EFFECTIVE USE OF ECONOMETRIC MODELS IN

MACROECONOMIC POLICY FORMULATION

by

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Gregory C. Chow [1]
Princeton University

At the beginning of each year, the Economic Report of the President of the
United States makes projections of GNP in nominal and real terms for the coming
year, the unemployment rate and the inflation rate and states the major fiscal
and monetary policies required to achieve these target rates. For example, the
Report of January 1976 estimates real GNP to be 6 to 6.5 per cent higher in 1976
than in 1975 (p.19), the unemployment rate to fall by almost a full percentage
point and the inflation rate measured by the rise in the GNP deflator to be about
6 per cent (p.24). The associated fiscal policies include a proposed Federal out-
lays in fiscal 1977 of $394 billion, a cut in taxes beginning in July 1976 of
about $28 billion relative to what they would be under the 1974 law (p.22). The
rate of growth in the money supply M_1, as announced by the Federal Reserve, ranges
between 5 1/2 and 7 1/2 per cent, but the Report asserts that maintaining a rate
of money growth at the upper limit of this range would hinder the progress toward
lower inflation rates (pp.21-22). Assuming that econometric models are being used
for policy analysis, this paper presents a systematic approach to apply some re-
cently developed techniques of stochastic control to improve the formulation of
macroeconomic policies and the accompanying economic projections.

The analysis starts with the tentative paths for the policy variables[2] which
result from the existing procedure without the benefits of stochastic control
methods. Although we assume that an econometric model is used, its inaccuracies
will be duly considered. The recommended procedure consists of twelve steps.

1. Insert the tentative paths of the policy variables and the best available
estimates of the exogenous variables not subject to control into the econometric
model to obtain projections of the key economic variables for 8 quarters. This
step is already being performed in Great Britain, since the Treasury is required
to maintain an econometric model in the public domain and to make and publish

projections from the model given the current policy proposals.

2. Modify the econometric model, the estimates of the uncontrollable exogenous variables, and/or the economic projections if the projections from step 1 differ from those obtained from whatever existing procedure used in the formulation of macroeconomic policies. When making forecasts, econometric forecasters in the United States adjust the constants in their model utilizing observations of the equation residuals in recent quarters and other information. Others might insist on forecasting without adjustment of the model, in which case only the estimates of the uncontrollable exogenous variables and the final economic projections can be changed. Whatever adjustments of model parameters and economic projections are made, the essence of step 2 is to arrive at a set of forecasts of the important endogenous variables y_t^o, a set of estimates for the future uncontrollable variables z_t^o and an econometric model which are consistent with one another, given the tentative paths for the control variables x_t^o. Thus these variables satisfy each of the p simultaneous structural equations in the model

$$(1) \qquad y_{it}^o = \Phi_i(y_t^o, \ y_{t-1}^o, \ x_t^o, \ z_t^o) + \ \varepsilon_{it} \qquad\qquad (i = 1,\ldots,p)$$

if the random residual ε_{it} is set equal to zero.

3. Set target values for the future unemployment rate, inflation rate, real GNP, measures of balance of payments and possibly other important economic variables which are somewhat more desirable than the values given by y_t^o in step 2. The motivation here is to find out whether the tentative path x_t^o for the policy variables can be improved upon by performing optimal control calculations. To do so, we choose a quadratic loss function and use the above target values as elements in the vector a_t:

$$(2) \qquad W = \sum_{t=1}^{T} (y_t - a_t)' K_t (y_t - a_t)$$

where K_t is diagonal matrix giving weights according to the relative importance of the target variables, and the planning horizon T can be set equal to about 20 quarters.[3]

4. Linearize the equations (1) about the tentative paths y_t^0 and x_t^0, given z_t^0, obtaining a linear model of time-dependent coefficients, and compute the optimal feedback control equations

$$(3) \qquad x_t = G_t \, y_{t-1} + g_t$$

which minimize the expectation of the loss function (2) subject to the constraint of the linear stochastic model. A computer program is available for this purpose, as described in Chow (1976b).

Briefly, the computer program applies the Gauss-Seidel iterative method to solve the possible nonlinear econometric model for y_t^0, given z_t^0 and x_t^0, as required in step 1 above. It automatically linearizes the nonlinear structural equations (1) which are input to the program in Fortran code, and solves the resulting linear structural equations to obtain a set of linear reduced-from equations

$$(4) \qquad y_t = A_t \, y_{t-1} + C_t \, x_t + b_t + u_t$$

where the intercepts b_t incorporate the effects of z_t and the vectors of random residuals u_t are related to the residuals ε_{it} of (1) in a well-known manner. Then the coefficients G_t and g_t of the optimal feedback control equations (3) are computed. In the above notation, the vector y_t includes variables introduced to eliminate endogenous variables lagged more than one period and includes x_t as a subvector so that the loss function (2) has only y_t as argument.

By the use of this computer program after a set of optimal feedback control equations is obtained, a new set of y_t^0 will be calculated to correspond to the new policies, and the nonlinear model will be linearized around the new tentative paths

for y_t^0 and x_t^0, yielding a new set of linear reduced-form equations (4). Another set of optimal feedback control equations are obtained, and the computations are repeated until the process converges. Our experience with several U.S. models, including the Klein-Goldberger model, the St. Louis model and the University of Michigan Quarterly Econometric Model, is that it takes about 3 rounds of linearizations to converge. The Michigan model contains 61 endogenous variables from the original simultaneous equations plus 71 more new endogenous variables to convert the system into first-order, plus 3 variables $y_{133,t} = x_{1,t}, \ldots, y_{135,t} = x_{3,t}$ which are equal to the 3 control variables selected for our experiments, giving a vector of 135 elements for y_t in (1). To compute the optimal solution in one round of linearizations using the Michigan model for a 17-period control problem with 3 control variables, it costs about \$20.00 at the Princeton University Computer Center equipped with an IBM 360-91 Computer. The cost is expected to be about 4 times (somewhat less than 2^2) if the size of the model doubles. If the number of planning periods changes, the cost will change linearly because the program takes advantage of the time structure of the problem and computes the feedback coefficients G_t and g_t period by period. Control algorithms which treat a minimization problem with respect to the total number of variables (equal to the number of control variables times the number of periods) without regard to the time structure of the optimization problem will become much more than twice as expensive when the number of variables doubles. Our program has the additional property that its cost will hardly increase at all when the number of control variables increases.

5. Change systematically the weighting matrix K_t and the targets a_t in the loss function (2) and reoptimize in order to trace out the best combinations of the future inflation rates and unemployment rates attainalbe given the econometric model. The procedure is described in detail in Chow and Megdal (1976). Essentially, if the weight $k_{11,t}$ corresponding to the unemployment rate y_{1t} is very large as compared with the weight $k_{22,t}$ for the inflation rate y_{2t}, and if the target a_{1t}

for y_{1t} is set at 5 per cent and the target for the inflation rate is set low enough, the solution will give the lowest inflation rate attainable for a 5 per cent unemployment. By varying a_{1t} from 4 to 8 per cent, one can compute the optimum solutions to find out the best inflation rates corresponding to these various unemployment rates and the associated policies required to achieve them. Since we are dealing with T periods, it may be useful to plot the mean unemployment and inflation rates over these periods, or to plot the root mean squared deviations of these rates form their targets.

6. Present the results of step 5 to the policy makers who will then make a choice among the best feasible combinations of unemployment and inflation. It is quite likely that the unemployment and inflation rates from the tentative solution in step 1 are dominated by the solutions obtained in step 5. If the solution for the unemployment rate is around 6 per cent in step 1, say, the solution in step 5 using a_{1t}=6 per cent guarantees that the resulting inflation rates are the lowest possible as a consequence of optimization. The choice made here and the corresponding optimal policy will constitute set of intermediate solution paths for y_t and x_t for further analysis and improvement.

7. If the solution paths for the policy variables in step 6 drift very far away from the paths in step 1 or show severe fluctuations, impose penalties in the loss function for them and reoptimize. The weights in the K_t matrix may be assigned to the levels of the policy variables which are given certain reasonable target paths. Or the quarter-to-quarter changes in some policy variables can be dampened by introducing the first differences as new variables which are then given appropriate weights in the K_t matrix and steered toward the target zero. Perhaps trials and errors are required in this step to obtain reasonable solution paths for the control variables.

8. Examine the reasonableness of the new solutions for y_t and x_t in step 7 using any outside information available. Adjust the econometric model and re-optimize if necessary. The need to adjust the econometric model and/or the estimates of the uncontrollable exogenous variables may arise at this stage because the new solutions in step 7 may be quite far from the solutions in step 1, affecting the accuracy of the econometric model as an approximation of reality and even conceivably affecting the values of some variables which have been treated as exogenous but may indeed react to sizable changes in policies. Reoptimize after the model is adjusted.

9. If a second reasonable econometric model is available, it would be useful to apply the policy paths in step 8 to it and compare its projections of unemployment, inflation and real GNP with those obtained from the first model in step 8. If the two sets of projections are similar, or if the second set is as satisfactory as the first set from the original model (so that there is no risk of very bad performance if the alternative model is true), conclude the search for optimal policies and to to step 11. Otherwise, go to step 10.

10. Examine the consequences of at least three policies, (a) the optimal policy based on the first model as obtained in step 9, (b) the optimal policy based on the second model using the same loss function to be similarly computed, and (c) the originally proposed policy used in step 1, under the alternative assumptions that one of the two models is correct. Here a 3 by 2 payoff matrix can be utilized, with 3 policies combined with two possible states of the world or models. By applying the three policies to the two models, we can compute the total expected losses for 8 quarters, say, to be entered in the above payoff matrix. If policy (a) or (b) dominates policy (c), as shown by the first or second row of the matrix having smaller losses than the third row, we have found an improvement over the policy originally proposed. If neither policy (a) nor (b) dominates (c),

the payoff matrix will still serve as a useful tool of analysis. If one takes the Bayesian approach, he assigns probabilities to the two models and chooses that policy which minimizes the expected loss obtained by weighing the losses from the policy by the probabilities. If one is conservative, he may choose the minimax stratety. An illustrative analysis using such a payoff matrix can be found in Chow (1976c).

What if the two models disagree, as shown by large expected losses in the 1-2 and 2-1 entries in the above matrix, and one is unwilling to take the Bayesian approach to resolve the conflict? A further analysis can be performed. It is based on the idea that policies are made sequentially period by period, and that the policy maker does not have to follow the policy recommendations computed from one model for many future periods after he decides to follow it for a quarter or two. The analysis described in the last paragraph ignores the possibility of shifting and revising models as it examines the expected total loss for many periods when the policy recommendations from one model are followed throughout. The dis-agreements between the policy recommendations from two different models would be reduced and the difficulties in choosing between conflicting policies would dimish if this possibility is taken into account. The first-period policies from the two models may not differ by very much even if following the recommendations from the two models for many periods would lead to very different consequences. Furthermore, assuming that the first-period policies based on the two models differ greatly, and that their multi-period expected losses also differ, the policy maker would still not face a serious dilemma if he knows that following the policies from model 1 for one or two quarters and shifting to the policies from model 2 afterwards will be nearly as good as following the policies from model 2 for all periods when model 2 happens to be the true model.

In essence the i-j entry of payoff matrix in this analysis should show the total expected loss for many periods if the policy recommendation from model i is followed only for period 1 but the policies from model j will be followed afterwards. This construction is based on the notion that the decision for the first quarter, even if it is mistaken, can be corrected in the following quarters. Therefore, the damage done in this quarter is measured by the difference between the multiperiod losses incurred when (1) following the wrong policy of model i for one quarter but the correct policies of the right model j afterwards and (2) following the policies of the correct model j all through, the latter being given by the j-j entry of the payoff matrix. Such a matrix is quite easy to compute if the optimal stochastic control algorithm described in step 4 is used. This algorithm is derived from the method of dynamic programming (Chow, 1975, Chapter 8) by which one reduces successively the problem of minimizing the expected loss for T periods to the problem of minimizing the expected loss for one period, starting with the problem for period T, and then the problem for the last 2 periods, etc., until the problem for all T periods is solved. The final problem amounts to minimizing the expectation of a quadratic function $y_1'Hy_1 - 2y_1'h + c$ of only the variables y_1 in period one with respect to the first-period policy x_1, it being understood that, whatever the outcome y_1 for period one turns out to be, the future policies $x_2, \ldots, x_T$ shall be optimally chosen. See Chow (1975, pp. 178-179). Using the right-hand side of equation (4) to substitute for y_1 in the above quadratic loss function and taking its expectation, we find the total expected loss for T periods to be a quadratic function of x_1, say $x_1'Q_jx_1 - 2x_1'q_j + d_j$, where the subscript j indicates that the optimal control calculations are performed using model j. This function gives the expected T-period loss if x_1 is applied in the first period and x_2 to x_T shall be optimally chosen according to model j, under the assumption that model j is true. If we minimize this function with respect

to x_1, we obtain the optimal first-period policy according to model j. If we apply the three different first-period policies x_1 used in the construction of the 3 by 2 payoff matrix to evaluate this function, we will obtain the entries for the j-th column of the required payoff matrix.

The purpose of step 10 is to arrive at a final policy recommendation for the current quarter. Even the payoff matrix constructed in the last paragraph may show seriously conflicting first-period redommendations from the two alternative models, but a decision has to be reached by the Bayesian, Minimax or some other criterion. It is better to know the various risks involved under the alternative states of the world when making a decision than not to know them at all. When faced with conflicting recommendations, one may attempt to find a robust policy which would work reasonably well under the alternative models. This is a subject requiring further research. One approach is to modify the optimal policies by allowing for the uncertainty in the estimated parameters of the econometric models used, as described in Chow (1976a).

11. Calculate the mean paths and the covariance matrix of the major economic variables using a reasonable model and the optimal feedback control policy chosen above. The decision makers should be informed of the likely consequences in the future when the recommended policy is applied. Using equations (3) and (4) obtained in step 4 above, we obtain a linear approximation of the dynamic stochastic system under control

$$(5) \qquad y_t = (A_t + C_t G_t) y_{t-1} + (b_t + C_t g_t) + u_t$$

$$= R_t y_{t-1} + r_t + u_t$$

The mean path of this system is given by

$$(6) \qquad \bar{y}_t = R_t y_{t-1} + r_t$$

Using $y_t^* = y_t - \overline{y}_t = R_t\, y_{t-1}^* + u_t$, we can compute the covariance matrix by

$$Ey_t^*\, y_t^{*'} = R_t\,(Ey_{t-1}^*\, y_{t-1}^{*'})\, R_t' + Eu_t u_t'$$

where the covariance matrix $Eu_t u_t'$ of the reduced form residuals are calculated from the estimated covariance matrix of the residual ε_t in the structural equations (1).

12. If the above steps are followed each quarter, the econometric models used will be revised and improved, and more weights will eventually be given to the recommendations from the models that have shown a better tracking record.

Why should the procedure outline above be adopted? It will make explicit the underlying rationale in the making of macroeconomic policies. If such an approach is not used, one would wonder on what basis government macroeconomic decisions are reached, what dynamic relationships among the important economic variables are assumed in policy making, and what objectives the government is trying to achieve. Once these questions are answered explicitly and quantitatively, the logical approach is to write down the dynamic economic equations and the objective function, and to find the policies that would best achieve the objectives. This is precisely our recommendation. We have simply filled in the details in implementing such an approach by bringing the available econometric knowledge to bear and by designing a computationally efficient procedure to find optimal policies which are to be made sequentially and to ascertain the economic consequences of such policies.

The reader will have recognized that, although we suggest the use of stochastic control techniques for policy analysis, we are far from advocating the automatic use of these techniques without the intervention of human judgment

and political considerations. Needless to say, poor policy recommendations are likely to follow from poor econometric models, no matter whether optimization techniques are used or not. In reality, technical economic advice may play only a limited role in the formulation of economic policies. Whatever its limited role, the current practice has already incorporated the use of econometric models to simulate the likely outcomes of alternative policy proposals. We merely suggest a computationally more efficient way to obtain good policy proposals and to deduce the likely consequences of the proposed policies as indicated in step 11 above. Furthermore, by subjecting the econometric models to more serious scrutiny through the optimal control solution in step 8 and to continuous reexamination in step 12, it is hoped that the quality of econometric models will be improved in the process.

Footnotes

[1] I would like to thank Burton G. Malkiel for very helpful comments and the National Science Foundation for financial support.

[2] The determination of which variables are the policy variables subject to the control of government authorities is often a difficult problem in practice. We will not discuss this issue because any policy analysis using an econometric model has already faced this issue and the purpose of our paper is to introduce stochastic control techniques to implement such policy analyses.

[3] In using a planning horizon as long as 20 quarters, we are not assuming that the econometric model will be very accurate in making projections that far ahead, but we have to anticipate and incorporate the delayed effects of current policy in order to avoid recommending policies which will yield desirable results in the near future but undesirable consequences later on.

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