OPTIMAL FOREIGN EXCHANGE MARKET INTERVENTION

WITH RATIONAL EXPECTATIONS

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I. INTRODUCTION

Prices Versus Quantities

This paper deals with a number of related issues. The main substantive problem analyzed is the derivation of the optimal foreign exchange market intervention policy from the point of view of a small open economy by using simple stochastic control theory. The extensive literature on the comparative merits of fixed and floating exchange rate regimes[1] and the proliferation of proposals for gliding, sliding and crawling pegs, intervention bands, snakes and other animals, all deal with special cases of this general problem. We can distinguish between "price setting policies" or <u>exchange rate management policies</u>, and "quantity setting policies" or <u>reserve management policies</u>.[2] With exchange rate management policies a value for the exchange rate is chosen each period (at each point in time in the continuous case) by the policy authority, which then buys or sells reserves in the amount required to realize that exchange rate given the behaviour of the domestic private sector and the external sector.[3] The fixed exchange rate regime is the special case where the exchange rate is set at the same level each period (at each point in time). With reserve management policies the authority decides each period how much it wishes to add to or subtract from its stock of foreign exchange reserves. The exchange rate is then left free to find the level required to achieve that objective. Freely floating exchange rates are the special case where the amount of reserves bought or sold each period is zero.

Clearly the issue of price management versus quantity management is a trivial one in a world without risk or uncertainty. Only differences in the costs associated with using the two instruments can lead one to favour the one over the other. The question as to what constitutes the best exchange market intervention policy can therefore only be analyzed properly in an explicitly stochastic model. This paper considers a number of very simple stochastic open economy models. The uncertainty is due to a variety of domestic and foreign disturbances.

Rules Versus Discretion

In addition to analyzing the "prices versus quantities" issue in the context of foreign exchange market intervention, the paper also addresses the "rules versus discretion" issue. This is rather a misnomer. "Open-loop" or "no-feedback" policies versus "non-trivial closed-loop" or "feedback" policies would be a more appropriate description of this debate. With open-loop policies the values of the time paths of the policy variables are specified at the beginning of a planning period. These paths are to be followed by the policy maker without regard to future events. Closed-loop policies specify the policy variables as functions of observations yet to be made, i.e. the values of the policy variables in the future will depend on future observations, which will reflect the results of current policy.[4] The best-known example of an open-loop policy prescription is Milton Friedman's advocacy of a fixed growth rate for some monetary aggregate.[5] Formally, consider a first order system of linear difference equations

1)
$$Y_t = AY_{t-1} + CX_t + b + U_t \quad .$$

Y_t is a vector of endogenous variables, X_t a vector of controls and U_t a serially uncorrelated vector with mean 0 and covariance matrix V.

A policy rule:

2)
$$X_t = G_t Y_{t-1} + g_t$$

is a non-trivial closed-loop policy if $G_t \neq 0$ and an open-loop policy if

$$G_t = 0.$$

In this paper, a result favourable to the "fixed rule" school of thought would be that the optimal exchange rate management policy is the fixed exchange rate and that the optimal reserve management policy is the freely floating exchange rate.

Two comparatively new themes in the economic policy literature appear to be giving different answers to the open-loop versus feedback control question. The optimal control literature seems to suggest that even for imperfect econometric models, optimal policy rules can be derived that will significantly improve performance in terms of some explicit objective functional over any fixed rule. These optimal rules can, in the case of stochastic models, only be expressed in terms of feedback control rules, which make explicit the dependence of the optimal values assumed by the policy instruments on the past states of the economy.[6] This paper uses the linear-quadratic framework of optimal control: a linear model with additive noise and a (finite) multi-period quadratic objective functional.

In contrast, the rational expectations literature seems to suggest that if economic agents form expectations in a way that is rational (i.e., using all "available" information optimally),[7] the scope for policy makers

to exert a beneficial impact upon economic welfare through stabilization policy is pretty well eliminated.[8/] Rational expectations are the stochastic analogue of perfect foresight. Rational expectations are optimal predictions conditional on all the information "available" to the economic agent in question. In practice, unbiased (least squares) estimators are always used. Rational expectations do not have to be good forecasts although they are the best forecasts. In the simple models considered in this paper, rational expectations will indeed be very good forecasts; the information assumed to be available to private economic agents is considerable: they know the structure of the economy, including the stochastic processes and the government's objective function. They can solve simultaneously for their expectations of the behaviour of the economy and for the behaviour of the economy that is affected by these expectations. The main implication of the rational expectations hypothesis for macromodelling is this necessity to solve simultaneously for the currently anticipated value of a variable and its future value calculated from the model.

There are a host of practical and conceptual problems associated with rational expectations theory. The assumption of perfect knowledge of probability distributions of outcomes is substituted for the assumption of perfect knowledge of the outcomes themselves. Obvious questions are whether these probability distributions are stable (stationary) and can be estimated. Problems of model selection do not arise in most rational expectations papers because the "correct" model used for forecasting is the author's model in the paper. In practical applications this will be a major problem. The issue of learning behaviour when the "environment" of an economic agent (the model) changes is another critical issue.[9/] Economic agents do not just have to know the objective model; they must believe in rational expectations theory itself for it to work!

The problem of non-existence of a rational expectations equilibrium arises when the same information is not available to all economic agents and strategic behaviour is permitted: economic agents may allow for the fact that their actions will affect the expectations and actions of other economic agents, and deliberately act so as to influence expectations. Situations of inconsistent expectations, e.g. a bargaining stand-off in which each party confidently expects the other(s) to cave in, cannot, by definition be modelled with rational expectations. Non-uniqueness of the rational expectations equilibrium trajectory raises serious conceptual problems. It is especially likely to arise when economic agents forecast many periods into the future.[10/]

A practical problem is that while rational expectations allow one to get away from the usual ad-hoc distributed lag expectation functions, adhoc-cery re-enters with a vengeance through the specification of the information set assumed to be available to economic agents. In other words, the hypothesis that economic agents use all available information optimally cannot be tested independently of a hypothesis about the economic agents' information set.

In addition to rational expectations, the models that have generated results that suggest the irrelevance of optimal feedback controls incorporate a number of further assumptions, each one of which is necessary for those results. They are:

1) Absence of static or dynamic money illusion.

2) Instantaneous market clearing. More precisely, all markets are efficient auction markets: prices adjust to clear markets with a frequency no less than the frequency with which new information accrues to private and public economic agents. The "market period" for each good or service equals the "information period". In reality, many implicit or explicit contracts extending over time intervals much longer than the "information period" exist and these

contracts are not complete contingent contracts in these sense of contingent on any new information that might accrue.

3) No differential access to (and capacity to utilize) different kinds of information as between private and public economic agents. This does not only rule out the possibility that uniformly better information is available to the government, it also excludes the possibility of private and public economic agents having comparative advantages in gathering, processing and analyzing different kinds of information.

4) The frequency with which prices adjust to clear markets (or more generally the frequency with which private economic agents can react to new events) is no less than the frequency with which the public authority can adjust at least one of its controls and influence some economic variable that affects private sector behaviour.[12]

In this paper, all four of these further assumptions are made. The result is that while the choice between price setting (exchange rate management) policies and quantity setting (reserve management) policies does affect the stochastic process governing the behaviour of real output, the actual values assigned to the exchange rate (if an exchange rate management is chosen) or to reserve sales or purchases (if a reserve management policy is chosen) will have no further effect on the probability distribution of real output. In Appendix I it is shown that if output is also a function of the stock of real reproducible capital, its probability distribution is affected by the values assigned to the foreign exchange market instruments when there is no perfect capital mobility. The external disequilibrium and the variability of the price level, however, will be always affected by the time path of the exchange rate or of reserve sales and purchases, as will real absorption. That this

is possible under rational expectations is a consequence of the inability of the private sector to "undo" every government action through market actions. While there are many plausible ways of ensuring that there is no "Modigliani-Miller Theorem" for the private sector vis a vis the public sector,[13] in our models the not unreasonable assumption that the policy authority controls the nominal stock of money and the quite common assumption that foreign exchange is held by the policy authority only provide two reasons -- each sufficient by itself -- for the non-neutrality of foreign exchange market intervention. In two recent papers, Fisher [1977] and Phelps and Taylor [1977] have demonstrated how multi-period wage contracts or price setting in advance of the period to which the price will apply may cause the information set available at the time of the current money supply decision to be larger than the information set available when the current wage or price was decided on. Monetary feedback rules can then affect the probability distributions of real output and employment even with rational expectations.

The paper considers different domestic and foreign sources of disturbances. Three models are analyzed. In order of increasing complexity they are:

1. A model without international financial flows and without an explicitly specified domestic financial sector.

2. A model without international financial flows but with an explicit domestic financial sector.

3. A model with an explicit domestic financial sector and international financial flows.

Results

The main results of the paper are the following.

1) In the absence of international financial flows, freely floating exchange rates and, more in general, reserve manangement policies, insulate the domestic economy from foreign disturbances. In the one commodity models of the paper, foreign disturbances take the form of random variations in the world price level. The result, however, can be extended to a world with import and export demand functions and random shifts in the export demand function as long as relative prices are not affected. This insulation of the domestic economy from foreign disturbances is a mixed blessing. "Locking out" foreign disturbances means "locking in" or "bottling up" domestic disturbances, whose full effect will be felt in the home economy without any spillover abroad through external leakages. If domestic and foreign disturbances are appropriately correlated, the total variability of the target variables may be less if the foreign disturbances are allowed to enter freely.

When there is international integration of financial markets[14/] reserve management policies (including a freely floating exchange rate) no longer insulate the domestic economy from foreign disturbances. Variations in "the" world interest rate will have obvious direct effects, but world price disturbances too will lead to changes in the real rate of interest. The exchange rate still adjusts to the level required to generate the policy-determined change in reserves, but this exchange rate adjustment cannot fully negate the effect of world price changes on the rate of return.

2) Open-loop controls are in general inferior to closed-loop feedback rules that can take into account new information as it becomes available. Once the price versus quantity management choice has been made, the probability distribution of real output is unaffected by the values assigned to the exchange rate or the reserve sales or purchases except in the case considered

in Appendix I, but the price level, the external position and real absorption can be stabilized by "active" optimal feedback controls.

3) The assumption of rational expectations (and the exact specification of the information available to the private and public economic agents) is crucial. Substituting ad-hoc expectations functions for rational expectations will give the policy authority scope to affect the probability distribution of real output as well.

4) Minor changes in the specification of the objective functional will significantly affect the relative desirability of fixed and floating exchange rates. E.g. having the level of the stock of foreign exchange reserves rather than the change in the level as an argument in the objective functional tilts the comparison against the fixed exchange rate regime.[15/]

5) While the optimal reserve management policy will in general be superior to the freely floating exchange rate policy and the optimal exchange rate management policy superior to the fixed exchange rate, we cannot rank the optimal reserve management policy and the optimal exchange rate management policy without additional information about the exact numerical values of the coefficients of both the deterministic and the stochastic parts of the model. The ranking of fixed and freely floating exchange rate regimes depends in a more transparent manner on the structural coefficients of the model.

Notation

Y — real output

A — real absorption

X — net exports

R — reserves

p — log of the price level

e — log of the exchange rate

i — interest rate

m — log of the nominal stock of money balances

B — stock of real-valued bonds

$\hat{p}(t-1,t)$ — expected value, as of t-1, of p at t.

$\hat{e}(t-1,t)$ — expected value, as of t-1, of e at t.

$\varepsilon_s(t)$ — domestic supply disturbance

$\varepsilon_d(t)$ — domestic demand disturbance

$\varepsilon_m(t)$ — domestic monetary disturbance

$\eta_p(t)$ — foreign price disturbance

$\eta_f(t)$ — foreign interest rate disturbance

σ_i^2 : variance of i, $i = \varepsilon_s,\ \varepsilon_d,\ \varepsilon_m,\ \eta_p,\ \eta_f$

ρ_{ij} : correlation coefficient between i and j, $i,j = \varepsilon_s,\ \varepsilon_d,\ \varepsilon_m,\ \eta_p,\ \eta_f$

E : the mathematical expectation operator.

II. THE SIMPLEST MODEL

In this section we consider the simplest small country open economy model. There is one traded commodity whose world price in terms of foreign currency is given. There are no international financial flows. The domestic financial sector is not represented explicitly (the LM curve is solved for the interest rate which is substituted into the IS curve). There is only one domestic disturbance, an aggregate demand shock, ε_d and one foreign disturbance, a world price shock, η_p. Throughout the paper we assume that there is complete sterilization of reserve flows, e.g. by open market operations. In addition, the government always balances its budget. The nominal quantity of money balances is therefore constant throughout.[16/] All foreign exchange is held by the government.[17/] As we are assuming that the government holds all its monetary and fiscal policy instruments constant, any "optimal" foreign exchange market intervention rule we may derive will be optimal only in a very limited sense. The pursuit of a multi-dimensional vector of objectives through the manipulation of a multi-dimensional vector of controls, while not conceptually more complicated than the single instrument -- multiple objective case analyzed in the paper is too unwieldy for analytical treatment.[18/]

The model can be summarized as follows:

1) $Y(t) = a(p(t) - \hat{p}(t-1,t))$ $a > o$

2) $A(t) = b_1(m - p(t)) + \varepsilon_d(t)$ $b_1 > 0$

3) $p(t) = e(t) + \eta_p(t)$

4) $X(t) = Y(t) - A(t)$

5) $\Delta R(t) \equiv R(t+1) - R(t) = X(t)$

6) $\hat{p}(t-1,t) = E_{t-1}(p(t) | I(t-1))$

7a) $E\varepsilon_d(t) = E\eta_p(t) = 0$

7b) $D(\varepsilon_d(t), \eta_p(t))' (\varepsilon_d(s), \eta_p(s)) = \begin{cases} \begin{bmatrix} \sigma^2_{\varepsilon_d} & \rho_{\varepsilon_d \eta_p} \sigma_{\varepsilon_d} \sigma_{\eta_p} \\ \rho_{\varepsilon_d \eta_p} \sigma_{\varepsilon_d} \sigma_{\eta_p} & \sigma^2_{\eta_p} \end{bmatrix} & \text{if } t=s \\[2em] \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} & \text{if } t \neq s . \end{cases}$

The government's objective is to minimize

8) $L = E_0 \sum_{t=1}^{T} \{ w_1 Y(t)^2 + w_2 (\Delta R(t))^2 + w_3 (\Delta p(t))^2 \}$. [19]

Equation 1) is a Phelps-Friedman[20] aggregate supply function. Current output, $Y(t)$, expressed in terms of the deviation from its normal or natural level is an increasing function of the excess of this period's price over the price that last period was expected to prevail during the current period. $p(t)$ is the natural log of the price level, $\hat{p}(t-1,t)$ is the expectation, as of $t-1$, of the natural log of the price level at t. The natural level of output is independent of the capital stock. The role of the Phelps-Friedman production function in generating the result that the probability distribution of real output is not affected by foreign exchange market intervention is crucial. With a Phelps-Friedman production function deviations of actual output from its natural level occur if and only if the actual price differes from the expected price. Given expectations, demand equals supply in the output market [and in the suppressed

labour market]. Notional and effective demand and supply schedules coincide. Perfectly competitive markets clear instantaneously and all of the time. Thus neither "target" output nor variations about that target level can be affected by government policy.

Equation 2) specifies aggregate demand or real domestic absorption (expressed in terms of deviations from the normal or natural level of output) as an increasing function of the real money stock. m is the natural log of the constant nominal supply of money balances. Aggregate demand is affected by a random error term ε_d . Expenditure could be made to depend on actual income, with a marginal propensity to spend between zero and one without changing any of the results in the paper. Our choice of units to measure Y and A permits us to economize on the number of constants to be carried around. It leads to a few prima facie unfamiliar results which shall be pointed out below.

Equation 3) is the "law of one price"; perfect commodity arbitrage equates the domestic currency price of output, P(t), to the foreign currency price $P^*(t)$ multiplied by the foreign exchange rate, $\Pi(t)$; $P(t) = \Pi(t) P^*(t)$. Taking logs on both sides, we get $p(t) = e(t) + p^*(t)$. The log of the world price level is assumed to be a random variable $\eta_p(t)$.

Equation 4) states that net exports are the excess of domestic production over domestic absorption.

Equation 5) states that in the absence of international capital flows the real value of the net increase in reserves is equal to the trade balance surplus.

Equation 6) is the rational expectations hypothesis. The price level expected in period t-1 to prevail in period t is the mathematical expectation of the price level at period t, conditional on the information available to the economic agents at t-1: I(t-1).

Equations 7a) and 7b) give the known mean vector and variance covariance matrix of the disturbances. Both have zero means and are mutually serially independent.[21]

The specification of the objective function (8) requires some justification. It is a special case of the T period sum of quadratic functions:

$$E_0 \sum_{t=1}^{T} (Y(t) - a(t))' K(t) (Y(t) - a(t)) .$$

The objections to a quadratic single period loss function are well known. It exhibits "satiation", i.e. it achieves a minimum or maximum at $a(t)$, and it is "symmetric", i.e. it assigns the same cost to a positive deviation from the ideal path, $a(t)$, as to a negative deviation of equal magnitude. The satiation problem can be avoided by setting the optimum, $a(t)$, at so unrealistic a level that the actual solution will always stay on the "right" side of the ideal path. The symmetry problem has been tackled by B. Friedman. He generalized the linear constraint-quadratic objective optimal control framework to piecewise linear constraints and piecewise quadratic objectives without losing the major advantage of the linear-quadratic framework: the simplicity of the optimal linear feedback rules it generates.[22]

It is not true, as is sometimes argued, that a quadratic specification means that the policy authority attaches utility only to reducing the variance of its objectives. While utility is attached to stabilization, the ideal path around which target variables are to be stabilized (which could, e.g. in the case of absorption be chosen to be the golden rule path) and the weights attached to deviations from the ideal path at different dates, given by the

weighting matrices $K(t)$, are equally important in determining the optimal trajectory. It is appropriate to view the quadratic function as a (second order) approximation to a more general non-linear objective function.

If we accept the quadratic specification, the choice of variables to be included in the objective function, of ideal paths for these variables and of weights to be attached to deviations of the objectives from their ideal or target values still remains to be made. While our objectives may have comparatively weak "welfare foundations" in the rather limited sense of being different from the most common arguments found in the normative micro-economic literature (essentially current and future (per capita) consumption paths) they are descriptively realistic in the sense that they correspond to the well-publicized proximate aims of demand management policy in advanced capitalist economies.

The government has 3 objectives. Two "internal balance" objectives: stabilization of output (employment) at normal capacity (at the "natural" rate of unemployment) and price level stability, and one external balance objective. We specify this in terms of minimization of the balance of payments deficit or surplus. If the level of the stock of reserves relative to some target level, $R-R^*$, rather than the change in the reserves, ΔR, is the external balance objective, fixed exchange rates become a less attractive proposition as will be demonstrated below. The government minimizes the expectation, as of time o , of the T period sum of squared deviations of the objectives, Y, ΔR and Δp from their target values (zero for all three). Without in any way weakening the main conclusions of the paper, we can substitute deviations of domestic absorption (A) from its target (zero) for deviations of Y from its target. Indeed, our conclusions as regards the existence of non-trivial

optimal feedback controls would be strengthened, as the stochastic process for $A(t)$, unlike that for $Y(t)$, is not independent of the values assumed by the controls, $\Delta R(t)$ or $e(t)$.

The information available to economic agents when forming their expectations about $p(t)$ during period $t-1$ is very rich. They know the structure of the model, and the joint density of the disturbances [only the first two moments of which matter for our analysis]. In addition, they know the government's objective function. They also know all past values of the endogenous variables and the policy instruments, up to and including those in period $t-1$. This allows them to derive the values of the disturbances in period $t-1$ and in all earlier periods.

When the government chooses, at the beginning of period t, the value for its control during period t it does not know the current (period t) values of the disturbances, but it does know all past states of the model.

The Fixed Exchange Rate Regime

Consider the case where $e(t)$ is set at the same level $\bar{e}$ each period. $\bar{e}$ has been chosen optimally so as to minimize 8). The private sector knows that the government follows a fixed exchange rate policy, believes it to be viable because $\bar{e}$ has been chosen optimally and reserves are large enough to withstand a run of bad luck, and knows that the price level is generated according to 3). Thus

$$\hat{p}(t-1,t) = E_{t-1}\, p(t) = E_{t-1}(e(t) + \eta_p(t)) = \bar{e} \ .$$

With a fixed exchange rate the behaviour of the target variables is given by

9) $Y(t) = a\eta_p(t)$

10) $\Delta R(t) = -b_1(m-\bar{e}) + \Psi \eta_p(t) - \varepsilon_d(t)$

11) $\Delta p(t) = \eta_p(t) - \eta_p(t-1)$,

where $\Psi = a + b_1$. Neither the behaviour of output nor that of the price level can be affected by the choice of the fixed value $\bar{e}$. Real output and rate of inflation are given by stochastic processes determined only by the foreign price disturbances.[23]

From equation 10) and the objective function, it is clear that $\bar{e} = m$ is the optimal fixed exchange rate; when $\bar{e} = m$ the expected external deficit $(E\,\Delta R(t))$ equals zero. This justifies the assumption that the private sector has confidence that the fixed exchange rate regime will not break down. The coefficient of unity on m is a result of our choice of units for Y and A .

The minimum expected loss equals

12) $L^*_{Fixed} = T\{(w_1 a^2 + w_2 \Psi^2 + 2w_3)\, \sigma^2_{\eta_p} + w_2\, \sigma^2_{\varepsilon_d} - 2w_2\, \Psi\, \rho_{\eta_p \varepsilon_d}\, \sigma_{\eta_p}\, \sigma_{\varepsilon_d}\}$.

The Freely Floating Exchange Rate Regime

When $\Delta R(t)$ is set equal to zero each period, domestic production equals domestic absorption and $Y(t) = A(t)$ can be solved for the domestic price level. The exchange rate required to clear the foreign exchange market is then determined via equation 3. By equating 1) and 2), we obtain:

$$p(t) = \frac{b_1}{\psi} m + \frac{a}{\psi} \hat{p}(t-1,t) + \frac{1}{\psi} \varepsilon_d(t) \quad .$$

According to the rational expectations assumption, private economic agents know this equation. Using $p(t-1,t) = E_{t-1} p(t)$, we find:

$$p(t-1,t) = m \quad .$$

Our choice of units for Y and A accounts for the coefficient of unity on m here as well. Substituting this into the previous equation, we get

$$p(t) = m + \frac{1}{\psi} \varepsilon_d(t) \quad .$$

The floating rate model can now be summarized as:

13) $$Y(t) = \frac{a}{\psi} \varepsilon_d(t)$$

14) $$\Delta R(t) = o$$

15) $$p(t) = \frac{1}{\psi} (\varepsilon_d(t) - \varepsilon_d(t-1)) \quad .$$

The economy is completely insulated from foreign disturbances. The freely floating exchange rate policy is simpler to execute than the fixed exchange rate policy as the level of the fixed rate needs to be chosen optimally while all the government has to do under floating rates is announce its absence from the foreign exchange market.

The expected loss is

16) $$L^{*}_{Float} = T(\frac{w_1 a^2 + 2w_3}{\psi^2}) \sigma^2_{\varepsilon_d} \quad .$$

Y(t), the state vector, can contain current and lagged target variables, current and lagged intervening or intermediate variables (those endogenous variables that are neither targets nor instruments), current and lagged values of the instruments and current and lagged disturbances. X(t) is the vector of current instruments, b(t) is a vector of exogenous variables and U(t) is a vector of mutually serially independent disturbances. A(t) and C(t) are known coefficient matrices.[25/] The K(t) are known symmetric (usually diagonal) positive semi-definite matrices and the a(t) are known vectors of desired values for the target variables.

The optimal control rule that minimizes 17) subject to 18) is:[26/]

19a) $X(t) = G(t) Y(t-1) + g(t)$

19b) $G(t) = - [C^(t) H(t) C(t)]^{-1} C^(t) H(t) A(t)$

19c) $H(t-1) = K(t-1) + A^(t) H(t) [A(t) + C(t) G(t)]$

19d) $g(t) = - [C^(t) H(t) C(t)]^{-1} C^(t)[H(t) b(t) - h(t)]$

19e) $h(t-1) = K(t-1) a(t-1) - A^(t) H(t) (b(t) + C(t) g(t)) + A(t)^h(t)$

19f) $H(T) = K(T)$

19g) $h(T) = K(T) a(T) .$

This solution is identical to the one we obtain when the random disturbance vector is ignored (replaced by its expected value). When we minimize the expectation of a multi-period quadratic loss function subject to a known linear model with additive random disturbances, multi-period certainty equivalence exists if the solution is expressed in the form of feedback control equations. The importance of equations 19a) - 19g) for our problem is that

the current period value of the optimal control, $X(t)$, depends only on last period's state of the model, $Y(t-1)$, which we assume to be known when economic agents form their expectations, at time $t-1$, about the current period value of the control, and on the known coefficient matrices. In other words, $X(t)$ is known with certainty to private economic agents at time $t-1$. Any effect that the active use of these controls may have can therefore certainly not be attributed to the government "fooling" the people. In our models the foregoing implies $E_{t-1} e(t) = e(t)$ and $E_{t-1} \Delta R(t) = \Delta R(t)$.

The Optimal Exchange Rate Management Policy

With rational expectations the general exchange rate management model can be written as:

20) $\quad Y(t) = a \eta_p(t)$

21) $\quad \Delta R(t) = - b_1 m + b_1 e(t) + \Psi \eta_p(t) - \varepsilon_d(t)$

22) $\quad \Delta p(t) = e(t) - e(t-1) + \eta_p(t) - \eta_p(t-1)$.

The objective is to minimize by choosing $e(t)$, $t=1,2,\ldots,T$

$$E_0 \sum_{t=1}^{T} [w_1 Y(t)^2 + w_2 (\Delta R(t))^2 + w_3 (\Delta p(t))^2] \ .$$

If we rewrite this in the format provided by equations 17) and 18), we obtain the following. Minimize

23) $\quad E_0 \sum_{t=1}^{T}$
$$
\begin{bmatrix} Y(t) \\ \Delta R(t) \\ \Delta p(t) \\ e(t) \\ \eta_p(t) \end{bmatrix}'
\begin{bmatrix} w_1 & 0 & 0 & 0 & 0 \\ 0 & w_2 & 0 & 0 & 0 \\ 0 & 0 & w_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} Y(t) \\ \Delta R(t) \\ \Delta p(t) \\ e(t) \\ \eta_p(t) \end{bmatrix}
$$

Subject to:

$$
24) \quad
\begin{bmatrix} Y(t) \\ \Delta R(t) \\ \Delta p(t) \\ e(t) \\ \eta_p(t) \end{bmatrix}
=
\begin{bmatrix}
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -1 & -1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} Y(t-1) \\ \Delta R(t-1) \\ \Delta p(t-1) \\ e(t-1) \\ \eta_p(t-1) \end{bmatrix}
+
\begin{bmatrix} 0 \\ b_1 \\ 1 \\ 1 \\ 0 \end{bmatrix} e(t)
+
\begin{bmatrix} 0 \\ -b_1 m \\ 0 \\ 0 \\ 0 \end{bmatrix}
$$

$$
+
\begin{bmatrix}
a\eta_p(t) \\
\eta_p(t) - \varepsilon_d(t) \\
\eta_p(t) \\
0 \\
\eta_p(t)
\end{bmatrix}
\quad .
$$

23) and 24) represent a simple version of 17) and 18) with all the $A(t)$, $C(t)$, $b(t)$ and $K(t)$ matrices time-invariant and the $a(t)$ vectors all equal to zero.

A hard look at 23) - 24) suggests immediately that the fixed exchange rate policy $e(t) = e(t-1) = m$ will not be optimal, because the authorities at time t know $\eta_p(t-1)$ and can therefore stabilize the price level by adjusting $e(t)$ upwards whenever $\eta_p(t-1)$ is positive and vice versa. [If $e(t)$ were kept constant when $\eta_p(t-1)$ is positive, the price level in period t would be expected to fall as $E_{t-1} \eta_p(t) = 0$]. It also suggests that if no weight were attached to price stability $(w_3 = 0)$, the optimal exchange rate management policy will in fact be the fixed exchange rate policy. The formal analysis bears this out. The explicit solutions of $g(t)$ and $G(t)$

for the last two periods (T and T-1) are given below:

25a) $\qquad G(T) = [\ 0\ \ 0\ \ 0\ \ \dfrac{w_3}{\Psi_o}\ \ \dfrac{w_3}{\Psi_o}\ ]$

25b) $\qquad g(T) = \dfrac{b_1^2\ w_2\ m}{\Psi_o}$

i.e.

26) $\qquad e(T) = \dfrac{w_3}{\Psi_o}\ e(T-1) + \dfrac{w_3}{\Psi_o}\ \eta_p(T-1) + \dfrac{b_1^2\ w_2\ m}{\Psi_o}$

as $\eta_p(T-1) = p(T-1) - e(T-1)$, 26) can be rewritten as:

26') $\qquad e(T) = \dfrac{w_3}{\Psi_o}\ p(T-1) + \dfrac{b_1^2\ w_2\ m}{\Psi_o}$

$\qquad\qquad \Psi_o = b_1^2\ w_2 + w_3\ .$

Note that $e(T)$ increases with $\eta_p(T-1)$ and that if $w_3 = o$, $e(T) = m$, as expected.

27a) $\qquad G(T-1) = [\ 0\ \ 0\ \ 0\ \ \dfrac{w_3\ \Psi_o}{\Psi_1}\ \ \dfrac{w_3\ \Psi_o}{\Psi_1}$

27b) $\qquad g(T-1) = \dfrac{b_1^2\ m\ \Psi_2}{\Psi_1}$

i.e.

28) $\qquad e(T-1) = (\dfrac{w_3\ \Psi_o}{\Psi_1})\ e(T-2) + (\dfrac{w_3\ \Psi_o}{\Psi_1})\ \eta_p(t-2) + \dfrac{b_1^2\ w_2\ \Psi_2}{\Psi_1}\ .$

Using $\eta_p(T-2) = p(T-2) - e(T-2)$, 28) can be rewritten as:

$$28') \qquad e(T-1) \;=\; \frac{w_3 \, \Psi_0}{\Psi_1} \; p(T-2) \;+\; \frac{b_1^2 w_2 \, \Psi_2 \, m}{\Psi_1}$$

$$\Psi_1 \;=\; b_1^4 \, w_2^2 + 3 \, w_2 \, w_3 \, b_1^2 + w_3^2 \; ; \qquad \Psi_2 \;=\; b_1^2 \, w_2 + 2 \, w_3 \; .$$

Again we note that $e(T-1)$ moves with $\eta_p(T-2)$ and that when $w_3 = 0$, $e(T-1) = m$.

The Optimal Reserve Management Policy

With rational expectations we can solve the equation $Y(t) - A(t) = \Delta R(t)$ for the actual and expected price level. We find

$$\hat{p}(t-1, \, t) \;=\; m + \frac{\Delta R(t)}{b_1}$$

$$p(t) \;=\; m + \frac{\Delta R(t)}{b_1} + \frac{\varepsilon_d(t)}{\Psi} \; .$$

Under general reserve management policies, the model can then be written as:

$$29) \qquad Y(t) \;=\; \frac{a}{\Psi} \, \varepsilon_d(t)$$

$$30) \qquad \Delta R(t) \;=\; \Delta R(t)$$

$$31) \qquad \Delta p(t) \;=\; \frac{1}{b_1} (\Delta R(t) - \Delta R(t-1)) + \frac{1}{\Psi} (\varepsilon_d(t) - \varepsilon_d(t-1)).$$

Note that the economy is still insulated from foreign disturbances.

The first-order representation of this system with serially independent disturbances is:

$$
32)\quad
\begin{bmatrix} Y(t) \\ \Delta R(t) \\ \Delta p(t) \\ \frac{a}{\psi}\varepsilon_d(t) \end{bmatrix}
=
\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -\frac{1}{b_1} & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} Y(t-1) \\ \Delta R(t-1) \\ \Delta p(t-1) \\ \frac{a}{\psi}\varepsilon_d(t-1) \end{bmatrix}
+
\begin{bmatrix} 0 \\ 1 \\ \frac{1}{b_1} \\ 0 \end{bmatrix}
\Delta R(t)
+
\begin{bmatrix} \frac{a}{\psi}\varepsilon_d(t) \\ 0 \\ \frac{1}{\psi}\varepsilon_d(t) \\ \frac{a}{\psi}\varepsilon_d(t) \end{bmatrix} .
$$

The objective functional is: minimize

$$
33)\quad E_o \sum_{t=1}^{T}
\begin{bmatrix} Y(t) \\ \Delta R(t) \\ \Delta p(t) \\ \frac{a}{\psi}\varepsilon_d(t) \end{bmatrix}'
\begin{bmatrix} w_1 & 0 & 0 & 0 \\ 0 & w_2 & 0 & 0 \\ 0 & 0 & w_3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}
\begin{bmatrix} Y(t) \\ \Delta R(t) \\ \Delta p(t) \\ \frac{a}{\psi}\varepsilon_d(t) \end{bmatrix} .
$$

Casual inspection of equations 30) and 31) suggests that the optimal value of $\Delta R(t)$ will move positively with $\varepsilon_d(t-1)$, but that if no weight is attached to price instability $(w_3 = o)$, the optimal foreign exchange reserve management policy is the floating exchange rate. This is again confirmed by the formal solution for $\Delta R(t)$ for the last two periods. The intuitive reason is that a positive domestic demand shock in period $t-1$ will, given $\Delta R(t-1)$, cause a depreciation of the exchange rate (a rise in the price level). As $E_{t-1}\varepsilon_d(t) = o$, we would expect an appreciation of the exchange rate (a fall in the price level) during period t. The policy authority should therefore purchase foreign exchange to prevent this appreciation of the exchange rate.

$$
34a)\qquad G(T) = \begin{bmatrix} o & \dfrac{w_3}{\psi_o} & o & \dfrac{w_3\, b_1}{a\, \psi_o} \end{bmatrix}
$$

34b) $\quad g(T) = 0$

i.e.

35) $\quad \Delta R(T) = \dfrac{w_3}{\Psi_0} \Delta R(T-1) + \dfrac{w_3 \, b_1}{\Psi \, \Psi_0} \varepsilon_d(T-1)$

as $\quad \dfrac{\varepsilon_d(T-1)}{\Psi} = p(T-1) - m - \dfrac{\Delta R(T-1)}{b_1}$

we can rewrite this as:

35) $\quad \Delta R(t) = \dfrac{w_3 \, b_1}{\Psi_0} (p(T-1) - m) \quad ,$

35) shows that $\varepsilon_d(t)$ and $\Delta R(T)$ will move together and that $\Delta R(T) = 0$ if $w_3 = 0$.

36a) $\quad G(T-1) = \left[\; 0 \;\; \dfrac{w_3 \, \Psi_0}{\Psi_3} \;\; 0 \;\; \dfrac{w_3 \, b_1 \, 0}{a \, \Psi_3} \; \right]$

36b) $\quad g(T-1) = 0 \; ,$

i.e.,

37) $\quad \Delta R(T-1) = \dfrac{w_3 \, \Psi_0}{\Psi_3} \Delta R(T-2) + \dfrac{w_3 \, b_1 \, \Psi_0}{\Psi \, \Psi_3} \varepsilon_d(T-2) \quad ,$

as $\quad \dfrac{\varepsilon_d(T-2)}{\Psi} = p(T-2) - m - \dfrac{\Delta R(T-2)}{b_1}$, we can rewrite this as:

37') $\quad \Delta R(t-1) = \dfrac{w_3 \, b_1 \, \Psi_1}{\Psi_3} (p(T-2) - m)$

$\Psi_3 = b_1^4 \, w_2^2 + 2w_2 \, w_3 \, b_1^2 + w_2 \, w_3 \, b_1 + w_3^2 \quad .$

37) shows, as expected, that $\varepsilon_d(T-2)$ and $\Delta R(T-1)$ move together and that $\Delta R(T-1) = 0$ if $w_3 = 0$.

Non-Rational Expectations

The rational expectations assumption is central and the consequences of abandoning it are far reaching. In general, the trajectory (the probability distribution function) or real output will now be affected by the values assigned to the policy instrument. The optimal feedback control will depend on the lagged value of $Y(t)$ and on w_1, the weight attached to output instability in the objective function. A frequently used ad-hoc expectations mechanism is the following:

$$\hat{p}(t-1), \; t) = p(t-1) + v(p(t-1) - \hat{p}(t-2, \; t-1)) \; .$$

This includes as special cases, static expectations $(v = 0)$, mildly regressive expectations $(-1 < v < o)$, strongly regressive expectations $(v < -1)$, mildly extrapolative expectations $(o < v < 1)$ and strongly extrapolative expectations $(v > 1)$.

The first order representation with serially independent disturbances of the exchange rate management model is:

$$
\begin{bmatrix} Y(t) \\ \hat{p}(t,t+1) \\ \Delta R(t) \\ \Delta p(t) \\ e(t) \\ \eta_p(t) \end{bmatrix}
=
\begin{bmatrix} o & -a & o & o & o & o \\ o & -v & o & o & o & o \\ o & -a & o & o & o & o \\ o & o & o & o & -1 & -1 \\ o & o & o & o & o & o \\ o & o & o & o & o & o \end{bmatrix}
\begin{bmatrix} Y(t-1) \\ \hat{p}(t-1,t) \\ \Delta R(t-1) \\ \Delta p(t-1) \\ e(t-1) \\ \eta_p(t-1) \end{bmatrix}
+
\begin{bmatrix} a \\ 1+v \\ a+b_1 \\ 1 \\ 1 \\ o \end{bmatrix} e(t)
+
\begin{bmatrix} o \\ o \\ -b_1 m \\ o \\ o \\ o \end{bmatrix}
+
\begin{bmatrix} a\eta_p(t) \\ (1+a_1)\eta_p(t) \\ (a+b_1)\eta_p(t) - \varepsilon_d(t) \\ \eta_p(t) \\ o \\ \eta_p(t) \end{bmatrix}
\; .
$$

With $Y(t)$ depending on current and lagged values of the instrument $e(t)$, the scope for stabilization policy is greatly enhanced. With a fixed exchange rate, the only non-zero characteristic root of the system is $-v$. Static expectations and mildly regressive or extrapolative expectations are therefore necessary and sufficient for stability under a fixed exchange rate regime.

The first order representation with serially independent disturbances of the reserve management model is:

$$
\begin{bmatrix} Y(t) \\ \hat{p}(t,t+1) \\ \hat{p}(t-1,t) \\ \Delta p(t) \\ \Delta R(t) \\ \varepsilon_d(t) \end{bmatrix}
\begin{bmatrix}
0 & \frac{-ab_1}{a+b_1} & 0 & 0 & 0 & 0 \\
0 & \frac{a-vb_1}{a+b_1} & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & \frac{a}{a+b_1} & \frac{-a}{a+b_1} & 0 & \frac{-1}{a+b_1} & \frac{-1}{a+b_1} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\begin{bmatrix} Y(t-1) \\ \hat{p}(t-1,t) \\ \hat{p}(t-2,t-1) \\ \Delta p(t-1) \\ \Delta R(t-1) \\ \varepsilon_d(t-1) \end{bmatrix}
+ \begin{bmatrix} \frac{a}{a+b_1} \\ \frac{1+v}{a+b_1} \\ 0 \\ \frac{1}{a+b_1} \\ 1 \\ 0 \end{bmatrix} \Delta R(t)
+ \begin{bmatrix} \frac{ab_1 m}{a+b_1} \\ \frac{b_1(1+v)m}{a+b_1} \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}
+ \begin{bmatrix} \frac{a}{a+b_1}\varepsilon_d(t) \\ \frac{(1+v)}{a+b_1}\varepsilon_d(t) \\ 0 \\ \frac{1}{a+b_1}\varepsilon_d(t) \\ 0 \\ \varepsilon_d(t) \end{bmatrix}
$$

Again, the probability distribution of real output depends on current and lagged values of $\Delta R(t)$, enhancing the scope for active feedback stabilization policy. It is also clear that the insulation of the domestic economy from foreign disturbances under reserve management policies in the absence of international financial flows does not depend on the rational expectations hypothesis. With a freely floating exchange rate, the only non-zero characteristic root is $\frac{a-vb_1}{a+b_1}$. The necessary and sufficient condition for

stability, $|v| < 1$, is the same as under a fixed rate. Even without going through the unrewarding hard slog of deriving the expected T-period loss under fixed and freely floating exchange rates, it would seem to be clear that the introduction of ad-hoc expectations mechanisms per se will not lead one to favour the one regime over the other.

While we shall not pursue the issue of non-rational expectations any further here, it is important to be aware of the tremendous power of the rational expectations assumption in "auction market" models.[27]

Alternative Specifications of the Objective Function

The existence of non-trivial feedback control rules is not affected by substituting R-R* for ΔR in the objective function, although the form of the feedback control will of course be different.

If real absorption rather than real output is included as an argument in the objective function, the scope for active feedback stabilization policy is enhanced. With exchange rate management policies, $A(t)$ is given by:

$$A(t) = b_1 m - b_1 e(t) - b_1 n_p(t) + \varepsilon_d(t) \quad ,$$

while with reserve management policies we get:

$$A(t) = -\Delta R(t) + \frac{a}{\psi} \varepsilon_d(t) \quad .$$

In the absence of international capital mobility, domestic absorption will only depend on the current value of the controls. With international capital mobility, current absorption will depend on current and lagged values of controls, as will be shown below.

III. A MODEL WITH AN EXPLICIT DOMESTIC FINANCIAL SECTOR

In this section of the paper, three domestic sources of disturbances are considered: ε_s, supply shocks, which can be interpreted as random events affecting the production function or the labour market, ε_d, demand shocks, the random shifts of the IS curve considered in the previous section and ε_m: financial market disturbances, or random shifts of the LM curve. These disturbances are assumed to have zero means and to be mutually serially independent. The assumptions of complete sterilization of reserve flows and absence of international financial flows continue to be maintained. $i(t)$ denotes the real rate of interest.

38) $\quad Y(t) = a(p(t) - \hat{p}(t-1,t)) + \varepsilon_s(t)$ $\qquad a > o$

39) $\quad A(t) = b_1(m-p(t)) + b_2\, i(t) + \varepsilon_d(t)$ $\qquad b_1 > o;\quad b_2 < o$

40) $\quad m-p(t) = c_1\, i(t) + c_2\, Y(t) + \varepsilon_m(t)$ $\qquad c_1 < o;\quad c_2 < o$

41) $\quad p(t) = e(t) + \eta_p(t)$

42) $\quad X(t) = Y(t) - A(t)$

43) $\quad \Delta R(t) = X(t)$

Domestic absorption varies inversely with the rate of interest (equation 39, $b_2 < o$). Equation 40 is the LM curve.

Under general exchange rate management policies, the model can be summarized as follows:

$$Y(t) = a\,\eta_p(t) + \varepsilon_s(t)\ ,$$

$$\Delta R(t) = -\Psi_4\, m + \Psi_4\, e(t) + \Psi_5\varepsilon_s(t) - \varepsilon_d(t) + \frac{b_1}{c_1}\varepsilon_m(t) + \Psi_6\,\eta_p(t)$$

$$\Delta p(t) = e(t) - e(t-1) + \eta_p(t) - \eta_p(t-1)\ .$$

The fixed exchange rate model is the special case when $e(t) = e(t-1) = \bar{e}$. [28/]

Comparing this model with the general exchange rate management model in the absence of an explicit domestic financial sector (20-22), we see that here too the authorities' knowledge of $\eta_p(t-1)$ when they decide on $e(t)$ permits them to improve on the fixed exchange rate policy.

To obtain the model for general reserve management policies, we solve the equation $\Delta R(t) = Y(t) - A(t)$ for $\hat{p}(t-1, t)$ and $p(t)$, obtaining:

$$\hat{p}(t-1, t) = m + \frac{\Delta R(t)}{\Psi_4}$$

$$p(t) = m + \frac{\Delta R(t)}{\Psi_4} - \frac{\Psi_5}{\Psi_6} \varepsilon_s(t) + \frac{1}{\Psi_6} \varepsilon_d(t) - \frac{b_2/c_1}{\Psi_6} \varepsilon_m(t) \ .$$

This permits us to summarize the model as:

$$Y(t) = \frac{\Psi_4}{\Psi_6} \varepsilon_s(t) - \frac{ab_2/c_1}{\Psi_6} \varepsilon_m(t) + \frac{a}{\Psi_6} \varepsilon_d(t)$$

$$\Delta R(t) = \Delta R(t)$$

$$p(t) = \frac{1}{\Psi_4} (\Delta R(t) - \Delta R(t-1)) - \frac{\Psi_5(\varepsilon_s(t)-\varepsilon_s(t-1))}{\Psi_6} + \frac{\varepsilon_d(t)-\varepsilon_d(t-1)}{\Psi_6}$$
$$- \frac{b_2/c_1(\varepsilon_m(t)-\varepsilon_m(t-1))}{\Psi_6}$$

The freely floating exchange rate model is the special case when $\Delta R(t) = \Delta R(t-1) = 0$. With any reserve management policy, the economy is insulated from foreign disturbances, in the absence of international capital flows. Comparing this model with the reserve management model without an

explicit domestic financial sector (equations 29-31), we can conclude that because $\varepsilon_s(t-1)$, $\varepsilon_d(t-1)$ and $\varepsilon_m(t-1)$ are known to the policy authority when it decides on $\Delta R(t)$, a closed loop (feedback) control rule that permits such new information to be taken into account, will dominate the freely floating exchange rate policy.[29]/

In the final section of this paper, I shall deal with a model in which there are international financial flows. The special case considered is that of "perfect capital mobility": there exists a foreign asset that is a perfect substitute in private portfolios for a domestic bond.

IV. A MODEL WITH INTERNATIONAL FINANCIAL FLOWS

Equations 44) - 51) represent the model under "perfect capital mobility".

44) $\quad Y(t) = a(p(t) - \hat{p}(t-1, t)) + \varepsilon_s(t)$ $\qquad a > 0$

45) $\quad A(t) = b_1(m-p(t)) + b_2 B(t) + b_3 i(t) + \varepsilon_d(t)$ $\qquad b_1 > 0; \quad b_2 > 0; \quad b_3 < 0$

46) $\quad m-p(t) = c_1 i(t) + c_2 Y(t) + c_3 B(t) + \varepsilon_m(t)$ $\qquad c_1 < 0; \quad c_2 > 0; \quad c_3 > 0$

47) $\quad X(t) = Y(t) - A(t)$.

48) $\quad p(t) = e(t) + n_p(t)$

49) $\quad i(t) = \bar{i}^* + n_f(t) + \hat{e}(t, t+1) - e(t) - (\hat{p}(t, t+1) - p(t))$

50) $\quad \Delta R(t) = X(t) + rB(t-1) - \Delta B(t)$ $\qquad r > 0 \quad$ [30]/

51a) $\quad \hat{p}(t-1, t) = E_{t-1}(p(t) | I(t-1))$

51b) $\quad \hat{p}(t, t+1) = E_t(p(t+1) | I(t))$

51c) $\quad \hat{e}(t, t+1) = E_t(e(t+1) | I(t))$.

Domestic bonds are real-valued short bonds. $B(t)$ denotes net domestic claims on the rest of the world by the private sector. $i(t)$ is the real rate of return on these bonds. $i^*(t)$ is the exogenously given world rate of return, in terms of foreign exchange, on a claim that is a perfect substitute in private portfolios for the real-valued bond. For simplicity we assume that the domestic private sector does not hold claims denominated in foreign exchange. The rest of the world holds the mixed portfolio that permits it to conduct the arbitrage operations reflected in the international interest arbitrage condition 49). $i^*(t) = \bar{i}^* + \eta_f(t)$. $\eta_f(t)$ is a foreign financial disturbance. All disturbances, the three domestic ones, $\varepsilon_s(t)$, $\varepsilon_d(t)$ and $\varepsilon_m(t)$ and the two foreign ones, $\eta_p(t)$ and $\eta_f(t)$ have zero means and are mutually serially independent.

(50) states that the official financing balance is the sum of the current account $X(t) + rB(t)$ and the capital account $-\Delta B(t)$. $rB(t)$, net interest income from abroad, should be $i(t) B(t)$. To preserve linearity, $i(t)$ is replaced by the constant r. r can be taken to be positive; $r = i^*$ might be an attractive compromise. The substitution of r for i is just a fudge for technical reasons, but could be "justified" by postulating a sufficiently bizarre policy of official transfers and remittances.

Under rational expectations, $\hat{p}(t, t+1) = E_t(e(t+1) + \eta_p(t+1)) = E_t e(t+1)$. Equation 49) therefore reduces to:

49) $\qquad i(t) = \bar{i}^* + \eta_f(t) + \eta_p(t)$.

The interest parity condition 49), which holds under both exchange rate management and reserve management policies, makes it clear that with international mobility of financial claims, reserve stock management policies (including a freely floating exchange rate) no longer insulate the domestic economy from foreign disturbances. Not only foreign interest rate disturbances,

but foreign price disturbances as well are transmitted to the domestic economy. The reason is that a random increase in the world price level, $\eta_p(t) > 0$, will raise the current domestic price level relative to the current exchange rate, as $p(t) = e(t) + \eta_p(t)$. Since these foreign price disturbances are serially uncorrelated with mean zero, next period's expected price level will be lower, relative to next period's expected exchange rate, than the current price level is relative to the current exchange rate $(\hat{p}(t, t+1) = \hat{e}(t, t+1))$. To satisfy the interest parity condition, which can be re-written as

$$i(t) + (\hat{p}(t, t+1) - \hat{e}(t, t+1)) - (p(t) - e(t)) = \bar{i}^* + \eta_f(t) \quad,$$

the domestic real rate of interest will have to increase. This will affect both real output and the rate of inflation under reserve management policies.

The trade account surplus under exchange rate management policies is given by:

52a)
$$X(t) = Y(t) - A(t) = (\frac{b_2 c_1}{c_3} - b_3) \bar{i}^* - (b_1 + \frac{b_2}{c_3}) m + (b_1 + \frac{b_2}{c_3}) e(t)$$

$$+ (1 + \frac{b_2 c_2}{c_3}) \varepsilon_s(t) - \varepsilon_d(t) + \frac{b_2}{c_3} \varepsilon_m(t) + [a + b_1 - b_3 + \frac{b_2}{c_3} \Psi_7] \eta_p(t)$$

$$+ (\frac{b_2 c_1}{c_3} - b_3) \eta_f(t)$$

$$\Psi_7 = 1 + c_1 + ac_2 \quad .$$

The current account surplus is given by:

52b) $\quad X(t) + rB(t) = [(b_2 - r)\dfrac{c_1}{c_3} - b_3]\bar{i}^* - (b_1 + \dfrac{b_2 - r}{c_3})m + (b_1 + \dfrac{b_2}{c_3})\,e(t)$

$\qquad - \dfrac{r}{c_3}\,e(t-1) + (1 + \dfrac{b_2 c_2}{c_3})\,\varepsilon_s(t) - r\dfrac{c_2}{c_3}\,\varepsilon_s(t-1) - \varepsilon_d(t) + \dfrac{b_2}{b_3}\,\varepsilon_m(t)$

$\qquad - \dfrac{r}{c_3}\,\varepsilon_m(t-1) + [a + b_1 - b_3 + \dfrac{b_2}{c_3}\,\Psi_7]\,\eta_p(t) - \dfrac{r}{c_3}\,\Psi_7\,\eta_p(t-1)$

$\qquad + (\dfrac{b_2 c_1}{c_3} - b_3)\,\eta_f(t) - r\dfrac{c_1}{c_3}\,\eta_f(t-1).$

The capital account surplus is given by:

52c) $\quad -\Delta B(t) = \dfrac{1}{c_3}\,e(t) - \dfrac{1}{c_3}\,e(t-1) + \dfrac{\Psi_7}{c_3}\,\eta_p(t) - \dfrac{\Psi_7}{c_3}\,\eta_p(t-1) + \dfrac{c_1}{c_3}\,\eta_f(t)$

$\qquad - \dfrac{c_1}{c_3}\,\eta_f(t-1) + \dfrac{c_2}{c_3}\,\varepsilon_s(t) - \dfrac{c_2}{c_3}\,\varepsilon_s(t-1) + \dfrac{\varepsilon_m(t)}{c_3} - \dfrac{\varepsilon_m(t-1)}{c_3}\,.$

The overall balance of payment surplus is $\Delta R(t) = X(t) + rB(t) - \Delta B(t)$. Under general exchange rate management policies, the model with perfect international capital mobility can be summarized as follows:

53) $\quad Y(t) = a\eta_p(t) + \varepsilon_s(t)$

54) $\quad \Delta R(t) = [(b_2 - r)\dfrac{c_1}{c_3} - b_3]\bar{i}^* - (b_1 + \dfrac{b_2 - r}{c_r})m + (b_1 + \dfrac{1 + b_2}{c_3})e(t) - \dfrac{(1+r)}{c_3}\,e(t-1)$

$\qquad + (1 + \dfrac{(1+b_2)}{c_3}\,c_2)\,\varepsilon_s(t) - \dfrac{(c_2+r)}{c_3}\,\varepsilon_s(t) - \varepsilon_d(t) + \dfrac{(1+b_2)}{c_3}\,\varepsilon_m(t)$

$\qquad - \dfrac{(1+r)}{c_3}\,\varepsilon_m(t-1) + [a + b_1 - b_3 + \dfrac{(1+b_2)}{c_3}\,\Psi_7]\,\eta_p(t) - \dfrac{(1+r)\,\Psi_7}{c_3}\,\eta_p(t-1)$

$\qquad + [\dfrac{(1+b_2)}{c_3}\,c_1 - b_3]\,\eta_f(t) - (1+r)\dfrac{c_1}{c_3}\,\eta_f(t-1)$

55) $\quad \Delta p(t) = e(t) - e(t-1) + \eta_p(t) - \eta_p(t-1)$.

If current account equilibrium is considered a more appropriate long run policy objective than equilibrium in the official financing account, 52b) could be substituted for 54).

The fixed exchange rate is the special case when $e(t) = \bar{e}$ for all t. The optimal fixed exchange rate is given by

$$\bar{e} = m - [\frac{(b_2-r)c_1-b_3c_3}{b_1c_3+b_2-r}] \bar{i}^*$$.

With international capital mobility, a change in the exchange rate affects the external equilibrium in the current period and, by changing net private sector claims on the rest of the world, also in the next period. Without international financial flows, an "active" feedback stabilization policy can only stabilize the price level and does so at the expense of some increase in the degree of external imbalance. With international financial flows, "active" stabilization policy will be optimal even if we assign zero weight to stabilization of the price level.

Real output again cannot be stabilized. It is shown in Appendix I that when we introduce durable capital into the model and make output dependent on the stock of capital, the probability distribution of real output will be affected by the value assumed by the exchange rate if there is no perfect international capital mobility. Through sterilization policies and exchange rate management, the government can determine the real quantity of money and thus the interest rate and the volume of investment. With perfect international capital mobility, the real interest rate is not under the control of the policy authority.

With general reserve management policies, we use the identity

$$\Delta R(t) = X(t) + rB(t-1) - \Delta B(t)$$

to solve for $p(t)$ and $\hat{p}(t-1, t)$. This gives us:

$$\hat{p}(t-1), t) = m + \frac{\Delta R(t)}{\Psi_8} + \frac{-\Psi_9}{\Psi_8} \bar{i}^* - \frac{(1+r)}{\Psi_8} B(t-1)$$

and

$$p(t) = m + \frac{\Delta R(t)}{\Psi_8} + (\frac{-\Psi_9}{\Psi_8})\bar{i}^* - \frac{(1+r)}{\Psi_8} B(t-1) - [\frac{\Psi_{10}}{\Psi_{11}}] \varepsilon_s(t) + \frac{1}{\Psi_{11}} \varepsilon_d(t)$$

$$- \frac{(b_2+1)}{\Psi_{11}c_3} \varepsilon_m(t) - \frac{\Psi_9}{\Psi_{11}} \eta_p(t) - \frac{\Psi_9}{\Psi_{11}} \eta_f(t)$$

where

$$\Psi_8 = b_1 + \frac{b_2+1}{c_3} \; ; \quad \Psi_9 = (b_2+1) \frac{c_1}{c_3} - b_3; \quad \Psi_{10} = 1 + \frac{(b_2+1)}{c_3} c_2 \; ;$$

$$\Psi_{11} = a+b_1 + \frac{(b_2+1)}{c_3} (1+c_2 a) \quad .$$

The behaviour of the model is summarized in equations 56) - 59).

56) $$Y(t) = (1 - \frac{a\Psi_{10}}{\Psi_{11}}) \varepsilon_s(t) + \frac{a}{\Psi_{11}} \varepsilon_d(t) - \frac{a(b_2+1)}{\Psi_{11}c_3} \varepsilon_m(t)$$

$$- \frac{a\Psi_9}{\Psi_{11}} \eta_p(t) - \frac{\Psi_9}{\Psi_{11}} \eta_f(t)$$

57) $$\Delta R(t) = \Delta R(t)$$

58) $$\Delta p(t) = \frac{\Delta R(t)}{\Psi_8} - \frac{\Delta R(t-1)}{\Psi_8} - \frac{(1+r)}{\Psi_8} \Delta B(t-1) - \frac{\Psi_{10}}{\Psi_{11}} \varepsilon_s(t) + \frac{1}{\Psi_{11}} \varepsilon_d(t)$$

$$- \frac{(b_2+1)}{\Psi_{11}c_3} \varepsilon_m(t) - \frac{\Psi_9}{\Psi_{11}} \eta_p(t) - \frac{\Psi_9}{\Psi_{11}} \eta_f(t) + \frac{\Psi_{10}}{\Psi_{11}} \varepsilon_s(t-1) - \frac{1}{\Psi_{11}} \varepsilon_d(t-1)$$

$$+ \frac{(b_2+1)}{\Psi_{11}c_3} \varepsilon_m(t-1) + \frac{\Psi_9}{\Psi_{11}} \eta_p(t-1) + \frac{\Psi_9}{\Psi_{11}} \eta_f(t-1)$$

59) $\Delta B(t) = -\frac{1}{c_3} \Delta p(t) - \frac{c_2}{c_3} Y(t) + \frac{c_2}{c_3} Y(t-1) - \frac{c_1}{c_3} \eta_p(t) - \frac{c_1}{c_3} \eta_f(t) - \frac{1}{c_3} \varepsilon_m(t)$

$$+ \frac{c_1}{c_3} \eta_p(t-1) + \frac{c_1}{c_3} \eta_f(t-1) + \frac{1}{c_3} \varepsilon_m(t-1) \; .$$

We can substitute from equation 56) for $Y(t)$ and $Y(t-1)$ and from equation 58) for $\Delta p(t)$ to obtain $\Delta B(t)$ exclusively in terms of its own lagged value and current and lagged values of the disturbances and the instruments.

Both foreign disturbances, $\eta_p(t)$ and $\eta_f(t)$ will affect the domestic economy under reserve management policies (including the freely floating rate given by $\Delta R(t) = 0$, for all t), when there is international capital mobility. The insulation provided by reserve management is gone, for better or worse.

Conclusion

The scope for active feedback stabilization policies is enhanced when the extent to which past events affect present and future states of the economy is increased by the existence of international capital market integration. International financial integration permits foreign exchange market intervention to have a favourable effect on external stability. In its absence, the only reason for not sticking to open-loop rules is price stabilization. Real output continues to be unaffected by the values assigned to the controls but real absorption depends on both current and lagged values of the controls (see equations 45, 58 and 59). The assumption of complete sterilization of reserve flows, maintained throughout the paper, is one of convenience only. In Appendix I, our qualitative results as regards open-loop versus closed-loop policies are confirmed for a model with no reserve flow sterilization.

APPENDIX I

When there is durable capital in the model, the conclusion that the probability distribution function of output is independent of the values assigned to the policy instruments no longer holds. Consider the model given by equation 38 - 43, modified to include durable reproducible capital. $K(t)$ denotes the stock of capital at the beginning of period t, $Z(t)$ denotes net investment and $C(t)$, consumption.

$$Y(t) = a_1(p(t) - \hat{p}(t-1, t)) + a_2 K(t) + \varepsilon_s(t); \quad a_1 > 0 \quad a_2 > 0$$

$$A(t) \equiv C(t) + Z(t) + \varepsilon_d(t)$$

$$C(t) = b_0 + b_1(m - p(t)) \qquad\qquad\qquad b_0 > 0 \quad b_1 > 0$$

$$Z(t) = z_0 + z_1 i(t) \qquad\qquad\qquad\qquad z_0 > 0 \quad z_1 < 0$$

$$m - p(t) = c_1 i(t) + c_2 Y(t) + \varepsilon_m(t) \qquad\qquad c_1 < 0 \quad c_2 > 0$$

$$p(t) = e(t) + \eta_p(t)$$

$$X(t) = Y(t) - A(t)$$

$$\Delta R(t) = Y(t)$$

$$K(t) = K(t-1) + Z(t-1)$$

Under an exchange rate management regime

$$Y(t) = a_1 \eta_p(t) + a_2 K(t) + \varepsilon_s(t) \quad .$$

Since $\quad K(t) = K(t-1) + z_0 + z_1 i(t-1)$

and

$$i(t-1) = \frac{m}{c_1} - \frac{1}{c_1} e(t-1) - \frac{(1+c_2 a_1)}{c_1} \eta_p(t-1) - \frac{1}{c_1} \varepsilon_m(t-1) - \frac{c_2}{c_1} \varepsilon_s(t-1)$$

$$- \frac{c_2}{c_1} a_2 K(t-1) \quad ,$$

it is clear that the probability distribution function of the stock of capital depends on the lagged values of the exchange rate. Changes in $e(t)$ and the associated changes in $p(t)$ will change the real quantity of money (given m), the rate of interest and the rate of investment. Current output is not affected by the current choice of $e(t)$, but next period's capital stock and level of real output will be affected. Similar conclusions can be drawn for reserve management policies. The reason for this result, which contrasts with those obtained for a closed economy by e.g. Barro (1976), is that through sterilization and exchange market operations the policy authority can control the real quantity of money. In an economy without perfect international mobility of capital, this permits the authorities to influence the real rate of interest. With perfect capital mobility, $i(t) = \bar{i}^* + \eta_f t) + \eta_p(t)$ and the behaviour of the capital stock cannot be affected by the policy authority.

APPENDIX II

If reserve flows are not sterilized, we can represent the simplest model by the following equations: (It is convenient to represent the external balance target in terms of $\Delta m(t)$, as there is no domestic credit creation.)

$$Y(t) = a(p(t) - \hat{p}(t-1, t)) \qquad\qquad a > 0$$

$$\Delta m(t) = d_1(p(t) - \hat{p}(t-1, t)) + d_2(m(t) - p(t) - \varepsilon_d(t)) \qquad d_1 > 0; \quad d_2 < 0$$

$$p(t) = e(t) + \eta_p(t)$$

$$m(t) = m(t-1) + \Delta m(t-1)$$

$$\min E_0 \sum_{t=1}^{T} [w_1(Y(t))^2 + w_2(\Delta m(t))^2 + w_3(\Delta p(t))^2]$$

With general exchange rate management policies, this can be summarized as:

$$Y(t) = a\eta_p(t)$$

$$\Delta m(t) = (d_1 - d_2)\eta_p(t) - d_2 e(t) - \varepsilon_d(t) + d_2 \sum_{i=0}^{t-2} (1 + d_2)^i (d_1-d_2)\eta_p(t-i-1)$$

$$- d_2 e(t-i-1) - \varepsilon_d(t-i-1)] + d_2(1 + d_2)^t m_0$$

$$+ d_2(1 + d_2)^{t-1} (d_0 - d_2)(\eta_p(0) - \varepsilon_d(0)) \ .$$

$$\Delta p(t) = e(t) - e(t-1) + \eta_p(t) - \eta_p(t-1) \ .$$

The current external deficit or surplus depends on current and lagged values of the disturbances and the exchange rate, and on the initial condition, m_0 . "Active" feedback controls will again be optimal.

With general reserve management policies, or more precisely, general money management policies the model becomes

(using $p(t-1, t) = -\frac{1}{d_2} m(t+1) + (1 + \frac{1}{d_2}) m(t)$ and

$$p(t) = -\frac{1}{d_2} m(t+1) + (1 + \frac{1}{d_2}) m(t) + \frac{\varepsilon_d(t)}{d_1 - d_2}):$$

$$Y(t) = \frac{a}{d_1 - d_2} \varepsilon_d(t)$$

$$\Delta m(t) = \Delta m(t)$$

$$\Delta p(t) = -\frac{1}{d_2} \Delta m(t) + (1 + \frac{1}{d_2}) \Delta m(t-1) + \frac{\varepsilon_d(t)}{d_1 - d_2} - \frac{\varepsilon_d(t-1)}{d_1 - d_2} .$$

As in the case of complete sterilization of reserve flows, active feedback control can be used to stabilize the price level. With international capital mobility, both the price level and the external deficit can be stabilized.

FOOTNOTES

1. Some well-known references are: Fellner (1966), Friedman (1953), and Meade (1955). Some recent contributions are Kindleberger, (1972), Argy and Porter (1972), Fischer (1974) and Turnovsky (1976).

2. This "prices versus quantities" approach was first developed formally by Weitzman (1974).

3. The official stock of foreign exchange reserves must be adequate to meet any likely demand on it.

4. Chow (1975) and Aoki (1976).

5. Friedman (1968).

6. Chow (1975), Cooper and Fischer (1972a), (1972b), (1972c), B. Friedman (1973), Turnovsky (1975), Pindyck (1973).

7. "Available" is unsatisfactorily vague. It can only be defined with reference to the costs of and uncertain return to gathering additional information.

8. See Muth (1961), Lucas (1972a), (1972b), (1973), (1975), (1976), Sargent and Wallace (1975, 1976), Sargent (1976), Gordon (1976), Barro (1976), Barro and Fischer (1976), and Poole (1976).

9. Taylor (1975).

10. See Black (1974), Taylor (1977).

11. Baily (1974), (1976), Azariadis (1975), D.F. Gordon (1974).

12. Both inside and outside policy lags must be taken into account. I am indebted to William Branson for this point.

13. See Modigliani and Miller (1958).

14. For a useful survey of the theory of international financial integration, see Kenen (1976). The paper considers the case in which there exists a foreign asset that is a perfect substitute in private portfolios for the domestic asset, i.e. "perfect" international capital mobility.

15. See Levin (1977).

16. Government interest-bearing debt is assumed not to be part of perceived private sector net worth.

17. Public sector holdings of foreign exchange reserves are assumed not to be part of perceived private sector net worth.

18. The supposed advantage of floating over fixed exchange rates in restoring the power of the domestic monetary authority to influence the domestic level of economic activity is not analyzed in this model, which treats all policy instruments other than the foreign exchange market intervention instrument, as constant.

19. $\Delta p(t) = p(t) - p(t-1)$.

20. See Phelps (1970) and Friedman (1968).

21. More complicated mixed moving average- autoregressive schemes for the disturbances could be incorporated at the cost of greater notational and arithmetic complexity without altering any of the main conclusions. Consider, e.g., the simple first-order autoregressive scheme:

$$\eta_p(t) = \rho\eta_p(t-1) + v(t); \quad Ev(t) = 0; \quad Ev(t)\,v(s) = \begin{cases} \sigma_v^2 & \text{if } t = s \\ 0 & \text{if } t \neq s \end{cases}.$$

In this case, $E_{t-1}\,\eta_p(t) = \rho_{\eta_p}(t-1)$ if $\eta_p(t-1)$ is known at $t-1$, or $\rho_{\eta_p}^2(t-2)$ if $\eta_p(t-1)$ is not known, but $\eta_p(t-2)$ is, etc. ($v(t)$ is assumed to be independent of all other disturbances).

22. B. Friedman (1972).

23. Output will of course also be affected by domestic supply disturbances, if there are any. See Section III.

24. See Chow (1975).

25. Chow (1975) also treats the case where $A(t)$ and $C(t)$ are stochastic.

26. For a discussion of the existence of uniqueness of this solution, see Garbade (1976).

27. The term is from Okun (1975).

28. A detailed discussion of the behaviour of the model under fixed and freely floating exchange rates is available from the author on request.

29. The explicit solution is omitted for reasons of space.

30. $\Delta B(t) \equiv B(t) - B(t-1)$. Note that $B(t)$ is an endogenous variable in period t. Under exchange rate management policies, it adjusts instantaneously to clear the money market. If we substitute $rB(t)$ for $rB(t-1)$ in (54), nothing essential changes.

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