

AN APPROACH TO THE STUDY OF
INCOME, UTILITY, AND HORIZONTAL EQUITY*

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I. Introduction

Recent contributions to the theory of 'optimal' income taxation have focused upon the tradeoff between efficiency and vertical equity.¹ The view in this literature is that the best tax system is one which maximizes a utilitarian social welfare function. The shape of the social welfare function reflects the planner's value judgments about the proper distribution of income, and the effects of taxes on work incentives are introduced via assumptions on the form of individual utility functions.

It has been pointed out ([10], [19]) that such a framework neglects an important concern of students of the tax system, horizontal equity. As traditionally defined, horizontal equity is the notion that "...people in equal positions should be treated equally." [18, p. 160] (Customarily, "equal positions" are defined in terms of some observable index of ability to pay such as income, expenditure, or wealth.) The injunction to treat equals the same appears neither as a constraint in the maximization problem, nor as an argument in the objective function. Therefore, such optimal tax designs will in general² fail to provide horizontal equity. To the extent one views horizontal equity as an important ethical precept, these optimal tax systems are unsatisfactory.

In order to put a discussion of horizontal equity on the same plane as the optimal taxation literature, it is useful to define it in terms of utility rather than ability to pay. Such a formulation has been suggested by Feldstein:

- i) If two individuals would be equally well off (have the same utility level) in the absence of taxation, they should also be equally well off if there is taxation.

- ii) [Furthermore,] taxes should not alter the utility ordering [10, p. 10].

From here on, we will refer to this as the 'utility definition' of horizontal equity.

A tax system that is perfectly fair according to the orthodox definition may fail the utility definition. Consider, for example, a global income³ tax system with no loopholes and no privileged sources of income. Such a system is near ideal in terms of the orthodox definition. However, to the extent that individuals vary in their tastes for leisure and consumption, the tax fails the utility definition. In particular, such a system lowers the utility of a 'consumption lover' more than that of a 'leisure lover.'

More generally, unlike the traditional definition, the utility formulation explicitly recognizes that as long as labor supply is endogeneous, income is not an unambiguous measure of welfare. Equal position must be defined with reference to the parameters of the individual's opportunity locus, not the particular point on the locus that is chosen. In light of this consideration, one might be tempted to define horizontal equity in terms of opportunity sets. (See, e.g., [13].) However, in general the opportunity set cannot be characterized by a single parameter. Even in the simplest of cases, one needs to know both the wage and nonlabor income. Therefore, the parameters of the opportunity locus must somehow be aggregated in order to compute an index of economic position. The appropriate way to do this is with the indirect utility function.

Can the utility definition of horizontal equity be made operational? The purpose of the present essay is to demonstrate how the utility definition can be used to evaluate an existing tax

structure.⁴ Although the methodology we develop is applied to actual data, the particular results should be regarded mainly as illustrative. Our intent is to show that the utility formulation has interesting empirical content, rather than to arrive at a definitive answer to the question of whether or not the income tax is fair.

In Section II we discuss the difficulties involved in making operational the utility definition of horizontal equity. These include the problems of measuring differences in 'tastes', i.e., estimating utility function parameters which may differ across individuals. Section III employs the utilities generated by these parameters as the basis for constructing two measures of departure from horizontal equity. A concluding section provides a summary and a discussion of some of the study's limitations.

II. Utility Functions and Differences in Tastes

In order to implement the utility definition of horizontal equity, we must postulate the existence of comparable family utility functions, and estimate their parameters for a sample of families. Using these parameters, family utilities with and without taxes then need to be calculated. We now discuss each of these issues in turn.

UTILITY FUNCTIONS

The utility formulation of horizontal equity cannot be implemented unless comparable utility functions between families are postulated.⁵ The assumption of comparability is common to virtually all the recent studies on the theory of optimal income taxation. (See [3], [8], or [17].) As Stern [23] and others have emphasized, it is futile to debate whether or not such an assumption is

'scientific.' Rather, it should be viewed as a value judgment without which it is difficult to say much of interest about this or most other equity issues.⁶

Assume, then, that the utility of each family depends upon the husband's leisure, the wife's leisure, and family consumption according to the generalized constant elasticity of substitution (CES) functional form:

$$(1) \quad U_i = [\alpha_{1i}(T-L_{Hi})^{-\mu_i} + \alpha_{2i}(T-L_{fi})^{-\mu_i} + (1-\alpha_{2i}-\alpha_{1i})Y_i^{-\mu_i}]^{-1/\mu_i}$$

where

U_i = utility of the i th family

L_{Hi} = i th husband's hours of market activity per year

L_{fi} = i th wife's hours of market activity per year

Y_i = family income

T = time endowment (assumed to be 5824 hours⁷ for all individuals)

and α_{1i} , α_{2i} and μ_i are parameters of the i th family's utility function. The α 's determine in part the shares of leisure and money income in full income, and the Allen partial elasticity of substitution between any two arguments (ϵ) is a function of μ , $\epsilon = 1/(1+\mu)$. (See [25].)

Equation (1) is more general than the popular Cobb-Douglas formulation in that it does not constrain the elasticity of substitution between the arguments of the utility function to be unity. On the other hand, it does constrain the elasticity of substitution between any two arguments to be the same. Unfortunately, use of more general utility functions leads to quite formidable nonlinear estimation problems. (See, e.g., [1] or [6].) For our purposes, the CES is probably sufficient to get 'ballpark' estimates

of how differences in tastes across the population can effect horizontal equity. Thus, the CES specification is taken to be a maintained hypothesis in this paper; no attempt is made to test it.

The discussion of Section I emphasized that the impact of the tax system on utilities may differ from that on incomes. This is true even if utility function parameters are identical for all families. However, as also mentioned above, it is conceivable that tastes for leisure and income vary, possibly creating further differences in the impact of taxes on individual utilities. The framework we are presently building allows exploration of both these issues. Constraining the α_i 's and the μ_i 's of (1) to be equal for all families permits investigation of the first issue, while allowing them to differ allows us to examine the second.

In practice, one cannot estimate a unique set of utility function parameters for each family. Standard statistical procedures would simply fail due to the excess of the number of parameters over the number of observations. Therefore, the parameters of the utility function are estimated separately for different groups in the sample. Although there are a number of possible demographic dimensions along which tastes for leisure and income might vary, it was decided that race and number of pre-school children are probably most important.⁸ Therefore, four different sets of parameters are estimated, depending on whether the families are black or white, and whether or not they have children under six years of age. A set of parameters is also estimated pooling all the observations, i.e., assuming that tastes are uniform across the population.

The estimation procedure takes advantage of the first order conditions for utility maximization in much the same way that the

necessary conditions for profit maximization have been used to estimate CES production functions. (See [4].) If (1) is maximized subject to the family budget constraint,

$$(2) \quad y = w_H L_H + w_f L_f + A$$

where w_H is the husband's net wage, w_f is the wife's net wage, and A is net non-labor income,⁹ then the necessary conditions

for a utility maximum imply:

$$(3) \quad \ln\left(\frac{T-L_H}{y}\right) = -\epsilon \ln w_H - \epsilon \ln\left(\frac{1-\alpha_1-\alpha_2}{\alpha_1}\right)$$

$$(4) \quad \ln\left(\frac{T-L_f}{y}\right) = -\epsilon \ln w_f - \epsilon \ln\left(\frac{1-\alpha_1-\alpha_2}{\alpha_2}\right)$$

where $\epsilon = 1/(1+\mu)$.¹⁰

Equations (3) and (4) suggest a possible estimation procedure: for each spouse attach an additive error term and regress the logarithm of leisure divided by family income on the logarithm of the spouse's wage and a constant. However, such a procedure fails to take into account the constraint that both $\ln w_H$ and $\ln w_f$ appear with the same coefficient. Neither does it correct for heteroscedasticity or correlations between the error terms for husbands and wives. In order to take these considerations into account and thus gain efficiency, the system (3)-(4) is estimated using a full information maximum likelihood technique. Note that with the constraint across equations, there are three coefficients from which to extract the three utility function parameters.

Before equations (3) and (4) can be estimated, one further problem remains. For individuals absent from the labor force, $\ln w_D$ is not observed. In order to deal with this problem, we adopt a procedure which has been suggested by Robert Hall. For both the

husbands and wives with observed wages, a regression of the log of the net wage on various market characteristics of the individuals is estimated.¹¹ Then the fitted values of these wage generating equations are used in (3)-(4).¹² The properties of the estimates yielded by this procedure are discussed by Hall [15].

SAMPLE

Ideally, we should have information on tax induced utility changes for members of a random sample representative of the entire U.S. population. For purposes of the illustration in this paper, however, the sample is much more limited. It consists of 2510 black and white married families in which the wife is between the ages of 30 and 44 for the year 1967.¹³ The reason for the selection of this sample is chiefly expediency. Excellent data on their wages and work histories are available in the National Longitudinal Sample for Mature Women 1967 (see [24]), and their work-leisure behavior has already been studied intensively (e.g., see [15], [22]). In 1967 there were about nine and one half million families with these characteristics.

PARAMETER ESTIMATES

The parameter estimates are reported in Table II.1; the implied utility function parameters are in Table II.2. (Since the α 's and μ 's are highly non-linear functions of the regression coefficients,¹⁴ their standard errors could be computed only as approximations, and have not been calculated.)

For both black and white families, the presence of pre-school children changes the elasticity of substitution, decreasing it for whites and increasing it for blacks. Hall's careful study of labor

TABLE II.1

COEFFICIENT ESTIMATES*

Group	$-\epsilon$	$-\epsilon \ln \left(\frac{1-\alpha_1-\alpha_2}{\alpha_1} \right)$	$-\epsilon \ln \left(\frac{1-\alpha_1-\alpha_2}{\alpha_2} \right)$	L/RATIO
white 0 children (1151 families)	-.463 (.0351)	-.410 (.0356)	-.365 (.0192)	4.40
white ≥ 1 children (859 families)	-.288 (.0397)	-.544 (.0447)	-.278 (.0213)	5.19
black 0 children (255 families)	-.247 (.0475)	-.364 (.0446)	-.241 (.0413)	2.90
black ≥ 1 children (245 families)	-.352 (.0678)	-.0383 (.0581)	-.0811 (.0489)	2.57
pooled	-.467 (.0211)	-.333 (.0222)	-.259 (.0218)	4.15

*Variables are defined in the text. Numbers in parentheses are asymptotic standard errors. Last column has value of loglikelihood ratio.

TABLE II.2
 IMPLIED UTILITY FUNCTION PARAMETERS

<u>No. of Children</u>	<u>Race</u>	
	<u>White</u>	<u>Black</u>
0	u = 1.16	u = 3.05
	$\alpha_1 = .221$	$\alpha_1 = .143$
	$\alpha_2 = .243$	$\alpha_2 = .235$
≥ 1	u = 2.47	u = 1.84
	$\alpha_1 = .099$	$\alpha_1 = .284$
	$\alpha_2 = .249$	$\alpha_2 = .399$
pooled	u = 1.14	
	$\alpha_1 = .237$	
	$\alpha_2 = .278$	

supply [14] also suggests that the presence of children affects the income-leisure choices of blacks and whites differently, although he calculates no implied elasticities of substitution.

Further interpretation of the coefficients is possible when we note that with the first order conditions and the parameters of Table II.2, hours of work for husbands and wives can be calculated for any given values of family wages and non-labor income:

$$(5) \quad L_H = \frac{-C_H^A + T + C_f^{w_f} T - TC_H^{w_f}}{1 + C_f^{w_f} + C_H^{w_H}}$$

$$(6) \quad L_f = \frac{-C_f^A + T + C_H^{w_H} T - TC_f^{w_H}}{1 + C_f^{w_f} + C_H^{w_H}}$$

where

$$C_H \equiv (w_H \left(\frac{1 - \alpha_1 - \alpha_2}{\alpha_1} \right))^{-\epsilon}$$

$$C_f \equiv (w_f \left(\frac{1 - \alpha_1 - \alpha_2}{\alpha_2} \right))^{-\epsilon}$$

and the other variables are as defined above.

It is clear from an examination of (5) and (6) that labor supply elasticities for individuals depend not only on their utility function parameters, but upon the levels of w_H , w_f and A as well. Thus, even in the case where 'tastes' are identical, individuals will differ in their responses to tax induced changes in the wage. In order to develop a sense for whether or not the parameters of Table II.2 are 'reasonable,' we report the implied elasticities for one particular case: $w_H = \$2.50$, $w_f = \$1.50$, $A = \$400.0$. To be more specific, for each group, (5) and (6) are evaluated for these values. Then w_f is incremented by 10% in equation (6), w_H is

incremented by 1% in equation (5), and the new labor supplies calculated. The elasticities are reported in Table II.3. They are in agreement with the results of earlier econometric studies of yearly hours of work: small elasticities in absolute value for husbands¹⁵ and larger positive elasticities for the wives (see, e.g., [14]). It should be noted, however, that for some configurations of $\{w_f, w_H, A\}$ quite 'unusual' elasticities can be generated.

III. Measuring Horizontal Equity

Using the estimated utility function parameters and equations (5) and (6), hours of work and income in the absence of the income and payroll taxes can be calculated. With figures on leisure and income with and without tax, family utilities in both situations are then computed by substituting into (1). The question then becomes how to use these utilities to make inferences about horizontal equity. We first discuss this issue, and then present some numerical results.

CONCEPTUAL PROBLEMS IN MEASUREMENT

Up to this point, essentially what we have done is to generate two vectors, one of family utilities before tax (U_b) and one of family utilities after tax (U_a). The real problem in measuring horizontal equity is to summarize the differences between these vectors in a meaningful way.

As Feldstein [10] has pointed out, there seems to be no simple or obvious way to do this. He does, however, discuss a suggestive measure which is particularly appropriate for the second part of the utility definition of horizontal equity, the rank correlation between

TABLE II.3
ELASTICITY CALCULATIONS FOR

$$A=400, w_f=1.50, w_H=2.50$$

<u>Group</u>	<u>Wife's Own Wage Elasticity</u>	<u>Husband's Own Wage Elasticity</u>
white 0 children	.96	.07
white ≥ 1 children	1.06	-.10
black 0 children	.57	-.12
black ≥ 1 children	2.36	.02
pooled	1.28	.08

U_b and U_a . The degree to which the ordering of the two vectors differs can be measured by the Spearman rank correlation (r_s):

$$r_s = 1 - \frac{6\sum d^2}{n(n^2-1)}$$

where d denotes the differences between the ranks of U_b and U_a , and n is the number of elements in the vector. [26, p. 435]. This measure has two attractive features. The first is that it is a non-parametric statistic; i.e., it requires no assumptions on the distribution of U_a and U_b . Secondly, r_s is invariant with respect to any monotonic transformation of the utility functions we estimated in Section II, provided the same transformation applies to all. In this context, equation (1) is not nearly as restrictive as it might appear at first glance. In particular, the 'constant returns' assumption is of absolutely no importance.

Unfortunately, it is clear that even if the ranking of utilities were left completely unchanged, horizontal equity could still be violated. Imagine, for example, a two person society in which the individual with slightly higher utility does not have his welfare changed by the tax, while the welfare of a second individual falls substantially. The rank correlation would be 1.0, but such a situation violates the spirit of the utility definition of horizontal equity.¹⁶ If two individuals' utilities were close before the tax was imposed, they should be close afterwards.

It is clearly desirable to construct an alternative measure of departure from horizontal equity which does take 'closeness' into account. To accomplish this, randomize the elements of U_b , and then arrange the elements of U_a in the same order, so that the i th

family in U_a is the same as the i th family in U_b for all i . Now form the vectors DU_b and DU_a whose i th elements are defined as

$$DU_{b,i} = | U_{b,i} - U_{b,i-1} |$$

$$DU_{a,i} = | U_{a,i} - U_{a,i-1} |$$

Then a measure of departure from horizontal equity is the simple correlation (ρ) between DU_b and DU_a . If ρ is near unity, individuals whose positions initially were closer (farther) than average remain closer (farther) than average under the tax,¹⁷ suggesting that horizontal equity has been maintained.

The obvious problem with this measure is that ρ is not invariant with respect to any monotonic transformation of the utility functions. It remains unchanged only under the linear monotonic transformations. However, we can investigate one particularly interesting set of non-linear transformations, that which alters the concavity of the utility function by raising it to some power, v , ($0 < v \leq 1$). Rather than constrain v to unity, ρ is calculated for a number of different values of v (1, .8, .5, .2). The impact on ρ of changing the degree of homogeneity of (1) can then be ascertained. In essence, by this device we are trying to find the extent to which our results are sensitive to different assumptions on the elasticity of the marginal utility of income.¹⁸

An additional problem arises with respect to the interpretation of both r_s and ρ : it is not obvious how low they must be in order to characterize a tax system as unambiguously inequitable. In order to develop such a benchmark, it is necessary to determine what r_s and ρ would be under a tax regime that grossly violated horizontal equity.

Of the many possible candidates for such a distinction, we choose a random head tax. Under this tax regime, if there are n families, each pays a lump sum equal to $1/(n/2)$ times total tax revenues with a probability of one-half, and zero with a probability of one half. The values of r_s and ρ for the random head tax will provide the bottoms of the scales along which we measure horizontal equity.

RESULTS

Under the assumption that 'tastes' are uniform across the population, the rank correlation between utilities with and without tax is .9951. If utility function parameters are allowed to vary as in Table II.2, the rank correlation is .9972. Do these figures represent significant changes in the ordering of utilities? To answer this question, we first note that the standard deviation for the rank correlation is given by $\sqrt{1/n-1}$, where n is the sample size. Unfortunately, not enough is known about the distribution of r_s to make a formal test of whether or not it differs significantly from unity (see [16], p. 74). However, given that the standard deviation is about .02, informal inspection suggests that the ranking of utilities has not been appreciably altered.

As mentioned above, changes in incomes may not be good proxies for changes in utilities. It is therefore informative to compare the rank correlation for utilities with and without tax to the same measure applied to incomes. When individuals have identical utility function parameters, r_s for incomes is .9955. It is .9941 if we allow them to differ. As was the case for utilities, the rank ordering of incomes appears essentially unchanged.

The extent to which the tax system leaves the ordering of both utilities and incomes unaltered is striking. In light of this, it might legitimately be asked if any tax system could budge the values of r_s for either utilities or income a significant distance from one. In response, we present the values of r_s for the random head tax discussed above. Under that regime, when all individuals in the sample have identical tastes, r_s for utilities is .7527; when tastes are allowed to vary, it is .8337. Thus, the high values for r_s under the income tax are not merely phenomena due to inherent difficulties of changing the rank order. It is also of interest to note that under the random head tax, the rank correlation of incomes is no longer a good proxy for that of utilities. For the case where tastes are the same, r_s for incomes is .8628; when they differ, it is .9082. The victims of the lump sum tax increase their work effort substantially (leisure is a normal good), so that the ordering of incomes is rearranged to a lesser extent than that of utilities.

Turning now to the simple correlation between DU_b and DU_a we find much the same story. The first row of Table III.1a shows the correlations when the utility function parameters are allowed to vary as discussed above; the second row has the results when they are constrained to be equal. In all cases, regardless of the choice of v , ρ exceeds .99. Fortunately, then, these results are quite insensitive to assumptions on the concavity of individuals' utility functions. For the sake of comparison, we have computed the analogue of ρ for incomes rather than utilities. In the case where utility function parameters vary, the correlation coefficient is .9988; for the constrained case, it is .9993.

TABLE III.1a

SIMPLE CORRELATIONS BETWEEN DU_a AND DU_b

	$v = 1$	$v = .8$	$v = .5$	$v = .2$
α, μ differ among families	.9925	.9923	.9919	.9917
α, μ constant	.9951	.9949	.9945	.9942

TABLE III.1b

SIMPLE CORRELATIONS BETWEEN DU_a AND DU_b UNDER

A RANDOM HEAD TAX

	$v = 1$	$v = .8$	$v = .5$	$v = .2$
α, μ differ among families	.6972	.6823	.6608	.6397
α, μ constant	.6234	.6033	.5727	.5380

TABLE III.1c

SIMPLE CORRELATIONS BETWEEN DU_a AND DU_b UNDER A

REGIME WITH HIGHER MARGINAL TAX RATES

	$v = 1$	$v = .8$	$v = .5$	$v = .2$
α, μ differ among families	.9758	.9743	.9653	.9430
α, μ constant	.9770	.9800	.9791	.9685

As before, the values of ρ for the random head tax (see Table III.1b) are much lower. Again we have a standard for a 'bad' tax that is considerably greater than zero, but to which the results for the actual tax system are significantly superior.

Our framework can also be used to assess the impact on horizontal equity of various changes in the tax laws. For example, it might be asked if a tax schedule with higher marginal rates would violate the utility definition of horizontal equity much more than the present one. In order to investigate this possibility, we calculated r_s and ρ for a tax system with marginal tax rates 50% higher than those of the actual system. The rank correlations were only slightly less than those of the actual tax system: .9813 and .9740 for the cases where utility function parameters are constrained to be equal and where they are allowed to vary, respectively. Similarly, the simple correlation coefficients in Table III.1c decrease only slightly in value from their counterparts in Table III.1a. It appears that given the behavioral responses estimated from this particular sample, even an income tax with much higher rates would not cause major diminutions in either r_s or ρ .

So far we have avoided discussing whether or not our calculations suggest the presence of socially important amounts of horizontal inequity. A judgment cannot be made unless ethical beliefs about horizontal equity are explicitly formulated. This could be accomplished by specifying a social welfare function which depends not only upon individuals' utilities, but also some parameters which measure the degree of horizontal inequity (e.g., r_s , ρ). In an individualistic framework, such a social welfare function

requires that families value horizontal equity, so that it appears as an argument in their utility functions.¹⁹ One can imagine varying the weights with which the measures of horizontal inequity appear in the social welfare function. In this way, the tradeoff between individuals' 'utils' and horizontal equity would be made explicit. To give the analysis more concreteness, the number of 'utils' needed to balance increases in horizontal inequity (i.e., the marginal rate of substitution between utils and horizontal equity) could be translated into uniformly distributed dollars across the population.²⁰ Such an exercise is beyond the scope of this paper, but our discussion does provide a framework for investigating the welfare loss of horizontal inequity.

IV. Qualifications

Our purpose has been to investigate the empirical consequences of a definition of horizontal equity which is framed in terms of utility rather than ability to pay. To accomplish this, we have computed family utilities using estimates of the parameters of a generalized CES utility function. For the sample analyzed, three principle results emerge: 1) Examination of both indices of departure from horizontal equity suggests that in some cases tax induced changes in utilities can differ from changes in incomes. 2) Differences in tastes of the magnitudes we have estimated do not seem to have a large impact on the indices of departure from horizontal equity. 3) For those features of the tax system that have been studied, there do not seem to be major departures from horizontal equity, although there has been no attempt to quantify their social importance. These results do not appear sensitive to the

particular cardinalizations examined.

As has been stressed above, these results are illustrative and must be regarded with great caution. They might be understating departures from horizontal equity because the sample analyzed is not representative of the entire population. For example, if single individuals were included along with married couples, more changes in the ordering of utilities would have been detected. With more complete data, it might be found that there were greater differences in tastes than suggested by our utility function estimates.

Similarly, the assumption that all individuals take the standard deduction probably influenced the outcome, although this assumption would probably be more important in a study of vertical rather than horizontal equity. (See [10, pp. 94-97].) With less restrictive assumptions on the form of the utility function, we might have isolated more differences in tax impacts on utilities versus incomes.

There are a number of less technical but perhaps more fundamental problems associated with the study. The possible effects of government expenditures on utility have been ignored due to the extreme difficulty of determining the value households place on public goods.²¹ We have also assumed that gross wages remain constant as the tax changes, system/ and that there is complete flexibility in choice of hours of work. Indeed, the very notion that family utility depends only on leisure time and income is an enormous simplification. (Of course, the use of all these assumptions is widespread.) Perhaps more important in the present context is that we have not dealt with the fact that non-pecuniary rewards vary between jobs, a phenomenon that has received insufficient econometric investigation.

Thus, this study must be viewed as only a preliminary empirical exploration of the important issues raised by the new interpretation of horizontal equity. However, at least one lesson seems clear. If our ultimate concern is the impact of taxes on the magnitude and distribution of welfare, then even very careful investigations of changes in income may not be enough. Individual utilities must be scrutinized, despite economists' traditional reluctance to do so.

FOOTNOTES

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¹See, e.g., [3], [8], or [17].

²It can be shown that if all individuals have identical tastes and there is only one type of ability, then horizontal equity will be satisfied by these schemes. (See [10] and the discussion below.) Such assumptions are built into a number of the optimal taxation studies.

³Of course, here and below the same considerations would apply to a tax system with some other index of equal position, e.g., consumption.

⁴No claim is being made that utility based taxation is a feasible goal of public policy. The issue is whether or not a utility definition can be used to evaluate a system based upon observable criteria. Note that opportunity taxation would be about as difficult to implement as utility taxation because of problems in measuring potential earnings. We do not consider another aspect of horizontal equity which has recently received some attention. This is the impact of the tax system on pre-tax rates of return on assets. See [5].

⁵In the U.S. tax system, the family rather than the individual is the unit of taxation, so it seems appropriate to have family rather than individual utility functions.

⁶The appropriate role for the economist in discussions of ethical questions is to draw out the different implications of alternative sets of value judgments. Clearly, a number of 'arbitrary' assumptions are needed in order to make interpersonal utility comparisons. However, the framework developed in the present essay is sufficiently flexible to allow investigation of the implications of other value judgments.

⁷This is based on sixteen hours per day, seven days per week and 52 weeks per year. There seemed to be no way to avoid setting an arbitrary time endowment. Hall [14] assumes 2000 hours per year available for work, but a number of individuals in our sample worked considerably more than this.

⁸Race and number of children have appeared as very important factors in a number of labor supply studies. See, for example, [7] or [14]. Ideally, we seek estimates of lifetime utility functions. The fact that in a given year one family has children and another

does not may mean less that they have different tastes than that they are in different stages of the life cycle. As more extensive longitudinal data become available, it will become possible to generate better estimates of utility function parameters. Also, in future work it would be of some interest to make finer classifications based on variables such as education, age and marital status.

⁹Taxes are calculated under the assumption that all families file joint returns. A payroll tax of 4.20% is added for earned income under \$6600. Our data do not permit exact determination of the tax rate. There is no information on capital gains, and it is necessary to assume that all families take the standard deduction. For further details on the rate schedule see [20, p. 56]. The husband's gross wage is calculated by dividing his earned income by number of hours worked per year. The wife's gross wage is reported directly. The variable A is corrected by an intercept adjustment associated with the linearization of the budget constraint. See [14] or [22] for a description of this procedure.

¹⁰The presence of strict equalities suggests the absence of corner solutions, thus ignoring the problems associated with those (particularly married women) who are absent from the labor force. The extent to which the concentration of observations at zero hours of work biases parameter estimates depends upon the structure of the sample. In an earlier study [22], I analyzed very much the same data set as that employed in the current paper, using both TOBIT and a conventional estimation technique. The differences in the implied behavioral elasticities were not great enough to merit using more expensive techniques, given the purposes of the present paper.

¹¹The limitations of this procedure have been discussed by Heckman [15], who shows that this kind of imputation procedure will in general yield biased predictions of wages for non-workers. In practice, it probably imports a downward bias to the wage elasticities. It would be of interest to attempt to re-do the estimation employing a variant of Heckman's procedure, although using an explicit utility function rather than ad hoc supply functions would complicate the analysis. Note that under a progressive tax system, the net wage varies with hours of work. Therefore, the wage is calculated at a standard number of hours rather than the actual number. Our results are not sensitive to the particular choice of standard number of hours.

¹²The wage generating equation takes a form similar to earnings functions found in the human capital literature. The logarithm of the net wage is a linear function of years of education, years of market experience, experience squared, and race. The wife's work experience is reported directly; for the husbands it is calculated as age minus years of education minus 5.

¹³The study omits families which are on welfare because they are less likely to behave according to the model outlined below. See [14]. Both spouses must be present in order to be included in the sample and neither can be self-employed. In addition, families with reported gross income of less than \$400 were excluded. It was felt that for such families there were either reporting errors, an extraordinarily large proportion of family income not earned in the market, or circumstances that lead to yearly income being a particularly poor proxy for permanent income.

¹⁴Let $\beta_1 = -\epsilon$, $\beta_2 = -\epsilon \ln((1-\alpha_1-\alpha_2)/\alpha_1)$ and $\beta_3 = -\epsilon \ln((1-\alpha_1-\alpha_2)/\alpha_2)$. Then $\alpha_1 = (\exp(\beta_3/\beta_1))/\Delta$, and $\mu = 1/\beta_1 - 1$, where $\Delta = (\exp(\beta_3/\beta_1)+1) \cdot (\exp(\beta_2/\beta_1)+1) - 1$. Note that if the β 's are consistent, so are the α 's and μ .

¹⁵The fact that their elasticities are small should not be interpreted as an assertion that taxes have no impact on the work behavior of married males. Other dimensions of labor supply such as years of education, occupational choice and time of retirement may very well be influenced by the tax system.

¹⁶Similarly, it is not difficult to construct an example in which equal utility families remain equal, but the rank ordering is completely reversed. One part of the utility definition being satisfied by a tax system is not sufficient to insure that the other will be.

¹⁷Of course, the value of ρ will change under different random orderings of the elements of U_b and U_a . The results presented below, however, were not very sensitive to several different random orderings of the U_{bi} and U_{ai} 's.

¹⁸This procedure is necessary in the absence of consensus on the concavity of individual utility functions. Perhaps the best known attempt to measure the elasticity of the marginal utility of income is that of Frisch [11]. Stern [23] discusses the Frisch study and a number of others.

¹⁹Alternatively, one could argue that 'society' values horizontal equity apart from the tastes of its members. Under this interpretation, the social welfare function would be non-paretian, but a number of writers (see, e.g., [10]) have suggested that there is nothing to prevent a reasonable set of value judgments from allowing for such a possibility. It is interesting in this context to note that Pigou [21] considered both equal absolute sacrifice and minimum burden as candidates for an ultimate principle of taxation, but did not consider the possibility of including both and trading them off against each other.

²⁰For applications of the uniformly distributed dollar concept, see [9].

²¹For an interesting attempt, see [12].

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