

UNEMPLOYMENT-INFLATION TRADE-OFFS
WITH
RATIONAL EXPECTATIONS IN AN OPEN ECONOMY

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Abstract

The paper established the existence of unemployment-inflation trade-offs with rational expectations in an open economy. In general, instantaneous competitive market clearing is required in addition to rational expectations for the unemployment rate to be independent of stabilization policy. By abandoning this "efficient auction market hypothesis" for the labour market and the domestic output market and by giving the home country some control over the world price of its exportable, it is possible to "buy" domestic inflation at a rate below the world rate of inflation by keeping the unemployment rate above the natural rate, or domestic unemployment below the natural rate by accepting a rate of inflation in excess of the world rate of inflation. The scope for stabilization policy to influence the internal and external targets [unemployment, inflation and balance of payment equilibrium] is characterized using the concepts of dynamic controllability and perfect dynamic controllability.

1) Introduction

During the past few years application of the concept of rational expectations to the area of stabilization policy has appeared to yield far-reaching implications.¹ Deterministic monetary (and fiscal²) policy rules have been argued to have no effect on the density functions of such real variables as output, employment and unemployment. In particular, there are no inflation-unemployment trade-offs, even in the short run, that can be exploited by demand management policies. The shakiness of the foundations of these new-fangled classical invariance theorems asserting the irrelevance of deterministic stabilization rules for output and employment has begun to be exposed in recent contributions by Fischer [1977], Phelps and Taylor [1977] and Taylor [1977]. The first two papers made the point that even if the same (correct) information set is available to both private and public economic agents, differences in private and public opportunity sets can generate scope for beneficial (or detrimental) deterministic monetary feedback policies. The specific examples that were analyzed were quite simple and involved multi-period money wage or price setting combined with a monetary policy instrument that could be adjusted each period. Realistic generalizations seem quite feasible, however. As long as the multi-period private contracts are not complete contingent forward contracts that turn out to be isomorphic to the sequence of contracts that would be concluded were all markets to re-open each period, non-trivial deterministic optimal stabilization rules in feedback form can be designed. The Taylor paper pointed out some conceptual problems associated with non-uniqueness of the rational expectations solution. None of these papers, however, brings out with

sufficient emphasis the second fundamental feature of the rational expectations literature models, a feature which is as essential for the "invariance theorems" as the rational expectations assumption itself. This second property I shall refer to as the efficient auction markets hypothesis. By this I mean the hypothesis that in each period (at each point in time in the continuous case), all market prices instantaneously assume the values required for full temporary competitive equilibrium. Given the endowments inherited from the past, given tastes and technology and given expectations about the future, utility maximizing households and profit maximizing firms determine notional demand and supply correspondences by acting as if, at the prevailing market prices, they can buy or sell any quantity of any good or service. Prices then always assume the values required to equate these notional demands and supplies in all markets. A more concise way of putting this is that the rational expectations literature considers only "flex-price"³ sequential competitive equilibrium models. In this category I include the stochastic-search-equilibrium-natural-rate models of Friedman [1968], Phelps [1970] and Lucas.

The consequence of modeling an economic system as an efficient auction market model is that stabilization policy changes can affect output and unemployment only to the extent that they are unanticipated. Let S_t denote the vector of values assumed by the stabilization policy instruments at time t . Efficient auction market models can always be rewritten in such a way that real variables are affected by stabilization policy only via terms like $S_t - {}_{t-1}S_t^e$, where ${}_{t-1}S_t^e$ denotes the anticipation formed at $t-1$ of the value of S at t . The consequence of adopting the rational expectations hypothesis is that ${}_{t-1}S_t^e = S_t + {}_{t-1}\epsilon_t$.

ε_t is the random forecast error that results when S_t^e is an optimal predictor of S_t , conditional on the information set available at $t-i: \phi_{t-i}$. The information set conditioning the forecast at $t-i$ is typically assumed to be the same for both public and private economic agents. It contains the correct model structure, including any deterministic policy rules adopted by the policy authorities. In the log-linear models that represent the bulk of the literature, unbiased (least squares) predictors are taken to be optimal; thus $S_t^e = E_{t-i}(S_t | \phi_{t-i})$, where E_{t-i} denotes the mathematical expectation operator as of $t-i$. As ε_t is the residual from an optimal forecast, it will be orthogonal to the information set conditioning that forecast; in particular it will be stochastically independent of the information contained in ϕ_{t-i} that determines the deterministic policy rules pursued by the policy authorities in periods $t-j, j \geq i$. With real variables influenced by stabilization policy only via one-period forecast errors of the values assumed by the policy instruments: $S_t - S_{t-1}^e, \dots, S_{t-j} - S_{t-j-1}^e, \dots$, etc. and with S_t set in a deterministic manner as a function of the elements of ϕ_{t-1} , and S_{t-j} as a function of the elements of ϕ_{t-j-1} etc., policy will only influence real variables via a series of independent forecast errors if expectations are formed rationally.⁴ The consequence therefore of combining the rational expectations hypothesis with the efficient auction market hypothesis is that real variables are no longer subject to any systematic influence from deterministic stabilization policy rules and by implication that there are no inflation-unemployment trade-offs offering a menu for policy choice, even in the short run.

There are many valid objections to the strong notion of rational expectations first advanced by Muth [1961]. I have listed and analyzed some of these elsewhere (Buiter [1977c]). In this paper I shall stay with

the strong notion of rational expectations--the formal model will in fact be a deterministic perfect foresight model--but I shall shy away from the efficient auction markets hypothesis for the labour market and the market for domestic output. The money wage and the price of domestic output do not at each point in time assume the values required to equate notional demand and supply in the labour market and output market.

Instead familiar and not implausible ad-hoc, sluggish wage and price adjustment functions are used to describe the dynamics of wages and prices over time. In a closed economy these disequilibrium specifications do not imply the existence of unemployment-inflation trade-offs when they are combined with rational expectations. In the open economy model the reverse is the case. The only exception occurs--not surprisingly--when one assumes a labour market disequilibrium adjustment mechanism that turns the model into a "natural rate with search" model à la Phelps, i.e. something equivalent to an efficient auction market model.⁵ The analysis of the scope for monetary and fiscal policy to influence the unemployment rate and other target variables like the inflation rate and the external deficit is organized around the concepts of controllability and perfect controllability.

Section 2 of the paper develops a wage-price sector for an open economy and establishes the existence of a short-run unemployment-inflation trade off under a fixed exchange rate. In Section 3 an IS-LM sector is added to complete the description of the momentary equilibrium of the system. The addition of asset dynamics via the open economy public sector financing identity and the investment function completes the dynamic specification of the model. Section 4 introduces the concept of dynamic controllability as a useful characterization of the scope for

economic policy in terms of the ability of the policy authority to achieve certain target states and to track certain target trajectories. This comparatively new tool of controllability analysis is then applied to the model. Section 5 looks at steady state behaviour under a fixed exchange rate. The existence of unemployment-inflation trade-offs under a freely floating exchange rate is discussed in Section 6.

2) The Wage-Price Sector of the Model.

The paper establishes the existence of unemployment-inflation trade-offs in an open economy, when expectations are formed rationally in the sense of being optimal forecasts using the correct structure of the economy.⁶ As the model does not explicitly incorporate uncertainty, rational expectations is equivalent to perfect foresight. The possibility of keeping the unemployment rate systematically above or below the fixed natural rate is most easily established under a fixed exchange rate regime and we shall concentrate on this case. Two things are required. First, there should be output differentiation between trading countries. Second, the home country should be able to influence the world price of at least one of the goods that it produces or consumes. With a freely floating exchange rate the analysis becomes less simple, but here too the unemployment rate can be made to diverge systematically from the natural rate.

The paper models domestic output (the exportable) as different from imported goods and assumes the home country to be "large" in the market for its exportable. The rate of change of the price of domestic output can therefore be systematically different from the rate of change of the price of imported commodities. Our result holds whether imports are final goods, imported intermediate inputs or both. For reasons of space only the intermediate import case is modeled here. Of course a systematic

divergence between the rate of change of the price of domestic output and the rate of change of import prices will lead to a growing deterioration or improvement in the terms of trade. This is inconsistent with the economy being in steady-state or long-run equilibrium. Our proposition concerning the existence of unemployment-inflation trade-offs and the possibility of sustained divergence of the actual rate of unemployment from the natural rate is therefore applicable to extended intervals of "real" i.e. calendar time, but not the timeless long run of the steady state. When we compare true long-run, steady state equilibria [sequences of momentary or short run equilibria in which expectations are realized and real stock-flow and stock-stock ratios are invariant over time] our model exhibits all the classical invariant natural rate properties that are associated with the balanced expansion paths of a traditional growth model.⁷

In order to demonstrate the existence of unemployment-inflation trade-offs with rational expectations as clearly as possible, a very simple model is presented. Labour force growth, technical change and international exchange of financial claims other than official reserves are abstracted from.⁸

There is one domestically produced commodity, with domestic currency price p_1 , which can either be consumed domestically [by either the public sector or the private sector], used as a capital good by the private sector or exported. There are no non-traded goods. The country specializes completely in the production of its exportable.

We only consider the case in which the imported commodity is an intermediate input into the production process for domestic output. Q denotes the volume of domestic output and M the volume of imports. Domestic output is produced using inputs of labour services N , capital services K and imported intermediate inputs. The production function is given by:

1. $Q = F(K,N)$

2. $M = Q$

$F(K,N)$ is a well-behaved linear homogeneous function in K and N . There are fixed proportions between output and the imported input, with the constant of proportionality set equal to unity by choice of units.⁹ p_2^* is the foreign currency price of the imported input. e is the spot price of foreign exchange. The "law of one price" holds for internationally traded goods. The domestic currency price of intermediate inputs, p_2 , is given by $p_2 = ep_2^*$. The home country is "large" in the world market for its exportable but small in the market for its import. This means that from the point of view of the home country p_2^* is parametric while from the point of view of the rest of the world p_1 is parametric.

In what follows we shall model p_1 as given in the short run (at a point in time) rather than as a competitive market clearing price. Its proportional rate of change is a weighted average of the proportional rates of change of unit labour cost and unit import cost, with some allowance for the state of excess demand or supply in the output market. Our results would be modified if we let p_1 be determined by an instantaneous equilibrium condition in the market for domestic output, but

the existence of unemployment-inflation trade-offs is not negated as long as the labour market fails to clear instantaneously. Real value added, Y , is given by

$$3. Y = Q - \frac{P_2}{P_1} M = \left(1 - \frac{P_2}{P_1}\right) Q$$

Our choice of units requires p_2/p_1 (the terms of trade) to be less than unity for value added to be positive.

In the short run output is demand-determined. Firms produce the effective demand level of output. They do this not only when they are sales-constrained and the value of the marginal product of labour exceeds the wage rate but also when they incur a short-run loss by meeting that demand, i.e. when the wage exceeds the value of the marginal product of labour. Fear of loss of goodwill or erosion of market share can be advanced as justification for such a departure from myopic profit maximizing behaviour. The excess of the real product wage over the marginal product of labour will be used to define an index of excess demand in the product market. Excess demand will lead firms to raise the rate at which they increase the price of their output above the rate of increase of their cost and conversely for excess supply.

$$4. m = m\left(\frac{w}{p_1} - F_2\left(1 - \frac{P_2}{P_1}\right)\right) \quad m(0) = 0 \quad m' > 0$$

The proportional rate of change of the money wage rate is given by an expectations-augmented wage-Phillips curve.

$$5. \frac{\dot{w}}{w} = j(u-\bar{u}) + \left(\frac{\dot{p}_1}{p_1}\right)^e \quad j' < 0 \quad j(0) = 0$$

u denotes the actual unemployment rate and \bar{u} the exogenous natural rate. The supply of labour is inelastic with respect to the real wage. Through choice of units we can write $N = 1-u$. With perfect foresight the expected proportional rate of inflation, $\left(\frac{\dot{p}_1}{p_1}\right)^e$ equals the actual one.

$$6. \left(\frac{\dot{p}_1}{p_1}\right)^e = \frac{\dot{p}_1}{p_1}$$

Equation 7 is the excess demand-adjusted mark-up equation determining the proportional rate of change in the price of domestic output.

$$7. \frac{\dot{p}_1}{p_1} = m + \varepsilon \frac{\dot{w}}{w} + (1-\varepsilon) \frac{\dot{p}_2}{p_2} \quad 0 < \varepsilon < 1 \quad 10$$

It is important to note that in this model neither the labour market nor the market for domestic output are efficient auction markets. At any given moment therefore, both the money wage rate and the price of domestic output are predetermined. They are not permitted to make discrete, discontinuous jumps at a point in time in response to current or expected future changes in any of the parameters of the models so as to maintain equilibrium in a market defined in terms of notional demand and supply schedules.¹¹

The unemployment inflation trade-off under a fixed exchange rate regime.

The wage inflation unemployment trade-off with rational expectations is obtained by substituting 6) and 7) into 5) and using the law of one price for p_2 . This yields:

$$8. \quad \frac{\dot{w}}{w} = \frac{1}{1-\epsilon} j(u-\bar{u}) + \frac{m}{1-\epsilon} + \frac{\dot{e}}{e} + \frac{\dot{p}_2^*}{p_2^*}$$

Under a fixed exchange rate regime, $\frac{\dot{e}}{e} = 0$ and the trade-off becomes

$$8'. \quad \frac{\dot{w}}{w} = \frac{1}{1-\epsilon} j(u-\bar{u}) + \frac{m}{1-\epsilon} + \frac{\dot{p}_2^*}{p_2^*}$$

It is shown in the appendix and is indeed intuitively plausible that in steady-state equilibrium $m = 0$ and $\frac{\dot{w}}{w} = \frac{\dot{p}_2^*}{p_2^*}$. This implies that $u = \bar{u}$.

There are no steady state trade-offs between inflation and unemployment. The long run equilibrium of the model has all the classical features.

The price Phillips curve is given by:

$$9. \quad \frac{\dot{p}_1}{p_1} = \frac{\epsilon}{1-\epsilon} j(u-\bar{u}) + \frac{m}{1-\epsilon} + \frac{\dot{e}}{e} + \frac{\dot{p}_2^*}{p_2^*}$$

Under a fixed exchange rate regime this becomes

$$9'. \quad \frac{\dot{p}_1}{p_1} = \frac{\epsilon}{1-\epsilon} j(u-\bar{u}) + \frac{m}{1-\epsilon} + \frac{\dot{p}_2^*}{p_2^*}$$

With a fixed exchange rate the slope of the short-run or momentary price inflation-unemployment trade-off is:

$$10. \quad \frac{d(\dot{p}_1/p_1)}{du} = \frac{\epsilon}{1-\epsilon} j' + \frac{m' F_{22} \left(1 - e \frac{\dot{p}_2^*}{p_2^*} \right)}{1-\epsilon}$$

With $0 < \epsilon < 1$ this will be negative. Note that when the share of

intermediate imports in total variable cost increases (b decreases) the Phillips curve becomes flatter. When we consider the special case of our model corresponding to a closed economy (b=1) the trade-off disappears and the momentary Phillips curve becomes vertical. In this simple model the opportunity for lowering the unemployment rate below the natural rate at the expense of a higher rate of wage inflation-- and thus, via the price mark-up equation, at the expense of a higher domestic rate of price inflation--does not exist when labour is the only short run variable input into the production function.¹² This is an instance of general point, discussed in greater detail at the end of the paper, that whenever the price expectations terms, $(\dot{p}_1/p_1)^e$ is equivalent to a wage expectations term $(\dot{w}/w)^e$, there can be no divergence between the natural and the actual rates of unemployment with rational expectations. If one period (in our model instantaneous) expectational errors are the only cause of divergence of the actual from the natural rate (rather than sluggish price and wage behaviour), assuming away these expectational errors will enthrone the natural rate as the only possible state.¹³

Analytically the fact that p_2^* (and, under a fixed exchange rate, p_2) is parametric to the home country fulfills a role similar to that of money illusion or of a rigid (and not merely sluggish) "Keynesian" money wage. There is a money price in the model whose behaviour is entirely independent of domestic economic forces. Thus changes in the behaviour of the domestic money wage and in the money price of domestic output can have real effects. The home country can "buy" domestic inflation at a rate below the world rate by keeping the unemployment rate above the natural rate, or domestic unemployment below the natural

rate by accepting a rate of inflation above the world rate. Doing so will of course lead to continuing changes in the terms of trade which will have cumulative effects on the trade balance and on real income. A position of full, long-run equilibrium will be characterized by constant relative prices and an unemployment rate equal to the natural rate. Even if we view the natural rate as the long run unemployment objective, it may be possible for the policy authority, when some disturbance has thrown the economic system off its long-run equilibrium path, to select trajectories for its controls that will lead to an adjustment path for the unemployment rate (and for other target variables such as the inflation rate and the external surplus) that is preferred to the adjustment path generated by the passive policy of maintaining the values of the policy controls at the steady state level. This model suggests that this is indeed possible, because of the existence of a short-run non-vertical Phillips curve. Both the inflation rate and the unemployment rate are endogenous variables, however. We therefore need to complete the model and relate the endogenous variables to the exogenous variables and the policy controls before we can determine the exact scope for the traditional instruments of stabilization policy to influence the behaviour of the unemployment rate. While this paper is mainly concerned with the ability of the policy authorities to influence the unemployment rate, we shall also consider the wider issue of the government's ability to simultaneously steer all three traditional internal and external balance objectives--unemployment, inflation and the external deficit--in a desired direction. The discussion will focus on the concept of controllability. This encompasses some properties of dynamic systems that are extremely

useful as a means of characterizing the options open to the policy authority.

3) The Complete Model.

We complete the model by adding to the wage-price block summarized by equations 5 and 7 an IS curve, an LM curve and the dynamic equations describing the behaviour of asset stocks over time.

The IS curve in an open economy with only intermediate imports is given by

11. $Q = C + I + G + X$

C denotes private consumption, I private investment, G government consumption and X exports. The LM curve equates the demand for real money balances, ℓ , to the supply H/p_1 .

12. $\ell = H/p_1$.¹⁴

Private consumption demand depends on real income and real non-human wealth. Real income is after-tax value added adjusted for capital gains or losses due to changes in the purchasing power of outside money balances. Taxes consist of a proportional income tax with rate θ and a lump-sum tax τ . Real non-human wealth is the real value of the stock of outside money H/p_1 plus the market value of claims to the stock of real reproducible capital. Government interest-bearing debt, B, is not perceived as net worth by the private sector and interest paid on government debt is not perceived as part of real income because the private sector anticipates a stream of future tax liabilities of equal magnitude. The only way in which bonds enter the model is therefore through the government financing requirement (Buiter [1977a]). K is the value of the capital stock evaluated at current reproduction costs. qK is the market value of claims to the

capital stock. "Tobin's q" (Tobin [1969]) is given by:

$$13. \quad q = \left[\left(Q \left(1 - \frac{ep_2^*}{P_1} \right) - \frac{w}{P_1} (1-u) \right) (1-\theta) \right] \left(r(1-\theta) - \frac{P_1}{P_1} \right) K^{-1}$$

r is the nominal interest rate on government interest-bearing debt.

Government bonds are assumed to be perfect substitutes in private portfolios for claims on capital. Investment is an increasing function of q .

Depreciation is ignored. Export demand depends on the relative price of imports and exports. Thus

$$14. \quad C = C \left(Y(1-\theta) - \tau - \frac{P_1}{P_1} \frac{H}{P_1}, \frac{H}{P_1} + qK \right) \quad 0 < C_1 < 1; C_2 > 0$$

$$15. \quad I = I(q) \quad I' > 0; I(1) = 0$$

$$16. \quad X = X \left(\frac{ep_2^*}{P_1} \right) \quad X' > 0.$$

The demand for real money balances is a decreasing function of the real rate of return differential between bonds and money, $r(1-\theta)$ and an increasing function of net worth.

$$17. \quad \ell = \ell \left(r(1-\theta), \frac{H}{P_1} + qK \right) \quad \ell_1 < 0; 0 < \ell_2 < 1$$

In the absence of international financial flows, the external surplus, δ , equals the trade surplus.

$$18. \quad \delta = X - \frac{ep_2^*}{P_1} M$$

The asset dynamics are given in equations 19, 20 and 21

$$19. \dot{K} = I(q)$$

$$20. \frac{\dot{H}}{P_1} = \lambda \left[G + \frac{rB}{P_1} (1-\theta) - \theta Y - \tau \right] + \sigma \delta$$

$$21. \frac{\dot{B}}{P_1} = (1-\lambda) \left[G + \frac{rB}{P_1} (1-\theta) - \theta Y - \tau \right] + (1-\sigma) \delta$$

19 states that in the absence of depreciation, the rate of change of the capital stock equals the rate of investment. 20 states that a fraction λ of the current public sector financing requirement is covered by issuing high-powered money and that a fraction σ of the balance of payments surplus is not sterilized. 21 shows what happens to the remainder of the public sector financing requirement and the balance of payments surplus. This completes the description of the model.

4) The controllability of unemployment, inflation and the external deficit under a fixed exchange rate.

Consider an $m \times 1$ vector y of short run endogenous variables or, in the language of control theory, output variables. Output controllability refers to the ability of the policy authority to transfer the output variables from any initial value to any target value in specified finite time through the choice of an appropriate trajectory for its $r \times 1$ vector x of controls. "Controllability is an existence condition of a dynamic policy."¹⁵ Definition 1 gives an exact statement of output controllability. z denotes an $n \times 1$ vector of short-run exogenous but long-run endogenous or predetermined variables or in the language of control theory state variables.

Definition 1.¹⁶ A deterministic dynamic system

22a. $y = g(t, z, x)$

22b. $\dot{z} = f(t, z, x)$

is output controllable at time t_0 , if for each pair of outputs, y_0 and y_1 , there exists a feasible instrument vector $x(\cdot)$ on some finite interval $t_0 < t < t_1$ such that the system moves from y_0 at t_0 to y_1 at t_1 .

Controllability concerns the ability of policy makers to achieve certain target values at a specified point in time. In practice policy makers are likely to be interested not merely in such "point" controllability but rather in "path" controllability, i.e. their ability to make the target variables follow some prescribed trajectory over a certain

time interval. This stronger notion of controllability is referred to as perfect output controllability or functional reproducibility.

Definition 2.¹⁷ The dynamic system described by equations 22a and 22b is perfectly output controllable at t_0 if there exists a feasible instrument vector $x(\cdot)$ on some finite interval $t_1 \leq t \leq t_1 + \tau$ ($t_1 \geq t_0$; $\tau \geq 0$) such that $y(t) = y^*(t)$ for $t_1 \leq t \leq t_1 + \tau$. $y^*(t)$ is the desired trajectory of the output vector.

A perfectly controllable dynamic system is one in which the policy authorities have the capability to exactly follow or track exogenously given target paths for the output variables. Note that nothing is said about the design of the policies that will allow the authorities to track these target trajectories. (Perfect) controllability only establishes the existence of such policies. Note also that these definitions do not place any constraints on the permissible values of the instruments. Nevertheless, establishing that the unemployment rate, the inflation rate and the external deficit are (locally) controllable and even perfectly (locally) controllable is an important first step towards the design of optimal stabilization policies. Proposition 1 below states necessary and sufficient conditions for output controllability in linear, time-invariant dynamic system. Proposition 2 states sufficient conditions for perfect output controllability for such systems. These global results for truly linear systems are then "scaled down" to local results for non-linear models. This permits their application to our open economy model under a fixed exchange rate regime.

Proposition 1¹⁸

The dynamic system

$$23a. \quad y = Cz + Dx$$

$$23b. \quad \dot{z} = Az + Bx$$

where y is an m -vector, z is an n -vector, x is an r -vector and A , B , C and D are constant matrices, is output controllable i.f.f. the rank of the $m \times (r+1)n$ matrix $[CB, CAB, \dots, CA^{n-1}B, D]$ is m .

Proposition 2¹⁹

The dynamic system given in 23a) and 23b) is perfectly output controllable if the matrix $[D, CB]$ has rank m .

Proposition 3²⁰

Consider the non-linear autonomous system

$$24a. \quad y = g(z, x)$$

$$24b. \quad \dot{z} = f(z, x)$$

Without loss of generality let $0 = f(0, 0)$. Let $A = \frac{\partial f}{\partial z}(0, 0)$,

$B = \frac{\partial f}{\partial x}(0, 0)$, $C = \frac{\partial g}{\partial z}(0, 0)$ and $D = \frac{\partial g}{\partial x}(0, 0)$. Then proposition 1

holds for $y_1 = 0$ and y_0 in some open neighborhood of $y_1 = 0$ [the set of all initial points y_0 that can be transferred to $y_1 = 0$ in some specified finite interval by an appropriate choice of $x(\cdot)$ over that interval contains an open neighborhood of $y_1 = 0$].

The system is then said to be locally output controllable near the origin. The definition of perfect local output controllability [near the origin] follows by analogy. Proposition 2 holds locally for non-linear systems [Aoki, 1976].

In the appendix it is shown that we can write the unemployment rate, the inflation rate and the external surplus as functions of a vector of state variables and a vector of control variables in the form of equation 24a. The rate of change of the state vector can also be written in the form of equation 24b, as a function of the level of the state vector and the control vector. To obtain the state-space representation of our system we supplement equations 1 - 7 (with $\dot{e}/e = 0$) with the IS and LM relations and the dynamic equations representing the behaviour of the stocks of high-powered money, government bonds and real reproducible capital, the only assets in the model. The output vector consists of the three target variables u , \dot{p}_1/p_1 and δ . The state variables are the real stock of money balances $h = \frac{H}{P_1}$, the real stock of bonds $b = \frac{B}{P_1}$, the capital stock K , the terms of trade $\psi = ep_2^*/p_1$ and the real wage $v = \frac{w}{P_1}$. The government controls are the volume of government consumption spending G , the marginal tax rate on factor income and bond interest income θ , a lump-sum tax τ , the fraction of the public sector deficit financed by money creation λ and the fraction of the balance of payments surplus that is not sterilized σ . Equation 25a represents the linearized version of the output equations of the model, evaluated at the steady state equilibrium corresponding to $G = G^*$, $\theta = \theta^*$, $\tau = \tau^*$, $\lambda = \lambda^*$ and $\sigma = \sigma^*$ and associated values of the state variables $h = h^*$, $b = b^*$, $K = K^*$, $\psi = \psi^*$, $v = v^*$ and of the outputs $u = \bar{u}$, $\dot{p}_1/p_1 = \Pi$ and $\sigma = 0$. [Π is the world rate of inflation \dot{p}_2^*/p_2^*]. It corresponds to equation 23a: $y = Cz + Dx$, and is derived in the appendix.

$$25a. \begin{bmatrix} u - \bar{u} \\ \frac{\dot{P}_1}{P_1} - \Pi \\ \sigma \end{bmatrix} = \begin{bmatrix} c_{11} & 0 & c_{13} & c_{14} & c_{15} \\ \alpha c_{11} & 0 & c_{23} & c_{24} & c_{25} \\ \beta c_{11} & 0 & c_{33} & c_{34} & \beta c_{15} \end{bmatrix} \begin{bmatrix} h-h^* \\ b-b^* \\ K-K^* \\ \psi-\psi^* \\ v-v^* \end{bmatrix} + \begin{bmatrix} d_{11} & d_{12} & -C_1 d_{11} & 0 & 0 \\ \alpha d_{11} & \alpha d_{12} & -\alpha C_1 d_{11} & 0 & 0 \\ \beta d_{11} & \beta d_{12} & -\beta C_1 d_{11} & 0 & 0 \end{bmatrix} \begin{bmatrix} G-G^* \\ \theta-\theta^* \\ \tau-\tau^* \\ \lambda-\lambda^* \\ \sigma-\sigma^* \end{bmatrix}$$

The equations of motion of the state variables are given by 25b. It corresponds to equation 23b): $\dot{z} = Az + Bx$ and is also derived in the appendix.

$$25b. \begin{bmatrix} \dot{h} \\ \dot{b} \\ \dot{K} \\ \dot{\psi} \\ \dot{v} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} & a_{15} \\ a_{21} & a_{22} & a_{23} & a_{24} & a_{25} \\ a_{31} & 0 & a_{33} & a_{34} & a_{35} \\ -\psi \alpha c_{11} & 0 & -\psi c_{23} & -\psi c_{24} & -\psi c_{25} \\ vj'c_{11} & 0 & vj'c_{13} & vj'c_{14} & vj'c_{15} \end{bmatrix} \begin{bmatrix} h-h^* \\ b-b^* \\ K-K^* \\ \psi-\psi^* \\ v-v^* \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} & b_{13} & b_{14} & 0 \\ b_{21} & b_{22} & b_{23} & -b_{14} & 0 \\ b_{31} & b_{32} & -C_1 b_{31} & 0 & 0 \\ -\psi \alpha d_{11} & -\psi \alpha d_{12} & +\psi \alpha C_1 d_{11} & 0 & 0 \\ vj'd_{11} & vj'd_{12} & -vj'C_1 d_{11} & 0 & 0 \end{bmatrix} \begin{bmatrix} G-G^* \\ \theta-\theta^* \\ \tau-\tau^* \\ \lambda-\lambda^* \\ \sigma-\sigma^* \end{bmatrix}$$

Before we consider the controllability properties of the dynamic system given by 25a) and 25b) we should explain the zeros in the coefficient matrices. The assumption that the private sector does not perceive government interest-bearing debt as net worth accounts for the fact that the second column of the C matrix in 25a) consists of zeros. As it is the stocks rather than the flows of assets that affect our target variables at any given point in time, the last two columns of D matrix in 25a) (which correspond to the money flow-bond flow composition of government financing and external financing) consist of zeros only. Equation 25b) shows that the money-bond financing mix only has a direct effect on the rates of change of two of the state variables: h and b. ($b_{i4} = 0, i = 3, 4, 5$ and $b_{i5} = 0, i = 1-5$ in 25b). At the long run equilibrium, $\delta = 0$. The sterilization coefficient $1-\sigma$ will therefore have no effect on the state variables in the neighborhood of the

steady state. We include σ as an instrument for completeness and because it would be important for the global properties of the model which are not studied here. Indirectly, by altering the stocks of h and b as time passes, the other state variables will also be affected by λ and, except at the long-run equilibrium, by σ . Changes in the value of the real stock of bonds, however, will only affect the behaviour of K , ψ and v by first having an effect on h [$a_{i2} = 0$, $i = 3, 4, 5$]. Thus government bonds have but a very tenuous impact on the evolution of the target variables, only exercising any influence by affecting the debt service component of the public sector borrowing requirement.

The D matrix in 25a) has rank 1. Given the values assumed by the state variables, the fiscal policy instruments G , θ and τ only affect the inflation rate and the external surplus via the unemployment rate. As regards the rate of inflation this follows directly from our specification of the Phillips curve. (equation 9'). The D matrix would have rank two if imports other than intermediate inputs had been modeled. With the volume of imports uniquely determined by the volume of domestic production and with exports not affected by domestic fiscal or financial policy the trade balance surplus will, given the state variables, be a function only of the unemployment rate. Even so, from propositions 1 and 2 it follows immediately that any one of u , $\frac{p_1}{p_1}$ and δ is perfectly locally controllable (and by implication locally controllable) using any one of the fiscal instruments. This is a consequence of the fact, established in the appendix, that d_{11} , d_{12} and d_{13} --- representing the impact effect on u of changes in G , θ and τ respectively--- are all non-zero. To assess the joint controllability and joint perfect controllability of the three target variables it suffices to evaluate the rank of the matrix $[D, CB]$. This matrix turns out to have rank three, as is shown in the appendix. The three target variables are perfectly locally controllable and therefore also locally controllable. The policy authority

can track any desired trajectory in $u - \frac{\dot{p}_1}{p_1} - \delta$ space in the neighborhood of $(\bar{u}, \Pi, 0)$. The rank deficiency of the D matrix, when contrasted with the fact that the perfect controllability conditions are satisfied, brings out an important feature of these dynamic notions of controllability. It is not necessary for policy to operate directly on the target variable [i.e. via the D matrix]. Policy can also affect the targets indirectly via the state variables. The impact multipliers could all be zero, yet the target vector might be perfectly controllable.

It could happen that the policy authority is able to track (according to the perfect controllability criteria) any desired trajectory for the target variables while relying exclusively on the manipulation of a subset of the available controls. In this model e.g. it is quite possible that the target vector is perfectly controllable using just the three fiscal instruments G , θ and τ while keeping the budget deficit financing mix λ and the sterilization coefficient $1-\sigma$ constant. To establish that the policy authorities can track any desired path for their internal and external target variables while relying exclusively on the adjustment of their fiscal controls is not to argue that it will be desirable to do so. Typically quite serious administrative and political costs [as well as micro-economic disruptions not modeled here] are associated with changes in G , θ and τ (and perhaps also with their levels). By embedding the model in an optimal control framework that allows explicitly for the differential costs associated with manipulating the various fiscal and monetary controls, optimal stabilization policies can be derived that may, in general, involve the use of all available instruments.

5) The steady state equilibrium under a fixed exchange rate

As there is no labour force growth or technical change in the model, the steady state will be a stationary state. From equations A 2a -A 2e in the appendix we derive the steady state conditions by setting the rates of change of the state variables equal to zero. This gives us equations 26 a-h.

$$26a. \quad u = \bar{u}$$

$$26b. \quad \dot{p}_1/p_1 = \Pi = \frac{\dot{w}}{w}$$

$$26c. \quad r(1-\theta) - \frac{\dot{p}_1}{p_1} = F_1(K, 1-u)(1-\psi)(1-\theta)$$

$$26d. \quad v = F_2(K, 1-u)(1-\psi)$$

$$26e. \quad \lambda[G+rb(1-\theta)-\theta F(K,1-u)(1-\psi)-\tau]+\sigma[X(\psi)-\psi F(K,1-u)] = \frac{\dot{p}_1}{p_1} h$$

$$26f. \quad (1-\lambda)[G+rb(1-\theta)-\theta F(K,1-u)(1-\psi)-\tau]+(1-\sigma)[X(\psi)-\psi F(K,1-u)] = \frac{\dot{p}_1}{p_1} b$$

$$26g. \quad \ell(r(1-\theta), h+K) = h$$

$$26h. \quad C(F(K,1-u)(1-\psi)(1-\theta)-\tau - \frac{\dot{p}_1}{p_1} h, h+K) + G + X(\psi) = F(k,1-u)$$

The long run equilibrium of the system has strictly classical properties. The unemployment rate equals the natural rate, irrespective of the values assigned to the fiscal or monetary instruments (26a). With a fixed exchange rate the long-run domestic rate of inflation equals the world rate of inflation (26b). In the absence of technical change the rate of change of money wages equals the rate of change of prices. The after-tax marginal product of capital equals the after-tax real rate of interest (26c) The real wage equals the marginal product of labour. The value of the total

increase in the stock of nominal money balances, from either the money financing of public sector budget deficits or from the non-sterilized part of the official settlements surplus should be equal to the reduction in the real value of outstanding stock of nominal money balances due to inflation (26e). Equation 26g is the portfolio balances condition or LM curve and 26h is the domestic output market equilibrium condition or IS curve. A strong argument can be made that there should be no net gain or loss of foreign exchange reserves in long-run equilibrium. Since the stock of foreign exchange reserves does not enter as an argument into any of the behavioural equations of the model, we have to impose this steady state balance of payments equilibrium condition in an ad-hoc manner.²¹

$$26i. X(\psi) = \psi F(K, 1-u)$$

With the current account balanced, the value of private saving \dot{p}_1/p_1 (h+b) just equals the public sector financing requirement. The additional steady-state condition 26i means that one of the government policy instruments (other than σ which now vanishes from the steady state conditions) has to be endogenously determined. This role could be assumed by any one of G , θ , τ , or λ . With 26i added, 26e and 26f become:

$$26e'. \lambda [G + rb(1-\theta) - \theta F(K, 1-u)(1-\psi) - \tau] = \frac{\dot{p}_1}{p_1} h$$

$$26f'. (1-\lambda) [G + rb(1-\theta) - \theta F(K, 1-u)(1-\psi) - \tau] = \frac{\dot{p}_1}{p_1} b$$

Equations 26a-d, e', f', g, h and i determine the steady state values of u , \dot{p}_1/p_1 , r , K , ψ , v , h , b and one of λ , G , θ or τ , given the values assumed by the remaining three controls and the world rate of inflation. Note that if λ is treated as a policy instrument, there can be no steady

state equilibrium if it is set equal to 0 or 1, unless the world rate of inflation (and therefore also the domestic rate of inflation) is equal to zero. With $\lambda = 0$ and a balanced external account, the nominal stock of money is constant. The real stock of money will not be constant unless the price level stays put. With $\lambda = 1$ the same applies to bonds. For λ different from zero or 1, we have $b = \frac{(1-\lambda)}{\lambda} h$ in the steady state. With a freely floating exchange rate the endogeneity of e [and \dot{e}/e] resolves the overdeterminacy of the fixed exchange rate regime.

It should be noted that the steady state values of all three targets: unemployment rate, inflation rate and external deficit, are independent of the values assigned to the government controls.

This absence of what may be called "static controllability" (or controllability across steady states) is quite consistent with the existence of considerable latitude for the management of these target variables outside the steady state. Tinbergen's theory of economic policy²² is concerned exclusively with this static controllability and is therefore not an appropriate vehicle for the study of non-steady state behaviour. The unemployment rate and (trivially) the external balance remain not statically controllable when we switch to a freely floating exchange rate regime but the domestic inflation rate becomes statically controllable.

6) Unemployment-inflation trade-offs in the short run and the long run under a freely floating exchange rate

With a freely floating exchange rate we cannot establish the existence of a short-run unemployment inflation trade-off by just considering the money wage or price adjustment equations 8 and 9 which are reproduced below.

$$8. \quad \frac{\dot{w}}{w} = j(u-\bar{u}) + m \left(\frac{w}{p_1} - F_2(K, 1-u) \left(1 - \frac{ep_2^*}{p_1} \right) \right) / (1-\epsilon) + \frac{\dot{e}}{e} + \frac{\dot{p}_2^*}{p_2^*}$$

$$9. \quad \frac{\dot{p}_1}{p_1} = \frac{\epsilon}{1-\epsilon} j(u-\bar{u}) + m \left(\frac{w}{p_1} - F_2(K, 1-u) \left(1 - \frac{ep_2^*}{p_1} \right) \right) / (1-\epsilon) + \frac{\dot{e}}{e} + \frac{\dot{p}_2^*}{p_2^*}$$

Both e and $\frac{\dot{e}}{e}$ must be known before we can determine the slope of the short-run inflation-unemployment trade-off. The exchange rate adjusts instantaneously, like a (relative) price in an efficient auction market. In our simple model without international financial flows the exchange rate is assumed to respond to current and expected future disturbances in such a way as to implement the policy authority's decision not to add to or subtract from its stock of foreign exchange reserves. Current account equilibrium is therefore added as a momentary equilibrium condition.

$$27. \quad X \left(\frac{ep_2^*}{p_1} \right) = \frac{ep_2^*}{p_1} F(K, 1-u)$$

We could solve equation 27 for e as a function of $\frac{p_2^*}{p_1}$, K and u . From this we can obtain \dot{e} as a function of $\frac{d}{dt} \left(\frac{p_2^*}{p_1} \right)$, \dot{K} and \dot{u} .²³ This rules out the derivation of a simple momentary Phillips curve trade-off as under a fixed exchange rate regime. The way to proceed is to turn to the complete model to determine the controllability of the unemployment rate and the other target variables. Even this is so much more complicated under the freely floating exchange rate regime that it will not be attempted here. The nature of the difficulty is the following e and \dot{e} both appear in the structural equations of the model. We cannot, however, treat e as a state variable. $e(t)$ is not a predetermined variable at time t whose value is "updated" by its derivative $\dot{e}(t)$, the value of which is determined

somewhere in the model. Being akin to an efficient auction market price, $e(t)$ is not necessarily a continuous, (let alone a differentiable) function of time. At t it can make a discontinuous jump in response to current or expected future parameter changes. $\dot{e}(t)$ should therefore be interpreted as just the right-hand derivative of $e(t)$ at t . To bring out the implications of this we paraphrase Sargent and Wallace (1973), who first analyzed this issue in detail. The exchange rate at t is determined (together with the other short-run endogenous variables u , $\frac{\dot{w}}{w}$, $\frac{p_1}{p_1}$ and r) by equations 8, 9, 27 and the IS and LM curves. This requires that $\dot{e}(t)$, the right-hand derivative of e , also be determined at t . However, to determine this derivative, we in effect have to determine e for the "next instant" which in turn depends on e for the following instant. Thus to determine \dot{e} we must pursue an infinite progression into the future and determine the entire path of the (expected) exchange rate from t to forever, with the expected exchange rate being conditional on (expected) future values of the policy instruments and of the state variables of the model.

We cannot determine the equilibrium value of the exchange rate at the current moment until we have specified the future path of all policy instruments from now until forever as well as initial conditions for the true, predetermined state variables, w , p_1 , p_2^* , K , M and B . Explicitly solving the model for all future time for alternative paths of the policy instruments to determine the effect of current and future policy changes on current and future values of the target variables is too vast a task to be undertaken here. It is worth making the point, however, that the two conditions required for the actual unemployment rate to coincide with the natural rate need not be automatically satisfied. These are that e takes on the value to equate the marginal product of labour

to the real wage [which implies $m = 0$] and that $\frac{\dot{e}}{e}$ assumes a value such that $\frac{\dot{e}}{e} + \frac{\dot{p}_2^*}{p_2^*} = \frac{\dot{w}}{w}$. There is nothing in the logic of the freely floating exchange rate regime that compels e and \dot{e} to behave in this manner all of the time or indeed at any time.

The long run equilibrium conditions for the freely floating exchange rate model are the same as those for the fixed exchange rate model, except that the rigid link between the domestic rate of inflation and the world rate is broken. Equation 26b is replaced by:

$$26b'. \quad \frac{\dot{p}_1}{p_1} - \frac{\dot{e}}{e} = \Pi = \frac{\dot{w}}{w} - \frac{\dot{e}}{e}$$

Any excess of the domestic rate of inflation over the world rate of inflation is reflected in exchange rate depreciation.

Equations 26a, b', c, d, e', f', g, h and i determine the steady state

values of u , $\frac{\dot{p}_1}{p_1}$, $\frac{\dot{e}}{e}$, r , K , h , b , ψ and v , given the values assumed by the policy instruments G , θ , τ and λ and the world rate of inflation, Π .

By having the proportional rate of exchange rate depreciation as an additional steady state endogenous variable, the "overdeterminacy" of the fixed exchange rate regime which compelled the endogenization of one of the policy controls in the steady state, is avoided.

7) Conclusion

The foregoing analysis can be generalized in a number of directions. Incorporating imported final goods and non-traded goods would be an obvious extension. The further analysis of the freely floating exchange rate case requires the use of numerical methods to solve iteratively for the perfect-foresight paths corresponding to different trajectories of the policy controls.

Under a fixed exchange rate regime there will be short-run unemployment-inflation trade-offs unless the labour market is an efficient auction market. Consider again the wage Phillips curve:

$$5. \quad \frac{\dot{w}}{w} = j(u-\bar{u}) + \left(\frac{\dot{p}_1}{p_1} \right)^e$$

A wage-inflation-unemployment trade-off will fail to exist under rational expectations if the price expectations term on the r.h.s. of 5) is equivalent to the expected rate of wage inflation: $\left(\frac{\dot{w}}{w} \right)^e$. While p_1 is the best price to be included in 5) if we view 5) as being "union-determined," it could be argued that some other price index $p = p(p_1, p_2)$ would be more appropriate if we view $\frac{\dot{w}}{w}$ as the outcome of some union-management bargaining process. Still, only if $\left(\frac{\dot{p}}{p} \right)^e = \left(\frac{\dot{w}}{w} \right)^e$ and if expectations are formed rationally, i.e. $\left(\frac{\dot{w}}{w} \right)^e = \frac{\dot{w}}{w}$, will there be no wage inflation unemployment trade-offs. The wage-Phillips curve $\frac{\dot{w}}{w} = j(u-\bar{u}) + \left(\frac{\dot{w}}{w} \right)^e$ has been advanced by Phelps [1970] without giving any convincing reasons for including the relative wage to the exclusion of the real wage in workers' utility functions. With the Phelps specification, the only source of departures from labour market equilibrium and the natural rate is incorrect instantaneous expectations. With perfect foresight, expectations are realized and there can be no wage inflation unemployment trade-off: $u = \bar{u}$. Whether or not the output market is an efficient auction market, to always have the unemployment rate at the natural rate also precludes the existence of price inflation-unemployment trade-offs.

The main theoretical and policy conclusion of the paper is that if we do not specify that the labour market is always in momentary competitive equilibrium but instead postulate plausible labour market disequilibrium adjustment mechanisms,²⁴ the assumption of rational expectations does not preclude the existence of unemployment-inflation trade-offs in open

economies when a wedge can be driven between the rate of change of domestic prices and the rate of change of world prices. The controllability analysis suggests in addition that the scope for stabilization policy aimed at the traditional internal and external balance targets may be considerable in such models.

Appendix. The state-space representation of the model.

Remembering that $Y = Q(1 - \frac{ep_2^*}{p_1})$, $Q = F(K, 1-u)$ and $M = Q$, we can, under a fixed exchange rate, solve the IS curve (11) the LM curve (12) the balance of payments equation (13) and the price Phillips curve (9') for $u, \dot{p}_1/p_1, \delta$ and r as functions of $h=H/p_1, K, \psi = \frac{ep_2^*}{p_1}, v = \frac{w}{p_1}, \Pi = \frac{\dot{p}_2^*}{p_2}$,

G, θ and τ . In order to obtain a state-space representation of the model, it is convenient to write $u, \dot{p}_1/p_1, \delta$ and r not only as functions of the variables just mentioned, but also as functions of $b = \frac{B}{p_1}, \lambda$ and σ with the corresponding partial derivatives identically equal to zero. λ denotes the fraction of the public sector deficit financed by printing money. σ is the fraction of the balance of payments surplus that is not sterilized.

We obtain:

$$\text{A1a. } u = g^1(h, b, K, \psi, v; G, \theta, \tau, \lambda, \sigma; \Pi)$$

$$\text{A1b. } \dot{p}_1/p_1 = g^2(h, b, K, \psi, v; G, \theta, \tau, \lambda, \sigma; \Pi)$$

$$\text{A1c. } \delta = g^3(h, b, K, \psi, v; G, \theta, \tau, \lambda, \sigma; \Pi)$$

$$\text{A1d. } r = g^4(h, b, K, \psi, v; G, \theta, \tau, \lambda, \sigma; \Pi)$$

$$g_b^i = g_\lambda^i = g_\sigma^i \equiv 0 \quad i = 1, 2, 3, 4.$$

To establish local controllability and local perfect controllability of the target variables $u, \dot{p}_1/p_1, \delta$ we first evaluate the matrix of partial derivatives of the target variables with respect to the instruments, i.e. the matrix corresponding to the D matrix in 12a).

$$D = [d_{ij}] = \begin{bmatrix} g_G^1 & g_\theta^1 & g_\tau^1 & g_\lambda^1 & g_\sigma^1 \\ g_G^2 & g_\theta^2 & g_\tau^2 & g_\lambda^2 & g_\sigma^2 \\ g_G^3 & g_\theta^3 & g_\tau^3 & g_\lambda^3 & g_\sigma^3 \end{bmatrix}$$

All partial derivatives are evaluated at the steady state equilibrium.

While we are not intrinsically interested in the impact effects of changes in G , θ , and τ on the interest rate, which is not a target variable, we need these impact multipliers [g_G^4 , g_θ^4 and g_τ^4] to study controllability.

At any point in time, the rate of inflation and the external deficit are a function of the unemployment rate alone. The matrix of the impact multipliers of fiscal policy is therefore singular and has rank 1 [provided g_G^1 , g_θ^1 and g_τ^1 are not all equal to zero]. An increase in government spending will in all likelihood reduce unemployment, increase the rate of inflation and increase the trade account deficit. An increase in the lump sum tax τ will increase unemployment reduce inflation and improve the trade balance. An increase in the income tax rate θ will have qualitatively similar effects. An increase in G will raise the interest rate, an increase in the lump-sum tax will lower it. An increase in θ will be contractionary in the output market. It will, by reducing the after-tax rate of return on bonds corresponding to any pre-tax r increase the demand for money. The effect on r is ambiguous. D would have rank 2 if imports were not confined to intermediate imports; imports of consumer goods would be subject to the effects of taxation just like the demand for domestic consumer goods.

The matrix corresponding to C in 23a, i.e. the matrix of partial derivatives of the target variables with respect to the state variables is given by:

$$C = [c_{ij}] = \begin{bmatrix} g_h^1 & g_b^1 & g_K^1 & g_\psi^1 & g_v^1 \\ g_h^2 & g_b^2 & g_K^2 & g_\psi^2 & g_v^2 \\ g_h^3 & g_b^3 & g_K^3 & g_\psi^3 & g_v^3 \end{bmatrix}$$

To analyze the equations of motion of the state variables we also need the reduced form derivatives of the interest rate with respect to the state variables: $g_h^4, g_b^4 = 0, g_k^4, g_\psi^4$ and g_v^4 .

The equations of motion of the state variables are (using 19, 20, and 21).

$$A2a. \quad \dot{h} = \lambda[G + rb(1-\theta) - \theta Y - \tau] + \sigma\delta - \frac{\dot{p}_1}{p_1} h$$

$$A2b. \quad \dot{b} = (1-\lambda)[G + rb(1-\theta) - \theta Y - \tau] + (1-\sigma)\delta - \frac{\dot{p}_1}{p_1} b$$

$$A2c. \quad \dot{K} = I(q)$$

$$A2d. \quad \dot{\psi} = \left(\Pi - \frac{\dot{p}_1}{p_1}\right)\psi$$

$$A2e. \quad \dot{v} = j(u - \bar{u})v$$

Substituting for $r, Y, \delta, \dot{p}_1/p_1, q$ and u in A2 we can rewrite this set of equations as:

$$A3a. \dot{h} = f^1(h, b, K, \psi, v; G, \theta, \tau, \lambda, \sigma; \Pi)$$

$$A3b. \dot{b} = f^2(h, b, K, \psi, v; G, \theta, \tau, \lambda, \sigma; \Pi)$$

$$A3c. \dot{K} = f^3(h, b, K, \psi, v; G, \theta, \tau, \lambda, \sigma; \Pi)$$

$$A3d. \dot{\psi} = f^4(h, b, K, \psi, v; G, \theta, \tau, \lambda, \sigma; \Pi)$$

$$A3e. \dot{v} = f^5(h, b, K, \psi, v; G, \theta, \tau, \lambda, \sigma; \Pi)$$

$$f_{\lambda}^i = f_{\sigma}^i = f_{\psi}^i = 0; i = 3, 4, 5$$

$$f_{\delta}^i = 0; i = 1, \dots, 5.$$

The matrices corresponding to the A and B matrices in equation 23b are

$$A = [a_{ij}] = \begin{bmatrix} f_h^1 & f_b^1 & f_K^1 & f_{\psi}^1 & f_v^1 \\ f_h^2 & f_b^2 & f_K^2 & f_{\psi}^2 & f_v^2 \\ f_h^3 & f_b^3 & f_K^3 & f_{\psi}^3 & f_v^3 \\ f_h^4 & f_b^4 & f_K^4 & f_{\psi}^4 & f_v^4 \\ f_h^5 & f_b^5 & f_K^5 & f_{\psi}^5 & f_v^5 \end{bmatrix} \text{ and } B = [b_{ij}] = \begin{bmatrix} f_G^1 & f_{\theta}^1 & f_{\tau}^1 & f_{\lambda}^1 & f_{\sigma}^1 \\ f_G^2 & f_{\theta}^2 & f_{\tau}^2 & f_{\lambda}^2 & f_{\sigma}^2 \\ f_G^3 & f_{\theta}^3 & f_{\tau}^3 & f_{\lambda}^3 & f_{\sigma}^3 \\ f_G^4 & f_{\theta}^4 & f_{\tau}^4 & f_{\lambda}^4 & f_{\sigma}^4 \\ f_G^5 & f_{\theta}^5 & f_{\tau}^5 & f_{\lambda}^5 & f_{\sigma}^5 \end{bmatrix}$$

The detailed evaluation of the A, B, C and D matrices in terms of the structural coefficients of the model is obtainable from the author.

The matrix [D, Γ] where $\Gamma = [\gamma_{ij}] = CB$ is written out in A4

$$A4. [D, \Gamma] = \begin{bmatrix} d_{11} & d_{12} & -c_1 d_{11} & 0 & 0 & \gamma_{11} & \gamma_{12} & \gamma_{13} & \gamma_{14} & 0 \\ \alpha d_{11} & \alpha d_{12} & -\alpha c_1 d_{11} & 0 & 0 & \gamma_{21} & \gamma_{22} & \gamma_{23} & \alpha \gamma_{14} & 0 \\ \beta d_{11} & \beta d_{12} & -\beta c_1 d_{11} & 0 & 0 & \gamma_{31} & \gamma_{32} & \gamma_{33} & \beta \gamma_{14} & 0 \end{bmatrix}$$

$$A5a. \gamma_{11} = c_{11} b_{11} + c_{13} b_{31} - c_{14} \psi \alpha d_{11} + c_{15} v j' d_{11}$$

$$A5b. \gamma_{12} = c_{11} b_{12} + c_{13} b_{32} - c_{14} \gamma \alpha d_{12} + c_{15} v j' d_{12}$$

$$A5c. \gamma_{13} = c_{11} b_{13} - c_{13} C_1 b_{31} + c_{14} \psi \alpha C_1 d_{11} - c_{15} v j' C_1 d_{11}$$

$$A5d. \gamma_{14} = c_{11} b_{14}$$

$$A5e. \gamma_{21} = \alpha c_{11} b_{11} + c_{23} b_{31} - c_{24} \psi \alpha d_{11} + c_{25} v j' d_{11}$$

$$A5f. \gamma_{22} = \alpha c_{11} b_{12} + c_{23} b_{32} - c_{24} \psi \alpha d_{12} + c_{25} v j' d_{12}$$

$$A5g. \gamma_{23} = \alpha c_{11} b_{13} - c_{23} C_1 b_{31} + c_{24} \psi \alpha C_1 d_{11} - c_{25} v j' C_1 d_{11}$$

$$A5h. \gamma_{24} = \alpha c_{11} b_{14}$$

$$A5i. \gamma_{31} = \beta c_{11} b_{11} + c_{33} b_{31} - c_{34} \psi \alpha d_{11} + \beta c_{15} v j' d_{11}$$

$$A5j. \gamma_{32} = \beta c_{11} b_{12} + c_{33} b_{32} - c_{34} \psi \alpha d_{12} + \beta c_{15} v_j' d_{12}$$

$$A5k. \gamma_{33} = \beta c_{11} b_{13} - c_{33} C_1 b_{31} + c_{34} \psi \alpha C_1 d_{11} - \beta c_{15} v_j' C_1 d_{11}$$

$$A5l. \gamma_{34} = \beta c_{11} b_{14}$$

Columns 6, 7 and 8 of the matrix $[D, \Gamma]$ are linearly independent. $[D, \Gamma]$ therefore has rank three. By proposition 2, unemployment, inflation and the external deficit are perfectly controllable (and by implication controllable) in the neighborhood of the steady state equilibrium.

FOOTNOTES

*Discussions with James Tobin on various aspects of this paper have, as always, been extremely valuable.

¹ e.g. Barro (1976), Lucas (1972a, 1972b, 1975, 1976, 1977), Sargent and Wallace (1975, 1976).

² The literature is mainly concerned with the invariance of the density functions of real variables under alternative deterministic monetary policy rules. For fiscal policy rules it is virtually impossible to separate the stabilization side from the structural or allocative side. Changes in taxes will affect notional demand and supply functions of labour services and other goods and services and will alter the composition of output (Buiter 1977a). Changes in exhaustive government spending will also have obvious real effects even if they do not affect the unemployment rate. Monetary policy changes too will have real effects in most classical macromodels if changes in the proportional rate of growth of the nominal money supply are involved. With the nominal interest rate on outside money balances institutionally determined, changes in the proportional rate of monetary expansion and associated changes in the rate of inflation will lower the real rate of return on money balances. This will alter portfolio equilibrium, feed through into capital formation and thus, as time passes, will alter the trajectory of the stock of capital and productive capacity. All this is well known and will be ignored in what follows. These "allocative" effects of monetary and fiscal policy are considered to be part of the structure of the real economy that is, by assumption, kept constant when monetary (or fiscal) stabilization policies are studied.

³ See Hicks (1965).

⁴ See Fischer (1977) and Sargent (1976a). If on the other hand, real variables are affected by stabilization policy via one and more period forecast errors of the values assumed by the policy instruments, non-trivial deterministic optimal stabilization rules in feedback form can often be derived. This means that stabilization instrument values enter the reduced form for the real variables via terms like:

$$s_t - {}_{t-1}S_t^e, s_t - {}_{t-2}S_t^e, \dots, s_t - {}_{t-j}S_t^e, \dots, \text{etc.}$$

Sluggish wage and price adjustment are one way of obtaining such reduced forms.

⁵ See Phelps (1970).

⁶ The existence of exploitable unemployment-inflation trade-offs with non-rational (e.g. adaptive) expectations is well established for both closed and open economies.

⁷ Different proportional rates of growth of the stock of outside money balances, while having no effect on the natural rate of unemployment, can affect the steady state capital-labour ratio and portfolio composition.

⁸ A more detailed model incorporating such features as labour force growth, technical change and internationally traded financial claims is presented in Buiter, (1977 C).

⁹ The fixed proportions assumption is merely a convenient simplification and is not essential for any of the results.

¹⁰ Ideally the weight ϵ would be endogenous, as it is supposed to represent the normal share of labour costs in total variable costs. The fixed weight in the paper represents a convenient simplification and is to be interpreted as a normal, long-run weight.

¹¹ For a model in which the money wage rate is predetermined but the price of output is determined in an efficient auction market which permits discontinuous price jumps at a point in time see Buiter and Lorie (1977). The theoretical issues involved in dealing with rational expectations in efficient auction market models were first discussed by Sargent and Wallace (1973). See also Sargent (1976b) and Calvo (1977).

¹² If there are imports of consumer goods whose world price is parametric to the home country, a trade-off will in general exist even when labour is the only variable input.

¹³ With uncertainty explicitly introduced into the model, random departures of the actual rate from the natural rate can occur, but the density function of the unemployment rate will be independent of all deterministic stabilization policy rules.

¹⁴ The financial market is modeled as an efficient auction market with the interest rate adjusting instantaneously to equate money demand and supply.

¹⁵ Aoki (1976), p. 70. See also Buiter (1975).

¹⁶ Aoki (1976).

¹⁷ Aoki (1976).

¹⁸ Aoki (1976), p. 29.

¹⁹ Aoki and Canzoneri, (1976).

²⁰ Aoki (1976), pp. 101-104.

²¹ The value of the exchange rate could be made to depend on the stock of reserves. With capital mobility, speculative inflows and outflows of capital in response to steadily growing or shrinking foreign exchange reserves would make steady state payments equilibrium into an endogenous rather than an ad-hoc constraint. Without international capital flows there seems to be no simple way to enter the stock of reserves into any behavioral relationship, except of course via ad-hoc government reaction functions.

²²See Tinbergen (1963) and Preston (1974).

²³ \dot{e} should be interpreted as the right side derivative of $e(t)$, i.e.

$$\dot{e}(t) = \lim_{\substack{h \rightarrow 0 \\ h > 0}} \left(\frac{e(t+h) - e(t)}{h} \right). \text{ As } e \text{ is like an efficient auction market}$$

price, it is permitted to make discrete jumps at a point in time.

²⁴The additional assumption of sluggish price adjustment is convenient but not essential.

²⁵We ignore the trivial steady state solution with $v = \psi = 0$.

BIBLIOGRAPHY

- Aoki, M. (1976), Optimal Control and System Theory in Dynamic Economic Analysis, North Holland, Amsterdam.
- Aoki, M. and M. Canzoneri (1976), "Sufficient Conditions for Control of Target Variables and Assignment of Instruments in Dynamic Macroeconomic Models," unpublished manuscript, Nov.
- Barro, R. J. (1976), "Rational Expectations and the Role of Monetary Policy," Journal of Monetary Economics, 4, April, pp. 103-124.
- Buiter, W. H. (1975), Temporary Equilibrium and Long-Run Equilibrium, Unpublished Ph.D. Thesis, Yale University, Ch. 6, "Controllability, Stabilization and Optimization."
- Buiter, W. H. (1977a), "Crowding Out and the Effectiveness of Fiscal Policy," Journal of Public Economics, 7, July, pp. 309-328.
- Buiter, W. H. (1977b), "Optimal Foreign Exchange Market Intervention with Rational Expectations," Econometric Research Program Research Memorandum, No. 216, November, Princeton University, forthcoming in Trade and Payments Adjustment under Flexible Exchange Rates, J. Martin and A. Smith eds. MacMillan, 1978.
- Buiter, W. H. (1977c), "Short-Run and Long-Run Effects of External Disturbances under a Floating Exchange Rate," Econometric Research Program Research Memorandum No. 217, November.
- Buiter, W. H. and H. Lorie (1977), "Some Unfamiliar Properties of a Familiar Macroeconomic Model," Economic Journal, 87, pp. 743-754.
- Calvo, G. A. (1977), "The Stability of Models of Money and Perfect Foresight: A Comment," Econometrica, 45, October, pp. 1737-1739.
- Fischer, S. (1977), "Long-Term Contracts, Rational Expectations, and the Optimal Money Supply Rule," Journal of Political Economy, 85, February, pp. 191-206.
- Friedman, M. (1968), "The Role of Monetary Policy," American Economic Review, 58, March, pp. 1-17.
- Hicks, J. (1965), Capital and Growth, Oxford, Clarendon Press.
- Lucas, R. E. (1972a), "Expectations and the Neutrality of Money," Journal of Economic Theory, 4, April, pp. 103-124.
- Lucas, R. E. (1972b), "Econometric Testing of the Natural Rate Hypothesis," in O. Eckstein, ed. The Econometrics of Price Determination, Board of Governors of the Federal Reserve System and S.S.R.C., pp. 50-59.

- Lucas, R. E. (1975), "An Equilibrium Model of the Business Cycle," Journal of Political Economy, 83, December, pp. 1113-1144.
- Lucas, R. E. (1976), "Econometric Policy Evaluation: A Critique," in K. Brunner and A. H. Meltzer eds. The Phillips Curve and Labour Markets, Amsterdam & New York, North Holland.
- Lucas, R. E. (1977), "Understanding Business Cycles," in K. Brunner and A. H. Meltzer eds. Stabilization of the Domestic and International Economy, Amsterdam & New York, North Holland.
- Muth, J. F. (1961), "Rational Expectations and the Theory of Price Movements," Econometrica, 29, July, pp. 315-335.
- Phelps, E. S. (1970), "Money Wage Dynamics and Labour Market Equilibrium," in E. S. Phelps et. al. Microeconomic Formulations of Employment and Inflation Theory, W. W. Norton & Co., Inc., New York, pp. 124-166.
- Phelps, E. S. and J. B. Taylor (1977), "Stabilizing Powers of Monetary Policy under Rational Expectations," Journal of Political Economy, 85, February, pp. 163-190.
- Preston, A. J. (1974), "A Dynamic Generalization of Tinbergen's Theory of Policy," Review of Economic Studies, Vol. 41, January, pp. 65-74.
- Sargent, T. J. (1976a), "The Observational Equivalence of Natural and Unnatural Rate Theories of Macroeconomics," Journal of Political Economy, 84, June, pp. 631-640.
- Sargent, T. J. (1976b), "Interest Rates and Expected Inflation: A Selective Summary of Recent Research," Explorations in Economic Research, Vol. 3, Summer, pp. 303-325.
- Sargent, T. J. and N. Wallace (1973), "The Stability of Models of Money and Growth with Perfect Foresight," Econometrica, 41, November, pp. 1043-1048.
- Sargent, T. J. and N. Wallace (1975), "Rational Expectations, the Optimal Monetary Instrument and the Optimal Money Supply Rule," Journal of Political Economy, 83, April, pp. 241-254.
- Sargent, T. J. and N. Wallace (1976), "Rational Expectations and the Theory of Economic Policy," Journal of Monetary Economics, 2, April, pp. 169-183.
- Taylor, J. (1977), "Conditions for Unique Solutions in Stochastic Macroeconomic Models with Rational Expectations," Econometrica, 45, September, pp. 1377-1385.
- Tinbergen, J. (1963), On the Theory of Economic Policy, North Holland, Amsterdam.