($\eta = 1$), and the situation where there is no income effect ($\eta = 0$).

The role of the constant marginal utility of income assumption may be clarified by noting the connection of this result with consumers' surplus theory. Condition (21) can be viewed as selecting the value of q

such that the change in the "area" under the aggregate demand function(s) is equated to the change in cost when there is a marginal variation in q. As with all efficiency conditions which apply to marginal changes, this condition can be given both a "willingness to pay" and "money equivalent of a utility change" interpretation. For non-marginal changes which affect premarginal units, the existance of income effects will normally lead to a difference between willingness to pay and the money equivalent of a utility change. In fact, it is this phenomenon which leads to the well-known complexities in consumer's surplus analysis.[9] Although the change in area associated with (21) results from a marginal change in q, factors relating to the premarginal price units are relevant. However, although the demand functions admit the possibility of income effects, because of the constancy assumption, there is for the change in surplus no difference between willingness to pay and the money equivalent of a utility change, both of which are represented by the change in the area(s) under the demand curves.

4. <u>Dropping the Assumption of Constant Marginal Utility of Income</u>

When one relaxes the assumption that the marginal utility of income is constant with respect to p and q, it is still possible to determine the marginal evaluation of public goods using the individuals' ordinary private good demand functions provided that a demand interdependency assumption is satisfied.

The important restriction is similar to the one made in Section 3: at every utility level there is a vector of private good consumption,

demanded at some price and income combination, such that with that consumption bundle the consumer is indifferent to the level of public good provision. More precisely, what is required is that there be for any utility level some vector of prices $\bar{p}^i$ at which public goods become valueless. (Note that this price vector can now be specific to the individual consumer.) Preferences for which very high prices drive to zero the consumption of private goods highly complementary to the public goods form a special case. We shall show that when preferences satisfy the described condition, all information needed to test for efficiency in public good provision is contained in individuals' ordinary demand functions.

We shall assume that the I individual demand functions $D^i(p,q,m_i)$ are observable, and drop the practice of singling out a numeraire good (so that the price vector p now includes all prices). In order to solve the Pareto efficiency problem, it is necessary to derive from the observable demand functions the preference indicators of the consumers. The key to this derivation is the income compensation function, $\mu^i(p,q/p^o,q^o,m^o)$, which is defined by

$$V^i(p,q,\mu^i(p,q/p^o,q^o,m_i^o)) \equiv V^i(p^o,q^o,m^o) \ .$$

The income compensation function determines the minimum level of money income which achieves the same level of utility obtained in a base situation parameterized by (p^o,q^o,m^o) when the consumer now faces prices p, and the vector of public goods, q.

For any q, the derivative of the income compensation function with respect to p equals the vector of compensated demand functions, i.e.,[10]

in vector notation,

$$\frac{\partial \mu^i}{\partial p} = D^i(p,q,\mu^i(p,q/p^o,q^o,m_i^o)) \qquad i=1,\ldots,I \ . \qquad (22)$$

The function, μ^i, transforms the ordinary demand function into a Hicksian compensated demand function by adjusting income in such a way that utility is held constant at the level associated with (p^o,q^o,m^o). In fact, (22) is a system of S partial differential equations for each consumer which can be used to solve for μ^i when the ordinary demand functions are known.[11]

Using (22) and the restriction assumed on preferences, we can construct the income compensation function. By definition of the income compensation function,

$$\mu^i(p^o,q^o/p^o,q^o,m^o) \equiv m^o \ .$$

Using (22) with this boundary condition we can obtain $\mu^i(\bar{p}^i,q^o/p^o,q^o,m_i^o)$, where $\bar{p}^i$ is a price vector resulting in a private good consumption vector at which the ith individual is indifferent about the choice of the vector q of public goods.[12] (Note that $\bar{p}^i$ might depend upon (p^o,q^o,m_i^o).) Then

$$\mu^i(\bar{p}^i,q^o/p^o,q^o,m_i^o) = \mu^i(\bar{p}^i,q/p^o,q^o,m_i^o)$$

for any q. Now we apply (22) again, integrating to obtain $\mu^i(p,q/p^o,q^o,m_i^o)$. It is thus possible to map out the entire five-variable income compensation function.

An important property of the income compensation function is that a relabeling of the parameters as variables (and vice-versa) transforms this function into a strictly monotone increasing function, g^i, of the indirect utility function.[13] Thus,

$$g^i(V^i(p,q,m_i)) = \mu^i(p^o,q^o/p,q,m_i)$$

where p^o and q^o can be viewed as either base or reference price and public good levels. Intuitively, this asserts that the more the individual must be compensated for the change from p,q to p^o,q^o, the higher the utility of p,q must be, other things being equal. Thus $\mu^i(\cdot)$ must be an ordinal utility function, correctly representing his indifference map.

Since we have shown that μ^i can be constructed from the private good demand functions of the ith individual, this completes the demonstration that the contours of the consumer's utility function, including its public good arguments, can be obtained from the observable ordinary demand functions for private goods.

Using $\mu^i(p^o,q^o/p,q,m_i)$ as an indirect utility function, we can repeat the analysis of Pareto optimality of Section 3. In particular, condition (17) becomes

$$\sum_{i=1}^{I} (\mu^i_q/\mu^i_{m_i}) = C_q , \tag{23}$$

where C_q is the vector of marginal costs of the public goods (in terms of the value of private goods foregone, measured at the prices appropriate for m_i). Condition (23) is recognizable as the requirement that the sum of individual marginal evaluations of public goods equal their marginal costs.

By virtue of (22) and the interdependency assumption that for some price vector $\bar{p}^i$ the value placed by the consumer on variations in the public goods drops to zero, we can write $\mu^i_q(p^o,q^o/p,q,m_i)$ as

$$\mu^i_q = \frac{d}{dq} [\int_{p}^{\bar{p}^i} D^i(\xi,q,\mu^i(\xi,q/p,q,m_i))d\xi]$$

which is the derivative of the area under a compensated demand function with respect to the level of provision of the public goods.[14] Therefore (23) can be written,

$$\sum_{i=1}^{I} (\frac{d}{dq}[\int_{p}^{\bar{p}^i} D^i(\xi,q,\mu^i(\xi,q /p,q,m_i))d\xi]/\mu^i_{m_i}) = C_q .$$

Although this condition has the general form as condition (12) of Section 2 and (21) of Section 3, the difference is that now we must know the individuals' ordinary demand functions are used rather than market aggregates. If $D^i_{m_i}$ is zero for all i then $\mu^i_{m_i} = 1$ everywhere and the no-income effect analysis emerges as a special case.

5. The Incentive Problem

The problem of preference revelation for public goods so emphasized by Samuelson can be thought of as that of creating just the right incentives for consumers to reveal their preferences. Thus, where the consumer perceives that the greater his revealed value for a public good the greater his share in its financing he has an incentive to understate that value. If the system attempting to ellicit the information places a disproportionate burden of financing the good on other consumers, he will have an incentive to overstate. If the use of the information is perceived as having no bearing on the outcome experienced by the consumer, utility theory by itself does not imply he will reveal his true preferences. Basically the theory is silent, although it suggests that such matters as the details of the information-gathering instrument--how long is the questionnaire, etc.--may become important.

The Dreze-de la Vallee Poussin and Groves-Ledyard approaches to this

problem, referred to in the introduction, both attempt to give the con-
sumer just the right incentive to reveal the truth by virtue of his stake
in the public good outcome. In the Dreze-de la Vallee Poussin mechanism
the influence that the individual has on the outcome as it applies to him
is great, since he substantially controls the share he bears of the costs
of provision. For example, a consumer who asserts he places zero value on
the public good will be assessed zero tax. The potential gain from
cheating is thus large and only an extremely cautious player would tell
the truth out of self-interest. In the Groves-Ledyard system, the conse-
quences for the individual of variations in his signaled preferences are
slight if it is a large system. Neither the equilibrium choice of public
good level nor anyone's share in its financing is much influenced by any
single consumer's report. Speaking loosely, one expects that as the
system is made larger (in number of consumers) the incentive for any
consumer to tell the truth tends to zero. Over the set of possible
answers to the hypothetical questionnaire the truth continues to have the
largest payoff, but the difference between it and other answers tends to
zero, so that other reasons for picking one answer over another (momentary
whim, neighbor's urging, etc.) are increasingly in danger of dominating.

These incentive problems appear to have been banished in the results
described here. The reward to the consumer from correctly revealing his
preferences in transactions on large markets is often regarded as self-
understood and it is this feature of the market system which we exploit.
However, the preference problem is lurking beneath the surface nonetheless.
Its full exploration would take us farther from our immediate subject than
seems warranted, but we shall attempt in this section to lay out the
issues informally.

The key assmption made in the sections above is that the consumer regards the level of public goods provided and his own budget as <u>parameters</u>. Granted this assumption the rest follows. Yet we know the same may be said of the Lindahl auction process when public good prices are parameters. How satisfactory is this assumption in the present context?

Let us take the hard case first, in which a benefit approach is to be taken in taxation, with individual demand data being used to estimate individual value received from a public good. We see that a satisfactory analysis requires us to specify the experiments by which the individual demand function is to be revealed. An example: the consumer participates in a Walrasian auction in which public good levels and private good price are being varied. The consumer's successive bids are recorded and used to estimate a demand function. The consumer knows that a certain area under his demand function will ultimately be used to calculate the amount of public good to be offered and, importantly, his contribution to its financing. On the other hand he knows that at any stage the auctioneer may declare markets cleared and he will be required to honor his bid at the most recent price-public good quotation. What should his bidding strategy be? What, if any, is the set of Nash equilibria of such a model? Clearly this example, which is probably not even a very satisfactory model, since it does not involve disequilibrium transactions, suggests the difficulty of drawing any very robust general conclusions about the incentive problem when benefit taxes at the individual level are to be obtained.

However, the possibility for gain from misrepresenting preferences seems slight when both public good levels and financing instruments are

general (apply to all consumers independently of information about preferences). An example would be a national road system financed out of income taxes. In this case the consumer's budget depends upon the level of provision of public goods, and the incentive problem arises normally because he will be aware of this functional dependence in declaring his preferences for collective consumption. To convey a false signal about preferences becomes extremely complicated and potentially costly under the procedures outlined in this paper. Again some sort of model is necessary of the way in which demand functions are estimated, and we may again use the illustrative case of a Walrasian auction.

Suppose that the public good in question can be offered at exactly two levels, 0 or 1. Assume the conditions of Section 1, so that the consumer's demand schedules for x, the complementary private good are as in Figure 1. In the example shown, the marginal cost of x is assumed independent of q, so that efficiency calls for the same price of x at both levels of q. The shaded region in the diagram represents the value the consumer attributes to the increase of q from 0 to 1.

Suppose, however, that the cost sharing arrangements, specified in advance, are such that if q is increased from 0 to 1 the consumer's tax bill will increase by more than the shaded area. Taking this into account he would prefer that q be provided at zero level. How does he manipulate his market signal to try to bring this about? To discourage the production of q, the consumer must reduce the benefit revealed in his market signal. For example, he might place his bids in the Walrasian auction according to the "false" demand curves, $D_f^{x_i}$, shown in Figure 2. Now, instead of the single-shaded area being conveyed as a positive value on the public good, the cross-
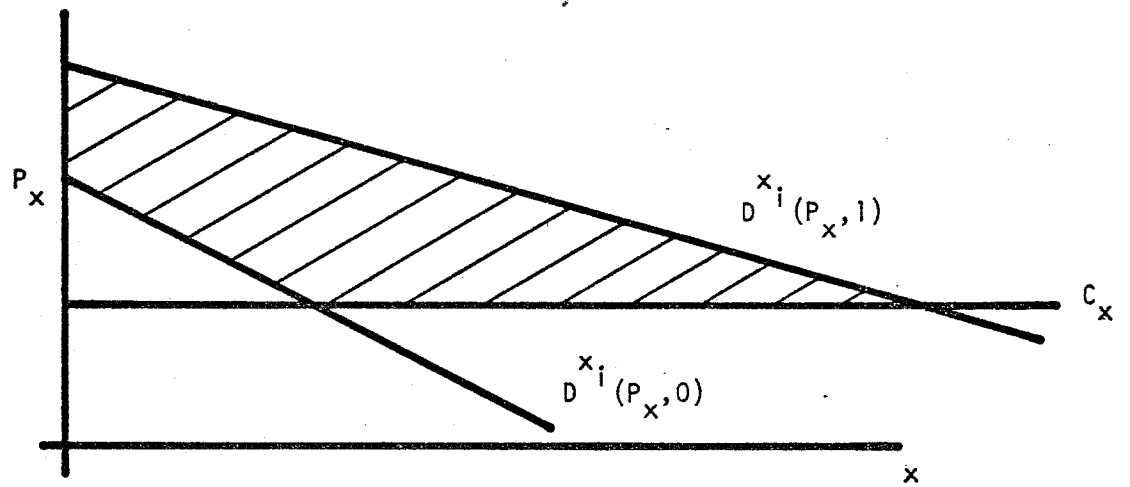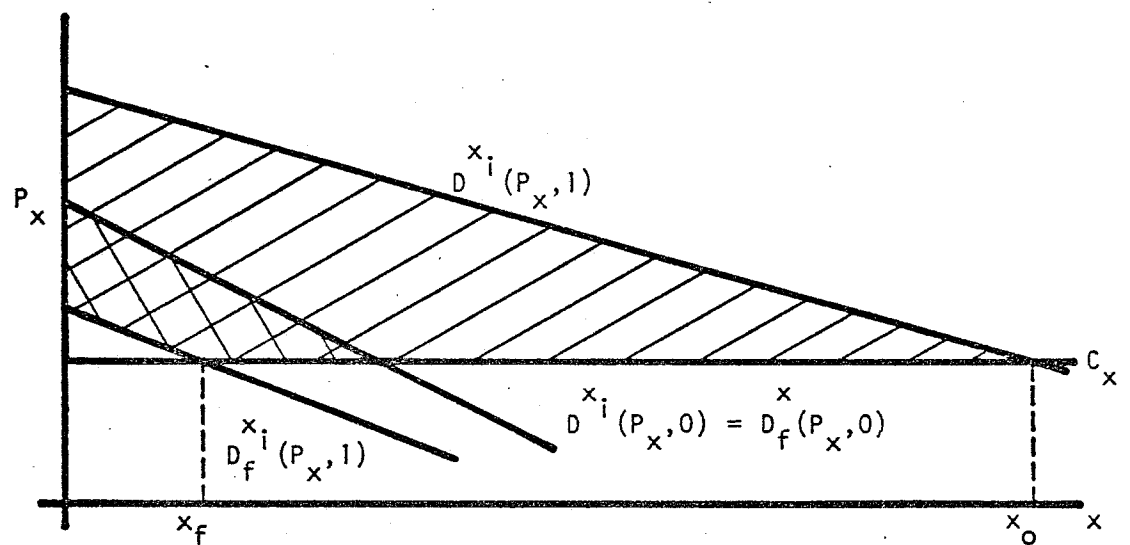
Figure 1. Revealing Value for One Unit of Public Good

Figure 2. False Preference Signaling

hatched area is conveyed as a *negative* value. The problem is that the evidence provided by the other consumers may result in q = 1, in which case he would obtain x_f units instead of his preferred quantity x_0 units of x at price P_x, foregoing a benefit equal to the consumer's surplus triangle under $D^{x_i}(P_x,1)$ between x_f and x_0 in Figure 2 (his taxes will be up anyway).

Without a more elaborate model of the process of demand function estimation in the context of optimizing and financing public goods it is not possible to make fully general statements about the incentive problem. However the example just discussed suggests the proposition that the incentive to distort demands to obtain gains from the resulting effect on public good provision is of the same order as the incentive to do so to obtain a lower price in a competitive market. That is, the same sort of approximation appears involved in the assumption that consumers regard public-good levels and taxes as parameters as that they regard prices as parameters, and we should look for problems under the same sorts of circumstances in both cases. As the number of consumers becomes larger the expected gain from false signalling (and thus influencing the result) appears to vanish while, and this is in contrast to the Groves-Ledyard mechanism, the expected cost (due to consuming a non-optimal amount of the private good, given the public good provision) does not.

6. Concluding Remarks

Just how general are the hypotheses of the propositions developed here (and we regard it as likely that substantial useful information about public goods can be extracted from private good demand functions even when the hypotheses are not satisfied) is an empirical issue. If a wide range

of collective consumption activities have associated with them comple-
mentary private goods of the sort described, for that wide range the pre-
ference revelation problem posed by Samuelson is solved in principle.
Unfortunately, this does not mean there is a decentralized system for
public good provision. There is no invisible hand to maximize consumers'
surplus in this case. It remains to be seen whether econometric work, on
data either naturally produced by the market or experimentally generated,
will enable this solution in principle to be a solution in practice.

FOOTNOTES

*U.S. Treasury and Princeton University. **U.S. Air Force Academy.
The work reported on here was stimulated by the analysis of A. M.
Spence [1973] of the problem of efficient choice among mutually
exclusive quality levels of a private good. As discussed in D.
Bradford [1970] there is a natural sense in which the choice of level
of provision of a public good is a quality rather than a quantity
decision. Since Spence's optimality condition involved only data
on observable demand functions, we were led to inquire whether the
same might not also hold for public goods. We would like to
acknowledge as well the helpful comments on an earlier draft of this
report by William Baumol, Jerry Green, Theodore Groves, Martin Hellwig,
and Robert Willig. The views expressed herein are those of the
authors and do not necessarily reflect the views of Princeton
University, the United States Air Force, the Department of Defense,
or the Department of the Treasury.

1. Another imaginative approach, utilizing sampling techniques, has
   been developed by Theodore C. Bergstrom [1974].

2. In written communication William Baumol has suggested that our
   result can be viewed as an implication of a sufficiently strong
   interdependency between the private good and the public good which
   insures that "the demand for one is automatically the demand for the
   other."

3. According to context sometimes notation such as U_i is used to refer
   to the partial derivative of a function with respect to its ith
   argument, while in other cases it is easier to keep track of the
   reasoning if, as in the case of the level q of provision of the
   public good, notation such as U_q is employed.

4. Note that these are the social efficiency conditions derived by
   Spence [1973] for the choice of a single quality level, q, to be
   embodied in the private good x. The equivalence derives from the
   fact that a single level of quality that is embodied in a quantity
   good is itself a good that enters more than one person's utility
   function, and thus is a public good. Jerry Green has pointed out
   to us that the existence of substantial fixed costs for either the
   private or public goods may result in the private complementary
   commodity being present only at some non-negligible level and above.
   Thus, there is the empirical problem of estimating $P(x,q)$ when there
   are few observations for negligible x. One can surmount this problem
   by making a priori assumptions about the functional form of $P(x,q)$.

5. A difficult problem may arise when it is not known in advance which
   of the R public goods will be consumed in positive amounts. If there
   are fixed costs associated with introducing a public good a nonconvex
   programming problem is involved. Note that R might represent the

characteristics of a single public good, e.g., grading, width, paving quality, etc., of a public road.

6. The expression on the right hand side of (19) is a line integral in vector notation. The fact that $V^i_{qp} = V^i_{pq}$ insures that the value of the integral is independent of the path of integration from vector $\bar{p}$ to vector p.

7. One can readily identify restrictions on the individual utility functions more general than (19) plus constancy of the marginal utility of income which are sufficient for using the aggregate demand function to implement condition (17). Remembering that

$$\sum_{i=1}^{I} D^i_q = D_q \quad,$$

and then dropping the individual index for convenience, we show that (i) $V_q(\bar{p},q,m)/V_m(\bar{p},q,m)$ and (ii) $V_q V_{mp} = V_p V_{mq}$ are the relevant restrictions. By differentiating V_q/V_m with respect to p, one obtains (with restriction (i))

$$V_q/V_m = \int_{\bar{p}}^{p} [(V_q V_{mp} - V_m V_{pq})/V_m^2]d\xi \quad.$$

Then, using (14), one can show that

$$\int_{\bar{p}}^{p} D_q\, d\xi = \int_{\bar{p}}^{p} [(V_q V_{mq} - V_m V_{pq})/V_m^2]d\xi \quad.$$

Thus, restrictions (i) and (ii) are sufficient and the problem becomes one of identifying the functional form of the ordinary demand functions implied by restriction (ii).

8. In written correspondence Robert Willig presented a proof that the marginal utility of income is constant with respect to p and q if the income elasticity of demand is (i) independent of price, (ii) independent of q, (iii) bounded above; (iv) if there exist $m,\bar{p}$ such that $D(\bar{m},\bar{p},q) = 0$; and (v) if $U_q(m,0,q) = 0$ where U is the direct utility function. Constancy with respect to p and q implies that the indirect utility function can be written in the form $V(m,p,q) = f(m) + b(p,q)$.

9. For a discussion of consumer's surplus which focuses on these two factors see Richter [1974].

10. The income compensation function, $\mu(p/p^o,m^o)$, has been studied definitively by Hurwicz and Uzawa [1971], and used as the building block of consumer surplus analysis by Willig [1973]. When written in the form, $E(p,\bar{U})$, where $\bar{U}$ is a specified level of direct utility, it is called the expenditure function. Thus, for the case at hand $\mu(p,q/p^o,q^o,m^o) = E(p,q,\bar{U})$. It is well known (see Diamond and McFadden [1973]) that the derivative of the expenditure function, $E(p,\bar{U})$, with respect to price equals the compensated demand function. This applies for any q and therefore (22) holds.

11. Differential equations which yield the income compensation function have been discussed by Mohring [1971] and Willig [1973].

12. Note that we have not proved that $\mu^i(\bar{p}^{-1}, q^o/p^o, q^o, m_i^o) = \mu^i(\bar{p}^{-1}, q/p^o, q^o, m_i^o)$ is equivalent to $V_q^i(\bar{p}^{-1}, q, m_i) = 0$.

13. By definition $V^i(p^o, q^o, \mu^i(p^o, q^o/p, q, m_i)) = V^i(p, q, m_i)$. It is assumed that V_3^i is everywhere strictly positive. Therefore, for any given p^o, q^o increases in the right hand side must be balanced by increases in μ^i, which is, therefore, regarded as a function of (p, q, m_i), a strictly positive monotone transformation of V^i, i.e., a utility function over its range of definition. We are grateful to Robert Willig for suggesting to us that a result by Hurwicz and Uzawa [1971] might be generalized.

14. Let $\bar{p}^{-i}$ be such a price vector. Then, dropping the individual super-script for convenience, $\mu(\bar{p}, q'/p, q, m) = \mu(\bar{p}, q^o/p, q, m)$ for all q' including $q' = q$. Thus, $\mu(\bar{p}, q/p, q, \underline{m})$ is a utility function. Now use the fact that $\mu(\bar{p}, q/p, q, m) = \mu(\bar{p}, q/p, q, \underline{m}) - \mu(p, q/p, q, m) + m$, where $\mu(p, q/p, q, m) = m$ by definition and $\mu(\bar{p}, q/p, q, m) - \mu(p, q/p, q, m)$ is the equivalent variation in income to the price change from p to $\bar{p}$.

(Note that the assumption is also being made implicitly that $\mu(\bar{p}, q/p, q, m)$ is finite.) Next use $\dfrac{\partial \mu}{\partial \bar{p}_s} = D^{x_s}(\bar{p}, q, \mu(\bar{p}, q/p, q, m))$, where D^{x_s} is the ordinary demand function for x_s. Then $\mu(\bar{p}, q/p, q, m)$ can be written as the sum of m and a line integral of the vector demand function D from p to $\bar{p}$, denoted by

$$\int_p^{\bar{p}} D(\xi, q, \mu(\xi, q/p, q, m)) d\xi \ .$$

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