PERFORMANCE INCENTIVES VERSUS PRICES VERSUS QUANTITIES*

by

Gregory G. Hildebrandt**

Laura D'Andrea Tyson***

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Princeton University

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**Gregory G. Hildebrandt, Major, USAF, Department of Economics, Geography and Management, United States Air Force Academy.

***Laura D'Andrea Tyson, Assistant Professor, Department of Economics, Princeton University.

Introduction

Models of economic planning have traditionally focused on the design of iterative procedures to achieve the optimal allocation and distribution of economic goods and resources. Models of this type generally assume that the iterative exchange of information between the center and its decentralized agents, the producers, can be continued until the center has enough information to choose an optimal plan. Suppose, however, that constraints on information exchange compel the center to choose a planning strategy in the absence of complete certainty about the conditions of production. What is the best strategy for the center to adopt under such circumstances? In a recent paper, Martin Weitzman (1974) sheds some light on this important question by contrasting the performance of two planning instruments, "prices and quantities," under conditions of uncertainty. Weitzman assumes that at the time the center chooses a planning instrument, it is uncertain about the actual conditions of production and consumption which will prevail when the plan is implemented. He then examines whether under such circumstances, the center would do better to set a quantity for producers or to set a price and allow the producers to set output according to profit-maximization.

Our purpose in this paper is to build upon the foundation laid by Weitzman by examining a third policy strategy available to the center, namely, the adoption of a general performance incentive function (PIF) or contract to guide the producer's output choice. In Section I, we define such a function or contract and demonstrate how it works as a planning tool. Section II demonstrates the superiority of a PIF over both prices and quantities in a simple planning model with a single producer. The analysis indicates that a properly specified PIF guarantees efficiency in production and requires less information processing by the center than either prices or quantities. Section III extends the examination of PIF's to cover the case of two or more producers and reaffirms the conclusion that PIF's do at least as well as prices and better than quantities as incentive mechanisms to achieve production efficiency. Finally, Section IV summarizes the results.

Before embarking upon a formal analysis of PIF's, it is important to recognize their potential applicability in a variety of circumstances. The idea of a "contractual incentive function" which specifies a mutually acceptable rule relating the monetary rewards paid by one decision maker to the subsequent performance of another is not new. Most of the existing work (see for example, Berhold (1971) and Wilson (1968)) has focused on performance contracts to motivate agents within an enterprise hierarchy to act in compliance with managerial goals or to motivate government contractors to meet their production commitments in the most efficient possible manner. Performance incentive contracts have actually been applied in both sets of circumstances.

Numerous enterprises have devised profit-sharing plans to motivate supervisory and managerial personnel, and the Department of Defense and NASA have relied on performance incentives to monitor the work of contractors in billions of dollars worth of government expenditure programs.[1] Recent innovations in the use of PIF's have appeared in the new Amtrak contract relating railroad payments to the quality of various railroad services (Baumol, 1975) and in a contractual arrangement in Orange, California linking the salaries of policemen to various indicators of crime prevention and control.[2]

Although the existing literature on central planning does not specifically mention the use of performance contracts, the concept does arise in discussions of "success indicators" in Soviet planning. The Soviets are noted for a system of planning in which enterprise agents are rewarded according to the degree to which certain plan targets are achieved and, more recently, according to the accuracy of the plan targets they project (Weitzman, 1974). By choosing enterprise success criteria and a related reward structure, the Soviet planners define a performance incentive system. In contrast to similar systems employed in the West, the Soviet system is not "contractual" in the sense that it is agreed upon by the planners and enterprise managers. Instead the state unilaterally chooses success indicators and rewards, and enterprise managers are expected to comply out of self-interest. This "non-contractual" incentive system is an example of the use of PIF's in central economic planning. Future examples will be forthcoming as more societies look toward some degree of planning to foster the efficient use of scarce resources.

I. Performance Incentive Functions in a Simple Planning Model

We begin with Weitzman's model of a single commodity and a single producer. We assume the existence of a cost function relating money costs to the level of output produced and a benefit function relating aggregate benefits measured in money terms[3] to the level of output consumed. In this simple model the planning problem is to achieve the level of production which just maximizes net benefits defined as

$$B(q) - C(q) \tag{1}$$

where by assumption, $B_{11} < 0$; $C_{11} > 0$; $B_1(0) > C_1(0)$; and $B_1(H) < C_1(H)$ for H sufficiently large; for PIF's, the less restrictive single condition $B_{11} - C_{11} < 0$ can replace the separate conditions on B_{11} and C_{11}, but for comparison purposes, we will assume these conditions to hold throughout the analysis.

As long as the center has complete knowledge of the benefit and cost functions, it can choose an optimal quantity directive q^* according to the first order condition:

$$B_1(q^*) = C_1(q^*) \tag{2}$$

or it can choose an optimal price directive p^* according to the first order condition

$$p^* = C_1(q^*) = B_1(q^*) \tag{3}$$

allowing the producer to choose the optimal quantity via profit maximization.

Now consider the more realistic and therefore more interesting case in which the center must make its planning decision in the presence of uncertainty. From the point of view of the center, the cost function is of the form $C(q,\theta)$ and the benefit function is of the form $B(q,n)$, where θ and n are independent random variables reflecting the center's informational uncertainty about costs and benefits at t_1 when the plan is formulated. At t_2, when the plan is implemented, the value of θ is revealed to the producer, and at t_3, when benefits are realized, the value of n is revealed to the center. At t_1, the center's planning problem is to choose an optimal planning strategy with certain information at its disposal. Since the optimal strategy depends on the center's objective function, it is first necessary to identify the function to be used. Weitzman assumes that the center, acting in accordance with the dictates of the consumer it represents, maximizes the expected value of net benefits or net social profits or rent at t_1. The optimal quantity directive $\hat{q}$ is then the solution to the following problem:

$$\max_q E\,\{B(q,n) - C(q,\theta)\}\,. \tag{4}$$

The first order condition for this problem is

$$EB_1(\hat{q}) = EC_1(\hat{q}) \tag{5}$$

indicating that at the optimal quantity, expected marginal benefits just equal expected marginal costs.

In the case of a quantity directive, the producer simply produces the level of output chosen by the center at t_1 regardless of the value of θ at t_2. In the case of a price directive, the producer chooses an output level to maximize profits given the price announced by the center at t_1 and the observed value of θ at t_2.

A reaction function of the form

$$q = h(p,\theta) \tag{6}$$

thus links profit-maximizing levels of output to price and θ at t_2. Given this reaction function, the optimal price directive $\tilde{p}$ or the price which maximizes expected net benefits at t_1 is the solution to the following problem:

$$\max_{p} E\,\{B(h(p,\theta),n) - C(h(p,\theta),\theta)\}\ . \tag{7}$$

This solution must satisfy the first order condition:

$$E\{B_1(h(\tilde{p},\theta),n)h_1(\tilde{p},\theta)\} = E\{C_1(h(\tilde{p},\theta),\theta)h_1(\tilde{p},\theta)\}\ . \tag{8}$$

Because at t_2, the producer equates $\tilde{p}$ and $C_1(q,\theta)$, this condition can be rewritten simply as

$$\tilde{p} = \frac{E\{B_1(h(\tilde{p},\theta),n)h_1(\tilde{p},\theta)\}}{E\{h_1(\tilde{p},\theta)\}}\ . \tag{9}$$

Instead of specifying a simple price or output target, the center may design a PIF, relating earned profits of the producer to certain characteristics of his performance. Such a function will be of the general form $\pi(q,c)$, where c represents actual production costs at t_2, and π is measured in the same units as benefits and costs. To maximize rewards, the producer will choose an output level which maximizes the PIF. Clearly, a special case of performance incentives is the use of the optimal price $\tilde{p}$ in a contract which specifies that the producer bears full production costs. The PIF then becomes

$$\pi(q,c) = \tilde{p}q - c . \tag{10}$$

To derive the optimal PIF,[4] suppose first that the center could choose an output level at t_2 when the actual value of θ is revealed. Under these circumstances, the center would want to maximize expected net benefits at t_2 and would find the optimal quantity by solving the following maximization problem

$$\max_{q} E\,\{B(q,n)\} - C(q,\theta) . \tag{11}$$

The first order condition for the optimal q^* from the center's perspective is therefore

$$EB_1(q^*,n) = C_1(q^*,\theta) . \tag{12}$$

This condition characterizes production efficiency at t_2 from the point of view of the center. However, because the center cannot observe θ at t_2, it cannot solve this problem directly. Instead, it can use a properly specified PIF to guarantee that the producer will choose q^* at t_2. Recall that the producer at t_2 will maximize the PIF. The associated first order condition for the producer can be written as:

$$\pi_1^*/\pi_2^* = C_1(q^*,\theta) \tag{13}$$

where π_1^* and π_2^* are the two first partial derivatives of the optimal PIF, and q^* is the optimal output level for the producer at t_2. Substituting this condition into the first order condition for the center's maximization problem yields the following condition for the optimal PIF:

$$EB_1(q^*,n) = C_1(q^*,\theta) = \pi_1^*/\pi_2^* \tag{14}$$

which can be rewritten as

$$\pi_1^* = -\pi_2^* EB_1(q) . \tag{15}$$

This condition indicates that the center has a degree of freedom in choosing the optimal PIF. It may specify any function whose partial derivatives satisfy expression (15). All such functions will generate the same total net benefits or social net rent. In terms of informational requirements, however, the center may prefer to choose a PIF for which $\pi_2^* = -1$, thereby forcing the enterprise to bear total actual production costs. Any other arrangement requires the center to measure actual production costs at t_2, to make sure that the desired sharing of total costs is realized.

If the center chooses $\pi_2^* = -1$ to minimize information gathering, then $\pi_1^* = EB_1(q)$ and integration yields a general optimal PIF of the form:

$$\pi^*(q,c) = G(q) - c \tag{16}$$

where $G(q)$ is equal to $EB(q)$ plus or minus some arbitrary constant of integration. In this case the center only needs to know $EB_1(q,n)$ to specify the optimal PIF.

A PIF of the form suggested in equations (15) and (16) has a simple intuitive explanation. If the center were certain of the actual cost conditions which would prevail at t_2, then it would set an output target at t_1 such that expected marginal benefits would just equal actual marginal costs. However, since the center cannot know actual costs at t_2, it cannot specify this output target at t_1. Nonetheless, it can still motivate the producer to choose this output level at t_2 by devising a contract so that the producer always operates at the point where actual marginal costs at t_2 just equal expected marginal benefits at t_1. The PIF corresponding to equation (15) yields a contract of this form. The producer is motivated to do the job which the center wants done at t_2 by a performance incentive contract specified at t_1. Moreover, the center does not need any information about the producer's cost function at either t_1 or t_2 to determine this performance contract. As far as the center is concerned, the producer's cost function is an unknown function whose first derivative is replaced by $EB_1(q,n)$ in the specification of the PIF. Thus, the center does not need to know either the cost function or the distribution function for θ, and privacy or the guarding of technological information by producer is maintained during the planning process. In contrast, both equations (5) and (9) indicate that the center must know $C(q,\theta)$ at t_1 to specify either the optimal quantity or the optimal price. Given the difficulties involved in information flow between the center and the producer and given the premium placed on "privacy" or the guarding of technological possibilities by the producer, the PIF is undeniably superior to both the optimal price and the optimal quantity tools.

II. The Allocational Efficiency of Performance Incentive Functions

Because the use of an optimal price or quantity at t_1 does not yield an optimal price-output configuration at t_2, when the value of θ is revealed, both price and quantity directives are second-best solutions to the problem of maximizing expected net benefits at t_2, and both involve a deadweight loss to society. In contrast, a properly specified PIF yields first-best optimization at t_2 by guaranteeing that expected marginal benefits just equal actual marginal costs at t_2, given the realized value of θ. The superiority of a PIF over prices and quantities can be illustrated using a technique suggested by Weitzman for the measurement of the comparative advantage of prices over quantities.

Weitzman defines the comparative advantage of prices over quantities as

$$\Delta \equiv E\{(B(\tilde{q}(\theta),n) - C(\tilde{q}(\theta),\theta)) - (B(\hat{q},n) - C(\hat{q},\theta))\} \quad (17)$$

where the loss function which the center wishes to minimize is the expected difference in gains between the two modes of control. Analogously, the comparative advantages of performance incentives over quantities can be defined as

$$A \equiv E\{(B(q^*(\theta),n) - C(q^*(\theta),\theta)) - (B(\hat{q},n) - C(\hat{q},\theta))\}. \quad (18)$$

To obtain some insight into what determines Δ and A, some additional assumptions are required about the shapes of the underlying cost and benefit functions. Weitzman assumes that the slopes of the marginal cost and marginal benefit functions are non-stochastic. Given these assumptions, he argues that it is reasonable to use stochastic linear approximations of the marginal cost and marginal benefit functions around $\hat{q}$, the prescribed quantity. Using these approximations and the associated stochastic quadratic approximations of the cost and benefit functions around $\hat{q}$, Weitzman derives the following approximation for Δ.

$$\Delta \cong \frac{\sigma^2(B_{11}+C_{11})}{2(C_{11})^2} \quad (19)$$

where $\sigma^2 \equiv E(C_1(q,\theta) - E(C_1(q,\theta)))^2$ and represents the variance of pure random shifts in the marginal cost function.

To compute the coefficient of comparative advantage of the PIF over the quantity control tool, it is necessary to further assume that a second order approximation of the optimal PIF of equation (16) is reasonably accurate. Combining this assumption with the Weitzman assumptions, a few simple manipulations shown in the Appendix yield the following approximation for A :

$$A \cong \frac{\sigma^2}{2(C_{11}-B_{11})} . \tag{20}$$

Since $C_{11} > 0$ and $B_{11} < 0$ by assumption, it follows that $A > 0$ or that the PIF is always superior to the quantity control tool. In fact, the analysis in the Appendix reveals that A is a measure of the deadweight loss caused by using a quantity control target instead of a PIF.

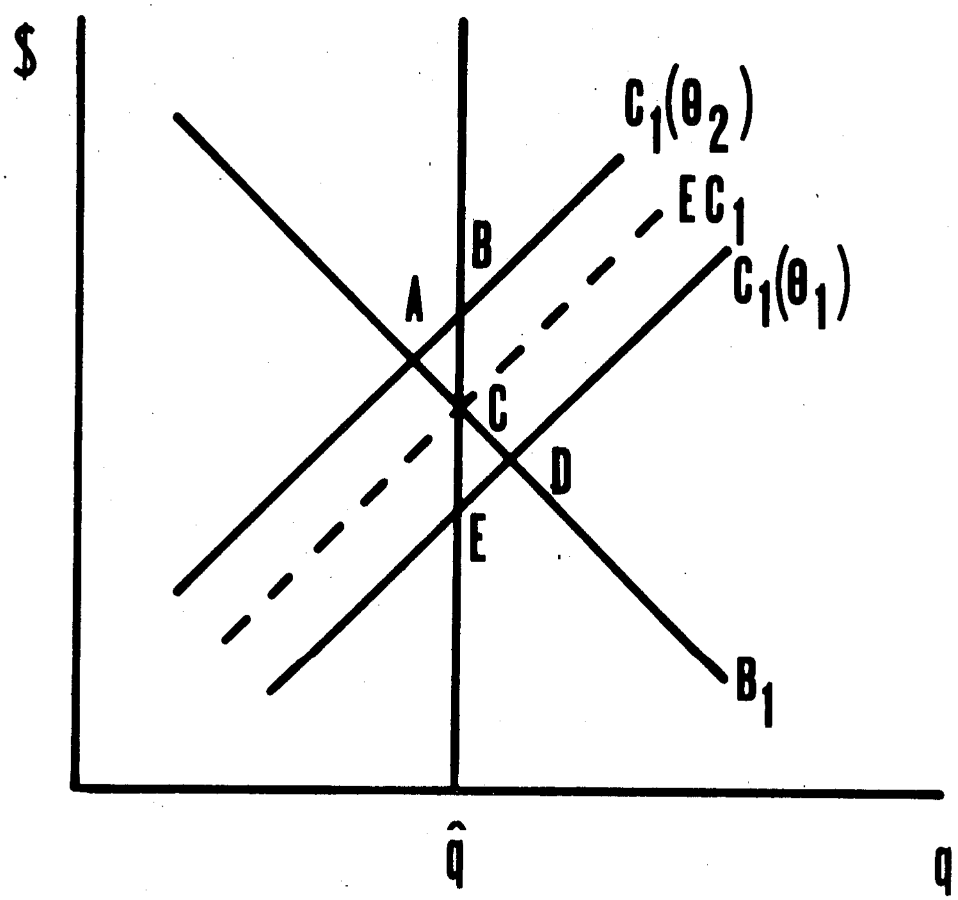
To compute the comparative advantage of the PIF over prices, Δ must be subtracted from A yielding the following relationship:

$$A - \Delta \cong \frac{\sigma^2}{2(C_{11}-B_{11})} - \frac{\sigma^2(B_{11}+C_{11})}{2C_{11}^2} . \tag{21}$$

Again, as long as $C_{11} > 0$ and $B_{11} < 0$, or, less restrictively, as long as $B_{11} - C_{11} < 0$, this expression is positive indicating that a PIF is also superior to a price control tool. The coefficient $A - \Delta$ is itself a measure of the deadweight loss caused by using a price control mechanism instead of a PIF.

The superiority of the PIF over prices and quantities can be graphically illustrated for the case in which the benefit function is deterministic and in which there are only two possible values for θ , θ_1 and θ_2 , each occurring with probability equal to 1/2.

Consider Graph 1. The optimal quantity tool $\hat{q}$ is chosen such that $EC_1 = B_1$



Graph 1

at t_1 . If at t_2 , $\theta = \theta_1$, then the optimal point for the producer is point D ; if at t_2 , $\theta = \theta_2$, then the optimal point for the producer is A . The expected deadweight loss resulting from the fact that given a quantity target, the producer must always produce at point C is given by 1/2(area ABC) + 1/2(area CED).

In the case of an optimal price tool, pictured in Graph 2, if θ_1 is the value of θ at t_2 , the producer chooses point L when point D is optimal. If, instead, θ_2 is the value of θ at t_2 , the producer chooses point F when point A is optimal. The expected deadweight loss in this case is 1/2(area FKA) + 1/2(area DHL). The Weitzman Δ can be calculated from the graphical analysis as

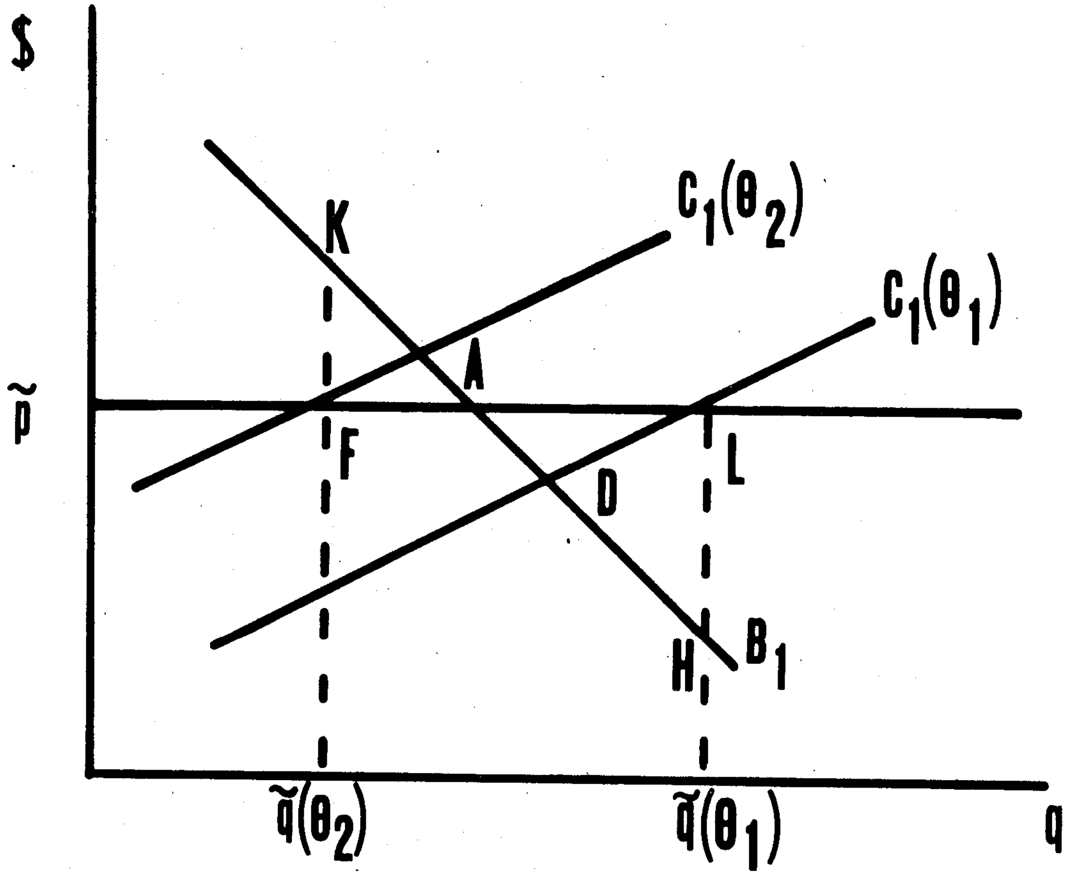
$$\Delta = \frac{1}{2}(FKA + DHL - ABC - CED) \; . \qquad (22)$$

In the case of the PIF, shown in Graph 3, the producer will operate at D when $\theta = \theta_1$ and at A when $\theta = \theta_2$. There is no deadweight loss and optimality is achieved at t_2 for the actual value of θ .

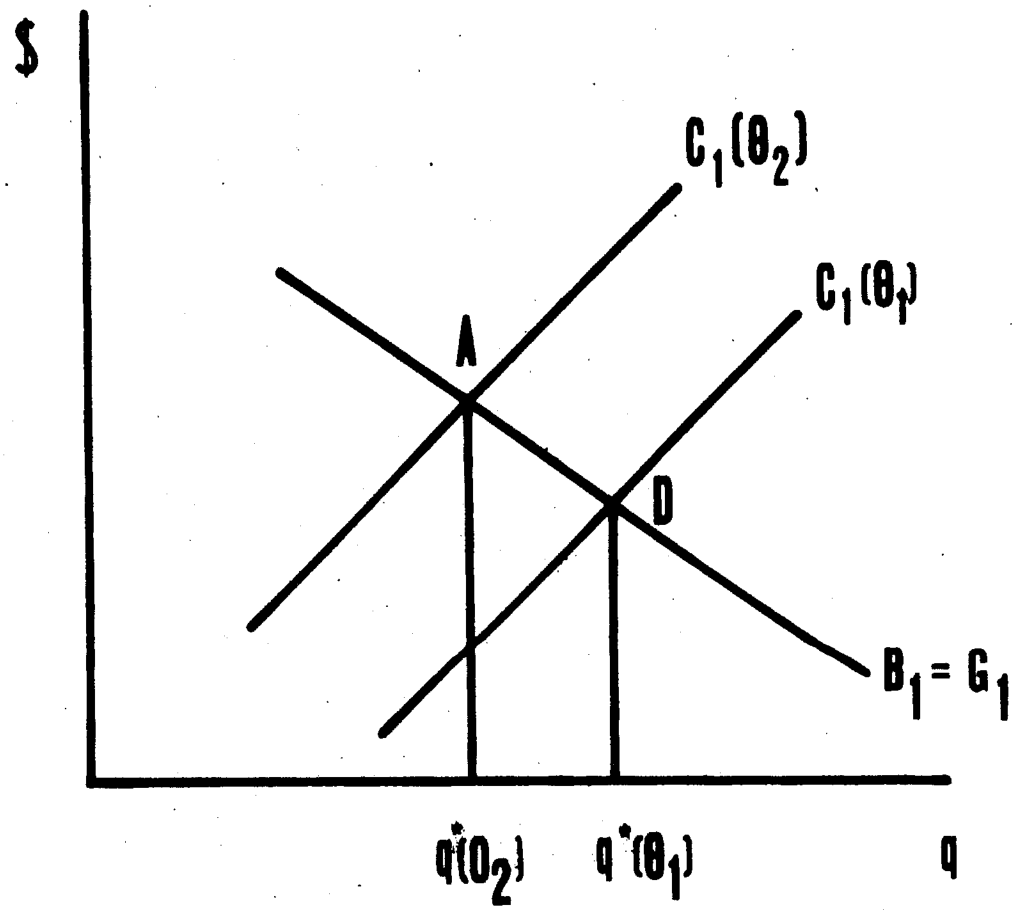
Summarizing the results derived in this section, we may conclude that an optimal PIF structured by the center at t_1 yields first best allocational decisions at t_2 when the true value of θ becomes known. Moreover, in structuring the optimal PIF, the center does not need to know the cost function of the producer. Thus, the PIF emerges as a powerful planning tool which satisfies the criteria of privacy and efficiency identified in the literature as desirable properties of planning procedures.

III. Performance Incentives and Two Producers

Suppose the center is trying to coordinate the activities of two producers, each of which produces a single output. In this case the benefit function is of



Graph 2



Graph 3

the form $B(q_1,q_2,n)$ where $B_{11} < 0$ and $B_{22} < 0$. Each producer in turn has its own cost function of the form $C^i(q_i,\theta_i)$, $i=1,2$, such that $C^i_{11} > 0$.[5]

To find the prescribed quantity for each producer, the center will maximize the total expected net benefits from the production of both goods. Thus, the center will solve the following maximization problem:

$$\underset{q_1,q_2}{\text{Max}} \; E\{B(q_1,q_2,n) - \sum_{i=1}^{2} C^i(q_i,\theta_i)\} . \tag{23}$$

To find the optimal price signal for each producer, the center will maximize the total expected net benefits from the production of both goods given the reaction functions $h_i(p_i,\theta_i)$, relating profit-maximizing output to price and θ values for each producer. With these reaction functions, the optimal price directive p_i for each producer or the price which maximizes expected net benefits from the production of both goods at t_1 is the solution to the following problem:

$$\underset{p_1,p_2}{\text{Max}} \; E\{B(h_1(p_1,\theta_1),h_2(p_2,\theta_2)) - \sum_{i=1}^{2} C^i(h_i(p_i,\theta_i),\theta_i)\} . \tag{24}$$

As derived by Weitzman, the coefficient of comparative advantage of prices over quantities in the two producer case can be expressed as:

$$\Delta_2 = E[(B(\tilde{q}_1,\tilde{q}_2,n) - \sum_{i=1}^{2} C^i(\tilde{q}_i,\theta_i)) - (B(\hat{q}_1,\hat{q}_2,n) - \sum_{i=1}^{2} C^i(\hat{q}_i,\theta_i))] . \tag{25}$$

Using an approximation approach analogous to the one used in the single good case, this coefficient is estimated by

$$\Delta_2 \cong \sum_{i=1}^{2} \sum_{j=1}^{2} \frac{B_{ij}\sigma_{ij}^2}{2C_{11}^i C_{11}^j} + \sum_{i=1}^{2} \frac{\sigma_{ii}^2}{2C_{11}^i} \qquad (26)$$

where σ_{ij}^2 is the covariance of pure random shifts in the marginal cost functions of the two producers.

If the center wants to use PIF's to guide production decisions, then by analogy with the one good case, we can show that it must specify functions of the general form:

$$\pi^i(q_i, C^i) = G^i(q_i) - C^i(q_i, \theta_i) \qquad i = 1, 2 . \qquad (27)$$

Profit maximization for each producer at t_2 implies:

$$G_1^i(q_i^*) = C_1^i(q_i^*, \theta_i) \qquad i = 1, 2 \qquad (28)$$

which in turn yields a reaction functional

$$q_i^* = F^i(G_1^i, \theta_i) \qquad i = 1, 2 \qquad (29)$$

relating the optimal output choice to the PIF and the actual value of θ .

To find the optimal PIF for each producer, the center must solve the following maximization problem:

$$\max_{G_1^1, G_1^2} E(B(F^1, F^2, n) - C^1(F^1, \theta_1) - C^2(F^2, \theta_2)) . \qquad (30)$$

The first best solution to this problem is characterized by the following relationships:

$$E(B_1(q_1^*, q_2^*, n)) - C_1^1(q_1^*, \theta_1) = 0 \qquad (31)$$

and

$$E(B_2(q_1^*,q_2^*,n)) - C_1^2(q_2^*,\theta_2) = 0 . \qquad (32)$$

These conditions imply that unless the benefit function is separable so EB_i does not depend on q_j $(i \neq j)$, the optimal PIF for producer i will depend on the output level of producer j. Realistically, of course, the center will never choose to make the reward structure for one producer contingent upon the performance of another unrelated producer. Consequently, the center's choice of a PIF is restricted by the constraint that the performance incentives offered to one producer do not vary with the other producer's output. This constraint renders the "first-best" optimal PIF's characterized by (31) and (32) infeasible, and the center must adopt alternative functions which fail to achieve efficiency at t_2.

One possible "second-best" solution under these conditions can be obtained by integrating θ_j out of the center's first best optimality condition for producer i. The relevant first order conditions for this solution then become:

$$\underset{\theta_2\ n}{E} (EB_1(q_1^*,F^2(G_1^2,\theta_2),n) - C_1^1(q_1^*,\theta_1) = 0 \qquad (33)$$

and

$$\underset{\theta_1\ n}{E} (EB_2(F^1(G_1^1,\theta_1),q_2^*,n) - C_1^2(q_2^*,\theta_2) = 0 . \qquad (34)$$

The constrained second best solution for the PIF's can then be achieved if we substitute G_1^i for C_1^i, thereby obtaining

$$G_1^1 = \underset{\theta_2\ n}{E} (EB_1(q_1^*,F^2(G_1^2,\theta_2),n)) \qquad (35)$$

and

$$G_1^2 = \underset{\theta_1\ n}{E} (EB_2(F^1(G_1^1,\theta_1),q_2^*,n)) . \qquad (36)$$

The relevant marginal benefit functions used in these conditions are obtained by averaging for each producer the output level chosen by the other producer.

It is significant to note that the center must have information about the reaction functionals F^1 and F^2 to find the second-best solution. Since the reaction functions in turn depend on the underlying cost functions, it is necessary for the center to have knowledge of each producer's cost conditions under these circumstances. Therefore, we can conclude that the center requires considerably more information in the two good case than in the one good case where the cost function need not be known by the center.

The second best PIF's identified here can be shown to be superior to quantities and to perform at least as well as prices on efficiency grounds. Using guadratic approximations of the cost and benefit functions and evaluating the expected benefits of one good at the prescribed quantity target of the other good, approximations to the second best PIF's satisfy the following conditions derived in the Appendix:

$$G_1^1 = EB_1(\hat{q}_1,\hat{q}_2,n) + (q_1-\hat{q}_1)EB_{11}(\hat{q}_1,\hat{q}_2,n) \qquad (37)$$

and

$$G_1^2 = EB_2(\hat{q}_1,\hat{q}_2,n) + (q_2-\hat{q}_2)EB_{22}(\hat{q}_1,\hat{q}_2,n) \ . \qquad (38)$$

These relationships suggest that in the more general case, when quadratic approximations are not employed, the marginal cost of producing each good should be set equal to the expected marginal benefits of that good, evaluated at the optimal quantity target for the other good.

Using the quadratic approximations of the cost and benefit functions, the comparative advantage of the second best PIF's over quantity targets is calculated in the Appendix as

$$A_2 = \frac{\sigma_{12}^2 B_{12}}{(C_{11}^1 - B_{11})(C_{11}^2 - B_{22})} + \frac{\sigma_{11}^2}{2(C_{11}^1 - B_{11})} + \frac{\sigma_{22}^2}{2(C_{11}^2 - B_{22})} > 0. \quad (39)$$

Under our assumptions about the signs of C_{11}^1, C_{11}^2, B_{11} and B_{22}, this expression is positive indicating that the second best PIF's are superior to prescribed quantities.

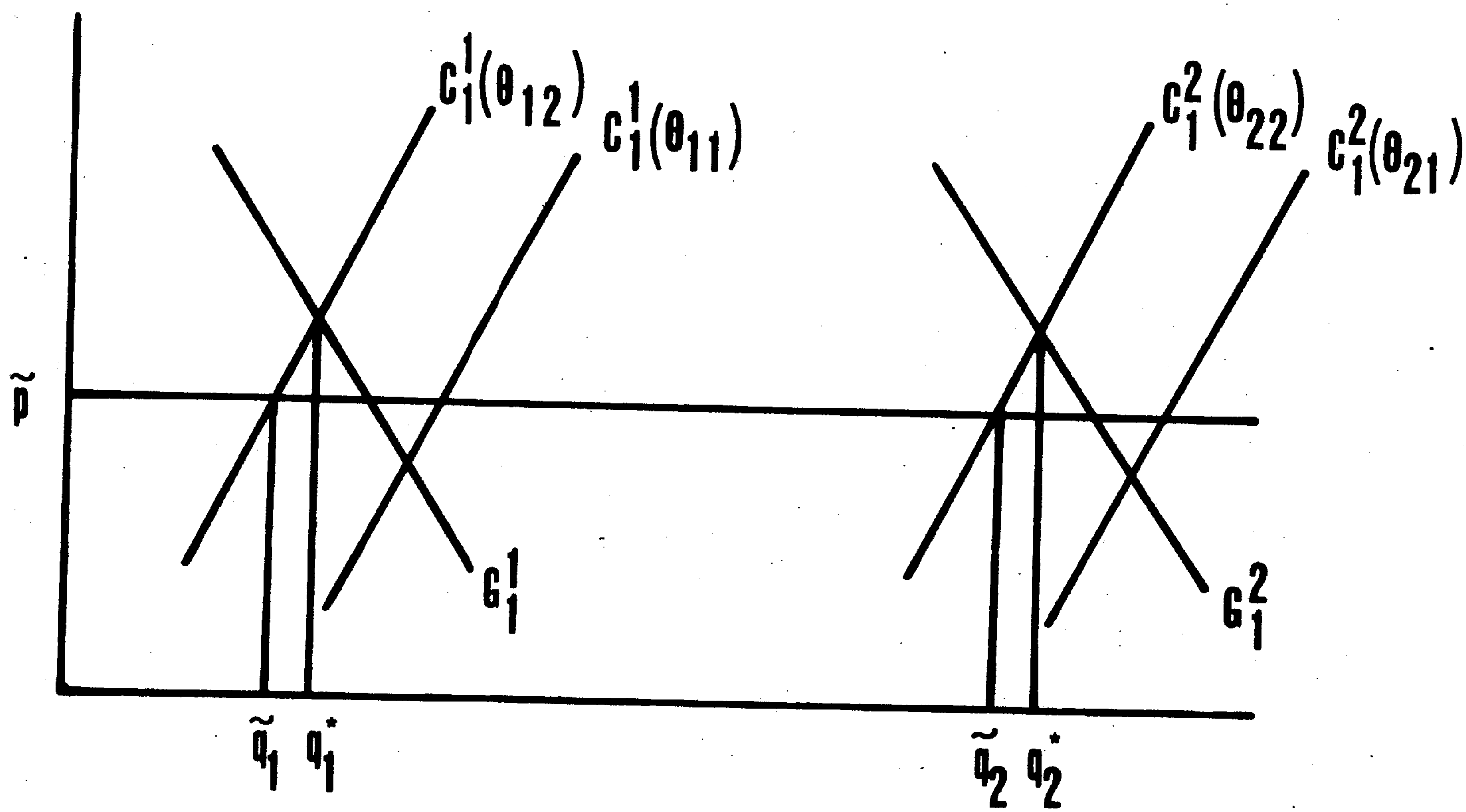
Subtracting the comparative advantage of prices over quantities in the two producer case from expression (39) in turn yields the comparative advantage of the second best PIF's over prices as:

$$A_2 - \Delta_2 = \frac{\sigma_{12}^2 B_{12}}{(C_{11}^1 - B_{11})(C_{11}^2 - B_{22})} + \frac{\sigma_{11}^2}{2(C_{11}^1 - B_{11})} + \frac{\sigma_{22}^2}{2(C_{11}^2 - B_{22})} - \frac{\sigma_{12}^2 B_{12}}{C_{11}^1 C_{11}^2} - \frac{\sigma_{11}^2 B_{11}}{2C_{11}^1} - \frac{\sigma_{22}^2 B_{22}}{2C_{11}^2} - \frac{\sigma_{11}^2}{2C_{11}^1} - \frac{\sigma_{22}^2}{2C_{11}^2}. \quad (40)$$

Under our assumptions, this expression is always nonnegative, indicating that PIF's perform at least as well as price signals. Intuitively, this conclusion is to be expected since price signals are just special cases of the more general PIF's. To see this conclusion more clearly, consider the case in which both producers provide the same output. In this case, the goal of the center is to solve the following maximization problem:

$$\max_{G_1^1, G_1^2} E(B(q_{11}+q_{21}), n) - \sum_{i=1}^{2} C^i(q_{i1}, \theta_i)). \quad (41)$$

The results of using optimal price signals are contrasted with the results of using optimal PIF's for this case in Graph 4. We make the further simplifying assumption that θ_i takes on only two possible values for each producer


$c_1^1(\theta_{12})$
$c_1^1(\theta_{11})$
$c_1^2(\theta_{22})$
$c_1^2(\theta_{21})$
$\tilde{p}$
G_1^1
G_1^2
$\tilde{q}_1$
q_1^*
$\tilde{q}_2$
q_2^*

Graph 4

$(\theta_{i1}, \theta_{i2}, i=1,2)$. A priori, the use of a price signal seems to be desirable in this case because marginal costs are equated to a single price and productive efficiency is thereby achieved. When a price is used, however, the wrong amount of output may be produced. For example, in Graph 4, if θ_{12} is the state of the world for producer 1, the price signal will lead producer 1 to output level $\tilde{q}_1$, when the optimal output level is q_1^* . Similarly, if θ_{22} is the state of the world for producer 2, prices will lead to an output level of $\tilde{q}_2$ when the optimal level is q_2^* . Although PIF's do not lead to productive efficiency by obtaining a least cost solution with equal marginal costs for both producers, they bring total industrial output on average closer to its socially correct level than a price signal does.

Finally, we conclude this section by noting that the generalization of the two-good, two-producer case to the n-good, m-producer case is straightforward. In this general case, we know that PIF's will do at least as well as prices. For example, in searching for the optimal PIF for producer 1 the center's maximization process scans over functions of the form

$$\pi^1 = \sum_{i=1}^{n} p_i q_{1i} - C^1(q_{11} \dots q_{1n}, \theta)$$

which yield the optimal price signal for producer 1.

IV. Conclusions

In this paper we extend Weitzman's analysis of the comparative usefulness of prices and quantities as planning instruments to include a third planning instrument, the so-called performance incentive function. Such a function relates the rewards of producers to certain characteristics of their output and cost performance. Using Weitzman's model we demonstrate the superiority of PIF's over both prices and quantities as a means for achieving socially optimal output decisions. In the one-good, one-producer case, the PIF is also shown to be superior in terms of the informational requirements imposed on the center. When the analysis is extended to more than one good, the center's problem for all of the control tools is finding a second best solution. Under these circumstances, PIF's are shown to lead to the same informational requirements as prices or quantities, but as price signals are a special case of PIF's, they do at least as well as prices and better than quantities.

It is necessary to realize that the optimal PIF results in a more complicated message being constructed and transmitted by the center and responded to by the producer than is the case with either prices or quantities. It has been implicitly assumed in the analysis that the construction and transmittal of messages by the center and the producer response to messages take place costlessly. In the real world, however, these activities carry significant costs, and only a comparison of these costs with the potential gains in productive efficiency and the potential savings in information gathering can indicate whether a PIF is the optimal planning tool under a given set of circumstances.

Appendix

I. The derivation of "A", the coefficient of comparative advantage of a PIF over a quantity tool.

Start with Weitzman's stochastic quadratic approximations of the benefit and cost functions around $\hat{q}$.

$$B(q,n) \cong b(n) + (B^1+\beta(n))(q-\hat{q}) + \frac{B^{11}}{2}(q-\hat{q})^2 \qquad (A.1)$$

$$C(q,\theta) \cong a(\theta) + (C^1+\alpha(\theta))(q-\hat{q}) + \frac{C^{11}}{2}(q-\hat{q})^2 \qquad (A.2)$$

where $B^1 = EB_1(\hat{q},n)$, $C^1 = EC_1(\hat{q},\theta)$; $B^{11} = EB_{11}(\hat{q},n) = B_{11}(\hat{q})$, $C^{11} = EC_{11}(\hat{q},\theta) = C_{11}(\hat{q})$. $b(n)$ and $a(\theta)$ respectively estimate the effect of the random variables on total benefits and total costs at $\hat{q}$ so $E(a(\theta)) = EC(\hat{q},\theta)$ and $E(b(n)) = EB(\hat{q},n)$; $\beta(n)$ and $\alpha(\theta)$ respectively estimate the effect of the random variables on marginal benefits and marginal costs at $\hat{q}$, and $E\alpha(\theta) = E\beta(n) = 0$ and $E(\alpha(\theta) \cdot \beta(n)) = 0$ by assumption.

In addition, we need Weitzman's approximation of the variance of marginal costs

$$\sigma^2 \equiv E(C_1(q,\theta) - E(C_1(q,\theta))^2 = E(\alpha(\theta)^2) .$$

Now consider a PIF of the form

$$\pi^*(q,c) = EB(q,n) - c + k = G(q) - c . \qquad (A.3)$$

Profit maximization by the producer implies that

$$C_1(q^*,\theta) = B_1(q^*,n) = G_1(q^*) . \qquad (A.4)$$

The quadratic approximation of this equality around $\hat{q}$ is

$$C^1 + \alpha(\theta) + C^{11}(q^*-\hat{q}) = G^1 + G^{11}(q^*-\hat{q}) \qquad (A.5)$$

where $G^1 = EG_1(\hat{q}) = EB_1(\hat{q},n)$ and $G^{11} = EG_{11}(\hat{q}) = EB_{11}(\hat{q},n) = B_{11}(\hat{q})$.

Now by Weitzman's analysis, it is true that for the prescribed quantity $\hat{q}$, $EC_1(\hat{q},\theta) = EB_1(\hat{q},n)$ which allows us to rewrite (A.5) as

$$\alpha(\theta) + C^{11}(q^*-\hat{q}) = G^{11}(q^*-\hat{q}) \qquad (A.6)$$

and to directly solve for the approximation of the reaction function as

$$q^* = \hat{q} - \frac{\alpha(\theta)}{C^{11}-G^{11}} . \qquad (A.7)$$

Substituting this expression into Weitzman's approximation of the cost function yields

$$C(q^*,\theta) \cong a(\theta) + (C^1+\alpha(\theta))(\hat{q}-\frac{\alpha(\theta)}{C^{11}-G^{11}}-\hat{q}) + \frac{C^{11}}{2}(\hat{q}-\frac{\alpha(\theta)}{C^{11}-G^{11}}-\hat{q})^2 . \qquad (A.8)$$

Taking expected values and using the definitions of $E(a(\theta))$ and σ^2 we derive

$$EC(q^*,\theta) = E(a(\theta)) - \frac{\sigma^2}{C^{11}-G^{11}} + \frac{C^{11}\sigma^2}{2(C^{11}-G^{11})^2} . \qquad (A.9)$$

Next substituting (A.7) into the quadratic approximation of the benefit function to yield

$$B(q^*,n) = b(n) + (B^1+\beta(n))(\hat{q}-\frac{\alpha(\theta)}{C^{11}-G^{11}}-\hat{q}) + \frac{B^{11}}{2}(\hat{q}-\frac{\alpha(\theta)}{C^{11}-G^{11}}-\hat{q})^2 . \qquad (A.10)$$

Using the assumption that θ and n are independent, the expected value of this expression is computed as

$$EB(q^*,n) = E(b(n)) + \frac{B^{11}\sigma^2}{2(C^{11}-G^{11})} . \qquad (A.11)$$

We now substitute (A.9) and (A.11) into equation (18) for A to derive

$$A = \frac{B^{11}-C^{11}}{2} \frac{\sigma^2}{(C^{11}-G^{11})^2} + \frac{\sigma^2}{C^{11}-G^{11}} . \qquad (A.12)$$

Finally, substituting B^{11} for G^{11} and recalling that $B^{11} = B_{11}(\hat{q})$ and $C^{11} = C_{11}(\hat{q})$ by assumption, we derive expression (20) in the text.

II. Interpretation of the A Coefficient.

"A" can be interpreted as the expected deadweight loss caused by using a quantity directive instead of a PIF. (A.7), the approximation of the reaction function, can be used to solve for the difference between the quantity target $\hat{q}$ and q^*, the quantity chosen by the producer maximizing the PIF. For any $\alpha(\theta)$, the deadweight loss involved can be computed as the triangle $\alpha(\theta)/2(C^{11}-G^{11})$. Taking the expectation of this expression and substituting B^{11} for G^{11} then yields expression (20) for "A".

III. The derivation of "A_2", the coefficient of comparative advantage of the second best PIF's over quantities in the case of two producers.

Begin with the quadratic approximation of the benefit function taken around the prescribed quantities.

$$B(q_1,q_2,n) = b(n) + (B^1+\beta_1(n))(q_1-\hat{q}_1) + (B^2+\beta_2(n))(q_2-\hat{q}_2)$$

$$+ 1/2d^2B \qquad (A.13)$$

where $B^2 = EB_2(\hat{q}_1,\hat{q}_2,n)$, $B^1 = EB_1(\hat{q}_1,\hat{q}_2,n)$; $\beta_1(n)$ and $\beta_2(n)$ represent the effect of random variable n on marginal benefits of good 1 and 2 at $\hat{q}_1$ and $\hat{q}_2$, respectively. $b(n)$ estimates the effect of random variable n on total benefits at $\hat{q}_1$ and $\hat{q}_2$ and $E(b(n)) = EB(\hat{q}_1,\hat{q}_2,n)$; and

$$d^2B = (q_1-\hat{q}_1)^2B^{11} + 2(q_1-\hat{q}_1)(q_2-\hat{q}_2)B^{12} + (q_2-\hat{q}_2)^2B^{22} ;$$

$$B^{11} = EB_{11}(\hat{q}_1,\hat{q}_2,n) = B_{11}(\hat{q}_1,\hat{q}_2)$$

$$B^{22} = EB_{22}(\hat{q}_1,\hat{q}_2,n) = B_{22}(\hat{q}_1,\hat{q}_2)$$

and

$$B^{12} = EB_{12}(\hat{q}_1,\hat{q}_2,n) = B_{12}(\hat{q}_1,\hat{q}_2) .$$

Taking the derivative of (A.13) with respect to q_1, we obtain

$$B_1 = B^1 + \beta_1(n) + (q_1-\hat{q}_1)B^{11} + (q_2-\hat{q}_2)B^{12} . \qquad \text{(A.14)}$$

From the one good analysis we know that the reaction function for q_2^* evaluated at the optimal quantity targets $\hat{q}_1$ and $\hat{q}_2$ is

$$q_2^* = \hat{q}_2 - \frac{\alpha(\theta_2)}{C^{112}-G^{112}} \qquad \text{(A.15)}$$

where $\alpha(\theta_2)$ represents the effect of θ_2 on producer 2's marginal costs at $\hat{q}_1$, $\hat{q}_2$ and $E(\alpha(\theta_2)) = 0$ by assumption; $C^{112} = EC^2_{11}(\hat{q}_2,\theta_2) = C^2_{11}(\hat{q}_2)$ equals the nonstochastic slope of producer 2's marginal cost curve at $\hat{q}_2$; and $G^{112} = EG^2_{11}(\hat{q}_1,\hat{q}_2) = EB_{11}(\hat{q}_1,\hat{q}_2,n) = B_{11}(\hat{q}_1,\hat{q}_2)$ equals the nonstochastic slope of the marginal benefit curve at $\hat{q}_1$, $\hat{q}_2$ with respect to output of producer 2.

Substituting (A.15) into (A.14) and taking expectations yields:

$$\underset{n}{E}B_1 = B^1 + (q_1-\hat{q}_1)B^{11} + \frac{\alpha(\theta_2)B^{12}}{C^{112}-G^{112}} . \qquad \text{(A.16)}$$

Recalling that $E(\alpha(\theta_2)) = 0$ by the construction of the quadratic approximation, we can integrate out θ_2 to obtain

$$\underset{\theta_2}{E}\,(\underset{n}{E}B_1) = B^1 + (q_1-\hat{q}_1)B^{11} . \qquad \text{(A.17)}$$

This allows us to ignore the cross-partial term B^{12} and permits us to structure a PIF by setting

$$G_1^1 = \underset{\theta_2\ n}{E}\ (EB_1) = B^1 + (q_1-\hat{q}_1)B^{11} \ . \qquad (A.18)$$

In words, the approximation of the first derivative of the optimal PIF is equal to the expected marginal benefit function of the center evaluated at the prescribed quantity of the other good.

Now to derive A_2 , first define it as

$$\begin{aligned} A_2 = & \ E(B(q_1^*(\theta_1),q_2^*(\theta_2),n) - C^1(q_1^*(\theta_1)\theta_1) \\ & - C^2(q_2^*(\theta_2),\theta_2)) - (B(\hat{q}_1,\hat{q}_2,n) \\ & - C^1(\hat{q}_1,\theta_1) - C^2(\hat{q}_2,\theta_2)) \ . \end{aligned} \qquad (A.19)$$

The own partial derivatives of this expression are the same as in the one good case, so the only additional computation required is the expected value of the cross partial term:

$$E((q_1-\hat{q}_1)(q_2-\hat{q}_2)B^{12} \ . \qquad (A.20)$$

Substituting the reaction functions (A.15), taking expected values, and replacing G^{111} by B^{11} and G^{112} by B^{22} one obtains

$$E((q_1-\hat{q}_1)(q_2-\hat{q}_2)B^{12}) = \frac{\sigma_{12}^2 B^{12}}{(C^{111}-B^{11})(C^{112}-B^{22})} \qquad (A.21)$$

where $\sigma_{12}^2 = E(\alpha_1(\theta_1) \cdot \alpha_2(\theta_2))$. Adding this to the results that apply to the "A" coefficient for the one good case (equation (20)), and using the definitions of C^{111} , B^{11} , C^{112} and B^{22} , we obtain expression (39) in the text.

Footnotes

[1]A rough estimate indicates that during the 1967-69 period, the Department of Defense used some form of a performance incentives contract in projects worth at least $27 billion. For more details on the use of PIF's in such projects see DOD and NASA Guide, Incentive Contracting Guide, October 1969.

[2]The Trenton Times, Sunday, December 15, 1974.

[3]A function of the form $B(q)$ assumes that there is no income effect in the consumption of good q since the willingness to spend on q does not depend on cost conditions. This assumption is implicit in Weitzman's analysis.

[4]It is possible to solve directly for the optimal PIF by setting the following problem for the center

$$\max_{\pi(q,c)} E\{B(q^*(\pi,\theta),n) - C(q^*(\pi,\theta),\theta)\}$$

where $q^*(\pi,\theta)$ is the reaction function relating the producer's choice of output at t_2 to the PIF. However, a complicated variational calculus procedure is needed to solve this maximization problem, and the method of deriving the optimal PIF presented in the text is much simpler and straightforward.

[5]For performance incentive functions, the conditions on B_{11}, B_{22} and C^i_{11} can be replaced by the less restrictive conditions $B_{11} - C^1_{11} < 0$ and $B_{22} - C^2_{11} < 0$. However, for comparison purposes we retain the more restrictive conditions throughout the analysis.

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