

STATISTICAL ANALYSES OF AIRCRAFT HIJACKINGS
AND POLITICAL ASSASSINATIONS

BY

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Abstract

In this article several time series of terrorist activity, namely of aircraft hijackings and political assassinations, are analyzed with emphasis on their stochastic behavior. It has been found that, based on the hijacking records of the U.S. as well as the whole world, from 1968 through 1972, the occurrences of hijackings follow a Poisson process with an interesting time pattern in its intensity parameter. Social and political interpretations are offered for the changes of the intensity. The second set of events analyzed is political assassinations in the U.S. from 1930 through 1968. Results of analysis indicate that occurrences of assassinations in the U.S. also follow a nonhomogeneous Poisson process.

STATISTICAL ANALYSES OF AIRCRAFT HIJACKINGS AND POLITICAL ASSASSINATIONS

1. Introduction

Numerous statistical models have been applied in recent years to better understand various social, economic and political phenomena and to explain apparent anomalies. For example, distribution models have been employed to describe the consequences of football games [14]; a probabilistic study has led researchers to develop a better strategy for pole vaulting [10] as well as weight lifting [5]; a stochastic model has been constructed to better interpret many features observed in recent U.S. elections [15]; a newly developed statistical detection technique has helped demonstrate the fact that the Watergate scandals and the related publicity in the news media are associated with a sizeable shift in the variance of stock-market prices over the period of 1973-74 [7]. In this article we study (1) aircraft hijackings in the period of 1968-1972 and (2) the political assassinations in the U.S. over a period of forty years with a view towards providing a better understanding of the underlying stochastic processes.

Although aircraft hijackings have essentially disappeared since early 1973, it is not only academic interest that prompts us to attempt to better understand the sequence of realizations observed from 1968 to 1972. A better understanding of the process may have significant policy implications, since the characteristics of the process may have implications for prevention and since the techniques applied here may well be relevant for analyzing other types of terrorism as well. We need only point out that the cost of pre-boarding checks in the U.S. per year is of the order of hundreds of millions of dollars; more than the total ransom money ever demanded by skyjacking

extortionists in any single year. Techniques designed to detect changes in the parameters of the underlying process might well be applied profitably to the series of bombings occurring in London between August and December 1975 and allegedly carried out for political reasons by members of the I.R.A. In Section 2, we present a statistical review of these hijacking incidents and suggest the possible factors which contributed to the formation and transformation of this unique series of events. We note that the occurrences of hijackings in the U.S. and the whole world, during the period of 1968-72, follow a Poisson process with a definite time pattern in its intensity parameter.

Another area of at least equal interest, one that includes several tragic events which have considerably changed the modern political history of the U.S., is that of the political assassinations and include those of President John F. Kennedy, Senator Robert F. Kennedy and Dr. Martin Luther King, Jr.; and also includes two recent assassination attempts on President Gerald Ford (September and October 1975). In Section 3 we investigate the probabilistic behavior of political assassinations in the U.S. from 1930 through 1968. We find that the occurrences of assassinations can be described by a Poisson process with step changes in the parameter.

A brief discussion on the implications of the empirical findings concludes this paper.

2. Statistical Analyses of Aircraft Hijackings

Aircraft hijacking is one of the modern phenomena that has been of much concern in the U.S. in the last fifteen years. On May 1, 1961, the seizure of a twin-engined CV-440 aircraft of National Airlines by a psychopath and its

diversion to Havana, Cuba, marked the start of the era of U.S. skyjacking. Between this date and the end of December 1972, a total of 159 U.S. registered aircraft were involved in skyjack incidents. Of these, 148 occurred in the period from 1968 through 1972. During the five peak years, skyjackings brought death and injury to many and fear and inconvenience to millions. The economic cost of skyjackings due to loss of life, property and time, must be valued in many millions of dollars.

Roughly up to the end of 1969, the desire to reach Cuba was the almost exclusive factor which caused skyjackings of U.S. aircraft. During the years of 1970 and 1971 this factor declined to account for roughly one half of all incidents. Finally in 1972, a new breed of skyjacking extortionists (called 'parajackers' in the jargon of hijacking analysts), who demanded parachutes for escaping from the seized airplane with the ransom money, has become the single most important group of air pirates in the U.S. Basically, there were three types of air pirates: the Cuban refugees and political dissidents, the mentally disturbed and the common criminals. A more detailed discussion of switching among the various types of air pirates will be presented after the results of a more formal analysis. In the next subsection we provide a detailed analysis of U.S. hijacking records; we test the hypothesis that it is a Poisson process and examine the stationarity of the intensity parameter. A brief analysis of the worldwide skyjacking data is reported in the following subsection.

The Hijacking of U.S. Registered Aircraft. The data used in our analysis, covering the years of 1968-1972, are taken from Chronology of Hijacking of U.S. Registered Aircraft, prepared by the Office of Air Transportation Security, Federal Aviation Administration. The data provide, among others, the

date of the incident, the type of flight (scheduled, chartered, private), the hijacker's boarding point and his or her destination. An attempt to separate the incidents into four partially overlapping categories according to the type of flight, was reported in a paper by Quandt [16]. However, from the results provided there, we felt that the series of overall hijackings is an adequate and convenient one for statistical analysis.

From the raw data we obtained, the interoccurrence times, denoted by t_i , $i=1,2,\dots$, in units of days¹ with the first interoccurrence time counted from January 1, 1968.

The Poisson model is the obvious first choice to fit the hijacking data. If a point process is Poisson, the distribution of the number of events occurring in a unit of time, whatever its length, is given by

$$\text{Prob (number of events} = i) = \frac{e^{-\lambda} \lambda^i}{i!}$$

where λ is the so-called intensity parameter, representing the expected number of occurrences in a time unit. Furthermore, the following are two relevant and well-known theorems:

Theorem 1. The interoccurrence times, t_i , of a Poisson process are independently distributed according to the exponential distribution $f(t_i) = \lambda e^{-\lambda t_i}$, $i=1,2,\dots$.

Theorem 2. Given the total number of events in an observation period of length T , elapsing from time 0 to T , the locations of the occurrences on the time axis are independently and uniformly distributed over the interval $(0,T)$,

¹Whenever two skyjacking incidents took place on the same day, we assumed that they occurred half a day apart, since the reported data do not provide the hour of the occurrence.

if the generating process is Poisson. In other words, the quantities $\tau_i = \sum_{j=1}^i t_j/T$ are distributed as a uniform random variable over the interval (0,1), conditional on the number of occurrences, n , being known. See Parzen [12], pp. 135-141.

The second theorem allows us to test the hypothesis that hijackings are generated by a Poisson process, by testing whether the τ_i are uniformly distributed over the (0,1) interval. Two standard tests with reasonably good power and computational simplicity are the Kolmogorov-Smirnov test and the Cramér-von Mises test. Further, the first theorem suggests a test of the hypothesis that the interoccurrence times t_i are exponentially distributed. The Moran test is adequate for this purpose.

The Kolmogorov-Smirnov statistic is $D = \sup_{-\infty < x < \infty} |F(x) - S(x)|$, where $F(x)$ is the hypothesized and $S(x)$ the sample cumulative distribution function. Exact significance levels have been tabulated by Birnbaum [2] who observed that the asymptotic distribution is quite accurate for a sample size $n = 80$. The Cramér-von Mises statistic is $W^2 = n \int_0^1 [S(X) - F(X)]^2 dF(X)$. For computational purposes we use the equivalent expression

$$W^2 = \sum_{i=1}^n (F(x_i) - \frac{i-0.5}{n})^2 + \frac{1}{12n}$$

where x_i are the i th order statistics in the sample of size n . Significance points have been tabulated by Anderson and Darling [1]. The third test derived by Moran [11] is asymptotically most powerful for the hypothesized exponential variables against the Gamma alternatives. The test statistic is:

$$L_n = 2n [\log \bar{t} - \frac{1}{n} \sum_{i=1}^n \log t_i] / [1 + \frac{n+1}{6n}]$$

where t_i are the observed interoccurrence times and \bar{t} is the mean of t_i . The statistic L_n is approximately distributed as $\chi^2(n-1)$.

Results of the tests performed on the entire records of 148 hijackings are displayed in Table 1.

Table 1

Kolmogorov-Smirnov, Cramér-von Mises and
Moran Statistics for U.S. Hijackings

	D	W^2	L_n
Statistic	0.0726	0.1196	164.04

None of the statistics is significant at the 0.2 level. On the basis of these observations, we tentatively conclude that the overall behavior of the hijackings is consistent with a Poisson model. However, more detailed investigation of specific types of potential departures from the hypothesis are required. The two types of particular interest are, first, the possible serial dependency between consecutive interoccurrence times, and, second, the suspected bunching of incidents due to a burst of imitative behavior.

An adequate test for the independence of consecutive interoccurrence times is to examine the significance of the first lag sample product-moment statistic, using the so-called exponential ordered scores [17]. The statistic is defined as

$$R_1 = \sum_{i=1}^{n-1} x_{i+1} x_i$$

where $x_i = \frac{1}{n} + \frac{1}{n-1} + \dots + \frac{1}{n-j+1}$ and j is the rank, in ascendant order, of the i th interoccurrence time. It has been proved [3, p.167] that approximations to the first two moments of R_1 can be expressed as:

$$E(R_1) = n - 2 + \frac{\log n + 0.5772}{n} + \frac{1}{2n^2} + O(1/n^2) ;$$

$$\text{Var}(R_1) = \frac{n^3 - 6n^2 + 24n}{(n-2)(n-3)} - 2\log n + O\left(\frac{\log^3 n}{n}\right) .$$

The proportionate error in the expression for $\text{Var}(R_1)$ will be small for $n > 100$.

R_1 is asymptotically normal, and the normalized statistic can be tested as a standard normal variable for large samples. For our series of hijackings,

$$R_1^* = \{R_1 - E(R_1)\} / \{\text{Var}(R_1)\}^{0.5} = -0.6722 \text{ which falls well short of the critical value of the 0.05 level.}$$

Another test developed by Hsu [8] examining the significance of the parameter in a first order autoregressive model which is fitted to the interoccurrence times through a transformation technique has furnished a conclusion consistent with that based on R_1 . A statistic comparable to a standard normal variable yields a value of -0.7852, which is well within acceptance region on the assumption of independence.

A test useful for examining the bunching of incidents is the Durbin-Knott decomposition [4] of the Cramér-von Mises statistic. For a series of length n , denote the components by z_{nj} ($j=1,2,\dots$) . Durbin and Knott show that the following decomposition holds:

$$W_n^2 = \sum_{j=1}^{\infty} \frac{z_{nj}^2}{j^2 \pi^2}$$

where the z_{nj} can be computed from

$$z_{nj} = \sqrt{2/n} \sum_{i=1}^n \cos(j\pi y_i), \quad j=1,2,\dots$$

Since the z_{nj} can also be represented as $z_{nj} = \sqrt{2n} \int_0^1 [S(x)-y] \sin(j\pi y) dy$, where $y = F(x)$, they turn out to be the Fourier sine coefficients in the

expansion of $S(x)-F(x) = S(x)-y$ and thus represent the correct frequency domain decomposition of a random function which vanishes at the ends of the range. The first 20 components for the skyjack series are displayed in Table 2.

Table 2

Components of the Cramér-von Mises Statistics for U.S. Hijackings

Component j	1	2	3	4	5	6	7	8	9	10
Value	-.423	-.187	-1.787*	-2.114 [†]	-1.395	-1.752*	1.549	-.404	1.253	-.456
Component j	11	12	13	14	15	16	17	18	19	20
Value	1.911*	-.013	-.837	-1.390*	-.232	-.674	1.309	1.972 [†]	1.310	.840

* Significant at the 0.10 level

† Significant at the 0.05 level

It may be verified that the number of significant components is significantly greater than would be expected under the null hypothesis. Under the null hypothesis and for large n , it seems reasonable to regard z_{nj} as approximately normal with mean zero and unit variance. Then defining

$$V_{nj} = \int_0^{|z_{nj}|} \left(\frac{2}{\pi}\right)^{0.5} e^{-u^2/2} du$$

the quantities V_{nj} are uniformly distributed on $(0,1)$ under the null hypothesis. Employing the one-sided Kolmogorov-Smirnov test, the null hypothesis is rejected at the 0.01 level. Although the standard Kolmogorov-Smirnov and Cramér-von Mises tests fail to reject the null hypothesis, the finer Durbin-Knott procedure does detect a departure from it. In the light of this finding, a visual inspection of the conventional cumulative plot of the number of events against time, useful for locating potential bunching of incidents, is desirable. In Figure 1, the cumulative occurrences of hijackings against time, in units of days, is displayed. The plot, which should be roughly a straight

line when the process is truly a homogeneous Poisson, contains a portion, corresponding to approximately the first four hundred days, with a significant departure from a straight-line configuration. In particular, between the 300th and the 400th days of the series, there is an unusual density of occurrences as indicated by the slope of the dotted curve.

The Durbin-Knott test and visual inspection of the cumulative plot both suggest a departure from a homogeneous Poisson process. To obtain further confirmation we performed tests by separating the observations into three non-overlapping and exhaustive groups, assuming each to represent a homogeneous Poisson process. The details of the groups of observations are as follows:

Table 3
Three Groups of Observations and
their Estimates of Intensity

Group i	Period Covered (M/D/Y)	Intensity Length of the Period in days (T_i)	Number of Hi- jackings (n_i)	Average Occurrences per day
1	1/1/68-10/26/68	300	14	0.0467
2	10/27/68-2/3/69	100	19	0.1900
3	2/4/69-11/14/72	1380	115	0.0833

It is well known that, if two processes are both homogeneous Poisson processes with intensity parameters, λ_i and λ_j respectively, the statistic

$$\hat{\lambda}_i - \hat{\lambda}_j = \frac{n_i}{T_i} - \frac{n_j}{T_j}$$

is an unbiased estimate of $\lambda_i - \lambda_j$, where n_k and T_k ($k=i, j$) are, respectively, the number of occurrences and the length of the observation time of the k th process. An unbiased estimate of the variance of this statistic is

$$\text{Var}(\hat{\lambda}_i - \hat{\lambda}_j) = \frac{n_i}{T_i^2} + \frac{n_j}{T_j^2}$$

Therefore,

$$\Lambda_{ij} = \{(\hat{\lambda}_i - \hat{\lambda}_j) - (\lambda_i - \lambda_j)\} / \{\text{Var}(\hat{\lambda}_i - \hat{\lambda}_j)\}^{1/2}$$

can, even when n_i and n_j are small, be treated as having a standard normal distribution [3, p.228]. Results of tests on the three periods under the assumption $\lambda_i = \lambda_j$ are listed in Table 4. All of the Λ_{ij} are significant at the 0.05 level and one pair of them (period 1 vs. 2) are even significant

Table 4

The Testing Statistics Λ_{ij} of
The Three Periods

i \ j	1	2	3
1	0.0	-3.1551	-2.4970
2	3.1551	0.0	2.3986
3	2.4970	-2.3986	0.0

at the 0.0027 level. These results indicate that the skyjack series is a nonhomogeneous Poisson process with a burst of occurrences falling in the three month period between the end of October 1968 and early February 1969.

However, the separation of the series into three segments is somewhat intuitive and arbitrary. A detection scheme designed to search for the potential locations at which the intensity parameter may have shifted is described below. A moving block scheme has been suggested by Hsu [6], in which a pair of blocks consisting of consecutive but nonoverlapping observations are tested for the equality of their parameters, and the location of the blocks is systematically moved throughout the entire series (e.g. from left to right

with each shift equal to a given distance). Since there are two different ways of characterizing the series of hijackings, one by the number of occurrences in a time period of given length, the other by the time elapsed before the i th occurrence, the moving block scheme was applied to two differently sorted records. The first series is the number of occurrences in a hundred-day period, the starting date of the period shifting 20 days at each move from left to right throughout the 1780 days in the data. The second series consists of the time intervals elapsed before the fifteenth occurrence, with the starting times of the intervals being at the occurrence times of the 0th, 5th, 10th, ... hijacking, where the 0th occurred at January 1, 1968. A sequence of systematic tests examining the equality of two Poisson processes corresponding to two blocks of consecutive but nonoverlapping observations, of which the essential statistics have been described above, were then performed. Results are displayed in Figures 2 and 3. For Figure 3 we note that the statistic used for the comparison of two nonoverlapping time intervals in both of which 15 events occurred, can be expressed as below:

$$R = \frac{15T_j \lambda_j}{15T_i \lambda_i} \sim F(30,30)$$

where i and j identify the blocks of observations from which the T 's were computed and " \sim " means "is distributed as." When λ_i and λ_j are assumed equal, the R statistic is the ratio T_j/T_i . For convenience, the smaller value of T was always placed in the denominator and thus only the upper tail was examined. From the results presented in Figures 2 and 3 we see that potential shifts of the Poisson parameter locate at the early part of the data, roughly before the 400th day in the record, and possibly at the very end of the series

as indicated in Figure 2. This latter phenomenon probably indicates the fact that during late 1972 some preventive measures were brought into effect and significantly cut back on hijacking. Although the detection power of this scheme against some vague alternatives is mostly unknown, it provides valuable information concerning the overall significance level and the rough locations at which the intensity parameter has possibly shifted.

Based on the preceding analysis, we now postulate a model for the intensity parameter which allows it to assume different values for each occurrence. Let λ_k indicate the value of the Poisson intensity parameter with which the k th interoccurrence time has been generated. It is reasonable to assume that the value of λ_k shifts either gradually or suddenly depending on the nature of the source which influences the intensity. Both types of movement can be described by a portion, either the increasing half or the decreasing half, of a cosine curve. A small angular shift in the cosine curve at each increment of k indicates a gradual change of the intensity, while a large angular shift does the opposite. Moreover, the cosine curve also indicates the starting and the end points of k between which the transition takes place. In the case of aircraft hijacking, the occurrence rate appears to increase gradually starting with the 300th day in the data and to decline rapidly at around the 400th day. Therefore, it is desirable to use two pieces of half cosine curves, one to describe the climb, the other the decline, for the expression of these transitions. In mathematical form, these can be written as follows:

$$\begin{aligned}\lambda_k &= a + [(1 - \cos g_1(k))/2] (b-a) \quad \text{for } 0 < k < K_2 \\ &= b + [(1 - \cos g_2(k))/2] (c-b) \quad \text{for } K_1 + \frac{\pi}{\theta_1} < k < n\end{aligned}$$

where, for $i=1,2$,

$$\begin{aligned} \varepsilon_i(k) &= \theta_i(k-K_i) & \text{if } 0 \leq \theta_i(k-K_i) \leq \pi \\ &= 0 & \text{if } \theta_i(k-K_i) < 0 \\ &= \pi & \text{if } \pi < \theta_i(k-K_i) \end{aligned}$$

$$\text{and } K_1 + \frac{\pi}{\theta_1} \leq K_2 .$$

In the above expression K_1 and K_2 are the indices at which a cosine curve starts to climb or decline from a and b , respectively, while θ_1 and θ_2 indicate the speed of change with respect to k . Incidentally, when $K_1 + \frac{\pi}{\theta_1} = K_2$ the expression forms a complete cosine curve. Given the 148 observed interoccurrence times of hijacking of U.S. registered aircraft, denoted by t_k , $k = 1, 2, \dots, 148$, maximum likelihood estimates of a , b , c , K_1 , K_2 , θ_1 and θ_2 can be obtained by numerically maximizing the following logarithmic likelihood function:

$$L(a, b, c, K_1, K_2, \theta_1, \theta_2) = \sum_{k=1}^{148} \log \lambda_k - \sum_{k=1}^{148} \lambda_k t_k$$

Solutions based on the sample are listed as follows:

$$\begin{aligned} \hat{a} &= 0.049, \quad \hat{b} = 0.300, \quad \hat{c} = 0.082, \quad \hat{K}_1 = 11.24 \\ \hat{\theta}_1 &= 0.1396, \quad \hat{K}_2 = 33.75 \quad \text{and} \quad \hat{\theta}_2 = 3.8410 \end{aligned}$$

A schematic diagram of the λ_k function is given in Figure 4. These results indicate that the intensity of hijacking kept climbing from the eleventh incident (August 4, 1968) through the 33rd (February 3, 1969) with the peak intensity equal to 0.30 which means that, on average, there was a hijacking every three days. Interestingly, right after the occurrence of the 33rd hijacking, since $\pi/\theta_2 = 0.82$, a rapid decline was observed and the transition

to a new lower level, equal to $\hat{c} = 0.082$, took only a single occurrence. In addition, the analysis indicates that the hijacking intensity remained at a level close to 0.082 from February 25, 1969 through late 1972.

In order to see whether this modelling of λ_k provides significant improvement over a homogeneous Poisson process, the conventional loglikelihood ratio statistic was computed. This is distributed as half of a χ^2 variable with the degrees of freedom equal to the number of parameters estimated minus one, in our case here equal to 6. The computed value of 8.427 is significant at the 0.01 level. This indicates that our specification of λ_k offers a better representation for the hijacking series than a simple Poisson model. Moreover, we re-examine the later part of the data, from the 34th through the 148th observations, by recomputing the statistics, D , W^2 , L_n , R_1 and z_{nj} previously defined. None of the results is significant at the 0.05 level. This finding suggests that, although the hijacking process is globally non-stationary, when adequately broken into some subperiods, it can be locally homogeneous Poisson.

The statistical evidence is considerably illuminated by social, political and psychological interpretation of the events under consideration. During early 1968, the hijacking of U.S. aircraft was almost exclusively undertaken by Cuban refugees who escaped from Cuba during the years of 1961-1963 and wished to return. In the absence of ordinary methods of transportation between the U.S. and Cuba, hijacking was indeed a convenient way to "charter" a plane to Cuba. However, the illusion that Cuba was a haven where pirates were treated royally as a reward for escaping from the "imperialist-capitalist" United States also induced a large number of non-Cubans, including the common

delinquents, mentally deranged and political dissidents, to divert U.S. planes to Havana during late 1968 and early 1969. This "epidemic" motive explains the increase in the hijacking intensity in that period. Then, in February 1969, various facts were released to the public at hearings before a Congressional committee, such as the facts that most of the hijackers were detained by Cuban government for various lengths of time for interrogation and, when released, were given "ordinary work, mostly at hard labor in agriculture" (see Phillips [13], p. 74-77 for more details). The apparent mistreatment of hijackers in Cuba presumably disillusioned many potential hijackers which explains the rapid decline in the hijacking density at that time. From then on the hijackers' purposes and destinations started to diverge. At the end of 1969, two incidents marked this divergence. One was an American Italian's hijacking of a TWA jet from the U.S. to Rome and the other was the blowing up of another TWA jet at Damascus by Arab guerrillas. While the hijacking of U.S. planes by Cubans continued in the years 1970 and 1971, other motivations explained to half of the incidents. On November 24, 1971 a hijacking extortionist seized a Northwest B-727 jet, demanded \$200,000 ransom and parachutes, jumped from the flying aircraft with the ransom and successfully disappeared. This inventive criminal was followed by dozens of imitators and made 1972 the year of hijacking extortionists. (As a digression, the extortionists appeared to have a preference on Friday. One may speculate that since Friday is a payday they counted on more cash being available in banks and useable for paying ransom. A binomial test confirms this hypothesis, for 10 out of 31 hijackings in 1972 were on Fridays. This indicates that the Poisson model needs a slight modification in this respect). By the end of 1972, many preventive measures were implemented both on ground and on board by the government and the airlines.

Passenger screening stations were established in most airports and skymarshals were travelling on numerous flights. These measures effectively reduced the number of hijacking attempts in late 1972. Eventually, the federal government's full enforcement of pre-boarding checks of passengers had completely ended the U.S. hijacking era at the start of 1973. Since then no successful hijacking occurred in the U.S.

Worldwide Hijacking. In the same period of 1968-72, there were 335 hijacking incidents in the world, of which 44 percent were associated with U.S. registered aircraft. Through an analysis similar to the one presented above, we found that the stochastic behavior of worldwide hijackings, as one can expect, is remarkably similar to that of the U.S. A fairly smooth take-off and a sudden decline in the intensity parameter were identified in late 1968 and the start of 1969. Also, the intensity is reasonably stable over the remaining period. A Poisson model with the occurrence rate equal to 0.2024 (per day) can satisfactorily represent the hijacking series in this period.

Since there is a variety of types of hijackers, countries of origin, and personal or political motivations, a thorough understanding of the worldwide hijacking series requires detailed investigation of many individual cases, in particular of the Arab hijacking war in which Arab guerrillas used air piracy for the purpose of political blackmail as well as a tactic of terrorism. Interested readers are referred to Phillips' book [13, Chapters 6 and 7] for a penetrating exposition.

3. Political Assassinations in the U.S.

Political assassination is a similar subject of considerable concern to all. Like aircraft hijacking, assassinations have been occurring in an

unpredictable manner and a statistical model which reasonably represents their stochastic behavior may be useful for many purposes.

The U.S. assassination data studied in this section were extracted from an appendix in Assassination and Political Violence (edited by J.F. Kirkham), a staff report to the National Commission on the Causes and Prevention of Violence, 1969. The records were collected, on a worldwide basis, by the so-called Leiden group for the period from 1919 to October 1968, listing each incident of attempted assassination, both successful and unsuccessful, arranged chronologically. Although there are many interesting aspects of assassinations in other countries that can be analyzed statistically, because of the U.S.'s high rank in assassination rate² and the special emphasis of this paper, we use here only the U.S. data to illustrate the analysis. Similar analysis can possibly be applied to the data of other nations. Moreover, for practical interest and for avoidance of unnecessary technical complication, we further confine ourselves to the analysis of the data in the period of 1930-1968; inclusion of the earlier years, due to the clustered occurrences of assassinations in 1926 and 1929 may considerably complicate our discussion here with no particular benefit for illuminating the general stochastic behavior.

Political assassination in the U.S. from 1930 to 1968 include fifty-seven incidents. From the raw data³ we constructed the sequence of interoccurrence

²We found that the total number of target persons, including office holders of all ranks, involved in the assassination attempts in the U.S., as shown by the Leiden data, is the third highest among 117 nations and areas, ranking just below those of Mexico and France, over the fifty-year period. For statistical tables otherwise compiled, see Chapter 3 of Kirkham (ed.) [9].

³The reported dates of three incidents are incomplete, two with the day missing, the other with both the month and the day missing. For the former case we assume the day to be the fifteenth of the month, and for the latter we assume it to be the thirtieth of June. These fillings of the missing data have only negligible effect on the results of analysis.

times in units of days⁴. To test whether the occurrences of assassinations again follow a Poisson model we computed the statistics: D , W^2 , L_n , R_1^* and the first 20 major components in the Cramér-von Mises statistic. The results are displayed in Tables 5 and 6. None of the statistics indicates a significant

Table 5
Test Statistics of U.S. Assassinations

	D	W^2	L_n	R_1^*
Statistic	0.1349	0.1412	71.18	-0.5686

Table 6
Components of the Cramér-von Mises Statistic
for U.S. Assassinations

Component j	1	2	3	4	5	6	7	8
Value	-0.3641	1.7293*	-1.5584	1.2427	-0.3199	-0.6194	-0.8852	0.3444
Component j	9	10	11	12	13	14	15	16
Value	0.4403	-0.4990	1.2096	0.9907	0.9594	0.8077	-0.0151	2.0939†
Component j	17	18	19	20				
Value	-1.5282	0.7578	0.5161	-0.0301				

* Significant at the 0.10 level.

† Significant at the 0.05 level.

departure from the Poisson model at the 0.10 level.

From the evidence presented, we are confident that the probabilistic structure of the occurrences of assassinations in the U.S. is approximately Poisson. However, to answer the question whether the intensity parameter is truly stationary in time requires further tests. The cumulative plot of counts against time and the moving block scheme described before are again

⁴Our time origin is November 25, 1929, since the last incident in the previous period occurred at this date.

employed for this purpose. Figures 5, 6 and 7 present the computed results. As one can easily recognize from Figure 5 that the plot of cumulative counts against time forms a kinky curve with two turning points located at roughly the 7,800th and the 12,000th days in the data, corresponding to the end of 1950 and the end of 1961 respectively. To investigate further, using the moving block procedure, we set the block size equal to 1,000 days and each stride equal to 200 days and compared the number of counts therein contained with that in the neighboring block. In Figure 6, where the computed results presented, two points significant at the 0.05 level, the 35th and the 56th tests, correspond precisely to the 7,800th and the 12,000th days in the data. Another set of moving block tests using the time intervals elapsed before the tenth occurrence, each stride equal to two occurrences, was performed and results shown in Figure 7. Again, the significant points correspond to the 32nd and the 42nd occurrences that locate at roughly the two time points found in Figure 6.

The model for λ_k expressed in Section 2 was then applied to fit the assassination data. The maximum likelihood estimates of the parameters are

$$\hat{a} = 0.004316 \quad \hat{b} = 0.002058 \quad \hat{c} = 0.007310$$

$$33 < \hat{K}_1 < 34, \quad \hat{\theta}_1 > \pi/(34 - \hat{K}_1), \quad 42 < \hat{K}_2 < 43, \quad \text{and} \quad \hat{\theta}_2 > \pi/(43 - \hat{K}_2)$$

The reason for having multiple values on the last four estimates is that the likelihood function is flat in the ranges specified. This implies that both of the two transitions in assassination intensity are of discrete type; they do not even take a single observation to finish the shifts. This appears to be consistent with our visual inspection on Figure 5 which indicates two sudden turns in the slope of the plot.

To provide a final check on the adequacy of this nonhomogeneous Poisson model for the assassinations, we separate the 57 interoccurrence times into three groups based on the estimates of the parameters K_1 and K_2 . Using the notations previously defined, we have

$$\begin{array}{lll} n_1 = 33 & n_2 = 9 & n_3 = 15 \\ T_1 = 7646 \text{ (days)} & T_2 = 4374 & T_3 = 2052 \end{array}$$

Results of F tests on the three groups are displayed below:

$$\frac{T_2}{n_2} / \frac{T_1}{n_1} = 2.0976 > F_{0.05}(18,66) = 1.79$$

$$\frac{T_1}{n_1} / \frac{T_3}{n_3} = 1.6937 \approx F_{0.05}(66,30) = 1.73$$

$$\frac{T_2}{n_2} / \frac{T_3}{n_3} = 3.5526 > F_{0.005}(18,30) = 2.90$$

The results suggest that the first transition in assassination intensity is significant at the 0.05 level while the second one at the 0.005 level.

The three subperiods cover, roughly speaking, the years of 1930-1950, 1951-1961, and 1962-1968 and the average number of occurrences per year for these periods are 1.57, 0.75 and 2.67, respectively. The assassination attempts against Presidents F.D. Roosevelt and H.S. Truman occurred in the first subperiod, while those of President J.F. Kennedy, Dr. Martin Luther King, Jr., and Senator R.F. Kennedy occurred in the third subperiod. The first period covers the great Depression and World War II and the last the Vietnam War and both are periods of political turmoil and/or severe social-economic change. One middle period, by contrast, is often thought to be a politically and socially

quiescent period. It is reassuring that the statistical tests confirm broad standard historical interpretations of the characters of the periods in question.

For studies concerning the probability of being assassinated by type of office holders in the U.S. and cross-national comparative studies of political assassination, the interested reader is again referred to Kirkham (3d.) [9].

4. Concluding Remarks

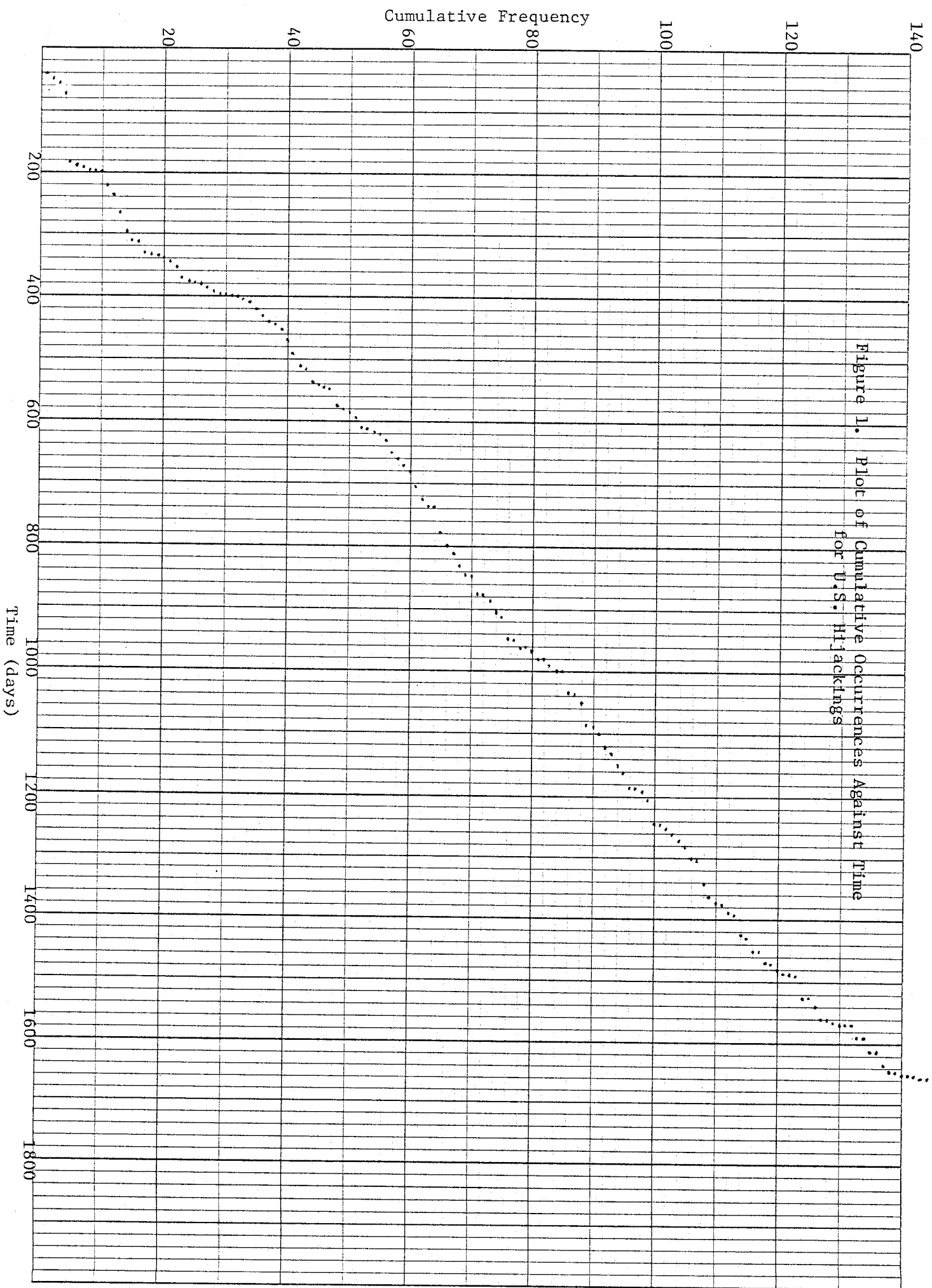
In this paper we have presented detailed models of the U.S. aircraft hijacking and political assassination. The results suggest that when approach in a systematic way, data which at first may appear to be lacking structure can be organized, modelled and interpreted through the use of statistical techniques. For policy purposes we find it particularly promising that one can identify changes in the parameters of the underlying stochastic process.

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Figure 1. Plot of Cumulative Occurrences Against Time for U.S. Hijackings



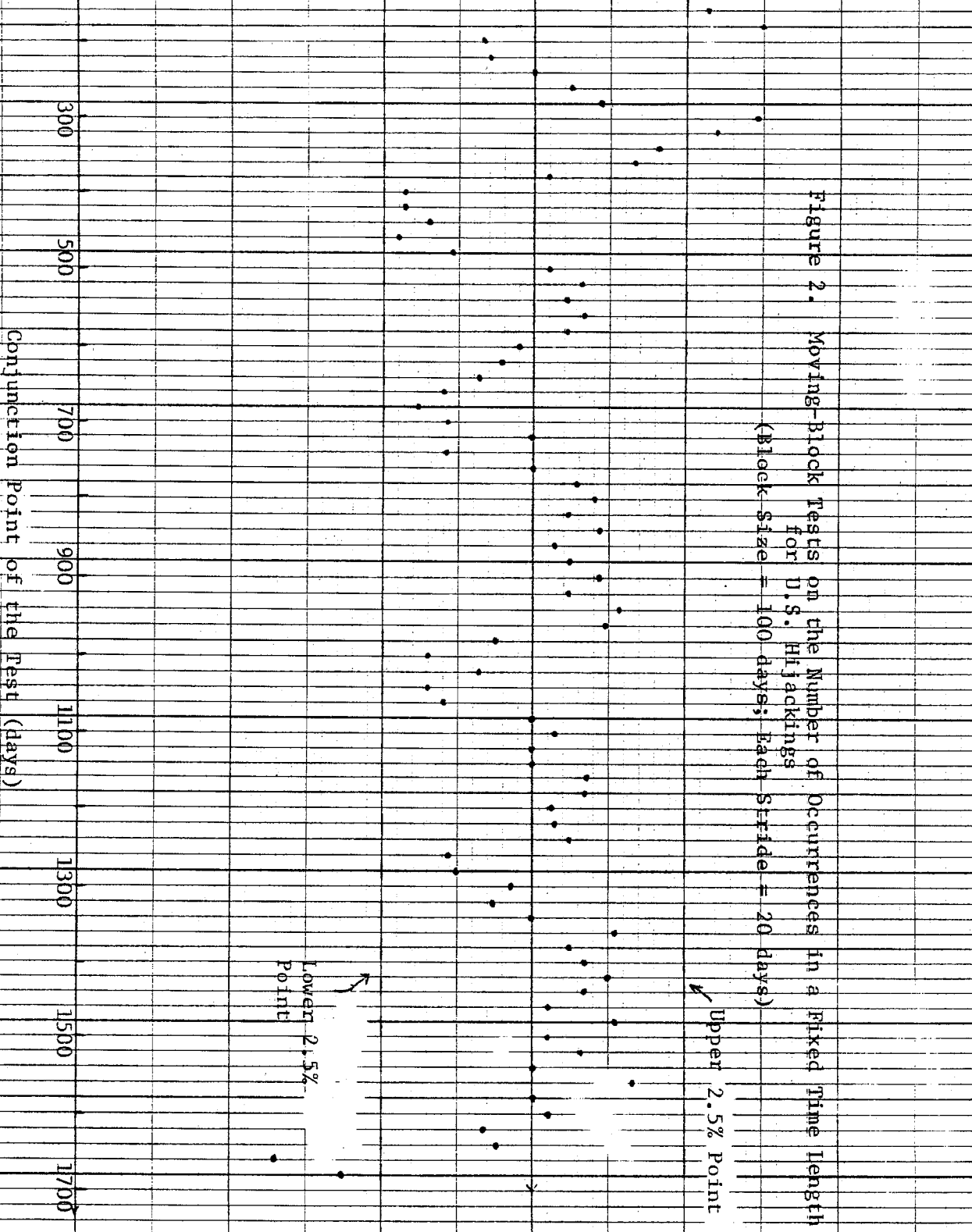


Figure 2. Moving-Block Tests on the Number of Occurrences in a Fixed Time Length for U.S. Hijackings (Block-Size = 100 days; Each Stride = 20 days)

Conjunction Point of the Test (days)

100 300 500 700 900 1100 1300 1500 1700

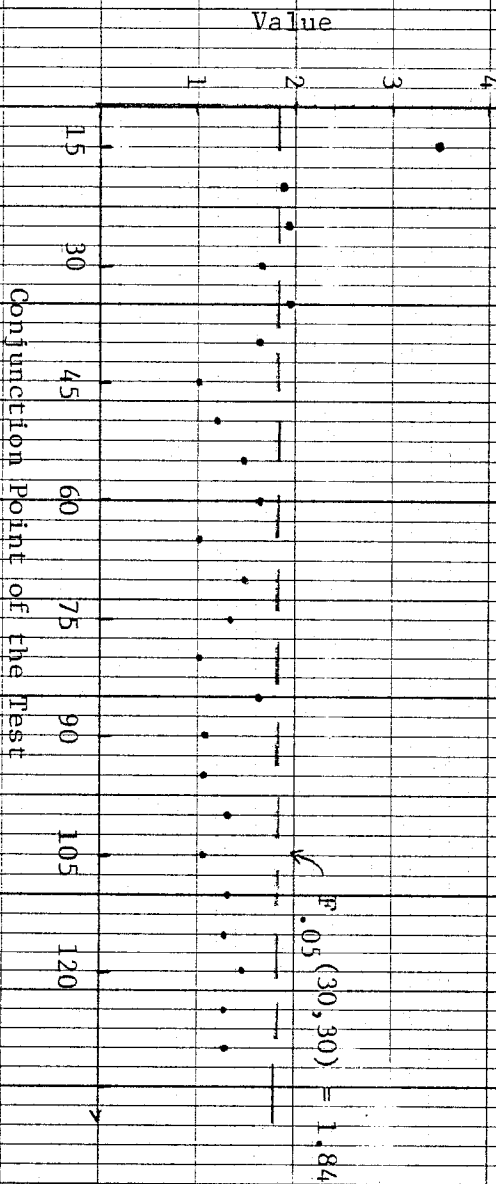
Value

4 3 2 1 0 -1 -2 -3

Upper 2.5% Point

Lower 2.5% Point

Figure 3. Moving-Block Tests on the Time Elapsed Before
 a Fixed Number of Occurrences
 for U.S. Hijackings
 (Block Size = 15; Each Stride = 5)



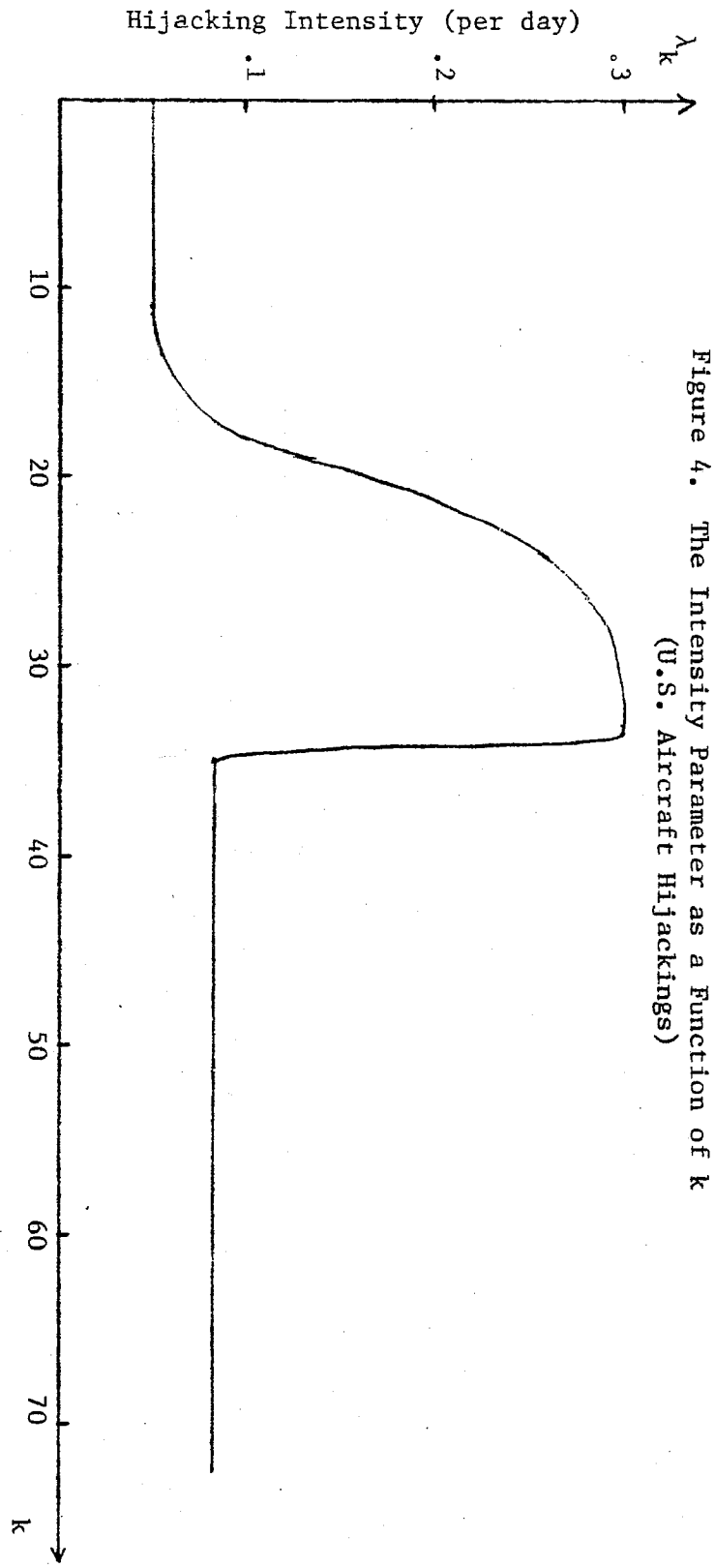


Figure 4. The Intensity Parameter as a Function of k (U.S. Aircraft Hijackings)

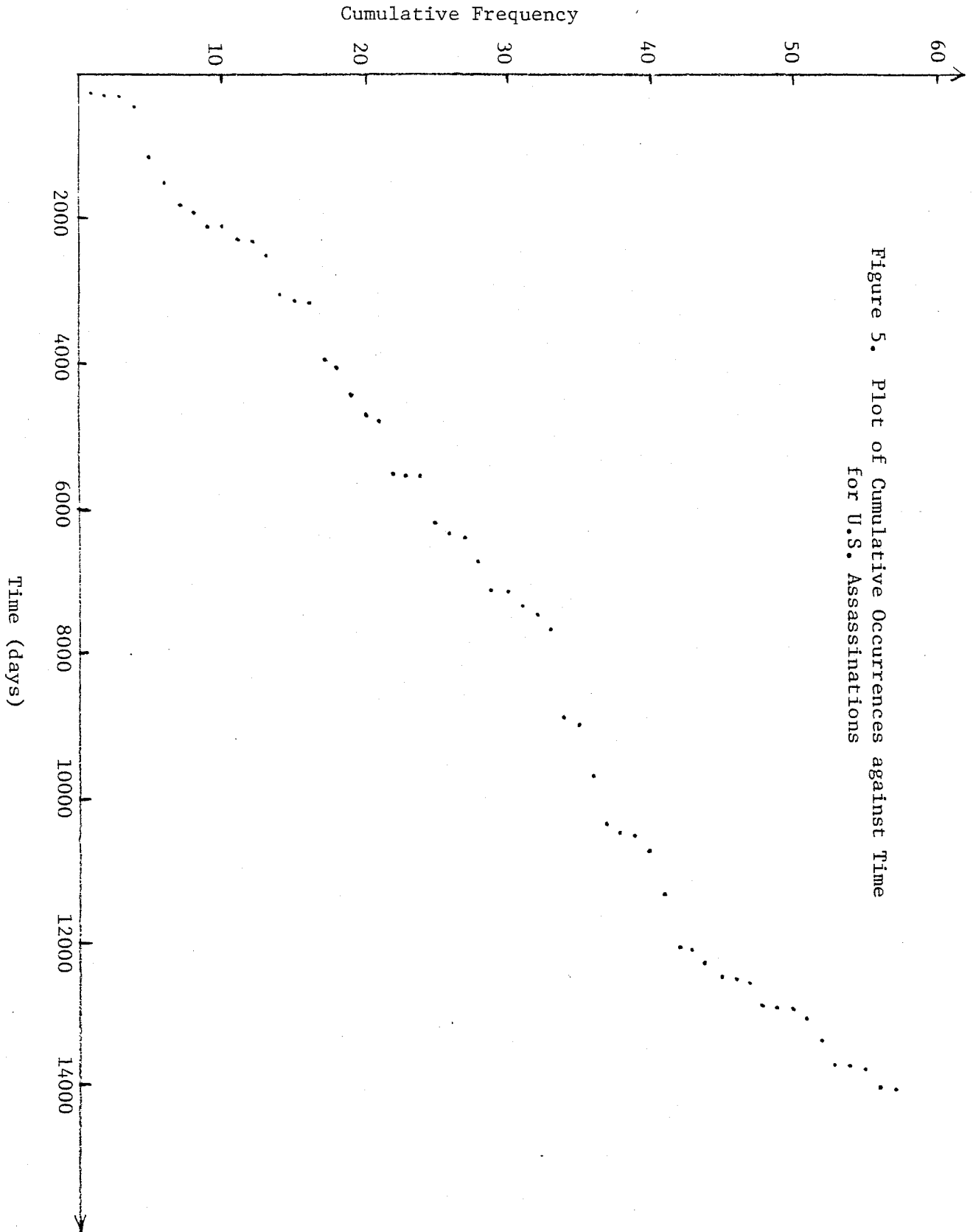


Figure 5. Plot of Cumulative Occurrences against Time for U.S. Assassinations

