USEFULNESS OF IMPERFECT MODELS
FOR THE FORMULATION OF STABILIZATION POLICIES

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By Gregory C. Chow[1]

1. Introduction

Econometric models are widely used to forecast the national economy. Are they accurate enough to be used by the government authorities for the formulation of macroeconomic policies? What kind of accuracy is required for them to be useful as a guide to policy? This paper provides a theoretical framework to answer this accuracy question, and applies it to ascertain the usefulness of two simplified models in the determination of stabilization policies.

Recently, through the efforts of the participants of the NBER-NSF seminar on the comparison of econometric models, interesting results have emerged on the comparative forecasting abilities and the multiplier effects of the major U.S. econometric models currently in use. Some of these results have appeared in the June and October, 1974, and the February, 1975, issues of the International Economic Review. According to the appraisal of Carl Christ [5 , p. 54], "though the models forecast well over horizons of four to six quarters, they disagree so strongly about the effects of important monetary and fiscal policies that they cannot be considered reliable guides to such policy effects, until it can be determined which of them are wrong in this respect and which (if any) are right." Christ goes on to exhibit in tables and charts the different multiplier effects of government purchases and unborrowed reserves based on the various models. The method of this paper can be applied to decide whether two models disagree significantly in terms of their policy recommendations. The existing models which imply different multiplier effects do "forecast well over horizons of four to six quarters." They do contain useful information, however imperfect, which can be exploited to make forecasts. Therefore, one cannot automatically assume that the same information is useless for the formulation of economic policy. In fact, sound economic policy is based on good economic forecasts made under the assumption of alternative policy proposals. Furthermore, just as two structures having different multiplier effects may produce similar forecasts in contrast with a naive forecast, they may also produce similar policy recommendations as least in comparison with some version of an inactive policy. It is necessary to find out how the entire set of dynamic multipliers is combined to yield a policy recommendation.

To show that two different models may yield the same or similar policy recommendations, consider the univariate difference equation

$$(1.1) \qquad y_t = ay_{t-1} + cx_t + u_t$$

where y_t is a dependent variable, x_t is a policy instrument or control variable and u_t is a serially independent random disturbance with mean zero and variance v. If the objective is to minimize the expectation Ey_t^2, then the optimal feedback policy is to set $ay_{t-1} + cx_t$ equal to zero, so that Ey_t^2 achieves its minimum $Eu_t^2 = v$. The policy is therefore

$$(1.2) \qquad x_t = (-c^{-1}a)y_{t-1} .$$

Another model, which has coefficients $\tilde{a}$ and $\tilde{c}$ instead of a and c, will yield the same policy provided that the ratio $\tilde{a}/\tilde{c}$ is the same as a/c. The multipliers $a^k c$ of x_{t-k} in the final form of model (1.1) could certainly be very different from those of the alternative model, as illustrated by $a = \cdot 9$, $c = 1$, $\tilde{a} = \cdot 09$ and $\tilde{c} = \cdot 1$. Thus an imperfect model, with coefficients ·09 and ·1, may yield a policy close to being optimal, if the true coefficients are ·9 and 1 respectively.

The key question concerning the usefulness of imperfect models, however, is not whether they will yield nearly optimal policies which could be obtained only by having perfect knowledge of the economic structure, but whether they can yield policies which are superior to an inactive policy allowing for no feedback. In the above example, an inactive policy is to set $x_t = 0$. Under this policy and assuming (1.1) to be the true model with $|a| < 1$, we can easily find the variance of y_t as t increases to be

$$Ey_t^2 = \frac{v}{1-a^2} . \tag{1.3}$$

If the government authority uses the inaccurate coefficients $\tilde{a}$ and $\tilde{c}$ and the resulting feedback policy, the system (1.1) will become

$$y_t = [a + c(-\tilde{c}^{-1}\tilde{a})]y_{t-1} + u_t , \tag{1.4}$$

which has the steady-state variance

$$Ey_t^2 = \frac{v}{1 - [a + c(-\tilde{c}^{-1}\tilde{a})]^2} . \tag{1.5}$$

This variance is smaller than the variance (1.3) prevailing under the inactive policy provided merely that $[a + c(-\tilde{c}^{-1}\tilde{a})]^2$ is smaller than a^2. Given a and c, a wide range of values for $\tilde{a}$ and $\tilde{c}$ will produce this required result.

Hence very imperfect models can still be better than using no models for the determination of macroeconomic policy.

We will generalize the above discussion in section 2 to treat dynamic econometric systems involving many variables and higher-order lags. Section 3 provides two illustrative models to be used for stabilization policy. Section 4 applies the method of section 2 to evaluate the usefulness of one of the models of section 3, assuming that the other model is the correct one. It illustrates how an imperfect model performs for the determination of policy as compared with using no model at all. Section 5 contains some concluding remarks.

2. Evaluation of Imperfect Models for Policy Analysis

Let the economy be governed by a time-varying linear system

$$y_t = A_t y_{t-1} + C_t x_t + b_t + u_t \tag{2.1}$$

where y_t is a vector of p endogenous variables, x_t is a vector of q policy or control variables with $q < p$, and u_t is a random vector independently distributed through time, having mean zero and covariance matrix V. The true parameters A_t, C_t, b_t and V are of course unknown to the policy maker. We will assume that the policy maker has available an imperfect model explaining a subset of the endogenous variables y_t. Written in the form (2.1), with appropriate zeros added, this imperfect model has coefficients $\tilde{A}_t$, $\tilde{C}_t$ and $\tilde{b}_t$. The question is how well a policy based on these inaccurate parameters would work, as compared with a policy of using no feedback, for certain hypothetical values of A_t, C_t and b_t. High-order lags in both the endogenous and policy variables are subsumed under the notation of (2.1) by suitable definitions, as illustrated by (3.2) in section 3 below. Nonlinear systems can be approximated by time-varying linear systems of the form (2.1) for our analysis, as will be explained later in this section.

The performance of the economy is measured by the expectation of the loss function

$$(2.2) \qquad \sum_{t=1}^{T} (y_t - a_t)' K_t (y_t - a_t)$$

where a_t are the targets and K_t are diagonal matrices giving the relative penalties of the squared deviations of the different variables from their targets. If the behavior of the policy variables also matters, they will be included in the vector y_t by appropriate definitions. We will be interested in comparing the performance of three policies. Policy I is the optimal policy assuming perfect knowledge of the true model (2.1). Policy II is obtained by minimizing the expectation of (2.2) under the assumption of an imperfect model, with coefficients $\tilde{A}_t$, $\tilde{C}_t$ and $\tilde{b}_t$. Policy III specifies a smooth time path for the policy variables which will not be altered by future observations of the economy.

As given in Chow [1 , Chapter 7], the optimal policy I is given by a set of linear feedback control equations

$$(2.3) \qquad x_t = G_t y_{t-1} + g_t \quad .$$

The coefficients G_t are obtained by solving

$$(2.4) \qquad G_t = - (C_t' H_t C_t)^{-1} C_t' H_t A_t \ ;$$

(2.5) $$H_{t-1} = K_{t-1} + (A_t + C_tG_t)'H_t(A_t + C_tG_t) \qquad (t = T, T-1, \ldots , 1)$$

with the initial condition $H_T = K_T$. The intercepts g_t are obtained by solving

(2.6) $$g_t = -(C_t'H_tC_t)^{-1}C_t'(H_tb_t - h_t) ;$$

(2.7) $$h_{t-1} = K_{t-1}a_{t-1} + (A_t + C_tG_t)'(h_t - H_tb_t)$$

with the initial condition $h_T = K_Ta_T$.

The economy under policy I will follow (2.1) and (2.3) which combine to yield

(2.8) $$y_t = R_ty_{t-1} + r_t + u_t$$

where

(2.9) $$R_t = (A_t + C_tG_t) ; \quad r_t = b_t + C_tg_t .$$

The mean path of the economy will follow

(2.10) $$\bar{y}_t = R_t\bar{y}_{t-1} + r_t .$$

By substracting (2.10) from (2.8) and defining the deviation from mean $y_t^* = y_t - \bar{y}_t$, we have

(2.11) $$y_t^* = R_t y_{t-1}^* + u_t$$

The covariance matrix of the system will therefore be

(2.12) $$Ey_t^* y_t^{*\prime} = R_t (Ey_{t-1}^* y_{t-1}^{*\prime}) R_t^{\prime} + V \qquad (t = 1, 2, \ldots, T)$$

with initial condition $Ey_0^* y_0^* = 0$ since y_0 is constant and $y_0^* = 0$.

By considering the deviation $y_t - a_t$ as the sum of $y_t - \bar{y}_t$ and $\bar{y}_t - a_t$, we will decompose the expectation of the loss function (2.2) into two parts,

(2.13) $$E \sum_{t=1}^{T} (y_t - \bar{y}_t)' K_t (y_t - \bar{y}_t) + E \sum_{t=1}^{T} (\bar{y}_t - a_t)' K_t (\bar{y}_t - a_t)$$

$$= \operatorname{tr} \sum_{t=1}^{T} K_t E y_t^* y_t^{*\prime} + \sum_{t=1}^{T} (\bar{y}_t - a_t)' K_t (\bar{y}_t - a_t) .$$

One part is a weighted sum of the variances of y_t, to be calculated by using the covariance matrix (2.12). The other is a weighted sum of the squared deviations of the means $\bar{y}_t$ from the targets a_t. This decomposition will be used to study the expected losses of policies II and III as well.

Policy II is obtained by minimizing the expectation of (2.2) subject to a model of the form (2.1) with coefficients $\tilde{A}_t$, $\tilde{C}_t$ and $\tilde{b}_t$. This policy is given by a feedback control equation of the form (2.3), with coefficients $\tilde{G}_t$ and $\tilde{g}_t$ which are computed from equations (2.4) to (2.7) using coefficients $\tilde{A}_t$, $\tilde{C}_t$ and $\tilde{b}_t$ instead. The economy under policy II will be governed by (2.1) and this feedback control equation, namely

(2.14) $$y_t = \tilde{R}_t y_{t-1} + \tilde{r}_t + u_t$$

where

(2.15) $$\tilde{R}_t = (A_t + C_t\tilde{G}_t) \; ; \quad \tilde{r}_t = b_t + C_t\tilde{g}_t \; .$$

The mean path and the covariance matrix of this system will be given respectively by (2.10) and (2.12) with $\tilde{R}_t$ and $\tilde{r}_t$ replacing R_t and r_t. The expected loss under this regime can be similarly decomposed as in (2.13).

Policy III allows for no feedback. If one refuses to use econometric models for the formulation of macroeconomic policy, what alternatives are available? One alternative is still to adjust the policy instruments according to the current state of the economy by some ad hoc rules which are not derived systematically from an econometric model. Such rules, once stated explicitly in the form of feedback control equations, can and should be evaluated by the method here proposed. Skeptics of the use of econometric models are under the obligation to show that their alternatives are no worse. The second alternative, which we will further examine, is not to use any feedback. It can always be written as $x_t = g_t^0$ for some fixed path g_t^0 to be specified without regard to the state of the economy. Under such a rule, which implies $G_t = 0$ in our notation, the mean and covariance matrix of the economic variables will be given by (2.10) and (2.12) respectively, with $R_t = A_t$ and $r_t = b_t + C_t g_t^0$. The two components of the expected loss can be computed by (2.13).

If the true model is nonlinear and consists of random disturbances, one cannot obtain analytically an optimal policy which would minimize the expectation of (2.2) under the assumption of perfect knowledge of the model parameters. However, for our analysis, policy I will be replaced by the following nearly optimal policy, which is described more fully in Chow [1, Chapter 12] and

[4]. First, ignoring the random disturbances in the model, one finds an optimal path to minimize (2.2) using the resulting deterministic model. One then linearizes the model about this path, producing a system of the form (2.1) with time-varying coefficients. The analysis suggested above can be carried out in exactly the same way. The feedback control coefficients $\tilde{G}_t$ and $\tilde{g}_t$ for policy II are obtained by employing an imperfect nonlinear model which is similarly linearized to yield the coefficients $\tilde{A}_t$, $\tilde{C}_t$ and $\tilde{b}_t$ needed for the calculations of (2.4) to (2.7). Policy III remains to be $x_t = g_t^0$. The two components of the expected loss resulting from each policy can be calculated as before.

In the special case of a linear model with constant coefficients A, C and b, and of a loss function with a constant target a and a constant matrix K , the above analysis may be simplified. Under appropriate conditions as described in Chow [1 , section 7.8], the solution of (2.4) to (2.7) as T increases may reach steady states G , H , g and h . If the characteristic roots of $R = (A + CG)$ are all smaller than one in absolute value, the mean and covariance matrix of the system will reach steady states given respectively by

(2.16) $$\bar{y} = (I - R)^{-1}r ;$$

(2.17) $$Ey_t^*y_t^{*\prime} = R(Ey_t^*y_t^{*\prime})R' + V .$$

It would be of interest to compare the three policies in terms of the two components of the expected loss per period in the steady state, i.e., $\text{tr } KEy_t^*y_t^{*\prime}$ and $(\bar{y} - a)'K(\bar{y} - a)$. The special case here treated is useful when the target variables are unemployment rate, inflation rate, or the rates of change in GNP and its major components, since the assumption of constant targets for these variables appears to be reasonable. If one employs a linear model with constant coefficients to explain the level of GNP and the price level, one may first-difference the model and assume constant targets for the changes in GNP and the price index. The method of this paragraph would apply if the residuals in the model explaining these first differences satisfy the assumptions stated for u_t in (2.1). As a generalization of the discussion of section 1, an imperfect model yielding the feedback coefficients $\tilde{G}$ can be used to stabilize the economy better than using no feedback provided that $R_t = (A + C\tilde{G})$ entering equation (2.12) will produce smaller variances than $R_t = A$.

In this section, we have suggested some analytical methods to evaluate policy recommendations derived from imperfect models. Without them, one would have to perform very expensive stochastic simulations to obtain sample paths of the economy under the assumptions of a hypothetically true model and alternative policy rules. The analytical methods can be used to deduce the means and covariance matrices of the sample paths without resort to the perhaps prohibitive computer simulations.

3. Fitting Two Illustrative Models

To illustrate the method of section 2, we will employ two hypothetical linear models. These models are derived from the multipliers reported in Christ [5] for the Michigan quarterly model and the Wharton Mark III model. Given the multipliers of the final form of an econometric model, the following procedure is applied to construct an approximate reduced form for policy analysis.

The procedure is based on the well-known relation between the reduced form and the final form. Let the reduced form be

(3.1) $$y_t = B_1 y_{t-1} + B_2 y_{t-2} + B_3 x_t + B_4 x_{t-1} + B_5 x_{t-2} + b_0 + v_t \, .$$

We will convert it to first order and eliminate the lagged control variables by writing

(3.2) $$\begin{bmatrix} y_t \\ y_{t-1} \\ x_t \\ x_{t-1} \end{bmatrix} = \begin{bmatrix} B_1 & B_2 & B_4 & B_5 \\ I & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & I & 0 \end{bmatrix} \begin{bmatrix} y_{t-1} \\ y_{t-2} \\ x_{t-1} \\ x_{t-2} \end{bmatrix} + \begin{bmatrix} B_3 \\ 0 \\ I \\ 0 \end{bmatrix} x_t + \begin{bmatrix} b_0 \\ 0 \\ 0 \\ 0 \end{bmatrix} + \begin{bmatrix} v_t \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

which will be rewritten simply as

(3.3) $$y_t = Ay_{t-1} + Cx_t + b + u_t \, .$$

Note that the new vector y_t of dependent variables includes the original dependent variables and control variables as subvectors. The matrices A and C and the vector b in (3.3) are defined by (3.2). By repeated elimination of lagged y's using (3.3), we obtain the final form

(3.4) $$\begin{aligned} y_t = {} & Cx_t + ACx_{t-1} + A^2Cx_{t-2} + \ldots + A^{t-1}Cx_1 \\ & + A^t y_0 + b + Ab + A^2 b + \ldots + A^{t-1} b \\ & + u_t + Au_{t-1} + A^2 u_{t-2} + \ldots + A^{t-1} u_1 \, . \end{aligned}$$

To construct a reduced form from the given final-form multipliers, we first make a tentative decision on the number of lagged y's and the number of lagged x's required as the reduced form was originally written in the form of equation (3.1). The coefficients B_i in (3.1) are related to A and C in (3.3) by definitions similar to those given in (3.2). The matrix C of impact multipliers are known. Denote the delayed multipliers AC, A^2C, ..., A^kC, respectively by M_1, M_2, ..., M_k which are also known. We will use the relations

(3.5) $$AC = M_1 \; ; \quad AM_1 = M_2 \; ; \quad AM_2 = M_3 \; ; \; \ldots \; ; \quad AM_{k-1} = M_k \; ,$$

$$\text{or} \quad A[C \; M_1 \; M_2 \; \ldots \; M_{k-1}] = [M_1 \; M_2 \; M_3 \; \ldots \; M_k] \; .$$

Each row a_i' of unknown elements in A will be chosen to minimize the sum of squares of the deviations of $a_i'[C \; M_1 \; \ldots \; M_{k-1}]$ from the i-th row m_i' of $[M_1 \; M_2 \; \ldots \; M_k]$. By the method of least squares,

(3.6) $$a_i = [C \; M_1 \; \ldots \; M_{k-1})(C \; M_1 \; \ldots \; M_{k-1})']^{-1}(C \; M_1 \; \ldots \; M_{k-1})m_i \; .$$

If the fit is poor, as judged by the sizes of the above deviations, we will increase the numbers of lagged y's and/or lagged x's in the reduced form (3.1).

For illustrative purpose, we have chosen two dependent variables, nominal and real GNP, and two instruments, Federal government non-defense purchases and unborrowed reserves. The multiplier effects of a \$1 billion increase in nominal government purchases on nominal and real GNP (in billions of 1958 dollars) are given in Table 3 of Christ [5 , pp. 66-67], lines 3 and 11 showing the effects for the Michigan Model and lines 5 and 13 for the Wharton Model. Similarly, the effects of a $1 billion increase in unborrowed reserves (or a cut of 50 basis point in the Treasury bill rate) are given in Table 4 of Christ [5 , pp. 68-69], lines 2 and 10 for the Michigan Model and lines 4 and 12 for the Wharton Model. The multipliers from the Michigan Model are based on simulations for the 40 quarters from 58.1 to 67.4. From the Wharton Model, they are based on simulations for the 16 quarters from 62.1 to 65.4. The results reported are the cumulative effects of a sustained increase in the instruments. In the notation of (3.5), they are the partial sums $M_1 + M_2 + \dots + M_i$ for different i. The 2×2 matrices M_i have been obtained from these cumulative effects by differencing. Since the cumulative effects were given in Christ [5] only for selected i, crude graphic interpolations have been employed to obtain the multipliers M_i for each quarter as given by the figures under the columns M_i in Tables 1A and 1B.

After some experimentation with different numbers of lagged dependent variables and lagged instruments, it was decided that a reduced form having dependent variables lagged 3 quarters and instruments lagged 9 quarters would fit the interpolated multipliers from the Michigan Model reasonably well; and that dependent variables lagged 3 quarters and instruments lagged 6 quarters would suffice to approximate the multipliers from the Wharton Model. Because of our crude graphic interpolation of the multipliers, our linearization of the models, and our somewhat arbitrary truncation of the number of lagged variables in the reduced forms, the resulting models, to be called M and W respectively, may behave quite differently from the original Michigan and Wharton models, but they serve to illustrate the possible value of the policy

Table 1A

Final Form Multipliers for Model M

Lag of x_1	Nominal GNP A^iC	Nominal GNP M_i	Real GNP A^iC	Real GNP M_i
0	.700	.700	.800	.800
1	.556	.556	.528	.528
2	.425	.425	.302	.302
3	.326	.326	.200	.200
4	.217	.217	.115	.115
5	.112	.112	.035	.035
6	.045	.045	-.029	-.029
7	-.005	-.005	-.088	-.088
8	-.059	-.059	-.137	-.137
10	-.108	-.123	-.167	-.175
12	-.096	-.137	-.144	-.185
14	-.075	-.127	-.110	-.173
16	-.053	-.081	-.075	-.130
18	-.033	.000	-.046	-.140
20	-.018	.067	-.024	.075
24	-.002	.112	-.002	.081
28	.003	.059	.004	.034
32	.003	.021	.004	.009
36	.002	.004	.002	.002

Lag of x_2	Nominal GNP A^iC	Nominal GNP M_i	Real GNP A^iC	Real GNP M_i
0	.100	.100	.100	.100
1	.300	.300	.300	.300
2	.500	.500	.500	.500
3	.700	.700	.700	.700
4	1.552	1.552	1.326	1.326
5	1.675	1.675	1.549	1.549
6	1.716	1.716	1.630	1.630
7	1.656	1.656	1.547	1.547
8	1.437	1.437	1.050	1.050
10	-.027	-.020	-.280	-.274
12	-.264	-.250	-.584	-.573
14	-.346	-.350	-.642	-.663
16	-.314	-.352	-.554	-.621
18	-.234	-.279	-.414	-.444
20	-.146	-.168	-.274	-.278
24	-.014	.069	-.075	-.020
28	-.044	.101	.012	.030
32	.057	.068	.033	.001
36	.051	.024	.028	.000

Table 1B

Final Form Multipliers for Model M

Lag of x_1	Nominal GNP A^iC	Nominal GNP M_i	Real GNP A^iC	Real GNP M_i	Lag of x_2	Nominal GNP A^iC	Nominal GNP M_i	Real GNP A^iC	Real GNP M_i
0	1.300	1.300	1.300	1.300	0	1.300	1.300	1.400	1.400
1	.258	.258	.983	.983	1	1.240	1.240	1.330	1.330
2	.205	.205	.750	.750	2	1.180	1.180	1.260	1.260
3	.161	.161	.529	.529	3	1.120	1.120	1.180	1.180
4	.132	.132	.351	.351	4	1.030	1.103	1.070	1.070
5	.107	.107	.202	.202	5	.930	.930	.969	.969
6	.084	.084	.100	.100	6	.800	.800	.817	.817
7	.064	.068	.028	.037	7	.601	.590	.626	.628
8	.046	.054	-.019	-.022	8	.389	.350	.385	.400
10	.012	.035	-.061	-.092	10	.068	.075	-.016	-.049
12	-.015	.020	-.068	-.128	12	-.106	-.090	-.200	-.181
14	-.029	.009	-.057	-.123	14	-.155	-.187	-.216	-.213
16	-.029	.000	-.038	-.083	16	-.131	-.220	-.145	-.171
18	-.022	-.002	-.018	-.040	18	-.077	-.157	-.058	-.086
20	-.011	-.003	-.002	.004	20	-.025	-.048	.008	-.027
23	.002	.000	.010	.000	23	.020	.000	.045	.000

recommendations from imperfect models. Note the differences between the multipliers in Tables 1A and 1B. For model M , the effects of government purchases on GNP become negative from period 7 on and are fairly large in absolute value; not so for model W . The multipliers of the monetary instrument increase in the first six quarters for model M while they decrease for model W . The reduced form coefficients obtained by our fitting procedure are given in Table 2; they are also fairly different for the two models. The final-form coefficients $A^i C$ of x_{t-i} deduced from the reduced form are given in Table 1; they resemble the observed coefficients M_i .

The intercepts of the reduced forms for M and W are assumed to be linear functions of time t , which takes the value 1 for 1966.1. Using the historical data[2] from 1966.1 to 1969.4 and the coefficients of Table 2, we have estimated the trend terms by least squares, as given in the lower right corner of Table 2. The covariance matrix of the residuals are estimated to be

$$(3.7) \qquad V_M = \begin{bmatrix} 16.605 & 13.170 \\ 13.170 & 11.569 \end{bmatrix} ; \qquad V_W = \begin{bmatrix} 22.012 & 16.914 \\ 16.914 & 40.524 \end{bmatrix} .$$

The GNP figures are in billions of current or 1958 dollars. The standard deviations of the residuals are between 3.4 and 6.4 billions.

4. Illustrative Evaluation of Two Imperfect Models

The method of section 2 is applied to the models of section 3 to evaluate the possible usefulness of imperfect models. Before applying any stabilization policy, be it derived from an imperfect econometric model or from some ad hoc reasoning, the government authorities should examine how it would perform under

Table 2

Reduced Form Coefficients for Models M and W

Model	$y_{1,t-1}$	$y_{2,t-1}$	$y_{1,t-2}$	$y_{2,t-2}$	$y_{1,t-3}$	$y_{2,t-3}$	x_{1t}	x_{2t}	$x_{1,t-1}$	$x_{2,t-1}$
M	.777	.592	-.023	-.354	.207	-.224	.700	.100	-.462	.163
	-.556	1.935	.430	-.788	.121	-.185	.800	.100	-.631	.162
W	1.545	-.392	-.604	.568	-.111	-.177	1.300	1.300	-1.241	-.219
	.801	1.468	-1.259	-.556	.493	-.011	1.300	1.400	-1.967	-1.767
	$x_{1,t-2}$	$x_{2,t-2}$	$x_{1,t-3}$	$x_{2,t-3}$	$x_{1,t-4}$	$x_{2,t-4}$	$x_{1,t-5}$	$x_{2,t-5}$	$x_{1,t-6}$	$x_{2,t-6}$
M	-.021	.127	.051	.130	-.035	.787	-.067	-.044	-.040	.014
	-.081	.122	.093	.124	-.005	.559	-.045	.128	-.039	-.014
W	.238	-.224	.110	.176	-.009	.132	-.007	.118	-.010	.090
	1.461	.730	-.490	.062	.005	.031	-.014	.073	.006	.003
	$x_{1,t-7}$	$x_{2,t-7}$	$x_{1,t-8}$	$x_{2,t-8}$	$x_{1,t-9}$	$x_{2,t-9}$	t	1	t	W 1
M	-.027	-.080	-.028	-.149	-.020	-.745	.717	21.296	1.863	87.162
	-.032	-.095	-.019	-.392	-.012	-.508	.282	29.176	-.472	22.373

reasonable assumptions about the dynamic structure of the economy. Although the true structure is unknown, it is necessary to assume hypothetical structures to test the performance of any policy being seriously considered for adoption. In this section, we use one of the models of section 3 as the hypothetical structure and evaluate the policy recommendations derived from using the other model. The planning horizon T is 32 quarters, with initial conditions given by historical data up to the last quarter of 1965. The target growth rates for nominal and real GNP are assumed to be .018 and .008 per quarter respectively; these are their average historical rates from 1966.1 to 1969.4. The diagonal elements of the K matrix are 1 and 1 for these target variables, and ·2 and ·2 for the instruments which are assigned growth rates of .011 and .013, their average historical rates from 1966 to 1969. Thus variations in the instruments will be restricted.

The inactive policy provides constant growth rates for the two instruments, the rates being calculated to minimize the given quadratic loss function. The method to compute these rates is given in Chow [1 , p. 218]. This procedure tends to favor the inactive policy. In practice, a nondiscretionary policy of maintaining constant growth rates for the instruments is hard to design partly because one does not know what growth rates are consistent with price stability and full employment. This difficult problem is assumed away in our analysis. A realistic evaluation of a nondiscretionary policy should utilize the growth rates proposed by its advocate. Given our loss function, the best growth rates for Federal government non-defense purchases and unborrowed reserves are -.057 and .025 respectively for model M; they are -.020 and .026 for model W.

Tables 3A and 3B give the main results of our illustrative calculations. For Table 3A, Model M is assumed to be true. Policy I is the optimal policy derived from using Model M. Policy II is the optimal policy for model W. Policy III uses

Table 3A

Components of Welfare Loss Assuming Model M To Be True

	Weighted Sum of Variances of GNP$ and GNP58			Weighted Sum of Sq. Deviations of Expectations from Targets		
Period	Policy I	Policy II	Policy III	Policy I	Policy II	Policy III
1	28.2	28.2	28.2	2.6	8.7	37.4
2	31.1	34.1	74.5	8.3	32.7	88.2
3	32.7	41.5	126.7	18.0	17.3	113.9
4	34.1	43.4	180.3	27.4	63.3	110.5
5	35.5	46.3	232.3	43.2	22.5	86.7
6	37.1	55.9	279.7	42.7	61.5	61.8
7	38.5	64.0	320.6	36.1	29.1	43.7
8	39.5	73.1	354.8	29.2	68.2	32.2
9	40.2	74.9	382.6	30.7	15.8	24.2
10	41.0	77.7	404.9	22.1	59.9	19.0
11	41.6	107.3	422.5	16.5	12.6	16.1
12	42.1	178.0	436.6	13.6	40.8	15.0
14	42.9	233.4	457.0	8.2	49.3	16.2
16	43.6	271.2	471.1	2.0	59.3	19.5
20	44.7	624.9	490.1	5.1	119.0	23.8
24	45.5	1,997.7	503.5	23.1	162.4	23.8
28	45.6	4,718.3	513.9	39.8	316.0	32.2
32	41.8	1,649.9	522.2	21.5	137.9	38.3
Sum to 12	441.6	824.4	3,243.7	290.4	432.4	648.7

Table 3B

Components of Welfare Loss Assuming Model W To Be True

	Weighted Sum of Variances of GNP$ and GNP58			Weighted Sum of Sq. Deviations of means from targets		
Period	Policy I	Policy II	Policy III	Policy I	Policy II	Policy III
1	62.5	62.5	62.5	30.1	96.0	113.1
2	119.8	326.9	242.0	14.9	50.2	251.0
3	135.6	540.1	420.6	4.4	129.1	297.2
4	138.5	917.6	544.3	1.3	139.6	302.4
5	139.2	1,349.7	627.1	0.5	73.8	289.6
6	139.4	1,693.8	680.9	0.3	24.9	267.5
7	139.5	1,909.9	714.2	.1	111.3	250.9
8	139.5	2,108.5	736.2	.0	121.6	246.5
9	139.5	2,438.9	753.3	.0	303.8	251.6
10	139.5	3,332.8	767.4	.1	303.3	262.0
11	139.5	4,577.6	778.7	.2	349.6	274.2
12	139.5	6,313.0	787.0	.3	464.7	285.6
14	139.5	8,843.6	795.6	.5	264.4	302.4
16	139.5	13,495.4	797.9	.6	1,709.8	311.4
20	139.5	62,353.7	800.8	.3	670.1	300.8
24	139.5	262,446.2	802.8	.0	17,938.4	225.5
28	139.5	2007,462.9	803.1	.3	40,839.7	102.2
32	136.3	930,934.7	803.2	3.1	25,677.3	35.5
Sum to 12	1572.0	25,571.3	7,114.2	52.2	2,167.9	3,091.6

the optimal but constant rates of change for the two instruments. For each policy and each period, we show separately the loss due to the variances of the variables and to the deviations of their means from the targets, as indicated by expression (2.13). Table 3B gives analogous results, assuming W to be the true model, with policy II being the optimal policy derived from model M. Without the stochastic control theory of section 2, one would have to solve an optimal control problem for 32 periods using the true model or the imperfect model as the case may be, and obtain the optimum values for the instruments in period 1; apply these values, together with a random drawing of the residuals u_1 in period 1 from the true model, to generate a set of dependent variables y_1 for period 1; using y_1 as the initial condition, solve a second optimal control problem for 31 periods, and obtain the optimum values of the instruments in period 2; apply these values to generate y_2 stochastically and so forth. This tedious process only provides one observation, covering 32 periods, of the stochastic time path for a hypothetically true model and a given strategy. The process has to be repeated many times in order to estimate the mean vector and the covariance matrix of the multivariate stochastic time series describing the economy under control. The analytical method of section 2 was used to calculate the means and variances for Tables 3A and 3B in lieu of such stochastic simulations and countless optimal control calculations.

Because the end of the time horizon is fixed, the policy recommendations for the later periods are subject to the well-known limitations of being myopic, and should therefore not be taken seriously. Furthermore, to evaluate the policy recommendations from an imperfect model realistically, one ought to allow for possible revisions of model parameters through time. For these two reasons, we consider the dynamic behavior of the economy described by Tables 3A and 3B only for the first 12 periods. The losses for selected later periods are shown partly to indicate how the economy would behave if the policy maker were to become more and more myopic and the model used were not to be modified at all according to future observations. The partial sum of each component of the loss function over the first 12 periods are given at the bottom of Tables 3A and 3B.

For each combination of the true world and the policy, the total expected loss due to both the variances and the squared deviations of means from targets is given in the following payoff matrix (negative sign omitted).

	True Model	
	M	W
Optimal Policy Derived From M	731.0	27,739.2
Optimal Policy Derived From W	1,256.8	1,624.2
Best Inactive Policy M	3,892.4	10,205.8

Thus the policy based on model W would be much better than the best inactive policy even if the true world were model M, and in spite of the apparent differences in the multipliers and the reduced form equations for the two models. The coefficients G_1 of the optimal feedback control equations derived from the two models for period 1 also differ, as partially shown below.

	$y_{1,t-1}$	$y_{2,t-1}$	$y_{1,t-2}$	$y_{2,t-2}$	$y_{1,t-3}$	$y_{2,t-3}$
G_1^M	-.267	-1.264	-.165	.564	-.200	.236
	.250	-.628	-.323	.504	-.025	.065
G_1^W	.074	-1.366	.985	.937	-.768	-.061
	-.896	.858	-.231	-.861	.568	.123

However, the policy derived from model M would be much worse than the best inactive policy if the true world were model W. The policy maker facing only these two possible states of the world should formulate his policy according to model W rather than following an inactive policy, which is dominated according to the payoff matrix. Note that our analysis favors the inactive policy because we have applied the best rates of change for the instruments according to the true model. In reality, these best rates would not be available without knowing the true model. Also, for the payoff matrix, the same policy (or the same rates of change in this case) should be applied to both of the hypothetical states M and W. For instance, simply applying the average historical growth rates of the instruments from 1966 to 1969 to both models M and W would yield much larger expected losses than given in the last row of the payoff matrix.

The calculations of this section are merely illustrative of the method of section 2. The results are not intended to apply to the original Michigan and Wharton models for obvious reasons. The method, however, applies to nonlinear models as pointed out in section 2, by using the (nearly) optimal feedback control equations of Chow [1, Chapter 12] and [4] for nonlinear models.

5. Concluding Remarks

In this paper, we have described a method to evaluate the performance of the optimal policy derived from an econometric model, and illustrate it with two simplified models. Although model W differs a great deal from model M in terms of the reduced forms and the multipliers, it can still be used effectively as a guide to policy even if the world is accurately described by model M. We propose to calculate the expected loss associated with an optimal policy derived from an imperfect model under different assumptions about the true state of the world. Certainly, from an imperfect econometric model, other rules can be derived than the optimal rule given by section 2 above. For example, uncertainty in the parameters can be allowed for as indicated in Chow [1, Chapter 10], [2] and [3]. Such a policy may perform better under the assumption that a different model is true. One may also devise a rule by somehow combining the parameter values from two different models so that it will behave reasonably well under both worlds. These matters are subjects for further research.

Econometric models are not perfect, but policy makers are benefiting from using them because there are no better tools to study the consequences of given policy proposals. Furthermore, given a loss function, the models themselves can be used to generate potentially useful policies by optimization. The policies so generated, like any other policy proposal, will have to be evaluated under hypothetical states of the world described by alternative econometric models. Hopefully, the method outlined in this paper will facilitate the evaluations of alternative policy recommendations and econometric models.

Footnotes

1. I would like to thank John J. Piderit and Ettie H. Butters for excellent research and programming assistance, and the National Science Foundation for financial support.

2. The time series used are quarterly data on nominal GNP, GNP in 1958 dollars, Federal givernment non-defense purchases of goods and services (all in billions of dollars at seasonally adjusted annual rates, from the Survey of Current Business), and nonborrowed member bank reserves in billions of dollars, (seasonally adjusted, from the Federal Reserve Bulletin).

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