PRICE VARIABILITY, UTILITY AND SAVINGS

Jonathan Eaton*

Econometric Research Program
PRINCETON UNIVERSITY
207 Dickinson Hall
Princeton, New Jersey

Abstract

The effects of relative price variability on the expected
utility and optimal savings of the consumer are explored in a
two-period, two-commodity model. Price variability is shown to
have an ambiguous effect on expected utility which depends on
the elasticity of substitution, the degree of risk aversion and
the availability of hedging opportunities. One may interpret
the effect of increased price variability on savings in terms of
its effect on the real interest rate, which is to raise both its
expected value and its variability. An extension of some of the
results to a simple general equilibrium context is provided.

PRICE VARIABILITY, UTILITY AND SAVINGS

by

Jonathan Eaton

1. Introduction

The observation that an increase in the variation of the price of a commodity about its mean raises the expected consumer surplus generated by that commodity originates with Waugh [1944]. Waugh and subsequent investigators (e.g., Massell [1969], Turnovsky [1976] and Bradford and Kelejian [1977]) use this result to derive propositions on the welfare and distributional effects of price stabilization schemes.[1] This approach ignores the uncertainty in real income generated by price variability. It also fails to consider the role of hedging against price variation through appropriate portfolio diversification. In this paper demand theory is used to show that relative price variability has an ambiguous effect on expected utility when only a single asset is available as a store of value. A positive effect is more likely when the demand elasticity is large relative to the degree of relative risk aversion and when the marginal propensity to consume the commodity in question is large. When an asset is available for each commodity certain in value in terms of the price of that commodity, the consumer can hedge against price variation to the extent that a positive effect is guaranteed. Increased variance in a log-normally distributed price unambiguously raises expected utility when the consumer hedges optimally.

A second purpose of this paper is to explore the implications of price variability for optimal savings as well as for expected utility. While the effects of uncertainty in the value of future endowments and assets on optimal savings have been explored extensively scant attention has been paid to the effects of relative price uncertainty.[2] The present paper attempts to fill this gap for the case in which utility is intertemporally additively separable.

Section 2 presents a general framework for analyzing expected utility maximization in a two-period, multi-commodity, multi-asset environment in which prices and income in the second period are stochastic. Section 3 analyzes the effect on expected utility of increased variability in the

relative price of a single commodity in a two-commodity context. Here and in section 5 the geometric mean preserving spread of Flemming, Turnovsky and Kemp [1977] provides a definition of increased variability. Section 4 derives optimal portfolio behavior for the special case in which the relative price is lognormally distributed. The effect of price variability on optimal savings in a two-period context is considered in section 5. Some concluding observations and an extension to a general equilibrium problem appear in section 6.

2. Utility Maximization under Relative Price Uncertainty

This section presents a formal model of consumer choice under price uncertainty.[3] The consumer chooses a vector of quantities $\underline{x}$ and a vector of quantities $\underline{y}$ sequentially to maximize an objective function $U(\underline{x},\underline{y})$. The consumer selects $\underline{x}$ knowing the prices of the elements of $\underline{x}$, denoted $\underline{p}$. Similarly, $\underline{q}$, the prices of $\underline{y}$, are known at the time $\underline{y}$ is chosen. The consumer may not, however, know the true value of $\underline{q}$ at the time $\underline{x}$ is selected (period 1), but has a subjective probability distribution for $\underline{q}$ denoted $F(\underline{q})$. Total exogenous income, M, may also be a random variable unrealized until period 2 when $\underline{y}$ is selected.

Denoting $I \equiv \underline{p} \cdot \underline{x}$ and $J \equiv M - I$ the consumer's maximization problem may be partitioned as follows. In period 2, given realizations of $\underline{q}$ and M and given the value of $\underline{x}$ selected in period 1 the consumer chooses $\underline{y}$ to attain

$$(2.1) \qquad V(\underline{q},J;\underline{x}) \equiv \max_{\underline{y}} U(\underline{x},\underline{y})$$

subject to

$$(2.2) \qquad \underline{q} \cdot \underline{y} \leq J = M - \underline{p} \cdot \underline{x} \,.[4]$$

Define $\underline{y}(\underline{q},J,\underline{x})$ as the value of $\underline{y}$ attaining V and let $\hat{\underline{y}} \equiv \underline{y}(\overline{\underline{q}},\overline{J},\underline{x})$, the value of $\underline{y}$ at $\overline{\underline{q}} \equiv E[\underline{q}]$ and $\overline{J} \equiv E[M] - I$.

In period 1 the consumer selects $\underline{x}$ to maximize

$$(2.3) \qquad E_{q,M}[V(\underline{q},M-\underline{p} \cdot \underline{x},\underline{x})] \,.$$

Let $\underset{=}{x}^*$ denote the value of $\underset{=}{x}$ maximizing (2.3).

If there exists an element $z_i \in \underset{=}{x}$ which is a perfect substitute for y_i then the consumer can hedge against or speculate on q_i by purchasing z_i in period 1. Let $\underset{=}{x} = [\underset{=}{x}', \underset{=}{z}]$ where $\underset{=}{z} = \{z_i\}$ and let $\underset{=}{p} = [\underset{=}{p}', \underset{=}{r}]$ where $\underset{=}{p}'$ and $\underset{=}{r}$ represent the vectors of prices corresponding to $\underset{=}{x}'$ and $\underset{=}{z}$ respectively. Then, defining $c_i \equiv z_i + y_i$, where c_i denotes consumption of good i,

$$(2.4) \qquad U(\underset{=}{x}, \underset{=}{y}) \equiv U(\underset{=}{x}', \underset{=}{c}) \ .$$

The consumer's maximization problem is equivalent to one of choosing, in period 2, $\underset{=}{c}$ to attain

$$(2.5) \qquad V(J, \underset{=}{q}; \underset{=}{x}', \underset{=}{z}) \equiv \max_{\underset{=}{c}} U(\underset{=}{x}'\underset{=}{c})$$

such that

$$(2.6) \qquad \underset{=}{q}(\underset{=}{z} - \underset{=}{c}) + J \geq 0, \quad c_i \geq 0 \ \forall i \ .$$

If $\hat{y}_i \leq 0$ or, equivalently, if $\hat{c}_i \leq z_i$ where $\hat{c}_i \equiv z_i + \hat{y}_i$, the consumer may be interpreted as having taken a long position in commodity i or as having speculated on commodity i.

In the analysis which follows savings S is defined as

$$(2.7) \qquad S \equiv J + \underset{=}{r} \cdot \underset{=}{z} = M - I'$$

where $I' = \underset{=}{p}' \cdot \underset{=}{x}'$. One may rewrite expression (2.1), then, as (2.1') $V(q, S, \underset{=}{x}', \underset{=}{z})$. This definition of savings is especially natural when U is of the form

$$(2.8) \qquad U(\underset{=}{x}', \underset{=}{c}) = U^1(\underset{=}{x}') + U^2(\underset{=}{c})$$

in which case one may write (2.1') as

$$(2.9) \qquad V(q; S, \underset{=}{x}', \underset{=}{z}) = V^1(\underset{=}{p}, M-S) + V^2(\underset{=}{q}, S, \underset{=}{z})$$

where

$$(2.10a) \qquad V^1 = \max_{\underset{=}{x}} U^1(\underset{=}{x}') \qquad \text{s.t.} \ \underset{=}{p}' \cdot \underset{=}{x}' \leq M - S$$

and

$$(2.10b) \quad v^2 = \max_{\underline{c}} U^2(\underline{c}) \qquad s.t. \underline{g}(\underline{c} - \underline{z}) \leq J .$$

To avoid complications introduced by intertemporal substitutability of consumption, the analysis of optimal savings in section 5 assumes U does indeed take the form of (2.8) .

Sections 3, 4 and 5 which follow consider the effect of uncertainty in the price of a particular commodity, called commodity 2, on the consumer's utility and on his optimal portfolio and savings decisions. Other period 2 prices are treated as certain in period 1, allowing the aggregation of y_i, $\forall i \neq 2$, into a single Hicks' composite commodity, commodity 1. For simplicity let both $q_1 = 1$ and $\underline{p} = \{1\}$. Thus one can denote q_2 simply as q . Income M is assumed certain in terms of q_1 in period 1 to focus attention on relative price variability.

In period 1 the consumer considers q a random variable with a mean of unity and with probability distribution $F(q)$. Integrability conditions on F sufficient to allow the analysis which follows are stated formally by Tesfatsion [1976, p. 301].

3. Price Variability and Welfare

Previous analysis of the effects of price uncertainty (e.g., Stiglitz [1972] and Turnovsky [1974]) have characterized increased price uncertainty as increased variance about a given mean or as an arithmetic mean preserving spread (AMPS) as defined by Rothschild and Stiglitz [1970] in the price distribution.[5] Flemming, Turnovsky and Kemp [1977] criticize such characterizations for their sensitivity to the choice of numeraire. They propose, instead, to represent increased uncertainty in q as a geometric mean preserving spread (GMPS). Their definition of a GMPS in q implies that it is equivalent to an AMPS in $\underline{q}$ where $\underline{q} = \ln q$.[6] A GMPS preserves the geometric mean of the relative price ratio regardless of the choice of numeraire. Increased price variability is therefore characterized in this section and in section 5 as a GMPS in the distribution of q.

The effects of an AMPS in $\underline{q}$ on expected utility in the absence and presence of hedging opportunities respectively are summarized in Propositions 3.1 and 3.2 below.

<u>Proposition 3.1.</u> If the consumer cannot hedge against relative price movements increased price uncertainty in the sense of an AMPS in $\underline{q}$ raises or lowers expected utility as

$$(3.1) \qquad (q\frac{\partial c_2}{\partial S} - \frac{\partial c_1}{\partial S} + E^D - \alpha R \gtrless 0$$

uniformly. When the sign of (3.1) is not uniform the effect is ambiguous. Here E^D represents the income-compensated price elasticity of demand for commodity 2, $\alpha \equiv qc_2/S$ expenditure on c_2 as a share of S, and $R \equiv -SV_{SS}/V_S$ the degree of relative risk aversion with respect to expenditure in period 2.[7]

Proof. For given $\underline{\underline{x}}$, Jensen's inequality implies that an AMPS in q increases or decreases expected utility as V is uniformly convex or concave in q. In the absence of hedging opportunities one may write expression (2.1') as

$$(3.2) \qquad V(q,S;\underline{\underline{x}}') .$$

Twice differentiating w.r.t. q yields

$$(3.3) \qquad V_q q + V_{qq} q^2 .$$

From Roy's identity,

$$(3.4) \qquad V_q \equiv -c_2 V_S ,$$

it may be shown that (3.3) is equivalent to

$$(3.5) \qquad [-q(\frac{\partial c_2}{\partial q})_S + c_2(\frac{\partial c_1}{\partial S} - q\frac{\partial c_2}{\partial S})]qV_S + q^2 c_2^2 V_{SS}$$

where $(\frac{\partial c_2}{\partial q})_S$ is the income-compensated price derivative of demand for commodity 2. Dividing (3.5) by $qc_2 V_S$ yields the left-hand side of (3.1).

An AMPS in q will typically affect $\underline{\underline{x}}^*$, however. Let $\underline{\underline{x}}_a^*$ denote the optimal value of $\underline{\underline{x}}$ given the original distribution of q and $\underline{\underline{x}}_b^*$ the optimal value subsequent to the AMPS in q. Similarly let E_a denote the expectations operator under the initial distribution and E_b under the riskier distribution. Then

$$(3.6) \qquad E_b[V(q,S;\underline{\underline{x}}_b^*)] \geq E_b[V(q,S;\underline{\underline{x}}_a^*)] \geq E_a[V(q,S;\underline{\underline{x}}_a^*)]$$

if V is convex in q while

$$(3.7) \qquad E_a[V(q,S;\underline{\underline{x}}_a^*)] \geq E_a[V(q,S;\underline{\underline{x}}_b^*)] \geq E_b[V(q,S;\underline{\underline{x}}_b^*)]$$

if V is concave in q . To determine the effect of an AMPS in q on expected utility it thus suffices to determine the concavity or convexity of V in q for given $\underline{x}$.

A sufficient condition for increased price variability to <u>raise</u> expected utility is that the marginal propensity to consume commodity 2 exceed the marginal propensity to consume commodity 1 and that the income-compensated elasticity of demand for commodity 2 exceed the degree of relative risk aversion times the share of commodity 2 in expenditure. If neither condition obtains increased price variability <u>lowers</u> expected utility. The roles of R and E^D in Proposition 3.1 are intuitively clear: the lower R the less the consumer suffers from variability in real income while the larger E^D the more easily can the consumer substitute commodity 1 for commodity 2 in response to price. A high marginal propensity to consume commodity 2 raises the utility-increasing effect of price variability: when $q\frac{\partial c_2}{\partial S} > \frac{\partial c_1}{\partial S}$ a low-price outcome generates a positive income effect raising demand more for the lower-priced commodity while a high-price outcome will create a negative income effect lowering demand more for the expensive commodity. Only the role of E^D in the relationship between price variability and expected utility is captured in the consumer-surplus analysis of Waugh [1944].

Consider now a situation in which the consumer may hedge against price uncertainty by buying or selling a perfect substitute for commodity 2 <u>ex ante</u> at price 1. Such hedging may take the form of an inventory stock or a future purchase of commodity 2, for example. Expression (2.1') now takes the form

$$(3.8) \qquad V(q, S; \underline{x}', z_2)$$

where z_2 , recall, denotes the consumer's position in commodity 2.

<u>Proposition 3.2.</u> Given z_2 the effect of an AMPS in q on expected utility has the same sign as

$$(3.9) \qquad (z_2 - c_2)\left(\frac{\partial c_1}{\partial S} - q\frac{\partial c_2}{\partial S}\right) - q(z_2 - c_2)^2 R/S + \sigma c_1 c_2 / S$$

where σ is the elasticity of substitution.

The proof is analogous to that of Proposition 3.1 and is not included here.

If the level of uncertainty is small deviations of c_2 from $\hat{c}_2$, the demand for c_2 at $q = 1$, will be small. For values of z_2 near $\hat{c}_2$, then, increased uncertainty is likely to raise expected utility:[8] choosing z_2 near $\hat{c}_2$ reduces the adverse real income effect of price variability while the favorable substitution effect remains. The consumer does not necessarily choose z_2 near $\hat{c}_2$, however, but may choose to speculate in one commodity or the other. The next section derives an expression for the optimal value of z_2 when q is lognormally distributed. Implications of increased price variability given optimizing portfolio behavior are then obtained.

4. Relative Price Variability and Portfolio Diversification

If q is lognormally distributed then an explicit approximation for optimal holdings of asset 2 may be derived. A Taylor-series expansion of (2.1) around $q = 1$ gives expected utility as

$$(4.1) \qquad V(S,\bar{q},z_2,\underline{x}') + \frac{1}{2}\,[V_{SS}(z_2)^2 + 2V_{Sq}z_2 + V_{qq} + V_S z_2 + V_q]\,\sigma_q^2$$

where σ_q represents the variance of q. Maximizing (4.1) w.r.t. z_2 yields, as an optimal value for z_2,

$$(4.2) \qquad z_2^* = \frac{-V_{Sq} - 1/2V_S}{V_{SS}}\,.$$

Substituting derivatives of Roy's identify and rearranging yields

$$(4.3) \qquad z_2^* = \hat{c}_2 + 1/2\left(\frac{\partial \hat{c}_1}{\partial S} - \frac{\partial \hat{c}_2}{\partial S}\right)S/R\,.$$

Denoting $z_1 \equiv S - z_2$ one may interpret the hedging problem as one of portfolio diversification. Given a level of wealth S the consumer must allocate his wealth between z_1 and z_2 to maximize expected utility; z_1 and z_2 may be interpreted as alternative assets transferring expenditure from period 1 to period 2.

From expression (4.3) follows:

Proposition 4.1. When prices are distributed lognormally the consumer speculates in the commodity for which he has a lower marginal propensity to consume.

To understand why this proposition is true consider its converse. If the consumer were to speculate in a commodity for which his marginal propensity to consume is high, high-price outcomes for that commodity would be met, via the income effect, with increased consumption mostly of the more expensive commodity. Conversely, low-price outcomes would lead to a greater reduction in consumption of the low-priced commodity. The benefits of a high return would be reduced while the costs of a low return would be exacerbated.

If tastes are homothetic then marginal and average propensities to consume are identical. A corollary to Proposition 4.1 is then:

Proposition 4.2. When prices are distributed lognormally and when tastes are homothetic the portfolio is more evenly divided between z_1 and z_2 than is consumption expenditure between $\hat{c}_1$ and $\hat{c}_2$ i.e., the consumer speculates in the commodity on which he spends less.

Compare expression (4.3) above and the subsequent discussion with the analysis of Stiglitz [1972, pp. 114-118]. Stiglitz considers portfolio diversification when, in present terminology, the relative price of commodity 2 is distributed normally rather than lognormally. He shows that the optimal level of z_2 never exceeds $\hat{c}_2$ in this case. Thus, the optimal holding of z_1 must always exceed $\hat{c}_1$. The asymmetry involved in treating the relative price distribution as normal is evident.

To consider the effect of increased price variability on expected utility when z_2 is chosen optimally substitute (4.2) into the second term of (4.1) to yield:

(4.4)
$$(\frac{\partial \hat{c}_1}{\partial S} - \frac{\partial \hat{c}_2}{\partial S})^2 S/4R + \sigma \hat{c}_1 \hat{c}_2 /S \ ,$$

the positivity of which implies:

Proposition 4.3. When the consumer may hedge against price movements an increase in the variance of a lognormally-distributed relative price raises expected utility. The increase in expected utility is greater the larger the elasticity of substitution and the larger the difference between the marginal propensities to consume commodities 1 and 2 in absolute value.

5. Relative Price Variability and Optimal Savings

To consider the impact of price variation on savings assume now that the utility function takes the form of (2.8). For the case in which hedging

is not possible the first-order condition for a maximum is

$$(5.1) \qquad -v_S^1 + E[v_S^2] = 0$$

Following the criterion of Rothschild and Stiglitz [1971] optimal savings will increase or decrease in response to an AMPS in q as $-v_S^1 + v_S^2$ is uniformly convex or concave in q . Differentiating (5.1) twice w.r.t. q yields

$$(5.2) \qquad [-q\frac{\partial^2 c_2}{\partial S \partial q} - (\frac{\partial c_1}{\partial S})(\frac{\partial c_2}{\partial S} + qc_2 \frac{\partial^2 c_2}{\partial S^2}]\, q\, v_S^2 +$$
$$[-c_2 - q\frac{\partial c_2}{\partial q} + 3qc_2(\frac{\partial c_2}{\partial S})]q\, v_{SS}^2 + (qc_2)^2 v_{SSS}^2\ ,$$

which is indeterminate in sign. The following two propositions help clarify (5.2):

Proposition 5.1. An AMPS in q <u>raises</u> optimal savings when the demand for commodity 2 is price and income inelastic and when absolute risk aversion (ARA) is positive and non-increasing.

Setting $\frac{\partial c_2}{\partial q} = \frac{\partial c_2}{\partial S} = 0$, (5.2) reduces to $qc_2(-v_{SS}^2 + q\, c_2\, v_{SSS}^2)$. Leland [1968] demonstrates that decreasing ARA implies $v_{SSS}^2 > 0$ while risk aversion implies $-v_{SS}^2 > 0$. When demand for commodity 2 is inelastic price variability is equivalent to income variability. Leland [1968] also shows that positive, decreasing ARA implies increased income variability will raise optimal savings.

Proposition 5.2. If relative risk aversion is constant for all values of S and q then increased price variability raises or lowers optimal savings as

$$(5.3) \qquad \alpha[2(\alpha - \frac{1}{2}) + E^D - \alpha R](1-R) \gtreqless 0$$

uniformly.

Proof. The condition for constant relative risk aversion (CRRA),

$$(5.4) \qquad d(-Sv_{SS}^2/v_S^2)/dS = 0\ ,$$

implies

$$(5.5) \qquad v_{SSS}^2 = (v_{SS}^2)^2/v_S^2 - v_{SS}^2/S\ .$$

Stiglitz [1969] has shown that global CRRA implies that commodity demand functions are of the form

$$(5.6) \qquad \underline{c} = g(\underline{q})S \ .$$

Substituting (5.5) and (5.6) into (5.2), yields an expression with the sign of (5.3) .

The effects of relative price variability on the rate of return on savings can explain Proposition 5.2. Price variability, by providing opportunities for the ex post substitution of less expensive for more expensive commodities, acts to raise the expected real return on savings. It also makes the real rate of return on savings more uncertain. In the one-commodity case, under CRRA, an increase in the expected rate of return raises or lower savings as $R \lesseqgtr 1$. An increase in uncertainty about the rate of return, generating "capital risk," raises or lower savings as $R \gtreqless 1$. (See Levhari and Srinivasan [1969] or Sandmo [1970], for instance.)

From (5.3) observe that the expected return effect of price variability dominates when $E^D > \alpha R$ and when $\alpha > 1/2$: price variability tends to raise the expected real return on savings more than demand for commodity 2 is price elastic. The capital uncertainty effect dominates when $E^D < \alpha R$ and $\alpha < 1/2$.

When the consumer may hedge against price variation he may exploit the expected return effect while reducing capital uncertainty. When the price of commodity 2 is lognormally distributed, under global CRRA the effect of increased price uncertainty is from (4.4), (5.5) and (5.6), given by the sign of

$$(5.7) \qquad (1-R)[(\frac{\partial \hat{c}_1}{\partial S} - \frac{\partial \hat{c}_2}{\partial S})^2/4R + \hat{c}_1 \hat{c}_2 \sigma/S^2] \ .$$

The term in square brackets is always positive. Thus follows:

Proposition 5.3. When the consumer may hedge against price movements an increase in the variance of a lognormally distributed relative price under global CRRA raises or lowers optimal savings as $R \lesseqgtr 1$.

Only the expected return effect is operative.

7. Concluding Remarks

The purpose of this paper has been twofold: to explore the consequences of relative price variability for expected utility and to consider

its effect on optimal savings. The first problem is analyzed in a quite general context while the second has been addressed for the case in which the utility function is intertemporally additively separable. The formal analysis for the nonseparable case is straightforward and simply adds to the conditions obtained for the separable case terms pertaining to the intertemporal substitutability of consumption.[9]

The analysis has restricted itself to consider the effects of price variability on consumer welfare and savings in partial equilibrium. It suggests, however, that inferences about the general equilibrium welfare and distributional consequences of price stabilization based on supplier and consumer surplus measures suffer from two omissions. First, the consumer's attitude toward the real income uncertainty generated by price variation is ignored. Secondly, the role of portfolio diversification, in the response to changes in the price distribution is not explicitly considered. Moreover, the results of the present analysis do generalize directly to a simple, general equilibrium context.

Consider, for example, a closed economy in which wheat and cloth are produced according to the labor theory of value. One worker is required to produce one square yard of cloth while producing a bushel of wheat requires P_t workers. P_t depends on rainfall in period t, a random variable. Workers are allocated between wheat and cloth production each period with knowledge of actual rainfall that period. All agents in the economy are workers sharing identical tastes.

Consider a costless irrigation scheme which reduces the variability of P_t around its geometric mean. Proposition 3.1 implies that, in the absence of storage, the effect on expected utility of such a scheme is ambiguous. If wheat may be stored costlessly and if P_t is lognormal then the effect is negative. While Samuelson [1972] argues that the ex post mobility of factors assumed here is unusual this example does suggest that, with accurate forecasting, flexibility of resource allocation, or hedging opportunities, natural variability can become an aspect of nature's bounty.

Footnotes

1. Samuelson [1972] has pointed out the limitations of extending Waugh's analysis to general equilibrium. He shows that randomizing the price level around a deterministic equilibrium can never improve average welfare.

2. The effect of increased uncertainty of the first type, that due to uncertainty about the value of future endowments, has been considered by Leland [1968] in a two-period model and by Sibley [1975] in a multiperiod context. Uncertainty about the value of savings relative to all consumption goods in future periods has been analyzed by Levhari and Srinivasan [1969], Hahn [1970], Sandmo [1970], Mirman [1971], Mirrlees [1971], Rothschild and Stiglitz [1971], Merton [1969 and 1971], Dreze and Modigliani [1972], and Kihlstrom and Mirman [1974]. Stiglitz [1972] and Fischer [1975] have analyzed optimal portfolio diversification when relative prices are uncertain but when there exists for each uncertain price an asset with rate of return perfectly correlated with it. Epstein [1975] presents a theoretical framework for treating the savings-consumption decision under uncertainty when only a single asset is available.

3. I am grateful to an anonymous referee for suggesting this particular formalization of the problem discussed in this section.

4. The utility function defined in (2.3) corresponds to Epstein's [1975] "variable indirect utility function."

5. A probability distribution $G(P)$ represents an AMPS in $F(P)$ if

$$\int_a^b [F(P) - G(P)]dP = 0 \quad \text{and}$$

$$\int_a^x [G(P) - F(P)]dP \geq 0 \quad \forall x \in [a,b] \ ,$$

where a and b represent the limits of integration.

6. Note that the relationship between the distributions F and G where G represents a GMPS in F is not necessarily one of second-degree stochastic dominance (SSD) as defined by Hadar and Russell [1971].

7. The measure R refers to attitudes toward temporal rather than timeless uncertainty in real income, as distinguished by Dreze and Modigliani [1972], since in the present model the savings decision occurs ex ante. The value of R , in contrast with the measure of absolute risk aversion, is insensitive to the choice of numeraire.

8. Stiglitz [1972] obtains a similar result in terms of the certainty-equivalent nominal income for a random, normally-distributed relative price.

9. See Leland [1968], Sandmo [1970], Dreze and Modigliani [1972], and Kihlstrom and Mirman [1974] for treatments of savings under uncertainty when utility is not intertemporally separable.

Bibliography

Bradford, D. F. and H. H. Kelejian [1977], "The Value of Information for Crop Forecasting in a Market System: Some Theoretical Issues," _Review of Economic Studies_, 64 (3), pp. 519-532.

Dreze, J. H. and F. Modigliani [1972], "Consumption Decisions Under Uncertainty," _Journal of Economic Theory_, 5, pp. 308-335.

Fischer, S. [1975], "The Demand for Index Bonds," _Journal of Political Economy_, 83, pp. 509-534.

Flemming, J. S., S. J. Turnovsky and M. C. Kemp [1977], "On the Choice of Numeraire and Certainty Price in General Equilibrium Models of Price Uncertainty," _Review of Economic Studies_, 64 (3), pp. 573-584.

Hadar, J. and W. Russell [1971], "Stochastic Dominance and Diversification," _Journal of Economic Theory_, 3, pp. 288-305.

Hahn, F. [1970], "Savings under Uncertainty," _Review of Economic Studies_, 37, pp. 21-24.

Kihlstrom, R. E. and L. J. Mirman [1974], "Risk Aversion with Many Commodities," _Journal of Economic Theory_, 8, pp. 361-388.

Leland, H. E. [1968], "Savings and Uncertainty: The Precautionary Demand for Savings," _Quarterly Journal of Economics_, 82, pp. 465-473.

Levhari, D. and T. N. Srinivasan [1969], "Optimal Savings under Uncertainty," _Review of Economic Studies_, 36, pp. 153-164.

Massell, B. J. [1969], "Price Stabilization and Welfare," _Quarterly Journal of Economics_, 83, pp. 284-298.

Merton, R. C. [1969], "Lifetime Portfolio Selection under Uncertainty: The Continuous-Time Case," _Review of Economics and Statistics_, 51, pp. 247-257.

_____ [1971], "Optimum Consumption and Portfolio Rules in a Continuous-Time Model," _Journal of Economic Theory_, 3, pp. 373-413.

Mirman, L. J. [1971], "Uncertainty and Optimal Consumption Decisions," _Econometrica_, 39, pp. 179-185.

Mirrlees, J. A. [1971], "Optimum Growth and Uncertainty," unpublished paper prepared for I. E. A. Workshop of Economic Theory, Bergen, 1971.

Rothschild, M. and J. E. Stiglitz [1970], "Increasing Risk: I. A Definition," _Journal of Economic Theory_, 2, pp. 225-243.

_____ [1971], "Increasing Risk: II. Its Economic Consequences," _Journal of Economic Theory_, 3, pp. 66-84.

Samuelson, P. A. [1972], "Feasible Price Stability," _Quarterly Journal of Economics_, 86, pp. 476-493.

Sandmo, A. [1970], "The Effect of Uncertainty on Saving Decisions," Review of Economic Studies, 37, pp. 353-360.

Sibley, D. S. [1975]., "Permanent and Transitory Income Effects in a Model of Optimal Consumption with Wage Income Uncertainty," Journal of Economic Theory, 11, pp. 68-82.

Stiglitz, J. E. [1969], "Behavior Towards Risk with Many Commodities," Econometrica, 37, pp. 660-667.

_____ [1972], "Portfolio Allocation with Many Risky Assets," Mathematical Methods in Investment and Finance. Edited by G. P. Szego and K. Shell (Amsterdam: North-Holland).

Tesfatsion, L. [1976], "Stochastic Dominance and the Maximization of Expected Utility," Review of Economic Studies 43 (2), pp. 302-315.

Turnovsky, S. J. [1974], "Technological and Price Uncertainty in a Ricardian Model of International Trade," Review of Economic Studies, 41, pp. 201-217.

_____ [1976], "The Distribution of Welfare Gains from Price Stabilization: The Case of Multiplicative Disturbances," International Economic Review, 17, pp. 133-148.

Waugh, F. V. [1944], "Does the Consumer Benefit from Price Instability?" Quarterly Journal of Economics, 58, pp. 602-614.