PERFORMANCE INCENTIVES AND PLANNING UNDER UNCERTAINTY*

by

Greogry Hildebrandt**

and

Laura D'Andrea Tyson***

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** Major USAF, Department of Economics, Georgraphy and Management, United States Air Force Academy.

*** Assistant Professor, Economics Department, Princeton University

Econometric Research Program
PRINCETON UNIVERSITY
207 Dickinson Hall
Princeton, New Jersey

Introduction

The study of economic planning has focused almost exclusively on the design of iterative procedures to achieve the optimal allocation and distribution of economic goods and resources. Planning procedures of this type explicitly assume that the iterative exchange of information between the center and the producer can be continued until the center is sufficiently cognizant of the technology to choose the optimal production plan. In actuality, however, constraints on the feasible number of information exchanges limit the information available to the center. Consequently, a realistic model of the planning process must confront the fact that the center devises a system of directives or incentives to motivate optimal production decisions in the presence of what Laffont (1975) has called subjective uncertainty. Subjective uncertainty which stems from information constraints at the center is to be distinguished from objective uncertainty or real randomness in technological production possibilities. Objective uncertainty is an inherent characteristic of the economic environment which planning cannot eliminate, while subjective uncertainty stems from the decentralization of information which can be reduced but is not eliminated by planning.

In the presence of subjective uncertainty the center's planning problem becomes one of devising incentives to permit the delegation of decision-making authority to the productive agents which have the information required to make optimal decisions at the time production is undertaken. One way the center can solve this problem is by structuring a so-called performance incentive function (PIF) relating the rewards of the producers to certain characteristics of their performance. The actual form of the PIF chosen by the center depends on both its allocational and distributional goals.

In the simplest case, where the distribution of societal profits or rent between the center and its agents does not matter, the optimal PIF depends solely on the allocational goals of the center. This simple case is examined in the first section of this paper. However, once we allow for distributional objectives, the selection of an optimal PIF becomes considerably more complicated. At first, this result may seem surprising. As long as lump sum transfers between the center and the producers are feasible, the center should be able to choose an optimal PIF on allocational grounds and to achieve its distributional goals by lump sum adjustments once production decisions have been made. However, several examples analyzed in the later sections of the paper indicate that in the presence of subjective and objective uncertainty, the separation of allocational and distributional goals is not always feasible.

For example, to achieve optimal production decisions under conditions of subjective uncertainty, the center may direct producers to maximize profits at a given price. However, realized profits may diverge from the profits consistent with the center's distributional goals. This divergence stems from the fact that profits actually fulfill two functions: they serve as an incentive or control tool by which the center can guide enterprise behavior; and they serve as a distributional tool by which the center can achieve the desired distribution of societal rent. To achieve its distributional goal the center may be forced to modify realized profits by some adjustment function. If producers are aware that such an adjustment function exists, but they are not cognizant of its form, then they are likely to guess its value, and any miscalculation on their part will reduce the efficiency of their production decisions. On the other hand, if the producers are aware of the adjustment function, then they will maximize their total rewards, including the adjustment function, and simple profit maximization at the given price will be a superfluous incentive tool.

One of the major findings of this paper is that there exists an optimal PIF which incorporates the adjustment function needed to achieve both distributional and allocational optimality. The existence of such a PIF is the major reason why distributional optimality is an important consideration at the time the center formulates its incentive structure. By announcing the optimal PIF, the center thereby avoids the potential efficiency losses caused by producer miscalculation of the adjustment function, and allocational and distributional goals are simultaneously guaranteed.

Before proceeding to examine the allocational and distributional consequences of PIF's in some simple economic models, it is important to realize their potential usefulness in a variety of circumstances. The idea of a so-called "contractual incentive function" which specifies a mutually acceptable rule relating the monetary rewards paid by one decision maker to the subsequent performance of another is not new. Most of the existing theoretical work, such as the studies of Bernhold (1971) and Wilson (1968, 1970), has focused on the use of incentive contracting to motivate decision makers within a firm hierarchy to act in accordance with managerial goals or to motivate government contractors to meet their production commitments in the most efficient possible manner. Performance incentives have actually been employed in both sets of circumstances. Numerous enterprises have devised profit-sharing formulae to motivate supervisory and managerial personnel, and the Department of Defense and NASA have relied on the use of performance incentives to monitor the work of major contractors. (Ackerman, 1966; Department of Defense, 1969). Recent innovations in the use of performance incentives have appeared in the new Amtrak contract which relates payments to the railroads to the quality of services they provide (Baumol, 1975) and in a contractual arrangement which guarantees a one percent increase in the salaries of the policemen of Orange, California to every three percent decline in rape, robbery, burglary and auto theft.[1]

Although the existing literature on economic planning does not specifically mention the use of performance incentive functions or incentive contracts, the concept does arise in discussions of "success indicators" in Soviet planning. The Soviets have become notorious for a system of planning in which enterprise agents are rewarded according to the degree to which certain plan targets are achieved. By choosing enterprise targets and a related reward structure, Soviet planners implicitly define a performance incentive system. In contrast to similar systems employed in the West, the Soviet system is not "contractual" in the sense that it is agreed upon by the planners and the enterprise managers. Instead, the state unilaterally chooses the plan targets and rewards, and the enterprise managers are expected to comply in an effort to attain their own maximum reward within the confines of the rules laid down by the planners. This "non-contractual" incentive system is clearly an example of the use of performance incentives in the implementation of economic planning. Future examples will be forthcoming as more societies look toward some degree of economic planning to foster the most efficient use of scarce resources.

I. Performance Incentive Functions and Resource Allocation -- The Basic Model

Following a recent paper by Weitzman (1974), we begin with a basic model in which there is only one commodity which is produced by a single producer. We assume the existence of a cost function relating money costs to the level of output produced and a benefit function relating aggregate societal benefits measured in monetary terms[2] to the level of output consumed. In the simple model the planning problem is to achieve the level of production which just maximizes net benefits defined as

$$B(q) - C(q) \quad (1)$$

where $B_{11} < 0$; $C_{11} > 0$; $B_1(0) > C_1(0)$; and $B_1(H) < C_1(H)$ for H sufficiently large. As long as the center has complete knowledge of the benefit and cost functions, it can choose an optimal quantity directive q^* according to the first order condition

$$B_1(q^*) = C_1(q^*) \quad (2)$$

or it can choose an optimal price directive p^* according to the first order condition

$$p^* = C_1(q^*) = B_1(q^*) \quad (3)$$

allowing the producer to choose the optimal quantity via profit maximization.

Now consider the case in which the center must make its planning decision in the presence of subjective uncertainty. In this case from the point of view of the planner the cost function is of the form $C(q,\theta)$ and the benefit function is of the form $B(q,n)$ where θ and n are independent random variables reflecting the center's informational uncertainty about costs and benefits. There are two different interpretations which can be given to θ and n. On the one hand, costs and benefits may be uncertain from the center's perspective but may be revealed in full certainty to the producer at t_1, the time the plan is formulated. On the other hand, costs and benefits may be uncertain for the center and the producer at the time the plan is formulated but may be fully revealed to the producer at t_2, the time the plan is implemented and to the society at t_3, the time societal benefits are realized. Following Weitzman, we assume that θ reflects the center's subjective uncertainty about the cost function at t_1 and t_2, while n reflects objective uncertainty in the benefit function. The true value of θ is known to the producer but not to the center at t_2, the time production is set, while the true value of n is only known by the center at t_3, sometime later.

In the presence of uncertainty the center must devise an incentive function to motivate optimal production decisions. Such a function will be of the general form $\pi(q,c)$ where c represents actual production costs at t_2, and π is measured in the same units as benefits and costs. To maximize earned rewards, the producer will choose an output level which maximizes the PIF. Clearly, a special case of a performance incentive function is the use of a price parameter in a contract which specifies that the producer earns net profits at that price after the payment of full production costs. The PIF then becomes

$$\pi(q,c) = pq - c \ . \tag{4}$$

Suppose, as seems reasonable, that the center wishes to maximize expected net benefits at t_2, given the actual value of θ. The center's task at t_2 is to find the solution to the following maximization problem.[3]

$$\max_q E\ \{B(q,n)\} - C(q,\theta) \ . \tag{5}$$

The first order condition for the optimal q^* from the center's point of view is therefore

$$E\ B_1(q^*,n) = C_1(q^*,\theta) \ . \tag{6}$$

Because the center cannot observe θ at t_2, it cannot solve this problem directly. However, it can utilize a properly specified PIF to guarantee that the producer will choose q^* at t_2. Recall that the producer at t_2 will maximize the PIF. The associated first order condition for the producer can be written as

$$\pi_1^*/\pi_2^* = C_1(q^*,\theta) \tag{7}$$

where π_1^* and π_2^* are the two first partial derivatives of the optimal PIF, and q^* is the optimal output level for the producer at t_2. Substituting this condition into the first order condition for the center's maximization problem yields the following condition for the optimal PIF:

$$E\ B_1(q^*,n) = C_1(q^*,\theta) = \pi_1^*/\pi_2^* . \tag{8}$$

This condition can be rewritten as

$$\pi_1^* = -\ \pi_2^*\ EB_1(q) \tag{9}$$

which indicates that the center has a degree of freedom in choosing the optimal PIF. It may specify any function whose partial derivatives satisfy condition (9). All such functions generate the same total net benefits, and therefore on rent maximization or allocational criteria, one such function is as good as any other. In terms of informational requirements, however, the center may prefer to choose a PIF for which $\pi_2^* = -1$, thereby forcing the enterprise to bear total production costs. Any other arrangement would require the center to measure actual production costs at t_2 in order to make sure that the desired sharing of total costs by the center and the producer is realized.

If the center chooses $\pi_2^* = -1$ to minimize information requirements, then $\pi_1^* = EB_1(q)$ and integration yields a general optimal PIF of the form

$$\pi^*(q,c) = G(q) - c \tag{10}$$

where $G(q)$ is equal to $EB(q)$ plus or minus some arbitrary constant of integration. In this simple case the center only needs to know $EB_1(q,n)$ to specify the optimal PIF.

A PIF of the form suggested in equations (9) and (10) has a simple intuitive explanation. If the center were certain of the actual cost conditions which would prevail at t_2, then it would set an output target at t_1 such that expected marginal benefits would just equal actual marginal costs. However, since the center does not know actual costs at t_2, it cannot specify this output target at t_1. Nonetheless, it can still motivate the producer to choose this output level at t_2 by devising a contract so that the producer always operates at the point where actual marginal cost at t_2 just equals expected marginal benefits at t_1. The PIF corresponding to equation (9) yields a contract of this form. The producer is motivated to do the job which the center wants done at t_2 by a performance incentive contract specified at t_1. Moreover, the center does not need any information about the producer's cost function at either t_1 or t_2 to determine this performance contract. As far as the center is concerned, the producer's cost function is an unknown function whose first derivative is replaced by $EB_1(q,n)$ in the specification of the PIF. Thus, the center does not need to know either the cost function or the distribution function for θ, and privacy or the guarding of technological information by the producer is maintained during the planning process.

II. Performance Incentive Functions and the Distribution of Societal Profits: The Case of Subjective Uncertainty

The rules specifying the optimal PIF in the last section are consistent with a number of different functions which have different implications for the share of societal profits remaining with the producer and the share of profits flowing to the center. Which of the many feasible PIFs will be chosen by the center depends on its distributional objectives. Given these objectives, a particular PIF yielding both allocational efficiency and distributional justice can be achieved.

To study the distribution of profits it is first necessary to specify utility functions for the producer and the center. To begin we assume that both possess risk averse (strictly concave) von Neumann-Morgenstern utility functions which depend on the profits they earn. These utility functions can be interpreted as individual utility functions or as group utility functions. For example, U_c, the center's utility function, may represent the utility of all consumers provided they display the same degree of "cautiousness." (Wilson, 1968).

To determine the optimal distribution of profits, the center must have a normative view about the relative weights to be accorded to its own utility and to the utility of the producer. Therefore, we assume the existence of social welfare weights λ^c and λ^d which reflect the center's distributional goals. It is simplest to interpret the welfare weight which the center accords to itself as the welfare weight which arises when the center maximizes a social welfare function the arguments of which are the utilities of all consumers and the producer. However, it is also possible to think of the welfare weights as shadow prices reflecting the fact that the producer must receive some minimum expected utility at t_1 before it is willing to accept a contract to produce at t_2. This latter interpretation emphasizes the contractual nature of the relationship between the center and the producer at t_1.

Given the utility functions and the welfare weights, the center can determine the optimal profit-sharing rule or the optimal amount of profits to be granted to the producer for a given amount of total profits by solving the following maximization problem:

$$\max_{s_c, s_d} \lambda^c U^c(s^c) + \lambda^d U^d(s^d) + u^0(R-(s^c+s^d)) \qquad (11)$$

where s^c is the amount of profits going to the center;

s^d is the amount of profits going to the producer;

u_0 is a Lagrange multiplier; and

R is the total amount of profits available for distribution.

The first order conditions for the optimal rent sharing rule are given by

$$\lambda^c U^{c^1}(s^{*c}) = \lambda^d U^{d^1}(s^{*d}) = u^0 \qquad (12)$$

where $U^{c^1} = dU^c/ds^{*c}$ and $U^{d^1} = dU^d/ds^{*d}$.

Now suppose the center tries to achieve its allocational and distributional goals in the following stepwise manner. At t_1 the center specifies a PIF which is optimal according to the allocational rule given by equation (6). Profit maximization by the producer at t_2 yields the first order condition (7) which generates a reaction functional of the form:

$$q^* = F(\pi_1, \pi_2, \theta) \qquad (13)$$

relating the optimal output choice to the PIF and the actual value of θ . The output decision at t_2 in turn gives rise to a total volume of societal profits $R(q^*) = B(q^*) - C(q^*)$ at t_3 . Clearly, since q^* depends directly on θ , so will the total volume of profits $R(q^*)$ and the share of profits allocated to the producer. Given $R(q^*)$, the center can solve for the optimal rent sharing rule and adjust the profits realized by the producer at t_2 by a lump sum transfer $K(\theta)$ which satisfies the condition:

$$\pi(q^*, c) + K(\theta) = s^{*d}(R(q^*)) \ . \qquad (14)$$

In this way the center appears able to guarantee allocational optimality at t_2 , by the selection of an optimal PIF at t_1 , and distributional optimality at t_3 , by the selection of the optimal transfer $K(\theta)$. Moreover, because $K(\theta)$ can be expressed in terms of $\pi(q^*, c)$ and $s^{*d}(R(q^*))$, variables that are observable by the center, the center need not observe θ at t_2 to guarantee distributional optimality.

There are two problems with this approach, however. First, institutional arrangements may render lump sum transfers from the center to the producer infeasible. Second, even if lump sum transfers are feasible, they may not be optimal if the producer modifies his behavior at t_2 in an effort to increase the transfer received at t_3 or later. Suppose, for example, that the center does not announce the adjustment function at t_2, but that the producer is aware that an adjustment will be made. In general, the producer will find it in his self interest to anticipate or guess the adjustment rule at t_2 and to maximize total profits modified according to this rule. Consequently, the anticipated adjustment rule will influence producer behavior, and to the extent that the producer miscalculates the adjustment function, losses in efficiency will occur. Miscalculations of this sort can be circumvented if the center announces the adjustment rule at t_1, but in this case the original PIF, $\pi(q,c)$, is superseded by a modified PIF of the form $\pi(q,c) + K(\theta)$ which the producer will maximize to choose its production level. The best strategy for the center under these circumstances is to specify a PIF at t_1 which achieves both allocational and distributional goals simultaneously. Such a PIF can be found by solving the following overall maximization problem at t_1 :

$$\max_{\pi(q^*,c)} E\,\{\lambda^c U^c(B(q^*) - C(q^*,\theta) - \pi(q^*,c)) + \lambda^d U^d(\pi(q^*,c))\} \quad (15)$$

where q^* is the optimal choice of the producer given the reaction functional of equation (13).

Using the reaction functional and a variational calculus technique described in the Appendix, it can be shown that if $\pi(q,c)$ is the function solving equation (15), then the following first order conditions apply:

the allocational condition: (16a)

$$B_1 = C_1$$

and

the distributional condition: (16b)

$$\lambda^c U_{c^1}^c = \lambda^d U_{d^1}^d .$$

It remains to be demonstrated that a PIF of the form $\pi(q,c)$ can be constructed to satisfy these conditions simultaneously. First, write the producer's rent sharing function in terms of q and c to parallel the form of the PIF:

$$s^d = s^d(R) = s^d(B(q) - c) = \pi(q,c) . \quad (17)$$

Equation (16b) can be used to solve for the optimal rent sharing function which must be equal to the optimal performance incentive function to yield:

$$\pi^*(q,c) = s^{*d}(B(q)-c) . \quad (18)$$

Taking partial derivatives of this relationship with respect to c and q provides the following conditions:

$$\pi_2^* = \partial s^{*d}/\partial c = -\partial s^{*d}/\partial B \quad (19)$$

and

$$\pi_1^* = \partial s^{*d}/\partial q = (\partial s^{*d}/\partial B) \cdot (\partial B/\partial q) \quad (20)$$

which in turn imply that it is possible to construct the optimal PIF such that

$$\pi_1^* = -\pi_2^* B_1 . \quad (21)$$

In view of the producer's profit maximization condition given in equation (7), the allocational condition (16a) is clearly satisfied. In the next section it is shown that the optimal rent sharing function must be a monotone increasing function of the total rent generated. The all-around convexity of the function $R(q,\theta)$, plus this monotonicity implies that equating $\pi^*(q,c)$ to s^{*d} is sufficient for distributional optimality to be achieved. The existence of a PIF which maximizes (15) and satisfies (16a) and (16b) permits us to state the following proposition which has been proved.

Proposition : When there is certainty at t_2, and both the producer and the center are risk averse, then the optimal PIF achieves both allocational and distributional efficiency at t_2.

III. Optimal Rent Sharing and Risk

The optimal rent sharing functions s^{*c} and s^{*d}, identified in equation (12), play such an important role in the previous analysis that it is appropriate to review some of their properties which have been derived by Wilson (1968). In particular, we shall demonstrate their dependence on the degree of risk aversion incorporated in the underlying utility functions.

Differentiating (12) with respect to R yields:

$$(\lambda^c U^{c^{11}}) \cdot (s_R^{*c}) = (\lambda^d U^{d^{11}}) \cdot (s_R^{*d}) = u_R^0 \qquad (22)$$

where $U^{c^{11}} = dU^{c^1}/ds^{*c}$, $U^{d^{11}} = dU^{d^1}/ds^{*d}$, $s_R^{*c} = \partial s^{*c}/\partial R$, and $s_R^{*d} = \partial s^{*d}/\partial R$. Combining this result with equation (12) allows us to write

$$(s_R^{*c}) \cdot \left(\frac{u^0}{u_R^0}\right) = \frac{U^{c^1}}{U^{c^{11}}} \qquad (23)$$

and

$$(s_R^{*d}) \cdot \left(\frac{u^0}{u_R^0}\right) = \frac{U^{d^1}}{U^{d^{11}}} . \qquad (24)$$

The terms $U^{c^1}/U^{c^{11}}$ and $U^{d^1}/U^{d^{11}}$ have been called risk tolerance functions ρ^c and ρ^d by Wilson (1968). These functions are the reciprocals of the well-known risk aversion functions analyzed by Pratt (1964). Summing conditions

(23) and (24) yields

$$\left(\frac{u^0}{u^0_R}\right)\cdot \left((s^{*c}_R) + (s^{*d}_R)\right) = \rho^c + \rho^d \ . \tag{25}$$

Since $(s^{*c}) + (s^{*d}) = R$, it follows that $(s^{*c}_R) + (s^{*d}_R) = 1$, in which case (25) can be rewritten as

$$\frac{u^0}{u^0_R} = \rho^c + \rho^d \ . \tag{26}$$

Using this expression and substituting for ρ^c and ρ^d from (23) and (24), we can write

$$s^{*c}_R = \partial(s^{*c})/\partial R(\theta) = \frac{\rho^c}{\rho^c+\rho^d} \tag{27}$$

and

$$s^{*d}_R = \partial(s^{*d})/\partial R(\theta) = \frac{\rho^d}{\rho^c+\rho^d} \ . \tag{28}$$

Under the assumption that the utility functions are both strictly concave, ρ^c and ρ^d are both positive which implies that

$$0 < s^{*c}_R < 1 \quad \text{and} \quad 0 < s^{*d}_R < 1 \ . \tag{29}$$

Therefore, we can conclude that for optimal distribution in this case, as total rent increases so does the actual rent received by both the producer and the center.

There are circumstances under which it is optimal for each agent to receive a lump sum amount plus a fixed proportion of the total rent. It is important to know when this type of linear arrangement is optimal because a linear sharing rule greatly simplifies the form of the PIF. Furthermore, it will be seen in the next section that when objective uncertainty exists, the optimality of linear sharing is the only situation when it is possible to achieve allocation and distribution goals simultaneously.

First define $\rho^c + \rho^d = \rho^0$, and rewrite equations (27) and (28) as

$$\rho^c = (\rho^0) \cdot (s_R^{*c}) \tag{30}$$

and

$$\rho^d = (\rho^0) \cdot (s_R^{*d}) \ . \tag{31}$$

Differentiating with respect to R yields

$$\rho_R^c s_R^{*c} = \rho_R^0 s_R^{*c} + \rho^0 s_{RR}^{*c} \tag{32}$$

and

$$\rho_R^d s_R^{*d} = \rho_R^0 s_R^{*d} + \rho^0 s_{RR}^{*d} \ . \tag{33}$$

Clearly $\rho_R^c = \rho_R^0 = \rho_R^d$, if and only if the optimal sharing rule is linear. For if the sharing ratio is linear, then $s_{RR}^{*c} = s_{RR}^{*d} = 0$, and if $\rho_R^c = \rho_R^0 = \rho_R^d$, then $s_{RR}^{*c} = s_{RR}^{*d} = 0$.

The terms ρ_R^c and ρ_R^d have been called the "cautiousness" of the center and the producer in the face of risk (Wilson, 1968). Thus, the optimal rent sharing rule will be linear if and only if the center and the producer exhibit the same degree of cautiousness. There are a number of important classes of utility functions for the producer and the center for which this condition is satisfied. For example, if risk tolerance functions are linear of the form $\rho^c(s^{*c}) = as^{*c} + b$ and $\rho^d(s^{*d}) = as^{*d} + b$, then $\rho_R^c = \rho_R^d = a$, and both the center and the producer have the same cautiousness. Special cases of utility functions which provide risk tolerance functions are of this form include (Wilson, 1968):

exponential:	$a = 0$
quadratic:	$a = -1$
logarithmic:	$a = 1$
power:	a = arbitrary constant.

IV. Exponential Utility Functions and the Optimal PIF

Using the general analysis developed in the last section we can now derive PIF's for a particular set of utility functions. For simplicity, we choose exponential functions of the form:

$$U^c = -K^c e^{-s^c/K^c} \quad \text{and} \quad U^d = -K^d e^{-s^d/K^d} \tag{34}$$

which we know will yield a simple linear sharing rule. Substituting these functions into equation (11) and performing the maximization yields

$$\lambda^d e^{-s^d/K^d} = \lambda^c e^{-(R(\theta)-s^d)/K^c} . \tag{35}$$

Taking logs of both sides we obtain:

$$s^{*d} = \left(\frac{K^d}{K^d+K^c}\right) R(\theta) + \left(\frac{K^c K^d}{K^d+K^c}\right) \log\left(\frac{\lambda^d}{\lambda^c}\right) . \tag{36}$$

If we now let

$$\gamma = \frac{K^d}{K^c+K^d} \text{ and } L = \left(\frac{K^c K^d}{K^d+K^c}\right) \log\left(\frac{\lambda^d}{\lambda^c}\right)$$

we can write

$$s^{*d} = \gamma R(\theta) + L \tag{37}$$

and

$$s^{*c} = (1-\gamma) R(\theta) - L . \tag{38}$$

Since s^{*d} , the optimal earnings of the producer, is generated by the optimal PIF at t_2 , it follows that this function should be of the form

$$\pi^*(q,c) = \gamma(B(q)-c) + L . \tag{39}$$

In words, the center contracts to pay the producer the difference between actual benefits and costs at t_2 plus a particular lump sum L which depends on the underlying utility functions and welfare weights. In the contract, the center must specify the entire benefit function $B(q)$, so that the producer will know the marginal return from a unit of output at each possible level of output. Given $B(q)$ and actual cost conditions at t_2 , the producer will select output where $B_1 = C_1$ or where the allocational objectives of the center are fully satisfied. Thus, optimal allocation and distribution will be simultaneously achieved.

V. Risk Neutrality and the Optimal PIF

In the discussion so far we have assumed that the center is risk averse. However, since the center's utility function may represent benefits derived by a very large number of consumers, it may seem reasonable to suppose that it approaches risk neutrality as a theorem derived by Arrow and Lind (1970) suggests. As risk neutrality is approached from risk averseness, ρ^c , the measure of the center's risk tolerance, approaches infinity. As long as the producer remains risk averse, equations (27) and (28) imply that

$$\lim_{\rho^c \to \infty} s_R^{*d} = 0 \quad \text{and} \quad \lim_{\rho^c \to \infty} s_R^{*c} = 1 \ .$$

Thus, in the limiting case, if the center is risk neutral, it should bear all of the risk, and the amount of profits received by the producer should be some lump sum transfer independent of θ . Under these circumstances, the incentive function of profits is lost, because the reward earned by the producer does not depend on his actions.[4]

To relate this conclusion to PIF's, observe from equations (19)-(21), that $\pi_2 = 0$ and $\pi_1 = 0$, when the center is risk neutral. Therefore, the profit received by the producer is independent of his action, and it is not possible to use a PIF to achieve efficiency and distributional goals simultaneously. Some other method of control or guidance of producer behavior must be devised. One such method would require the center to set some sharing rule which is indirectly dependent on θ at t_1 and then to offset the θ-dependent share of the rent earned by the producer at t_2 by an additional lump sum transfer. For example, if the center selects a PIF at t_1 with a sharing rate equal to a constant w and a lump sum transfer equal to Z, and if it is optimal for the producer to receive J for every state of the world, then the adjustment function $V(\theta)$ must satisfy

$$V(\theta) = J - wR(\theta) - Z . \qquad (40)$$

The problem with this approach is that if the producer knows that it will always receive the same amount of profits at t_2, then it has no incentive to maximize profits, and there is no guarantee that allocational efficiency will be achieved. Therefore, in order to use profits to motivate allocational efficiency, the center will have to sacrifice the optimal distribution required by its own risk neutrality. Under these circumstances, the center may sometimes decide to act as if it were risk averse, even if it is not, to sustain efficient decision-making at t_2.

The failure of the profits mechanism as a control tool when the center is risk averse is analogous to the "moral hazard" problem discussed in the insurance literature (Spence and Zeckhauser, 1971). This problem concerns the conflict between obtaining appropriate incentives for "correct" behavior by the insured and socially optimal sharing of risk between the insurer and the insured. The problem arises only when a risk neutral insurer cannot observe or monitor the actions of the insured and therefore cannot guarantee that the insured has behaved in the "correct" way. If the insurer can monitor the insured's behavior, then the incentives problem is eliminated because the insurer can structure the insurance payoff function so that the insured will always behave correctly. The analogy to the central planning problem is obvious. If the center could monitor production decisions at t_2, or what is equivalent, if the center could observe θ at t_2, then the incentives problem would be eliminated, and a rewards structure to guarantee efficiency and distributional equity could be devised.[5]

VI. Objective Uncertainty and Performance Incentives

In the cases examined so far the producer is assumed to act with full certainty at t_2 when production decisions are made. Such an assumption is consistent with the existence of subjective uncertainty of the center at t_1 and complete certainty of the producer at t_1 and t_2, or with subjective uncertainty of the center and objective uncertainty of the producer at t_1 and complete certainty of the producer at t_2. Now suppose that at t_2 the producer has only partial information about the relationship between output and costs, so production decisions are made under conditions of objective uncertainty. At t_1 there now exists a joint probability distribution $f(\theta,y)$ where θ is a random variable at t_1 reflecting the center's subjective uncertainty about information which will become available at t_2, and y represents uncertainty about the technological relationship between costs and output that remains at t_2. At t_2, when the value of θ is known to the enterprise, a conditional distribution function $g(y|\theta)$ guides enterprise decision-making.

At t_1, a PIF of the form $\pi(q,c)$ is announced by the center. At t_2, the producer is faced with the following problem

$$\max_{q} \; E_{y|\theta} \; U^d(\pi(q, C(q,y,\theta))) \; . \tag{41}$$

The first order condition applying to the solution of this problem is

$$\underset{y|\theta}{E} \{U^{d^1} \cdot (\pi_1+\pi_2 C_1)\} = 0 \qquad (42)$$

which yields a reaction function of the form

$$q^* = F(\pi,\pi_1,\pi_2,y) \qquad (43)$$

where π has been included because a risk averse producer's behavior will be affected by the level of total profits through its effect on U^{d^1} .

At t_1 , the center's problem is to solve

$$\underset{\pi}{\text{Max}} \{E\{ \underset{\theta\ y|\theta}{E} [\lambda^c U^c (B(q^*) - C(q^*,\theta, y) - \pi(q^*, C(q^*,\theta,y))) +$$

$$+ \lambda^d U^d (\pi(q^*,C(q^*,\theta,y)]\}\} . \qquad (44)$$

Assuming that the producer acts in accordance with (42), the variational calculus technique demonstrated in the Appendix yields the following first order condition for this problem

$$\underset{y|\theta}{E} [U^{c^1} \cdot [B_1-C_1-\pi_1^*-\pi_2^* C_1]] = 0 \qquad (45)$$

and

$$\lambda^c \underset{y|\theta}{E} U^{c^1} = \lambda^d \underset{y|\theta}{E} U^{d^1} . \qquad (46)$$

There are two important conclusions to be drawn from these conditions. First, the optimal rent sharing function s^{*d} which satisfies these conditions specifies distributional optimality as perceived at t_2 when objective uncertainty still prevails. Overall ex post distributional optimality under conditions of complete certainty and given by the condition $\lambda^c U^{c^1} = \lambda^d U^{d^1}$ is no longer guaranteed. Second, the presence of U^{c^1} in both conditions (45) and (46) prevents the separation of an allocational condition at t_2 and a distributional condition at t_2 which is possible in the absence of objective uncertainty. Thus, the fact that there is producer uncertainty at t_2 necessitates a balance between allocational and distributional goals, where a balance simply means that blend of the two goals which is the most desirable given the constraints imposed on the problem. It is significant that this balance can be achieved only when the PIF is properly constructed at t_1. When there is objective uncertainty at t_2, then even if lump sum transfers are feasible, and even if the producer does not miscalculate the center's adjustment function, a step-wise procedure to achieve optimal allocation and distribution will not work. The only way to guarantee overall optimality is to incorporate distributional goals at t_1, when the PIF is structured.

The question arises as to why the presence of objective uncertainty at t_2 makes the optimal allocational and distributional conditions simultaneous rather than independent. An intuitive answer suggests itself. At t_2, neither the center nor the producer knows the actual conditions of production with certainty. Therefore, the center cannot specify a PIF which is optimal from a distributional standpoint at t_1 and depend on the producer to independently satisfy the optimal allocational condition at t_2, because the producer does not have the necessary information to do so at that time. It is the lack of information which makes the distributional and allocational conditions interdependent and which constrains the center to search for the optimal balance between efficiency and equity simultaneously.

Although the center will always be constrained under these circumstances, there is one special case in which the satisfaction of conditions (45) and (46) will guarantee ex post distributional optimality as well as distributional optimality from the vantage point of t_2, when the value of y is not yet revealed. This case arises when a linear rent sharing rule is optimal. We first note that the ex post distributional condition $\lambda^c U^{c^1} = \lambda^d U^{d^1}$ implies that

$$\underset{y|\theta}{E} \lambda^c U^{c^1} (\pi_1+\pi_2 C_1) = \underset{y|\theta}{E} \lambda^d U^{d^1} (\pi_1+\pi_2 C_1) . \tag{47}$$

Expected utility maximization by the producer implies that the left side of this equality equals zero, and this permits us to rewrite equation (45) as

$$\underset{y|\theta}{E} U^{c^1} B_1 = \underset{y|\theta}{E} U^{c^1} C_1 = 0 . \tag{48}$$

Now if the optimal rent sharing rule is linear, then π_2 is a constant, and since both π_2 and λ^c are constants, equation (48) can be rewritten as

$$\underset{y|\theta}{E} \lambda^c U^{c^1} \pi_2 B_1 = \underset{y|\theta}{E} \lambda^c U^{c^1} \pi_2 C_1 = 0 . \tag{49}$$

Now consider the producer's expected utility maximization condition (42) multiplied by λ^d , another constant, to yield

$$\underset{y|\theta}{E} \lambda^d U^{d^1} \pi_1 + \underset{y|\theta}{E} \lambda^d U^{d^1} \pi_2 C_1 = 0 . \tag{50}$$

Clearly, if π_1 is set equal to $-\pi_2 B_1$ in the optimal PIF, then the utility maximization of the producer which guarantees equation (50) also guarantees that equation (49) will be satisfied, and, consequently, that ex post distributional optimality will be achieved. Using $\pi^*(q,c)$ which satisfies $\pi_1^* = -\pi_2^* B_1$, where π_2^* is the constant associated with the optimal linear sharing rule, will therefore yield ex post distributional optimality. Furthermore, since $\pi^*(q,c)$ is structured so that equation (48) is also satisfied, then the production level chosen at t_2 will be the level which the center would choose if it directly administered production. We can characterize this conclusion by stating that when the optimal rent sharing rule is linear, then the output chosen by the producer at t_2 is the output which would be chosen under a regime of direct controls in which the center itself made production decisions.

If the optimal sharing rule is non-linear, then the direct control solution and the solution obtained with a PIF are not the same. This seems to be an argument for using direct controls when the optimal sharing rule is non-linear and there is objective uncertainty at t_2. To fully analyze this question, however, requires the analysis of three kinds of costs. There are the transactions costs associated with setting up, administering and policing the sort of quasi-market environment associated with PIF's and other indirect controls; the welfare costs associated with the producer choosing the wrong output under such a regime; and the administrative costs associated with direct controls. Unless the administrative costs are less than the transactions and welfare costs, there is no prima facie case for direct controls.

VII. Conclusions

In this paper we have examined a major problem in economic planning, namely the structuring of incentives to guarantee optimal decision making by decentralized production agents. This problem arises whenever the center is forced to forego direct control over production decisions because of its lack of information about production technology. Under such circumstances, the center must search for a contractual arrangement which will motivate the producer to make production decisions which will be optimal given the center's allocational and distributional goals. Using a simple one good model, a particular type of contractual arrangement called a performance incentive function is derived. As long as the producer chooses an output level in an environment of subjective certainty, the PIF is demonstrated to simultaneously achieve optimality in allocation and distribution. In contrast, when production choice is subject to objective uncertainty, the optimal PIF is shown to involve a balance between allocational and distributional goals.

Overall, the analysis suggests that in an environment in which information is decentralized, the construction and transmittal of a complicated set of messages in the form of a PIF given by the center to the producer allows the center to delegate decision-making authority without completely sacrificing its control over the allocation and distribution of economic goods and resources.

Appendix

When there is subjective uncertainty, the center's problem at t_1 is to solve

$$\underset{\pi(q,c)}{\text{Max}} = \int_{\theta} [\lambda^c U^c (B(q^*) - C(q^*,\theta) - \pi(q^*,c)) +$$

$$+ \lambda^d U^d (\pi(q^*,c)] f(\theta) d\theta \qquad \text{(A.1)}$$

where q^* is the reaction functional for the producer from equation (13). In that this functional was obtained by inverting the producer's profit maximizing condition (7), it is assumed that q^* is monotone in θ . The function $f(\theta)$ represents the center's probability assessments of the subjective uncertainty.

The calculus of variations will be used to derive the center's first order conditions (16a) and (16b). First, a scaled differential functional variation, $\varepsilon t(q,c)$, is added to the solution (or optimal) performance incentive function, $\overset{*}{\pi}(q,c)$, yielding a new function

$$\pi = \overset{*}{\pi} + \varepsilon t \ . \qquad \text{(A.2)}$$

Simultaneously, one obtains

$$\pi_1 = \overset{*}{\pi}_1 + \varepsilon t_1 \qquad \text{(A.3)}$$

$$\pi_2 = \overset{*}{\pi}_2 + \varepsilon t_2 \ . \qquad \text{(A.4)}$$

The solution strategy is to substitute for π in (A.1) using (A.2), set the derivative of the center's maximand with respect to ε equal to zero, evaluate the result at ε equal to zero (where $\pi = \overset{*}{\pi}$), and then determine the conditions which must be satisfied for $\overset{*}{\pi}$ to solve (A.1). After setting the derivative evaluated at $\varepsilon = 0$ equal to zero, one obtains

$$\int_\theta [\lambda^c U^{c^1} ((B_1 - C_1) dq^*/d\varepsilon - (\overset{*}{\pi}_1 + \overset{*}{\pi}_2 C_1) dq^*/d\varepsilon - t) +$$

$$+ \lambda^d U^{d^1} ((\overset{*}{\pi}_1 + \overset{*}{\pi}_2 C_1) dq^*/d\varepsilon + t)] f(\theta) d\theta = 0 \ . \qquad \text{(A.5)}$$

To obtain the distributional condition (16b), select a function t such that $t_1/t_2 = \overset{*}{\pi}_1/\overset{*}{\pi}_2$. Such a function t implies that variations in ε do not affect the output level selected by the producer, i.e., $dq^*/d\varepsilon = 0$, and permits all of (A.5) to be eliminated but the expression

$$\int_\theta [(-\lambda^c U^{c^1} + \lambda^d U^{d^1}) t] f(\theta) d\theta = 0 \ .$$

As q^* is monotone in θ , the class of functions t with $t_1/t_2 = \overset{*}{\pi}_1/\overset{*}{\pi}_2$ is wide enough to obtain the condition (16b) which achieves pointwise distributional optimality.

The allocational condition (16a) can be obtained by using the assumption of profit maximization (7) and the distributional condition (16b) to write (A.5) as

$$\int_\theta [\lambda^c U^{c^1} (B_1 - C_1) dq^*/d\varepsilon] f(\theta) d\theta = 0 \ .$$

Now a function t is chosen such that $dq^*/d\varepsilon$ is non zero implying that pointwise optimality requires that the allocational condition (16a) be satisfied for all values of θ . Thus, if a function $\overset{*}{\pi}$ is optimal and achieves pointwise optimality, then the allocational and distributional conditions must be satisfied.

When there is objective uncertainty at t_2 , the reaction functional (42) applies, and the problem which must be solved by the center at t_1 can be written

$$\max_{\pi(q,c)} \iint_{\theta y} [\lambda^c U^c (B(q^*) - C(q^*,\theta,y) - \pi(q^*,c)) + \lambda^d U^d (\pi(q^*,c)] g(y|\theta) k(\theta) dy d\theta \tag{A.6}$$

where $k(\theta)$ is the marginal density function. The functions specified in (A.2) through (A.4) are again used as the derivative of (A.6) with respect to ε is equated to zero and evaluated at $\varepsilon = 0$. This procedure yields

$$\int_{\theta} \mathop{E}_{y|\theta} (\lambda^c U^{c^1} [(B_1 - C_1 - \overset{*}{\pi}_1 - \overset{*}{\pi}_2 C_1) dq^*/d\varepsilon - t] + \lambda^d U^{d^1} [(\overset{*}{\pi}_1 + \overset{*}{\pi}_2 C_1) dq^*/d\varepsilon + t]) k(\theta) d\theta = 0 \tag{A.7}$$

where the inside integral of (A.6) has been replaced with the conditional expectation operator. Then, the same argument as used in the subjective uncertainty case is repeated to obtain the first order conditions (45) and (46) which must necessarily be true if $\overset{*}{\pi}$ solves the problem (A.6).

Footnotes

[1] Trenton Times, Sunday, December 15, 1974.

[2] A function of the form $B(q)$ assumes that there is no income effect in the consumption of good q since the willingness to spend on q does not depend on cost conditions. This assumption is implicit in Weitzman's analysis.

[3] It is possible to solve directly for the optimal PIF by setting the following problem for the center

$$\max_{\pi(q,c)} E\{B(q^*(\pi,\theta),n) - C(q^*(\pi,\theta),\theta)\}$$

where $q^*(\pi,\theta)$ is the reaction function relating the producer's choice of output at t_2 to the PIF. However, a complicated variational calculus procedure is needed to solve this maximization problem, and the method of deriving the optimal PIF presented in the text is much simpler and straightforward.

[4] It is interesting to note that as center risk neutrality is approached by a sequence of decreasingly risk averse functions, the ability of profits to serve as an incentirve mechanism is retained for all but the limiting function of the sequence. This discontinuity arises because while only the slightest variation is needed for profits to influence behavior, the limiting function exhibits no variation.

[5] This rewards structure would be straightforward: the center would give the producer a fixed amount, J, for acting efficiently given the true value of θ and $-\infty$ otherwise. This is analogous to Case II in the Spence-Zeckhauser paper.

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