THE CONTROL OF LARGE SCALE NONLINEAR ECONOMETRIC SYSTEMS

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I. Nature of Econometric Models

This paper reports on the techniques of feedback control which are applicable to large-scale, nonlinear stochastic models of national economies. At the outset, several important characteristics of the existing econometric models should be mentioned.

First, they are systems of simultaneous stochastic difference equations. The i-th equation in a structure of simultaneous equations can be written as

$$y_{it} = \Phi_i \, (y_t, y_{t-1}, x_t, w_t) + \varepsilon_{it} \tag{1.1}$$

where y_{it} is the i-th element of the vector y_t of endogenous variables at time t which are explained by the model, x_t is a vector of control variables, w_t is a vector of exogenous variables not subject to control and ε_{it} is a random residual with zero mean, independently distributed through time but correlated with ε_{jt} of equation j. Denoting the vector of functions by Φ, we write the system of structural equations as

$$y_t = \Phi \, (y_t, y_{t-1}, x_t, w_t) + \varepsilon_t \tag{1.2}$$

Because the system is simultaneous, with y_t appearing on the right-hand side of (1.1), it has to be solved to obtain y_t, given y_{t-1}, x_t, w_t and ε_t. This feature is absent from most of the models used in engineering.

Second, the functions Φ_i are often nonlinear, but not highly nonlinear.

An example from the University of Michigan Quarterly Econometric Model [7] is the following equation for y_{16}, consumption expenditures on nondurable goods in billions of 1958 dollars (the numbering of the variables ours):

$$y_{16} = 94.8 + 7.59\ (x_1/y_{47}) + .140\ y_{50} - 74.2\ (y_5/y_{47}) \\ -1.22\ [\sum_{i=1}^{3} y_{30,-i}/3] + .558\ y_{16,-1} \tag{1.3}$$

The second subscript of each variable is omitted if it equals t, and is written -i for t-i. This equation includes 4 current endogenous variables, y_{16}, y_{47} (price index for personal consumption expenditures), y_{50} (disposable personal income in billions of 1958 dollars) and y_5 (price index for consumption expenditures on nondurable goods). The appearance of several current endogenous variables in one equation makes the model simultaneous. y_{30} is the interest rate on Aaa corporate bonds, entering with time lags from 1 to 3 quarters. x_1 is government transfer payments to persons in billions of current dollars, being treated as a control variable.

Third, the models are frequently large. The version of the Michigan Quarterly Econometric Model adopted for our control experiments consists of 61 endogenous variables and an equal number of simultaneous equations. Since there are accounting identities relating the endogenous variables, one could eliminate some unimportant endogenous variables using these identities and reduce the number of equations. On the other hand, to write the model as a first-order system for the control calculations, 71 additional variables (taking the form $y_{62} = y_{7,-1}$ for example) are added to the vector y_t to eliminate variables with time lags of two or more quarters. This makes a total of 132 elements in y_t, but a system of only 61 nonlinear simultaneous equations have to be solved to obtain y_t, given y_{t-1}, x_t and w_t. The Mark III version of the Wharton Quarterly Econometric Model (in use until 1972) consists of 201 simultaneous equations. The MIT-Penn-SSRC Model consists of 177 simultaneous equations. Although the number of simultaneous equations is large, the number of current endogenous variables appearing in each equation is small. For example, only 4 out of 61 variables actually appear in equation (1.3) for the Michigan Model. If the equations were linear and were to be solved by matrix inversion, the matrix involved is a sparse matrix. This feature can be exploited to economize computations.

Fourth, not only the unknown parameters in the structural equations (1.2) but the specification of these equations are subject to a high degree of uncertainty. The former uncertainty is inherent in the statistical estimation of a large number of parameters using a limited number of time-series observations. The latter uncertainty is due mainly to the insufficiency of economic theory in specifying the time pattern of responses and the suitable degrees of aggregation for different economic variables. Given these two types of uncertainty, econometric model builders have paid less attention to the problem of measurement errors which are explicitly dealt with by the control engineers. Uncertainty concerning the econometric models has important implications for the application of stochastic control techniques to economic policy formulation, partly to be discussed in section VI.

II. Use of Econometric Models for Policy Projections and Optimization

Ever since large econometric models were constructed in the 1950's, they have been used to make economic forecasts and to deduce the economic consequences of alternative paths for the policy variables x_t. To make a one-period projection of y_t, given y_{t-1}, x_t and w_t, the system of equations (1.2) is usually solved by the Gauss-Seidel method, with ε_t set equal to its expectation zero. This iterative method lists the equations in a certain order, applies the current set of trial values for the endogenous variables y_t to the right-hand-side of the i-th equation (1.1) to obtain a new trial value for y_{it}, and continues to use the next equation to revise the next endogenous variable until convergence. A damping factor is introduced if successive trial values of y_{it} oscillate. Experience has shown that this simple method works for nearly all econometric models. To make a projection of y_t for several periods, the system (1.2) is solved several times and the solution for each period is used as y_{t-1} in the calculation of y_t for the following period.

Although in the 1950's economists began to set up a quadratic loss function for the purpose of policy optimization and discovered that the certainty equivalence solution for the current period policy x_t is optimal for the multiperiod stochastic control problem of minimizing the expectation of a quadratic loss function given a linear model with additive random disturbances [10], [11], multiperiod policy optimization using a large-scale, nonlinear, simultaneous econometric model has occurred only in the 1970's. A popular approach is to convert a T-period stochastic control problem into a deterministic control problem by setting the random disturbances equal to their expectations (partly appealing to the possibly near optimality of the certainty-equivalence solution for nonlinear models). One then solves the deterministic control problem as an unconstrained minimization problem with respect to the policy path $x_1, x_2, \ldots, x_T$ since the multiperiod loss function can be regarded as a function of $x_1, \ldots, x_T$, given the nonstochastic econometric model. The number of unknowns equals the number q of control variables times the number T of periods. Various standard minimization algorithms have been applied [1], [2], [6], [8], [9]. An Econometric Model Comparison Seminar composed of the proprietors of the major U.S. econometric models, chaired by Lawrence Klein and sponsored by the National Bureau of Economic Research with a grant from the National Science Foundation, is currently comparing the deterministic control solutions from the different models (with the disturbances set at their historical values) for the 17 quarters beginning with 1971.1, using the same essentially quadratic loss function with the inflation rate, the unemployment rate, the GNP gap, and the balance of international payments as arguments. This exercise should reveal the differences among the econometric models in terms of their policy recommendations during a crucial historical period.

If the deterministic control solution is used to make recommendations to policy makers, one may appeal to the near optimality of the first-period solution x_1 and apply it to the first period. The policy in period two will be obtained as the first-period solution to a (T-1)-period deterministic control problem formulated at the end of period one after y_1 is observed, and so forth. Such a procedure may work well, but the dynamic characteristics of the system under control are extremely difficult and costly to ascertain because extensive stochastic simulations would be required. If the policy maker wishes to know not only the expected paths of y_t and x_t in the future when the economy is under control, but the covariance matrices of these time series and the expected total loss for T periods, the solution to the original stochastic control problem in the form of feedback control equations will be desirable. This paper describes and recommends a solution to stochastic control in feedback form using a large, nonlinear econometric model, and assuming the loss function to be quadratic.

III. A Feedback Control Algorithm

We will describe an approximate solution to the optimal stochastic control problem in feedback form, i.e., $x_t = G_t y_{t-1} + g_t$ where y_t incorporates x_t as a subvector to eliminate x_t from the quadratic loss function

$$W = \sum_{t=1}^{T} (y_t - a_t)' K_t (y_t - a_t) = \sum_{t=1}^{T} (y_t' K_t y_t - 2y_t' K_t a_t + a_t' K_t a_t) \tag{3.1}$$

The solution consists of the following steps.

(1) Starting with a tentative policy path $x_1^o, x_2^o, \ldots, x_T^o$, and given $w_1, w_2, \ldots, w_T$, we apply the Gauss-Seidel method to the model (1.2) with $\varepsilon_t = 0$ for T periods to obtain a solution path $y_1^o, y_2^o, \ldots, y_T^o$ of the endogenous variables. Thus for each period t, the following system of equation holds

$$y_t^o = \Phi (y_t^o, y_{t-1}^o, x_t^o, w_t) \qquad (t=1,\ldots,T) \tag{3.2}$$

(2) Equations (1.2) are linearized about the point y_t^o, y_{t-1}^o, and x_t^o to yield

$$y_t = y_t^o + B_{1t} (y_t - y_t^o) + B_{2t} (y_{t-1} - y_{t-1}^o) + B_{3t} (x_t - x_t^o) + \varepsilon_t \tag{3.3}$$

The i-j elements of B_{1t}, B_{2t} and B_{3t} are respectively the derivatives of Φ_i in (1.1) with respect to the j-th elements of y_t, y_{t-1} and x_t. They are easily computed numerically by changing the j-th element of y_t, y_{t-1} or x_t by a small amount and evaluating the resulting changes in Φ_i. For the illustrative equation (1.3) from the Michigan Model, the 16th row of B_{1t} has only 3 non-zero elements, in the 5th, 47th and 50th columns.

(3) The linear simultaneous equations (3.3) are solved to obtain the linearized state-space model (or the <u>reduced-form equations</u> in the economist's terminology)

$$y_t = A_t y_{t-1} + C_t x_t + b_t + u_t \tag{3.4}$$

where

$$(A_t;C_t;u_t) = (I - B_{1t})^{-1}(B_{2t};B_{3t};\varepsilon_t) \tag{3.5}$$

$$b_t = y_t^o - A_t y_{t-1}^o - C_t x_t^o$$

(4) Using the linear model (3.4) with additive random disturbances u_t and the quadratic loss function (3.1), we compute the optimal linear feedback control equations

$$\hat{x}_t = G_t y_{t-1} + g_t \tag{3.6}$$

by well known methods [4, pp. 178-179].

(5) A new tentative policy path $x_1^o,\ldots,x_T^o$ and solution path $y_1^o,\ldots,y_T^o$ are obtained by solving (3.6) and (1.2) with ε_t=0 consecutively for x_t and y_t (t=1,...,T).

(6) We go back to step (2) to linearize the model about the new solution path and then compute the optimal feedback solution for the linear-quadratic problem until the process converges. At the point of convergence, the solution vectors y_t and x_t satisfy both the nonlinear system (1.2) with ε_t=0 and the linearized model (3.3) with ε_t=0 and thus also the linearized reduced form (3.4) with u_t=0.

(7) The stochastic system under control is approximately described by (3.4) and (3.6), i.e.,

$$y_t = (A_t + C_t G_t)\, y_{t-1} + (b_t + C_t g_t) + u_t$$
$$= R_t y_{t-1} + r_t + u_t \qquad (3.7)$$

The mean path of this sytem is given by $\bar{y}_t = R_t \bar{y}_{t-1} + r_t$, or equivalently by the solution vector y_t^0 in the last iteration which also satisfies (3.6) and (3.4) with $u_t = 0$. The covariance matrix of the system is computed by, for $y_t^* = y_t - \bar{y}_t$,

$$E y_t^* y_t^{*\prime} = R_t (E y_{t-1}^* y_{t-1}^{*\prime})\, R_t' + E u_t u_t' \qquad (3.8)$$

where $E u_t u_t' = (I - B_{1t})^{-1} (E \varepsilon_t \varepsilon_t') (I - B_{1t})^{-1'}$ on account of (3.5). The covariance matrix of the random residuals in (1.2) is assumed to have been estimated together with the other unknown parameters in the econometric model. The expected total loss in T periods when the system is under feedback control can be calculated by a well-known formula [4 , p. 179].

IV. Further Details Concerning Computations

The first step in preparing the model for control using the computer program available at Princeton University [3] is to write in Fortran code the structural equations (1.1). To eliminate endogenous variables lagged more than one period, such as $y_{30,-2}$ in the consumption expenditures equating (1.3), we would introduce nid identities of the form Y(89) = YL(30) and Y(90) = YL(89), where YL stands for y lagged one period. These identities enable us to write $y_{30,-2}$ as $y_{89,-1}$ and $y_{30,-3}$ as $y_{90,-1}$. Furthermore, the program automatically makes up an identity of the form $y_{133,t} = x_{1,t}$ for each of the nx control variables, permitting the user to write the welfare loss (3.1) as a function of y_t alone. These nid + nx identities are combined with the original ns simultaneous structural equations, making a total of p = ns + nid + nx equations. In addition to the Fortran coding of the model the user of the algorithm must provide the variance - covariance matrix of the residuals, the weighting matrix K, the values of the vector y_o and, for each t in the control horizon (t = 1, ..., T), the target values a_t, the values for the exogenous variables w_t and the trial values for the nx control variables x_t.

As described in Section III, given the initial vector y_o and the trial solutions for x_t, the Gauss-Seidel method is used to obtain a solution path $y_1^o, y_2^o, \ldots, y_T^o$ about which the model will be linearized. Once this solution path is obtained, the elements of the matrices B_{1t}, B_{2t} and B_{3t} of equation (3.3) will be computed. The i,j element of B_{1t} is obtained by computing

$$\frac{\partial \Phi_{it}}{\partial y_{jt}} = \frac{y_{it}^{(1)} - y_{it}^{(2)}}{2\delta_{jt}} \tag{4.1}$$

where $y_{it}^{(1)}$ equals Φ_i evaluated at $y_{jt} + \delta_{jt}$, $y_{it}^{(2)}$ equals Φ_i evaluated at $y_{jt} - \delta_{jt}$ and $\delta_{jt} = \max(|dy.y_{jt}|, dmin)$. dy and dmin are set by the user. The default value for both is equal to .001. Similarly, we evaluate the i-k element of B_{2t} by perturbing $y_{k,t-1}$ and the i-j element of B_{3t} by perturbing $x_{j,t}$.

Due to the structure of the model there will be many zero elements in B_{1t}, B_{2t} and B_{3t}. In particular, B_{1t} will be a block diagonal matrix

$$B_{it} = \begin{bmatrix} B_{1t}^{*} & 0 \\ 0 & 0 \end{bmatrix} \tag{4.2}$$

where B_{it}^{*} is ns by ns whereas B_{1t} is a p by p matrix. Only the ns simultaneous equations, and not the identities introduced, have current values of the endogenous variables on the right hand side. Frequently there will be columns of zeroes within B_{1t}^*. This information will be utilized by the program and no derivatives will be computed for these columns. The matrix B_{2t} can also be partitioned:

$$B_{2t} = \begin{bmatrix} B_{2t}^* \\ D \\ 0 \end{bmatrix} \qquad (4.3)$$

where B_{2t}^* is ns by p, D is nid by p and 0 is nx by p. The kth column of B_{2t} will be a zero vector if $y_{k,t-1}$ does not appear in any of the ns structural equations. Again, this information will be utilized by the program. Each row d_i of the matrix D will be a vector with zero elements except for one unit element:

$$d_{ij} = \begin{cases} 1 & \text{if} \quad y_{ns+i} = y_{j,-1} \\ 0 & \text{otherwise.} \end{cases}$$

The user provides a vector which indicates the position of that unit element, and the matrix D will be automatically formed. B_{3t} can be represented in partitioned form as

$$\begin{bmatrix} B_{3t}^* \\ 0 \\ I \end{bmatrix} \qquad (4.4)$$

where B_{3t}^* is ns by nx, 0 is nid by nx and I is nx by nx. Only the elements of B_{3t}^* are computed. B_{1t}, B_{2t} and B_{3t} are used to obtain the reduced form coefficient matrices A_t, C_t and b_t in (3.4) and (3.5). Because of the special form of B_{1t} as given by (4.2), the inversion of $I - B_{1t}$ requires only the inversion of its upper left hand ns by ns submatrix $I - B_{1t}^*$. A sparse matrix inversion routine can be used for this purpose since, as pointed out in section I, B_{1t}^* is in general a sparse matrix.

Given the linearized reduced form equations (3.4), core space and computer time are saved by treating a smaller dynamic system formed by excluding those endogenous whose lagged values are absent from the original system and whose behavior is of no interest. If there are m such variables, the reduced form can be written as

$$\begin{bmatrix} y_t^a \\ y_t^b \end{bmatrix} = \begin{bmatrix} A_t^a & 0 \\ A_t^b & 0 \end{bmatrix} \begin{bmatrix} y_{t-1}^a \\ y_{t-1}^b \end{bmatrix} + \begin{bmatrix} C_t^a \\ C_t^b \end{bmatrix} x_t + \begin{bmatrix} b_t^a \\ b_t^b \end{bmatrix} \qquad (4.5)$$

where y_t^a is (p-m) by 1 and y_t^b is m by 1. To compute the feedback control equations (3.6) only the submodel

$$y_t^a = A_t^a y_{t-1}^a + C_t^a x_t + b_t^a \qquad (4.6)$$

is used.

The iterations as described by steps (1) through (6) in section III will terminate if, for each variable i included in the loss function the proportional change of y_{it} in each time period t (t=1, ..., T) between two successive iterations is smaller, in absolute value, than some preassigned number. The default value of this number is .001. There are two ways to speed up convergence. First, if the values of y_{it} oscillate in successive iterations, a damping factor between 0 and 1 can be introduced to dampen the change of each $\hat{x}_{it}$ from one iteration to the next. Second, for some control problems, the parameters in the loss function can be changed to facilitate convergence. For example, one may be interested in obtaining the lowest inflation rates y_{2t} (t=1, ..., T) that correspond to an unemployment rate y_{1t} equal to 4%. In specifying the matrix K in the loss function, a very large weight will be assigned to the unemployment variable whose target is set at 4% and a much smaller weight will be assigned to the inflation variable whose target is set at some unachievably low rate such as 2% per year (which could be further lowered if it turned out to be achievable). A weight combination of 10,000 and 1 may make the program converge more rapidly than a combination of 100 and 1.

To complete the optimal feedback control calculations for the Michigan Quarterly Econometric Model for 17 quarters using 3 control variables, the program takes 126 seconds of CPU time in the IBM 360-91 computer at Princeton University, for each iteration or linearization of the model costing $21.00 at the delay priority rate. It takes three iterations to converge. Increasing the number of control variables to 6 would have almost no effect on the computing time because the matrix $(C_t'H_tC_t)^{-1}$ required to compute the matrix G_t in the feedback control equating (3.6) would merely become 6 by 6 instead of 3 by 3. Increasing the number of time periods T would raise the computer time approximately linearly because similar calculations are performed for each period. Doubling the size of the model, as measured by the number ns of simultaneous equations would increase the computing cost by a factor of about 4, or ~~slightly less than~~ 2^2.

V. Application to the Michigan Quarterly Econometric Model

Analysis of a particular control problem will serve as an illustration of the application of the techniques described above as well as identify the role optimal control of econometric systems can play in the evaluation of economic policy. The loss function, except for the last term involving UR\$, is one used by the NBER - NSF Econometric Model Comparison Seminar mentioned in Section II. For the 17 quarters, 1971.1 through 1975.1 the objective is to minimize

$$\sum_{t=1}^{17} [.75\ (u_t - 4.0)^2 + (\dot{p}_t - \alpha_t)^2 + (TB_t - 0)^2 + .75\ (\text{GNP Gap}_t - 0)^2 + .1\ (UR\$_t - \gamma_t)^2] \tag{5.1}$$

where u = the unemployment rate

$\dot{p}$ = the annual rate of inflation measured by the GNP price deflator

α_t = 3.0 for t=1, ..., 12
7.0 for t=13, ..., 17

TB= the trade balance as a percentage of GNP in current dollars

GNP Gap = the percentage deviation of GNP in 1958 dollars (GNP58) from capacity output

UR\$ = unborrowed reserves in billions of current dollars

and γ represents a smooth expansionary path for UR\$. The instruments available to the policy maker are nondefense government purchases of goods and services in billions of current dollars (GFO\$) and UR\$, representing government fiscal and monetary policy, respectively. The UR\$ term in the loss function serves to prevent erratic behavior of the monetary instrument; however, the deviation of $UR\$_t$ from γ_t will not count as a part of the loss.

The purpose of this control problem is to determine if some politically feasible combination of fiscal and monetary policies could have improved the performance of the economy during the period under consideration by comparing the optimal and historical paths for the target variables and instruments. Examination of the results leads to an affirmative answer. Tables 1 and 2 present the historical and optimal paths, respectively, for certain key variables. Excluding the UR$ term, the historical value of the loss is 703.4 whereas the minimum value of 232.7. The optimal path for GFO$ is considerably more expansionary than the historical path, though it does fluctuate. The optimal path for UR$ is only mildly expansionary for the first eight quarters but becomes more expansionary in the later quarters. Monetary policy affects the values of the listed variables with a lag; thus, much of the behavior of the variables, especially during the first half of the control horizon, is attributable to the impact of GFO$. For each of the 17 quarters, real output, GNP58, is greater in the optimal control solution. This leads to lower values for the GNP gap throughout and a lower rate of unemployment in all quarters but one, reducing the loss contribution of the gap term by 421.7 and the unemployment term by 37.8. In the early quarters, however, the lower unemployment rate is achieved at the expense of a higher annual rate of inflation. For nine of the first 10 quarters of the control horizon, the historical rate of inflation is lower than that obtained under control, though for six of the remaining seven quarters the historical figure is higher. The overall contribution of the inflation term to the loss is reduced by implementation of the control solution. In the control solution the trade balance term deviates more from its target than it did historically but in neither case is the contribution to the overall loss significant.

In the above calculation, the estimated residuals in the structural equations were included so that the historical values of the endogenous variables would result if the actual values of the instruments were applied. The economist can use this type of analysis in the evaluation of past policies. More importantly, the policy maker can use the stochastic control techniques as described in sections III and IV to formulate and evaluate current and future policies. A number of objective functions could be minimized and the results compared to determine a policy mix which is politically feasible and would

TABLE 1

HISTORICAL PATHS OF SELECTED MACROECONOMIC VARIABLES

	$\dot{p}$	U	GNP Gap	TB	GNP58	UR$	GFO$
1971.1	4.69	5.95	5.01	.278	737	29.5	24.1
71.2	4.85	5.98	5.27	-.006	742	30.1	25.5
71.3	2.58	5.96	5.55	.009	747	30.6	27.9
71.4	1.90	5.94	4.99	-.312	759	31.2	28.5
72.1	5.49	5.84	4.45	-.631	771	31.9	29.7
72.2	1.92	5.69	3.44	-.602	787	32.9	30.0
72.3	3.32	5.57	3.01	-.406	798	32.9	30.1
72.4	4.05	5.29	2.01	-.440	814	30.4	30.5
73.1	5.49	5.04	.75	-.006	833	30.1	31.4
73.2	7.29	4.91	1.17	.033	837	30.6	32.2
73.3	8.26	4.74	1.74	.515	841	32.3	32.0
73.4	8.64	4.72	2.12	.696	846	33.8	33.1
74.1	12.31	5.20	4.82	.832	831	33.7	35.7
74.2	9.36	5.15	6.14	-.101	827	33.7	37.7
74.3	11.88	5.49	7.51	-.225	823	34.0	38.8
74.4	14.44	6.60	10.54	.123	804	36.2	40.6
75.1	7.96	8.35	14.06	.375	780	34.7	42.5
Contribution to Loss	214.2	44.5	441.7	3.0	Total: 703.4		

TABLE 2

OPTIMAL PATHS BASED ON THE MICHIGAN QUARTERLY MODEL

	$\dot{p}$	u	GNP Gap	TB	GNP58	UR$	GFO$
1971.1	6.07	5.08	.48	-.182	772	26.4	78.6
71.2	3.69	4.61	1.34	-.491	773	26.8	60.6
71.3	2.81	4.56	1.76	-.423	777	26.4	61.9
71.4	3.27	4.23	-.57	-.902	804	26.8	90.9
72.1	5.43	4.51	1.82	-.977	792	27.2	55.1
72.2	3.84	4.77	1.16	-.847	805	28.2	66.2
72.3	4.21	4.94	1.31	-.619	812	29.8	68.2
72.4	4.85	4.96	1.18	-.548	821	30.7	64.6
73.1	5.93	4.99	.45	-.108	835	32.4	60.9
73.2	7.61	4.96	.68	-.041	842	34.5	61.0
73.3	8.11	4.69	.63	.348	850	36.2	58.6
73.4	8.11	4.54	.71	.458	858	38.9	50.2
74.1	11.75	4.88	2.98	.485	847	40.9	46.9
74.2	9.63	4.08	.85	-.949	874	41.8	94.8
74.3	11.78	3.31	-.99	-1.660	899	41.8	131.3
74.4	12.62	3.70	1.11	-1.596	889	40.9	118.1
75.1	9.11	4.41	1.18	-1.931	897	40.3	172.2
Contribution to Loss	191.9	6.7	20.0	14.1		Total: 232.7	

lead to desired values for the endogenous variables. It should be stressed that analyses of both types, for the evaluation of historical policies and the formulation of current policies are based on the assumption that the model used is a reasonably good approximation of reality. In the next section, we will comment briefly on the methods to deal with uncertainty in the econometric models to be used in optimal control calculations.

VI. Further Research

There are three areas of research closely related to the techniques of feedback control as expounded in this paper. The first is to improve the computational efficiency and capability of the algorithm to deal with larger nonlinear econometric models. The second is to modify the control solution to account for the uncertainty in the statistical estimates of the model parameters. Although a solution to this problem has already been obtained [5], we have not completed a computer code for it. One would like to study the effect of parameter uncertainty on the (nearly) optimal policy and the associated expected welfare loss. Third, the problem of misspecification of econometric models has to be attacked by the effective use of two or more models.

The basic framework to deal with two or more models is a payoff matrix whose elements are the expected losses resulting from the different proposed policies when the alternative states of the world, or models, are true. The policies considered may include the (nearly) optimal policies based on the different models, with or without incorporating uncertainty in their parameters, and some passive policies of feedback control. The expected loss in each element of the payoff matrix need not be the expected multiperiod loss when the policy recommendations from one model are followed throughout all future periods. Because the decision maker can change the model employed for policy recommendations after one period, the entry in the payoff matrix corresponding to a given true state or model and a given strategy should be the expected loss resulting from applying the given strategy for only the first period but the strategies based on the true model for the remaining periods of the planning horizon. This would show the damage of following the incorrect model in the first period only, but not necessarily in the future periods. Such a payoff matrix can form the basis for deriving a Bayesian strategy, a minimax strategy and some robust strategies in the formulation of macroeconomic policies in the face of incomplete economic knowledge.

Footnote

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