RESIDENTIAL DEMAND FOR NATURAL GAS

by

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Much of the economic analysis of the natural gas market has focused on the supply side, particularly the effect on supply of deregulating the price of gas sold across state lines. The Federal Energy Administration estimated that 1985 domestic gas production would be 22.3 trillion cubic feet (TCF) with total deregulation, but only 15.8 TCF with a price ceiling on gas to be transported through interstate pipelines of \$1 per thousand cubic feet (MCF) at the wellhead. The difference between these two annual levels corresponds to the daily energy content of 3 million barrels of oil, roughly one-sixth of the oil consumed and half the oil imported in the United States in 1975.

In this context, long-run supply elasticities for natural gas have been estimated to be between .42 and .47 over the range of \$1/MCF to \$2/MCF depending on assumptions about oil prices.

This paper is a study of the demand side of the natural gas market, more specifically, of residential demand for natural gas used for space heating. The analysis pertains to a set of Twin Rivers, New Jersey families, part of a larger group being examined by the interdisciplinary Center for Environmental Studies at Princeton University.

Time series demand functions are estimated for the combined cross-

section of 151 Twin Rivers families and for each of the families individually. The considerable variation in estimated price elasticities across families is especially interesting, given the socio-economic homogeneity of these Twin Rivers residents. The estimated demand functions also permit testing of the hypothesis that demand for natural gas is homogeneous of degree zero in prices.

Section I of this paper is a discussion of the theoretical framework and the data used in the subsequent empirical analysis. The estimated demand functions based on combined data for all families are reported in Section II and those for the individual families in Section III. Conclusions are offered in Section IV.

The households analyzed in this paper are part of the community of Twin Rivers, New Jersey, a planned development consisting of approximately three thousand owner-occupied townhouses. The houses are made of masonry bearing-wall construction with wood framing, floors, and roofs. They use natural gas solely for space heating and rely on electricity for air conditioning and appliances. Each house has been occupied by only one family from October 1971 to January 1976, a period encompassing the twenty-seven winter (November-April) months under study. Less than twenty-seven gas consumption observations are available for some of these families, however, because gas personnel occasionally were unable to read meters and recorded "approximated" readings instead.

The Twin Rivers community is quite representative of much of the residential housing built in the past decade in the United States and the families themselves are typical members of the young, affluent suburban middle class. Most of the family heads have at least a bachelor's degree, ninety-six percent are white, and the mean age at present is about 35. Average family income was roughly \$20,000 per year in 1971. The homogeneity of these families and their housing permits researchers to avoid considering explicitly the relatively great variation in family and housing characteristics present in a more representative cross-section of American households. At the same time, however, this homogeneity implies that one may not be able to generalize results for these families to a wider cross-section of households.

Since the consumer price of gas is regulated, each family is a price-taker and the household demand functions estimated below are clearly 8/ identified. Monthly household consumption of natural gas (Q) is assumed

to be a function of the current price of natural gas (p_G) , prices of substitutes and complement goods, family income, and a set of non-economic characteristics, of which the weather is most important.

Time series observations on family income are unavailable and are not included in the equations. Prices of substitutes and complements are proxied by the consumer price index (P). The weather variable used is the number of heating degree days in a month (D) and is computed by summing over the days in a month the measure D for the ith day:

$$D_{i} = \left[(R - (T_{min} + T_{max})/2), 0 \right]$$

where R = the reference temperature (65° F in this study), T_{min} = the day's minimum temperature, and T_{max} = the day's maximum temperature. The degree days for the winter months studied in this paper are computed from data recorded at the weather station in Trenton, New Jersey, located about ten miles from Twin Rivers, and are listed in Table 1.

The marginal price of natural gas in any month is a non-increasing step function of the number of therms purchased by consumers. The last step of this function (50 therms) is well below the monthly gas consumption for each family in the sample and therefore its price can be used as the relevant (nominal) price of gas in the demand function. The real price of natural gas is obtained by dividing this nominal price by the corresponding consumer price index for the New York-Northeastern New Jersey region. Both nominal and real price series for natural gas are given in Table 2.

One form for the time series demand function for family i is

(1)
$$Q_{it} = a D_t^{\beta} \left(\frac{p_{ct}}{P_t}\right)^{\gamma}$$

a corresponding stochastic version of which is

(2)
$$\ln Q_{it} = \alpha + \beta \ln D_t + \gamma \ln \frac{P_{Gt}}{P_t} + u_{it}$$

where $\alpha = ln$ a, and u_{it} is a stochastic error term with

$$E(u_{it}) = 0$$
, $var(u_{it}) = \sigma_i^2$ for all t, and $cov(u_{it}, u_{is}) = 0$ for $t \neq s$.

This functional form is selected because the coefficients β and γ equal the monthly elasticities of natural gas consumption with respect to weather and the real price of gas. Two hypotheses are of special interest. The first is the null hypothesis $\beta = 1$, tested against the alternative $\beta \neq 1$. This null hypothesis states that a given percentage increase in degree days engenders an equal percentage increase in gas consumption and receives a priori support because much of the increase in household gas consumption will occur automatically since household furnaces are driven by the outside air temperature. The alternative subhypothesis $\beta > 1$ ($\beta < 1$) will hold if families increase gas consumption by a greater (lesser) percentage than a given percentage increase in degree days. Such would be the case if, ceteris paribus, families' preferred indoor temperatures and thermostat settings varied inversely (directly) with the outside temperature. The second basic hypothesis is the null hypothesis $\gamma = 0$ to be tested against the alternative that $\gamma < 0$. The alternative hypothesis states that the price elasticity of natural gas is negative.

An alternative to equation (2) substitutes the price of gas lagged one month for the current price, viz.,

(3)
$$\ln Q = \alpha + \beta \ln D + \gamma \ln \left(\frac{p_G}{P}\right)_{-1} + u.$$

Equation (3) assumes that consumers' decisions on gas consumption are based on the price of gas in the previous month, reflected in their previous month's gas bill. This may be viewed as a more reasonable assumption than that incorporated in equation (2) given that consumers are billed and become aware of the relevant price of gas after the corresponding period of consumption. Equation (2) is more appropriate if it is assumed that consumers can predict prices rather precisely as a function of past prices. The Twin Rivers families might be expected to be unusually accurate price predictors since they are a relatively well educated group and particularly well-informed about energy issues as a result of the Center for Environmental Studies' attention. Estimated equations obtained using current and lagged prices turned out to be virtually identical, however, and only those involving lagged prices are reported below.

The price series reported in Table 2 indicate a secular increase in the price of gas, more pronounced in the nominal than in the real series because of the secular increase in the consumer price index. These series suggest that estimates of the price elasticity obtained from equations (2) and (3) may be overstated in absolute terms, since the price coefficients may be capturing the secular decrease in gas consumption engendered by increased consumer awareness of the growing scarcity of energy. That is, households may have decreased natural gas consumption over time not only because of price increases, but rather because conservation of energy in general and natural gas in particular became more socially or politically acceptable. A demand function more appropriate than (1) may be

(la)
$$Q = a D^{\beta} \left(\frac{p_G}{P}\right)^{\gamma} e^{\rho t}$$
,

in which demand for natural gas is assumed to decrease over time at the rate $\rho \cdot \rho$ presumably negative. To the extent that the time term is proxies family real income growth, ρ will have a tendency to be positive. In any event, (2) and (3) can be modified in accord with (la) to

(2a)
$$\ln Q = \alpha + \beta \ln D + \gamma \ln \left(\frac{p_{G}}{P}\right) + \rho t + u$$

and

(3a)
$$\ln Q = \alpha + \beta \ln D + \gamma \ln \left(\frac{p_G}{P}\right)_{-1} + pt + u$$

and an additional hypothesis ρ = 0 can be tested against the alternative ρ < 0, which assumes a secular downward trend in household natural gas consumption, holding temperature and prices constant.

A final modification of the above equations can be used to test the hypothesis that the demand for gas is homogeneous of degree zero in the price of gas and the consumer price index. The following is a more general formulation of (1):

$$(4)$$
 Q = a $D^{\beta}p_{G}^{\gamma} P^{\delta}$.

Suppose that the price of gas and the consumer price index are multiplied by δ factor k. Then the new quantity of gas demanded $Q^* = a \ D^{\beta} (kp_G)^{\gamma} (kp)^{\delta}$ = $k^{\gamma+\delta}Q$, which equals Q only when $\gamma + \delta = 0$. This hypothesis can be tested in the equations

(5)
$$ln Q = \alpha + \beta ln D + \gamma ln p_G + \delta ln P + u$$

and

(6)
$$\ell n Q = \alpha + \beta \ell n D + \gamma \ell n \binom{p}{G}_{-1} + \delta \ell n (P)_{-1} + u$$

against the alternative γ + δ $\stackrel{1}{\Rightarrow}$ 0.

A time term can be added to these equation yielding equations (5a) and (6a):

(5a)
$$ln Q = \alpha + \beta ln D + \gamma ln p_G + \delta ln P + \rho t + u$$

(6a)
$$\ln Q = \alpha + \beta \ln D + \gamma \ln {p \choose G}_{-1} + \delta \ln {p \choose -1} + \rho t + u$$
.

This section is a discussion of the coefficients of the demand equations estimated from observations for all 151 families. Because of missing data on gas consumption, the number of observations, 3614, is less than the product of 151 families and 27 months, 4077. Table 3 contains estimated coefficients of equations (3), (3a), (6), and (6a). The time variable in equations (3a) and (6a) is a monthly index beginning with the value 1 for November 1971. Gaps in this time series variable reflect the omission of summer month observations.

The hypothesis that the coefficient of degree days equals one can be rejected at quite high levels of significance against both the alternative hypothesis that the coefficient is not equal to one and the alternative subhypothesis that the coefficient is greater than one. The demand equations therefore strongly support the hypothesis that percentage increases in natural gas consumption exceed given percentage increases in degree days and are consistent with families' preferred indoor temperatures and thermostat settings varying inversely with outside temperatures. These degree day coefficients also may be used to estimate the effect on natural gas consumption of lowered household thermostat settings. Decreasing indoor thermostat settings is essentially equivalent to reducing the reference temperature in the degree days formula and thus the effective, i.e., household perceived, level of monthly degree days. For a given reduction in the indoor thermostat setting, the percentage reduction in effective degree days is greater, the higher the outiside winter temperatures. For example, lowering the thermostat from 65° to 60° amounts to a 20% drop in effective degree days when the average of maximum and minimum daily outside temperatures is 40° but only a 10% drop when the daily average temperature is 15°. When the degree day elasticity is constant and greater than unity, lowered thermostat settings will induce relatively great percentage but low absolute savings in gas consumption in

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areas with relatively mild winters.

The hypothesis that the coefficient of the real price of natural gas equals zero can be strongly rejected against the alternative that the coefficient is negative in the equations both with and without the time 13/variable. Consistent with the discussion in Section I, the price elasticity is much larger in absolute value in equation (3) than in equation (3a). Depending on assumptions about a secular decrease in gas consumption over and above that induced by rising prices, the price elasticity may be either overstated in equation (3) or understated in equation (3a).

Equations (6) and (6a) are more general formulations of equations (3) and (3a). In each equation, the sign of the consumer price index coefficient is negative, contrary to the presumption in Section I, although in neither equation greater than its standard error. The hypothesis that the sum of the gas price and consumer price index coefficients equals zero, testing whether the demand function is homogeneous of degree zero in these two prices is strongly rejected in equation (6) ($\hat{\mathbf{t}} = 4.21$) but cannot be rejected even at rather low levels of significance in equation (6a) ($\hat{\mathbf{t}} = .870$). The standard error of the price index coefficient is much greater in equation (6a) than in equation (6) no doubt because of the strong multicollinearity between the consumer price index and time variables.

The hypothesis that the time coefficient equals zero is strongly rejected against the alternative that the coefficient is negative in equation (3a) but not in equation (6a). The multicollinearity between the price index and time noted above is reflected in the large standard error of the time coefficient in equation (6a).

The secular decline in gas consumption over and above that induced by rising prices need not be linear as is assumed in equations (3a) and (6a).

Alternative demand equations were estimated with various combinations of time

dummy variables. The resultant estimated degree day and price elasticities were very similar to those reported in Table 3 but there was some evidence that declines during and after the energy crisis winter of 1973-74 were larger than those before. For example, estimating equation (3) with added winter dummy variables, using the 1971-72 winter as a base, produced the following coefficients (standard errors) for the winter variables: 1972-73, .013 (.019); 1973-74, -.072 (.029); 1974-75, -.086 (.021); and 1975-76, -.121 (.035).

III.

This section is a discussion of the demand for natural gas equations estimated for each of the 151 Twin Rivers families individually. The analysis is again limited to those equations in which price is lagged one month. As was the case for the aggregate equation, results using current and lagged prices yielded very similar results.

To begin with, using the Chow test for equations (3), (3a), (6), and (6a), one can reject at very high levels of siginificance the hypothesis that the coefficients are equal across families. The dispersion of temperature and price elasticities in these individual equations is indeed quite remarkable, given the homogeneity of these families and their houses. Because of the relatively few observations in these individual equations, it is generally more difficult to reject the null hypotheses of Section I than was the case with the aggregate equations in Section II.

The hypothesis that the degree days coefficient equals one is not as strongly rejected in the individual regressions as it was in the aggregate equations. In equation (3), where time is excluded and the real price of gas is entered as a single variable, the hypothesis was rejected in 38, 55, and 67 equations at the 1%, 5%, and 10% levels of significance, respectively, using a two-tailed test. In 62 of these 67 and 119 of the 151 total cases, the degree day coefficient was greater than one. The size of the coefficients ranged from .539 to 1.723. These results were very similar to those obtained with other equation specifications and suggest wide variation in household thermostat settings in response to changes in the weather.

The price coefficients exhibit even more variation. Estimate coeff-cients for the 151 families for equation (3) are reported in Table 4. The mean and median price coefficients are -.666 and -.583, bracketing the aggregate estimate of -.596. The maximum and minimum coefficients are 1.112 and -3.120

and the coefficients' standard deviation is .636. One hundred and thirty three price coefficients are megative, seventy-six of which are significantly negative at the 1% level using a one-tailed test. These individual equations thus strongly support the proposition that the price elasticity of natural gas is negative.

As might be expected from Section II, support for the proposition that the price elasticity is negative is weaker when time is added to equation (3) yielding equation (3a). The results of this specification are summarized in Table 5. The mean and median price coefficients are -.260 and -.249, slightly greater in absolute value than -.224, the corresponding aggregate estimate. The variation in these coefficients is greater than those in equation (3a): the maximum and minimum coefficients are 4.016 and -5.954 and the standard deviation is .837. One hundred and eleven of the coefficients have a negative sign, but only 4, 17, and 30 of these are significant at the 1%, 5% and 10% level, respectively, using a one-tailed test.

The time coefficients in these equations provide somewhat weaker support than does aggregate equation (3a) for the hypothesis that there is a secular decline in gas consumption holding prices constant (Table 6). The mean and median time coefficients are -.0029 and -.0020, bracketing the aggregate estimate of -.00264. The maximum and minimum time coefficients are .0225 and -.0205 and the coefficients' standard deviation is .0053. One hundred and twelve of the coefficients are negative, 27, 42, and 51 of which are significant at the 1%, 5%, and 10% level, respectively, using a one-tailed test.

The hypothesis that these demand functions are homogeneous of degree zero in prices generally cannot be rejected. More specifically, in equation (6), the hypothesis was rejected at the 1%, 5%, and 10% level in 22, 37, and 47 cases, respectively, using a two-tailed test. In one hundred and nine

equations γ and δ were of opposite sign, but of these γ was negative and δ positive in only 66 cases. In all of the remaining 42 equations, γ and δ were both negative.

Similar results obtained in equation (6a). The hypothesis $\gamma + \delta = 0$ was rejected at the 1%, 5%, and 10% level in 7, 18, and 34 cases, respectively, using a two-tailed test. In seventy-nine equations γ and δ were of opposite sign, but of these γ was negative and δ positive in only 54 cases. In the remaining 72 equations, γ and δ were both negative 53 times and both positive 19 times.

IV.

The equations estimated in this paper strongly suggest that increases in natural gas prices will encourage families to reduce their natural gas consumption -- even in the short run. The significant price elasticities are especially impressive considering that the sample of Twin Rivers families lacks such long-run options as switching to alternative fuels or moving to homes with lower heating requirements.

Mean and median monthly price elasticity point estimates for individual families were -.666 and -.583 and the corresponding elasticity for the 151 household aggregate was -.596. If one assumes a secular decrease in gas consumption over and above that induced by price, then the mean and median price elasticity point estimates for individual families are only -.260 and -.249, and the corresponding aggregate elasticity is -.224.

The hypothesis that the demand for gas function is homogenous of degree zero in the price of gas and the consumer price index generally could not be rejected at conventional test levels. However, often the coefficients of these two variables were opposite in sign to those presumed in Section I. The signs and the relatively large standard errors of the coefficients of $\mathbf{p}_{\mathbf{G}}$ and P imply that the power of the homogeneity test against reasonable alternative hypotheses is rather low.

The extent to which one can generalize these results to a wider cross-section of American households is unclear because the Twin Rivers families are rather well educated, affluent, and well informed about energy issues. Their price elasticities may be relatively high because of their unusual ability to obtain, analyze, and react to economic information. On the other hand, their price elasticities may be relatively low since their affluence permits them the luxury of allocating income without close scrutiny of prevailing prices.

The wide variation in estimated price elasticities across apparently

homogeneous families is an especially interesting result of this paper. This individual variation suggests that functions of the sort estimated in Section II for aggregates mask a good deal of variance in the behavior of their constituents. Further research attempting to explain the variation in natural gas price elasticities across households might begin by ascertaining information on the number of people at home during the day. It is reasonable to suppose, for example, that childless working couples would turn their thermostats down during the day in response to gas price increases, but that families with infants at home would maintain high household temperatures even when price increases are great. Psychologists and sociologists might provide still other hypotheses to test in an attempt to explain this wide variation in economic behavior across apparently similar households.

Table 1

Degree Days for the Months Under Analysis

Month/Year	71-72	72-73	73-74	74-75	75-76
November	602	647	510	436	391
December	673	761	746	797	852
January	824	882	932	863	1077
February	1068	975	942	865	
March	726	533	631	801	
April	489	397	391	509	

Table 2

Prices of Natural Gas in Current and November 1971 Dollars

Month/Year	71-	72	72-7	3	73-7	4	74-75	
	Current	Nov.71	Current	Nov. 71	Current	Nov. 71	Current	Nov. 71
November	.127	.127	.158	.151	.175	.155	.210	.167
December	.148	.148	.161	.154	.175	.153	.215	.170
January	.132	.131	.147	.140	.17 5	.152	.203	.160
February	.128	.126	.144	.136	.166	.142	.203	.159
March	.146	.143	.147	.137	.175	.148	.203	.159
April	.139	.136	.148	.137	.182	.154	.219	.171

75-76

	Current	November 71
November	.229	.170
December	.229	.169
January	. 2 65	.196

Estimated Coefficients in Demand for Natural Gas
Equations for Group of Families

Equation		Coefficient	(Standard error) of		
	ln D	$\ln\left(\frac{P_G}{P}\right)$	ln P _G ln P	t	Sum of squared residuals
(3)	1.115	596 (.056)			356.250
(3a)	1.106	224 (.106)	:	00264 (.00064)	354.576
(6)	1.106		236161 (.102) (.188)		354.507
(6a)	1.106		223087 (.106) (.373)		354.502

All prices are lagged one month.

Table 4

Individual Family Price Coefficients Estimated From Equation (3)

Interval	Number	Signifi 10% level	5%	Negative 1% level	at
> .4 .24 02 -2. to 0 4 to2 6 to4 8 to6 -1.0 to8 -1.2 to -1.0 -1.4 to -1.2 -1.6 to -1.4 -1.8 to -1.6 -2.0 to -1.8 -2.2 to -2.0 -2.4 to -2.2 -2.6 to -2.4 -2.8 to -2.6 -3.0 to -2.8 < -3.0	5 3 10 11 19 29 22 19 12 10 4 1 0 0 2 1	1 10 27 20 17 11 10 4 1	0 5 26 20 17 11 9 4 1	0 3 17 17 13 10 7 4 1	
	151	106	98	76	

Interval	Number	10%	5%	Negative at
. 1 0		level	level	level
>1.0	3 · · · · · · · · · · · · · · · · · · ·			
.8 to 1.0	1			
.6 to .8	. 1			
.4 to .6	3			t .
.2 to .4	19	·		
0 to .2	13			
2 to 0	26			
4 to2	38	2	1	0
6 to4	16	6	1	0
8 to6	16	12	6	0
-1.0 to8	6	4	3	
-1.2 to -1.0	3	3	3 3	2 1
-1.4 to -1.2	0	_	•	-
-1.6 to -1.4	0			
-1.8 to -1.6	3	1	1	1
-2.0 to -1.8	. 0	-	_	
-2.2 to -2.0	0			
-2.4 to -2.2	0			,
-2.6 to -2.4	i	1	1	0
-2.8 to -2.6	0	, *	-	U i
-3.0 to -2.8	ŏ			
<-3.0	2	1	7	0
. 5.0				
	151	30	17	4

Table 6

Individual Family Time Coefficients Estimated From Equation (3a)

Interval $(x10^{-3})$	Number	Significantly Negative at
Interval (x10 ⁻³) >5 4 to 5 3 to 4 2 to 3 1 to 2 0 to 1 0 to -1 -1 to -2 -2 to -3 -3 to -4 -4 to -5 -5 to -6	Number 5 2 5 9 13 15 18 20 8 10 5	10% 5% 1% level level 1 1 0 1 1 0 4 2 0 3 1 0 9 6 4
-6 to -7 -7 to -8 -8 to -9 -9 to -10 -10 to -11 -11 to -12 -12 to -13 -13 to -14 -14 to -15 <-15	8 9 3 5 2 1 1 1 5	4 4 3 8 7 5 6 6 4 3 3 2 5 5 4 1 1 1 1 1 1 1 1 1 1 1 1 0 0 0 3 2 1 51 42 27

- $\frac{1}{A}$ American Gas Association, Gas Data Book, p. 9.
- Natural gas accounted for roughly 30% of the energy consumed in the United States in 1974, according to the U.S. Federal Energy Adminitration, 1976 Executive Summary National Energy Outlook, p. 2. The U.S. Federal Energy Administration's Natural Gas Facts and Figures for 1974, p. 6 lists the distribution of the 19.08 trillion cubic feet of natural gas to end users as follows: residential 4.79, commercial 2.26, industrial 8.31, electric utilities 3.43, and other (deliveries to municipalities, public authorities, street lighting, etc.) 0.29.
- 3/U.S. Federal Energy Administration, 1976 Executive Summary National Energy Outlook, p. 5.
- 4/U.S. Federal Energy Administration, Natural Gas Deregulation Analysis
 Technical Report FEA 76-3, p. 2.
- 5/U.S. Federal Energy Administration, <u>National Energy Outlook</u>, <u>op. cit.</u> p. 3.
- 6/U.S. Federal Energy Administration, <u>Natural Gas Deregulation</u>..., op. cit., p. 8.
- Their work is summarized in Newsweek, Vol. LXXXVIII No. 15, October 11, 1976, p. 54.
- $\frac{8}{\text{Months}}$ referred to in this paper are determined by meter reading dates, rather than the calendar.
- $\frac{9}{\text{At}}$ Twin Rivers, a therm is the energy content of (1/1.02) x 100 cubic feet of natural gas.
- 10/Hypotheses of homoskedastic and uncorrelated error terms could not be rejected at conventional test levels using Goldfeld-Quandt and Durbin-Watson tests in any of a randomly selected subsample of 20 individual family equations.
- Estimates of elasticities at the mean based on a corresponding equation linear in the variables were virtually identical to those estimated below. The specification used in this paper has the advantage of greater simplicity in computing elasticities.
- $\frac{12}{\text{From equation (1)}}$, dropping subscripts and normalizing the constant and price terms, $Q = D^{\beta}$. If R represents the thermostat setting and

$$\frac{T_{\min} + T_{\max}}{2} \leq R \leq 65^{\circ} \text{ for each day in the month, then } \frac{dD}{dR} = k,$$

a negative constant, and $\frac{\frac{dD}{D}}{dR} = \frac{k}{D}$, larger in absolute value, the smaller

is D . Now
$$\frac{\frac{\partial Q}{Q}}{\partial R} = \frac{\frac{\partial Q}{Q}}{\partial D}$$
 \cdot $\frac{dD}{dR} = \frac{k}{D^{\beta}}$ \cdot $\frac{\partial Q}{\partial D} = \frac{k}{D^{\beta}}$ $\beta D^{\beta-1} = \frac{\beta k}{D}$, larger

in absolute value, the smaller is D. Also, $\frac{\partial Q}{\partial R} = \frac{\partial Q}{\partial D} \, k = k\beta D^{\beta-1}$, larger in absolute value, the larger is D for $\beta > 1$.

- Without estimates of the income elasticity of natural gas and the share of income allocated by households to natural gas consumption, it is impossible to determine exactly how much of the price coefficient (η_{D}) represents the pure substitution effect of a price change on the demand for natural gas $((\eta_{D})_{\Delta U} = 0)$. The typical 1975-76 winter monthly gas bill for a Twin Rivers family was almost \$46. Assuming a six-month winter, the share of annual income allocated to natural gas consumption (k_{Q}) is therefore slightly more than 1%. The income elasticity of natural gas (η_{I}) should be approximately equal to that for housing space and therefore somewhat smaller than the income elasticity of housing, which measures an increase in both the quantity and quality of housing. Even if the income elasticity of natural gas is unity, however, it follows from the Slutsky equation $\eta_{I} = (\eta_{I})_{\Delta U} = 0^{-1} \lambda_{I} = 0^{-1}$, that the estimated coefficients overstate the pure substitution elasticity by only .010 to .015.
- The 99.5% critical values for each of the four equations lie between $F_{.995}$ (120, 120) = 1.61 and $F_{.995}$ (∞ , ∞). The computed values for the Chow test for the four equations were: (3), $\hat{F} = 7.63$; (3a), $\hat{F} = 5.96$; (6) $\hat{F} = 6.07$; and (6a), $\hat{F} = 4.81$.
- With no missing observations, significance tests on coefficient from equation (3) use the t-statistic with 24 degrees of freedom. Those from equations (3a) and (6) use the t-statistic with 23 degrees of freedom and those from equations (6a) the t-statistic with 22 degrees of freedom.