A REFORMULATION OF SIMULTANEOUS EQUATIONS MODELS FOR MARKETS IN DISEQUILIBRIUM

Gregory C. Chow

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PRINCETON UNIVERSITY
207 Dickinson Hall
Princeton, New Jersey

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1. Introduction

The purpose of this paper is to propose a reformulation of simultaneous equations models in econometrics which would allow for the possibility that many endogenous variables explained by a model may not be in equilibrium. When the simultaneous equations model was formulated by members of the Cowles Commission at the University of Chicago [8, 9], it was assumed that the solution values for the endogenous variables obtained from a system of simultaneous equations are observed directly in the market. That is, the market is always in equilibrium. For example, in a model of two equations explaining the price and output of a commodity through the interaction of demand and supply, it is assumed that the desired demand always equals the desired supply. This assumption is relaxed in the proposed reformulation, as given in section 2.

In the last several years, there has been some interest in formulating a disequilibrium model by assuming that the quantity purchased is the minimum of the desired demand and the desired supply given the market price [1, 3, 4, 5, 7, 10]. In section 3, we try to examine the rationale of this formulation and to explain why our formulation is the more reasonable one.

Long before the recent formulation of disequilibrium models, economists had developed techniques to deal with disequilibrium situations through the use of distributed lags. Our formulation is a natural generalization of the ideas of distributed lags as applied to a system of simultaneous equations.

In section 4, we will compare our formulation with the specification of distributed lags for individual equations in a system as is customarily done in the construction of simultaneous equations models. We will try to persuade the readers that our formulation is more appropriate for many problems.

Methods of estimating the parameters in a system of disequilibrium linear equations will be discussed in section 5. The incorporation of variables determined by the hypothesis of rational expectations will be studied in section 6. Some extensions of the model and some important unsolved problems will be given in section 7.

2. A Disequilibrium Simultaneous Equations Model

The existing linear simultaneous equations model is written as

$$(2.1) \qquad By_t = \Gamma x_t + u_t$$

where y_t, x_t and u_t are respectively vectors of endogenous variables, predetermined variables and stochastic disturbances; y_t is assumed to be observed. Our formulation is

$$(2.2) \qquad By_t^* = \Gamma x_t + u_t$$

where y_t^* is a vector of equilibrium or desired values of the endogenous variables which are not directly observed. The reduced form equations for y_t^* is

$$(2.3) \qquad y_t^* = B^{-1}\Gamma x_t + B^{-1}u_t = \Pi x_t + v_t$$

We assume that the vector y_t of observed values of the endogenous variables is given by

$$(2.4) \qquad y_t - y_{t-1} = D(y_t^* - y_{t-1})$$

where D is a matrix measuring the speeds of adjustments. Certainly, a more complicated adjustment mechanism than (2.4) can be introduced, for example, by assuming that $y_t - y_{t-1}$ is a nonlinear (vector) function of $(y_t^* - y_{t-1})$, or that the elements of D are functions of other variables, or that terms involving y_{t-k}^* $(k \geq 1)$ should be included. However, for most purposes the simple form (2.4) will suffice. In fact, we may often assume that D is a diagonal matrix, and that some of its diagonal elements are unity.

Substituting the right-hand side of (2.3) for y_t^* in (2.4), we write the model as

$$(2.5) \qquad y_t = DB^{-1}\Gamma x_t + (I-D)y_{t-1} + DB^{-1}u_t$$

This equation explains the observed values of the endogenous variables, as equation (2.3) explains the desired values y_t^* of the endogenous variables.

The rationale of the above formulation is that the system (2.2) of simultaneous equations formulated by the economist determines only the values of the endogenous variables in equilibrium. One cannot automatically assume that these equilibrium or desired values are directly observed in the market. The reasoning is exactly the same as that introduced to justify the use of partial adjustment to explain one dependent variable in a single equation. In that case, some equation, such as a demand equation for the stock of consumer durables, explains only the desired or equilibrium value. The change in the actual stock observed is then assumed to be a fraction of the difference between the desired stock and the stock existing in the last period.[2] When a vector of desired values for the endogenous variables is determined by a system of simultaneous equations, a natural generalization of the single-equation model is to assume that the first difference $y_t - y_{t-1}$ of the observed endogenous variables is a linear transformation D of the vector $y_t^* - y_{t-1}$ of differences between the equilibrium values and the actual values of the preceding period.

It should be recognized that the justification for our formulation of (2.4) is no more or no less valid than the justification usually given for the univariate case. We have not provided a deeper theoretical basis for this adjustment mechanism. However, we will argue in the following two sections that our proposed statistical model is more reasonable than the existing alternatives.

3. Why Not the Minimum of Two Desired Values?

An alternative model for markets in disequilibrium is to assume that the observed value of an endogenous variable equals the minimum of the values given by two equations. Consider the simple but basic example:

(3.1) $$D_t = D_t(p_t, X_{1t}) + u_{1t}$$

(3.2) $$S_t = S_t(p_t, X_{2,t}) + u_{2t}$$

(3.3) $$Q_t = \text{Min}(D_t, S_t)$$

where D_t and S_t stand for demand and supply which are not directly observed, and the observed quantity Q_t is the minimum of D_t and S_t, given the price p_t and the exogenous variables X_{1t} and X_{2t}.

Our main objection to the assumption (3.3) is that it is unlikely to prevail in the actual market. Consider the market to be in equilibrium in period 1, as depicted by the interaction of the solid D and S lines in Figure 1. Assume that in period 2 X_{1t} changes so that the new demand curve is depicted by the broken line D_2. If the market were always in equilibrium, P_2 and Q_2 in period 2 would be given by the intersection of D_2 and S. If disequilibrium is allowed, it seems reasonable to assume that observed point of price and quantity in period 2 will be part way between the observed point in period 1 and the new equilibrium point, as shown for instance by the point (Q_2, P_2) in Figure 1.[3]

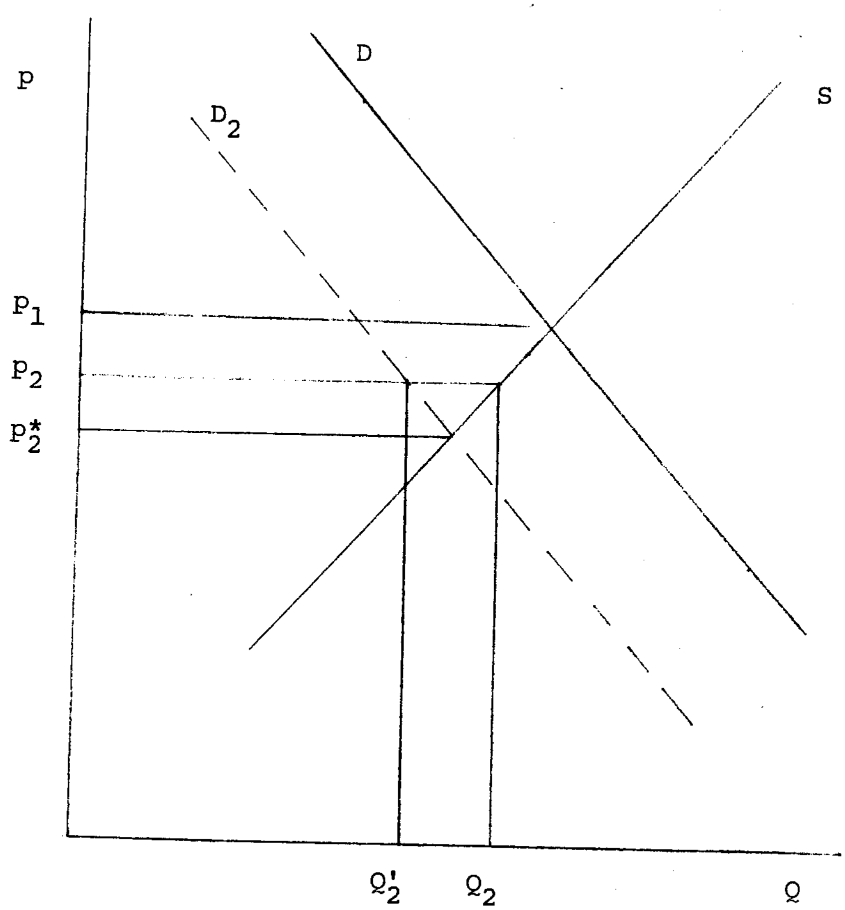

Figure 1.

Dynamic Adjustment in Disequilibrium

This is precisely what our formulation stipulates. On the other hand, the alternative formulation may permit the observed price to adjust part way toward equilibrium, say to p_2 in Figure 1, but it assumes that, at this price, the quantity will be equal to the minimum of the amount given by the D_2 and S curves, i.e., to Q_2'. This seems unreasonable because Q_2' is even smaller than the equilibrium quantity in period 2. The alternative formulation would imply wide oscillations in the adjustment of quantity as Q_2' indicates an overshoot from the new equilibrium.

It may be argued that the demand curve, by definition, shows the maximum amount that the consumer is willing to buy at a given price and therefore that, given price equal to p_2, demand cannot exceed Q_2'. The above argument is valid only if demand is defined as equilibrium demand in a static situation. If price were to stay at p_2, then equilibrium demand could not exceed Q_2'. Just for one period, demand may exceed equilibrium demand because of the costs associated with a change in demand or sheer habit. In the case of the demand for consumer durables, the existing stock cannot be reduced fast enough without incurring high transaction costs. In the case of the demand for a nondurable, there may be persistence of habit to prevent full adjustment to equilibrium. These reasons are well accepted when distributed lags are introduced into individual econometric equations. We simply apply the same reasoning to a system of equations.

In the alternative formulation of disequilibrium with the min condition (3.3), it is often further assumed that price adjustment takes the form

$$(3.4) \qquad p_t - p_{t-1} = \alpha(D_t - S_t)$$

Such a formulation may be attributed to Walras and might appear to be difficult to dispute. However, a closer examination reveals that it is not as reasonable as the formulation proposed in section 2. Refer again to Figure 1. Let the demand curve shift in period 2 to D_2. The formulation (3.4) would let the change $p_2 - p_1$ be a function of the difference $Q_2' - Q_2$. Our formulation would assume the change $p_2 - p_1$ to be a function of the difference $p_2^* - p_1$. Our formulation appears more reasonable because, given the same difference $D_t - S_t$ between the two desired quantities at the existing price, the rate of change in price would depend on how far the actual price is from equilibrium. For example, let the curves D, D_2 and S become much steeper than they are drawn in Figure 1, so that, starting from the same intersection point in period 1, the new equilibrium point would give a much lower equilibrium price than p_2^* shown in Figure 1. Although the horizontal distance between D_2 and S at the new observed price p_2 (lower than the p_2 actually drawn in Figure 1) may be the same as $Q_2' - Q_2$ given in Figure 1, the actual price change $p_2 - p_1$ would be much larger, in absolute value. The point is, why make the change in price be a function of the difference in quantities, and not of the difference between the existing price and the equilibrium price itself? The same difference in quantities is likely to produce a larger price change the less elastic the demand and supply curves are. The same criticism applies to a modification of the formulation (3.4) to make $p_t - p_{t-1}$ a function of $D_{t-1} - S_{t-1}$, i.e., of the difference between the desired quantities evaluated at p_{t-1}.

My last criticism of the formulation combining (3.3) and (3.4) is that it fails to provide a hypothesis of the actual change in quantity. Why is there no adjustment mechanism to explain $q_t - q_{t-1}$? Why is $q_t - q_{t-1}$ treated differently from $p_t - p_{t-1}$? Our formulation not only provides an adjustment mechanism for q_t. It treats the dynamic adjustments in q_t and p_t symmetrically. There is no need to choose between the Walrasian adjustment in price and the Marshallian adjustment in quantity.

4. Why Not Partial Adjustments for Individual Equations ?

As pointed out earlier, our formulation of section 2 is a natural generalization of partial adjustment models for individual equations. Once we accept the idea that the equilibrium values of the endogenous variables are determined by a system of simultaneous equations, it might be unreasonable to assume that the dynamic adjustment in each observed endogenous variable depends solely on a desired quantity as defined by one equation alone. As Zvi Griliches has pointed out in his survey of distributed lags [6, p. 45], if we apply separate formulations of distributed lags to the quantities demanded and supplied, there can be problems. If we let the quantities demanded and supplied to be respectively

$$Q_t - Q_{t-1} = \alpha_1 [D(p_t, X_{1t}) - Q_{t-1}] + u_{1t} \tag{4.1}$$

and

$$Q_t - Q_{t-1} = \alpha_2 [S(p_t, X_{2,t}) - Q_{t-1}] + u_{2t} \tag{4.2}$$

we are assuming, somewhat questionably, that only quantity adjusts slowly while price will reach equilibrium instantaneously. On the other hand, we may consider replacing equation (4.2) by

$$p_t - p_{t-1} = \alpha_2 [S^{-1}(Q_t, X_{2t}) - p_{t-1}] + u_{2t} \tag{4.3}$$

where $S^{-1}(Q_t)$ is the inverse function of $S(p_t)$, with X_{2t} treated as parameters. Although the combination (4.1) and (4.3) is logically consistent, its rationale is questionable. Why don't the suppliers adjust partially the quantities which they supply when we postulate that the demanders adjust the quantities they buy ? Why is price adjustment determined solely by the disequilibrium in the supply relation as given by (4.3) and quantity adjustment solely by the disequilibrium in the demand relation as given by (4.1) ? We do not have to face these difficult questions if our formulation of dynamic adjustments is adopted.

Thus it seems more reasonable to use the formulation of section 2 than to specify partial adjustments for the individual equations separately when we model a pair of demand and supply equations. How about the modelling of a system of macroeconometric equations ? To answer this question, consider first a simplified multiplier accelerator model consisting of the following consumption and investment equations,

$$(4.4) \qquad C_t = \alpha_1(C_t + I_t) + \alpha_2 C_{t-1} + u_{1t}$$

$$(4.5) \qquad I_t = \beta_1(C_t + I_t) - \beta_2(C_{t-1} + I_{t-1}) + \beta_3 I_{t-1} + u_{2t}$$

where, for simplicity, we have assumed income $Y_t = C_t + I_t$ while recognizing that replacing C_t+I_t by $C_t+I_t+G_t$, with G_t denoting exogenous government expenditures, would not affect the argument. Such a model of aggregate demand appears reasonable even when dynamic adjustments are introduced separately into the two endogenous variables C_t and I_t. This model would be considered an equilibrium model in the framework of section 2. It is a special case of, and is thus consistent with, our model of section 2 with the matrix D in equation (2.4) being specified as the identity matrix. Note that x_t in equation (2.1) denotes predetermined variables which include lagged endogenous variables. Our formulation is still valid, but does not add anything to the traditional treatment of simultaneous equations models. However, the difficult questions of the last paragraph would arise, and our more general formulation would become more attractive, once disequilibrium situations are introduced. We consider two important cases.

First, let us include the rate of interest r in the investment equation (4.5) and add a demand equation for money, say

$$(4.6) \qquad M_t = \gamma_1(C_t + I_t) - \gamma_2 r_t + \gamma_3 M_{t-1} + u_{3t}$$

where M_t is treated as exogenous. Here we are beginning to face some difficult questions concerning the formulation of distributed lags. For example, if M_t is exogenous, why not revise (4.6) to write

$$(4.7) \qquad r_t = \gamma_1(C_t + I_t) - \gamma_2 M_t + \gamma_3 r_{t-1} + u_{3t} \quad ?$$

The choice between (4.6) and (4.7) can be a subject of some debate. If (4.6) is accepted, for example, we consider the pair of equations, the revised (4.5) and (4.6), as a partial mechanism simultaneously determining I_t and r_t, given C_t. This is of course a slight modification of the IS-LM mechanism as we have replaced the IS curve by the investment equation alone. We could easily include the rate of interest r in the consumption function (4.4) without affecting the argument. There are now two equations explaining the rate of interest and investment (or total) expenditures. If we are willing to assume that, given the predetermined variables, the observed values of these two variables are determined as solutions to these two equations, i.e., that the market is always in equilibrium, then our formulation in section 2 need not be introduced. On the other hand, it might be reasonable to assume that the rate of interest as determined by these two equations is only an equilibrium rate. One would then introduce an adjustment equation of the form (2.4) to explain the observed rate of interest.[4]

In this example, when the rate of interest is included in equation (4.5), the resulting structural equation might still not explain directly the observed values of r_t in spite of the distributed lag mechanism for I_t already incorporated in the relation. This distributed lag mechanism may be due to the time delays between the planning and the actual expenditures for investment. Given this lag structure, the resulting relation between investment expenditures and the rate of interest may still depict only an equilibrium relationship and not an actually observed relationship as far as r_t is concerned. Similarly, equation (4.6) can be considered only as an equilibrium relationship for r_t. This formulation is better than using (4.7) alone to bear the entire burden of disequilibrium of r_t, as in the case of equation (4.3) for p_t.

For the second case, let us introduce an equation to explain the price level P in the above model. We first reinterpret the variables C_t and I_t to mean expenditure in constant dollars, and add the variable P_t in equations (4.4), (4.5) and (4.6) or (4.7). These equations are combined to yield an aggregate demand equation relating P_t and $C_t + I_t = Y_t$. For the missing equation, two possible choices will be considered: an aggregate supply equation and a price adjustment equation in the spirit of the Phillips curve. The former can be written as

$$(4.8) \qquad Y_t = \delta_0 + \delta_1 P_t + \delta_2 X_t + u_{4t}$$

The variable X_t may be expected price (in which case δ_2 may be negative) or some index of total productive capacity. We assume that X_t is predetermined, being a weighted sum of past values of an observed variable, say. The interesting case where X_t denotes expected price formed by rational expectations will be discussed in section 6. There are now two equations explaining aggregate output and the price level. It seems reasonable to apply our disequilibrium model of section 2 by hypothesizing that the solutions of these equations give only the equilibrium values of the endogenous variables and that the observed variables are generated by the adjustment equation (2.4). This formulation may capture some aspects of price rigidity.

A second possibility is to introduce a price adjustment equation of the form

$$(4.9) \qquad P_t - P_{t-1} = \alpha(Y_t^D - Y_t^S)$$

where $Y_t^D = C_t + I_t$ and Y_t^S denotes an exogeneously given capacity output. This equation is in the spirit of the Phillips hypothesis that the rate of change in the wage rate is a function of the rate of unemployment, although the functional form is different and we are dealing with the market for goods rather than labor. Since equation (4.9) is the same as equation (3.4), we have already presented in section 3 our criticism of it and our arguments in favor of the alternative of using a supply equation such as (4.8) to explain equilibrium price and output, and applying our disequilibrium model to explain observed price and output.

5. Methods of Estimation

Since the main purpose of this paper is to introduce a disequilibrium model of simultaneous equations and to explain why it is a useful model, we cannot possibly cover the subject of statistical estimation of its parameters. We will only show that well-known methods are available to provide consistent estimates, when the model is linear and the vector of random disturbances is independently and identically distributed through time and has a finite covariance matrix.

One immediately observes that the parameters $\Pi = B^{-1}\Gamma$ of the reduced form (2.3) and the matrix D of the adjustment equation (2.5) can be consistently estimated by applying least squares to the autoregressive equations (2.5) or [5]

(5.1) $$y_t = D\Pi x_t + (I-D)y_{t-1} + DB^{-1}u_t \quad .$$

Since the method of least squares provides consistent estimates of the parameter matrices $D\Pi$ and $I-D$ in equation (5.1) under the assumption that u_t is serially uncorrelated, one can solve for the parameters D and Π. Given a consistent estimate of Π, one can apply the method of indirect least squares to obtain consistent estimates of the structural parameters B and Γ, using the equation $B\Pi = \Gamma$. As it is well known, if the i-th structural equation is over-identified, there will be too many equations (too many columns of Π) to determine the unknown elements in the i-th row of $(B\ \Gamma)$. A consistent estimate of this row is still obtained by arbitrarily selecting just a sufficient number of columns of Π to solve for the unknown coefficients.

Another obvious method is two-stage least squares. We use the above consistent estimate of Π, say $\hat{\Pi}$, to form

(5.2) $$\hat{y}^*_t = \hat{\Pi}x_t \quad .$$

These "estimated" values of the equilibrium endogenous variables y^*_t are, for large samples, uncorrelated with the disturbance term u_t according to the reduced-form equations (2.3). We then use these "estimated" values of y^*_t to apply least squares, in the second stage, to an individual structural equation in (2.1).

The method of three-stage least squares is also applicable. We simply stack up the regression equations formulated for the application of the second stage of two-stage least squares, as indicated above for all structural equations, and apply generalized least squares to the resulting stacked regression equations, using an estimate of the covariance matrix of its residuals obtained from the two-stage least squares regression residuals. If u_t is further assumed to be multivariate normal, one can apply the method of maximum likelihood, but this subject will not be pursued here.

6. The Hypothesis of Rational Expectations

Since the hypothesis of rational expectations [11] has received much attention recently, it may be of interest to discuss how this hypothesis fits into our model.

The main feature of our model is to distinguish between the equilibrium values y_t^* generated by the model from the observed values y_t of the endogenous variables. The hypothesis of rational expectations would require another set of unobserved variables y_t^e, which are the expected values of the endogenous variables in period t formed by rational expectations. Without going into the more complicated case of requiring expected values of these variables in future periods, we can introduce y_t^e into the model (2.2) by writing

$$(6.1) \qquad By_t^* = \Gamma_1 y_t^e + \Gamma_2 x_t + u_t \ .$$

An example of this model was given in section 4 when we replaced x_t in (4.8) by the rationally formed expected price p_t^e and assumed that the structural equations determine only the equilibrium quantities, not the directly observed quantities of the endogenous variables. The reduced form equations for y_t^* are

$$(6.2) \qquad y_t^* = B^{-1}\Gamma_1 y_t^e + B^{-1}\Gamma_2 x_t + B^{-1}u_t = \theta y_t^e + \Pi x_t + v_t$$

The observed values are generated by the adjustment mechanism

(6.3) $$y_t = Dy_t^* + (I-D)y_{t-1}$$

After substitution of (6.2) it becomes

(6.4) $$y_t = D\theta y_t^e + D\Pi x_t + (I-D)y_{t-1} + Dv_t$$

To find an equation explaining y_t^e , we form conditional expectations of both sides of (6.4) given all available information at the beginning of period t , assuming x_t to be known for simplicity.

(6.5) $$y_t^e = D\theta y_t^e + D\Pi x_t + (I-D)y_{t-1}$$

Solving (6.5) for y_t^e yields

(6.6) $$y_t^e = (I-D\theta)^{-1}D\Pi x_t + (I-D\theta)^{-1}(I-D)y_{t-1}$$

By substituting the right-hand side of (6.6) for y_t^e in (6.4), we obtain an equation for all observables

(6.7) $$y_t = [I-D\theta]^{-1}D\Pi x_t + [I-D\theta]^{-1}(I-D)y_{t-1} + Dv_t$$

$$= Gx_t + Fy_{t-1} + Dv_t$$

It is apparent from (6.7) that identification problems arise in this formulation. Although the coefficients G and F in (6.7) can be consistently estimated by least squares under the assumptions stated in section 5, there are in general not enough equations in $G = [I-D\theta]^{-1}D$ and $F = [I-D\theta]^{-1}(I-D)$ to solve for the unknown parameters D, θ and Π. A thorough study of the identification problem for model (6.1) is beyond the scope of this paper. Just to show that this model can be manageable, consider a simple illustration given by the following restrictions

(6.8) $$B = \begin{bmatrix} B_{11} & 0 & 0 \\ B_{12} & B_{22} & 0 \\ B_{31} & B_{32} & B_{33} \end{bmatrix} \quad D = \begin{bmatrix} D_1 & 0 & 0 \\ 0 & I & 0 \\ 0 & 0 & I \end{bmatrix} \quad \Gamma_1 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & \Gamma_{1a} & 0 \\ 0 & \Gamma_{1b} & 0 \end{bmatrix}$$

Here we assume that only the first n endogenous variables can be in disequilibrium and that only the second n endogenous variables can have expected values in the model, with B_{11}, D_1, B_{22} and Γ_{1a} being n by n matrices. One can easily check that the adjustment matrix D_1 and the reduced form parameters $\theta = B^{-1}\Gamma_1$ and $\Pi = B^{-1}\Gamma_2$ are identifiable by solving the two equations relating them to G and F. Under the usual identification conditions for linear models, the unknown structural parameters B , Γ_1 and Γ_2 can be deduced from these reduced-form parameters.

Having established the formal relations between our disequilibrium model and the rational expectations hypothesis, at least under a set of simplifying assumptions, let us consider the economic relevance of this hypothesis in the light of our model. First, it appears that the need to introduce expected values in an econometric model would be reduced once it is recognized that the variables y_t^* in the structural equations (2.2) are equilibrium values. For example, if an expected or permanent income variable is needed to explain consumption because the consumers are assumed to make their decisions according to some equilibrium measure of income rather than the directly observed income, one may simply use an element y_{it}^* in y_t^* for this purpose. If the consumption variable in the model denotes permanent consumption, it may be represented by the equilibrium consumption in our model. If planned production is assumed to depend on some permanent or normal price, both variables may be represented by the equilibrium variables formulated in our disequilibrium model (2.2). In short, to the extent that expected variables are needed to represent something akin to permanent, equilibrium, or normal values, as distinguished from observed values, our disequilibrium formulation may have already satisfied some of this need. Of course, expected values may be introduced for other reasons.

Second, in those cases where the expected values y_{it}^e and our equilibrium values y_{it}^* are considered identical, the rational expectations hypothesis appears to be a less reasonable hypothesis for the determination of y_{it}^e. If an expected variable denotes some equilibrium or normal value, why should it be represented by the conditional expectation of the current variable, given that the same information is shared by both the econometric model builder and the economic agents ? Why shouldn't it be the conditional expectation of $y_{it}^e = y_{it}^*$ rather than of y_{it} ? If the conditional expectation of y_{it} is more reasonable than the expectation of y_{it}^*, it may or may not be more reasonable than the variable $y_{it}^* = y_{it}^e$ itself, because the variable y_{it}^* already incorporates the required equilibrium concept to some extent.

If one chooses to represent y_{it}^e by $E(y_{it}^*)$ rather than simply by y_{it}^* for all expected variables, then the model will be represented by equations (6.1), (6.2) and (6.3), but (6.5) will no longer be valid. Instead, one takes conditional expectations of both sides of (6.2) to get

$$(6.9) \qquad y_t^e = B^{-1}\Gamma_1 y_t^e + B^{-1}\Gamma_2 x_t = \theta y_t^e + \Pi x_t$$

the solution of which is

$$(6.10) \qquad y_t^e = (I-\theta)^{-1}\Pi x_t$$

Substituting (6.10) for y_t^e in (6.2) one obtains

(6.11) $$y_t^* = (I-\theta)^{-1}\Pi x_t + B^{-1}u_t \quad .$$

The right-hand side of this equation can be substituted for y_t^* in equation (6.3) for the purpose of estimation.

In short, our disequilibrium formulation weakens both the need for the introduction of additional expected variables in an econometric model and the rationale of the rational expectations hypothesis for the formation of expectations. It also raises interesting problems of identification and estimation in this context.

7. Conclusions

In this paper we have relaxed an important assumption of the classic simultaneous equations model, namely, that the solution values of such a system of simultaneous equations correspond directly to observed economic data. We consider the more general case where these values are interpreted as equilibrium values. We have suggested that, for representing a pair of demand and supply equations of an individual commodity, our model of disequilibrium seems to make more sense than the two existing alternatives. One uses the minimum of demand and supply for the quantity observed, and the other introduces an adjustment mechanism to each equation individually. For the application to macroeconometric models, we recognize that once distributed lags due to time delays are introduced into the individual equations, the resulting system may, in some cases, be reasonably assumed to generate the observed endogenous variables directly, without having to go through an additional adjustment mechanism because of disequilibrium. An example is the simplified multiplier-accelerator model of aggregate demand. However, for the determination of the general price level where disequilibrium appears to play an important role, our formulation may turn out to be a sensible one. Undoubtedly, no one can claim general applicability for any statistical model in economics, and the user has to make a seasoned judgment in each application. If one believes that several important variables in a macroeconomic model are in disequilibrium, one may choose to apply our model (2.4) for them, while assuming $y_{it} = y^*_{it}$ for the remaining variables. Furthermore, our model fulfills partially the need for using expected variables and weakens the rationale for the rational expectations hypothesis in some situations.

To the extent that our model is considered useful, we have left many problems for further research. The treatment of the estimation and identification problems in sections 5 and 6, the latter incorporating rational expectations, is very sketchy and many obvious statistical problems require further investigation. The role of expectations and the hypothesis of rational expectations including the related policy implications require further discussion in the context of our model. The model (2.2) may be assumed to be nonlinear. The disturbance term u_t may be serially correlated. The adjustment mechanism (2.4) may be more complicated, including nonlinearity, assymetry between positive and negative changes and dependency on some other variable. Since our model is essentially an application of partial adjustment to the multivariate situation, many interesting problems and solutions associated with the subject of distributed lags may carry over to our formulation as well. Hopefully our model will be useful for many empirical studies of disequilibrium in economics.

Footnotes

1. I would like to thank Stephen Goldfeld for helpful comments and to acknowledge financial support from the National Science Foundation.

2. See Chow [2] for a discussion and an empirical application of this mechanism.

3. We assume the speeds of adjustment to equilibrium for the two variables to be the same in this illustration, recognizing that they may be different.

4. One could also allow for disequilibrium in investment expenditures, but then the lag structure of the investment equation (4.5) would have to be revised to exclude I_{t-1} as an explanatory variable. Otherwise, there would be an identification problem. See footnote 5 below.

5. In this discussion we assume that x_t does not include any lagged endogenous variable which is also included in the vector y_{t-1} in equation (5.1). That is, if $y_{i,t-1}$ is an element of x_t, the elements of the i-th column of D are all zero except for the i-th element which is unity. If $y_{i,t-1}$ appears in the i-th equation for all equations i, the matrix $I-D$ in (5.1) will be zero. This corresponds to the system consisting of (4.1) and (4.3) which we have criticized. To distinguish this system from our disequilibrium formulation for the purpose of identification, we have to assume D to be a diagonal matrix, so that, in our formulation, only $y_{i,t-1}$ (and no $y_{j,t-1}$) appears in the reduced form equation for y_{it}.

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