# ISSUES IN CONTROLLABILITY AND THE THEORY OF ECONOMIC POLICY

Willem H. Buiter
Mark Gersovitz

Econometric Research Program
Research Memorandum No. 232

September 1978

Econometric Research Program
Princeton University
207 Dickinson Hall
Princeton, New Jersey

# ISSUES IN CONTROLLABILITY AND THE THEORY OF ECONOMIC POLICY

Willem H. Buiter
Mark Gersovitz
Princeton University

## ABSTRACT

The paper demonstrates that the concepts of dynamic controllability are useful for the theory of economic policy by establishing four propositions. First, dynamic controllability is a central concept in stabilization policy. Second, the ability to achieve a target state, even if it cannot be maintained, may be of economic interest. Third, dynamic controllability is of special interest for "historical" models. Fourth, the conditions for any notion of dynamic controllability are distinct from and weaker than those for Tinbergen static controllability.

# ISSUES IN CONTROLLABILITY AND THE THEORY OF ECONOMIC POLICY

Willem H. Buiter
Mark Gersovitz
Princeton University

## 1. Introduction

A considerable volume of research on extending the theory of economic policy to dynamic models has been summarized in a recent paper by Nyberg and Viotti [1978] (henceforth N-V). It is the purpose of our paper to emphasize certain issues not generally discussed in this literature, thereby supplementing and extending the discussion summarized in N-V. In particular, we focus on an assessment of the N-V conclusion that "the concept of [dynamic] controllability. . . is of limited interest for the theory of economic policy. . . ." We find four important reasons for qualifying this statement.

First, dynamic controllability provides a convenient sufficient criterion for determining whether the policy authority has the ability to steer the economy toward an equilibrium state. The consequent importance of controllability for stabilization policy is discussed in Section 2. Second, controllability can be relevant for policy even if the state to which the economy is moved cannot be maintained. We provide examples in Section 3. Third, controllability is especially interesting for models exhibiting hysteresis i.e. models for which the equilibrium depends on the initial conditions. An example of such a historical model is given in Section 4. Finally, N-V have inferred an overly strict requirement for a system to be perfectly controllable which has led them to an incorrect generalization of Tinbergen's static controllability condition to dynamic systems. This issue is discussed in Section 5. Before proceeding to Sections 2 to 5, we establish some terminological conventions and recall some important theorems on dynamic systems.

Dynamic Point Controllability[1/]

Consider a linear system with constant coefficients:

(1) $\dot{x} = Ax + Bu$

A is an nxn matrix and B is an nxr matrix of constants. x is an n-vector of state variables and u is an r-vector of instruments or controls. What N-V call controllability or dynamic controllability we call dynamic point controllability.

The system of (1) is dynamically point controllable iff there exists a path for the controls capable of moving the state vector from any initial state in the state space and from any initial time to any other terminal or target state in pre-assigned finite time. The necessary and sufficient condition for the system (1) to be dynamically point controllable is that the n x nr matrix $\phi$ have rank n where

(2) $\phi = [B, AB, A^2B \ldots, A^{n-1}B]$ [2/]

Dynamic Path Controllability

What N-V refer to as perfect controllability,[3/] we call dynamic path controllability. The system (1) is dynamically path controllable iff there exists a path for the controls capable of moving the state vector from any initial state and from any initial time along any pre-assigned (target) trajectory for any pre-assigned finite time interval. The necessary and sufficient condition for (1) to be dynamically path controllable is that the $n^2 \times (2n-1)r$ matrix $\Psi$ have rank $n^2$ where

(3) [4]

$$\Psi = \begin{bmatrix} B & AB & \cdot & \cdot & \cdot & \cdot & A^{2n-2}B \\ 0 & B & & & & & \cdot \\ \cdot & 0 & \cdot & & & & \cdot \\ \cdot & \cdot & & \cdot & & & \cdot \\ \cdot & \cdot & & & \cdot & & \cdot \\ 0 & 0 & & & & B & A^{n-1}B \end{bmatrix}$$

In what follows, A and B will be assumed to be of full rank.

## Static Controllability

An equilibrium of the system (1) is any $x^*$ such that

(4) $$0 = Ax^* + Bu$$

for constant u. The equilibrium of system (1) is statically controllable iff there exists a $\tilde{u}$ such that $0 = Ax^* + B\tilde{u}$ for all $x^*$. If A is of full rank (n), the equilibrium of (4) is statically controllable iff the rank of B = n, i.e. there should be as many linearly independent instruments as there are linearly independent targets.[5]

## 2. Dynamic Point Controllability of Target States that are Equilibria

N-V emphasize that if the target state is not an equilibrium of the system, dynamic point controllability only ensures that there is an adjustment path that will make the system pass through the target state at a pre-assigned point in time (T). Thus this concept does not indicate whether it is possible to keep the system at the target state beyond T. However, if the state is an equilibrium of the system, it is clearly possible to keep the system there.

From the viewpoint of stabilization policy, dynamic point controllability is therefore a more important property of the system than stability. If the system is stable, it returns to an equilibrium after a perturbation with the policy authority rigidly adhering to whatever fixed values of its controls are consistent with the original equilibrium. Stability analysis therefore assumes u is fixed and considers only the eigenvalues of the matrix A. Dynamic point controllability, by contrast, implies that there exists a trajectory for the u capable of returning the system to an equilibrium after a perturbation.[6/] Consequently, we consider dynamic point controllability to be a better characterization of the policy options potentially available to the policy authority. This interpretation is strengthened by the connection between dynamic point controllability and the stabilizability of a system.

## Stabilizability

A pair of matrices (A, B) is stabilizable if the range space of $[B, AB, \ldots, A^{n-1}B]$, i.e. the space spanned by its columns, contains the subspace spanned by the eigenvectors of A with non-negative real parts (Aoki [1973], p. 134). Intuitively, (A, B) are a stabilizable pair if all sources of instability in A can be eliminated by a control matrix B, as the following propositions, advanced without proof,[7/] indicate.

Proposition 1 If the dynamic system (1) is dynamically point controllable, (A, B) are a stabilizable pair.

In other words, dynamic point controllability implies stabilizability. Proposition 2 implies that a system which is stabilizable can always be stabilized in a simple manner.

Proposition 2: If (A, B) is a stabilizable pair, there exists an (r x n) matrix $\Gamma$ such that $A + B\Gamma$ is a stable matrix, i.e. all eigenvalues of $A + B\Gamma$ have negative real parts.

Consequently, dynamic controllability implies that there always exists a set of proportional feedback controls which stabilize the system. Proportional feedback is equivalent to policy behavior characterized by partial adjustment, the simplest and most intuitive form of response to disequilibrium. To know that any system that is dynamically point controllable can be stabilized in so simple a manner is clearly of great interest.

## 3. Dynamic Point Controllability of Disequilibrium States

The previous section focussed on the usefulness of the dynamic point controllability concept with respect to equilibrium states. In this section we briefly consider the usefulness of this concept when the state which is reached at T is not an equilibrium and it is therefore not possible to say, at least on the basis of point controllability [but see Section 5], whether the system can be made to stay in this state.

N-V dismiss the importance of reaching a state which cannot be maintained: "In economics, it is usually not sufficient to reach the desired position; we must also be able to stay there." While we are in general agreement with this statement we wish to emphasize exceptions to this position. For instance, models of the political business cycle emphasize that governments may try to bring the economy to a point on, or just before, the election which ensures re-election. Problems of sustainability after re-election may be of secondary importance, especially if the favorable situation can be reconstructed by the next election. Clearly, controllability is the natural analytical device for this purpose. Other examples of the usefulness of point controllability may well be developed by the consideration of other problems in political economy. For instance, tariff retaliation could be formulated as a dynamic game where the ability to reach a state in which one's opponent capitulates may be important.

## 4. Dynamic Point Controllability of Historical Systems

Hysteresis is the dependence of an equilibrium on the initial state and the path the economy experiences towards the equilibrium. Consider the system (1) when A is not of full rank. In this situation, $A^{-1}$ does not exist and the equilibrium of the system is not uniquely determined, for any given fixed u, by (4). Instead, if A has no roots with positive real parts,[8/] it will move from any particular initial state to an equilibrium determined by that initial state, the disequilibrium path of u and the final value of u. If the system is dynamically point controllable, the equilibrium values of the instruments can be used to control the equilibrium values of those state variables that are statically controllable; the initial values of the instruments and their values during the adjustment towards equilibrium can be used to control the equilibrium values of the remaining state variables.

A potentially important example of a historical model in which "the time path to equilibrium partially shapes that equilibrium" is mentioned in Phelps' Inflation Policy and Unemployment Theory (77-80, 256). If a temporary boom has permanent effects on the attitudes and or aptitudes of workers, i.e. if a departure from equilibrium produces effects which persist after the return to equilibrium, the natural or equilibrium rate of unemployment is not invariant to the expected inflation rate.

The following macrodynamic model provides an example of the role of point controllability in the analysis of a historical system. Let p denote the price level, $\Pi$ the <u>expected</u> rate of inflation, y real income, r the real interest rate, M the nominal stock of outside money and G real public spending on final goods and services and m = M-p the real money stock. All variables are measured in <u>natural logarithms</u>.

The structural equations are

(5a) $\dot{p} = \alpha(y-y^*) + \Pi$ $\qquad \alpha > 0$

(5b) $\dot{\Pi} = \eta(\dot{p}-\Pi)$ $\qquad \eta > 0$

(5c) $y = \beta r + \gamma G$ $\qquad \beta < 0, \gamma > 0$

(5d) $m = \delta r + \varepsilon y$ $\qquad \delta < 0, \varepsilon > 0$

(5e) $\dot{M} = \Pi$

Equation (5a) is an expectations-augmented price Phillips curve or a Phelps-Friedman-Lucas supply function. Equation (5b) gives the adaptive expectations mechanism governing inflation expectations. The IS curve is given by (5c), the LM curve by (5d). The government is assumed to pay interest on its monetary debt at a rate equal to the <u>expected</u> rate of inflation.[9/] The real interest rate is therefore the appropriate argument in the money demand function. Equation (5e) states that interest on the stock of money is paid for by running the government printing press. Any remaining public sector deficit or surplus is financed by borrowing. Government bonds are not perceived as net worth and play no further part in the model.

It is convenient to rewrite the model as

(6a) $$m = \delta r + \varepsilon y$$

(6b) $$y = \beta r + \gamma G$$

(6c) $$\dot{m} = -\alpha(y-y^*)$$

(6d) $$\dot{\Pi} = \eta\alpha(y-y^*)$$

(6e) $$\dot{p} = \alpha(y-y^*) + \Pi$$

Equations (6a - c) can be solved for the entire trajectories of m, y and r, given an initial value for m (i.e. for M and for p) and given the path of the policy instrument G. With the path of y given by the solution of (6a - c), and given an initial value for $\Pi$, equations (6d) can be solved for $\Pi$. Given the paths of y and $\Pi$, equation (6e) determines $\dot{p}$ at each point in time. An initial condition for p then determines the entire trajectory for p.

The equilibrium equations are

(7a) $$y = y^*$$

(7b) $r = (y^* - \gamma G)/\beta$

(7c) $m = [(\delta + \varepsilon\beta)y^* - \delta\gamma G]/\beta$

(7d) $\Pi = \dot{p} = \text{constant}$

The steady state rate of inflation (actual and expected) cannot be determined from the steady state conditions alone. Neither can the price level path or the path of the nominal stock of money. It is apparent from (7c) that the real stock of money is constant and well defined.

The state space representation of the system is, in terms of deviations from the steady state $(m^*, \Pi^*, G^*)$ :

(8) $$\begin{bmatrix} \dot{m} \\ \dot{\Pi} \end{bmatrix} = \begin{bmatrix} -\alpha\beta/(\delta+\varepsilon\beta) & 0 \\ \eta\alpha\beta/(\delta+\varepsilon\beta) & 0 \end{bmatrix} \begin{bmatrix} m-m^* \\ \Pi-\Pi^* \end{bmatrix} + \begin{bmatrix} -\alpha\gamma\delta/(\delta+\varepsilon\beta) \\ \eta\alpha\gamma\delta/(\delta+\varepsilon\beta) \end{bmatrix} [G-G^*]$$

The rank deficiency of the state matrix is a reflection of the <u>hysteresis</u> property of the model; the steady state equations (7c) and (7d) cannot by themselves determine the equilibrium rate of inflation. Consequently, the target vector $(m, \Pi)$ is not statically controllable. Only a one dimensional subspace of the state space (i.e. only m) is statically controllable. Given an initial condition for $\Pi$, however, any solution of (8) which converges to a steady state will generate a well-defined equilibrium rate of inflation. As the path of m is independent of that of $\Pi$, a different initial condition for $\Pi$ will generate a different solution trajectory for $\Pi$ and a different steady state rate of inflation.

If the system (8) is dynamically point controllable, the policy-maker can indeed choose any initial condition for the state vector. By varying this initial condition, the government can alter the steady state rate of inflation. The steady state value of m is, however, independent of the initial conditions and depends only on the steady state value of G. Thus, while the Tinbergen static controllability criterion states that only m is statically controllable, dynamic point controllability implies that the policy authority can control the steady state value of both m and $\Pi$. The latter, however, can be controlled only by leaving the steady state temporarily and taking advantage of the hysteresis property of the model by choosing a different initial condition.

The system in (8) will be dynamically point controllable if the rank of the matrix $\Omega$ is two.

(9)
$$\Omega = \begin{bmatrix} -\frac{\alpha\delta\gamma}{\beta\varepsilon+\delta} & \frac{\alpha^2\beta\gamma\delta}{(\beta\varepsilon+\delta)^2} \\ \frac{\eta\alpha\gamma\delta}{\beta\varepsilon+\delta} & \frac{\eta^2\alpha^2\beta\gamma\delta}{(\beta\varepsilon+\delta)^2} \end{bmatrix}$$

It is readily seen that, except by accident, $\Omega$ will indeed be of full rank.

5. Dynamic Path Controllability

The necessary and sufficient condition for the system $\dot{x} = Ax + Bu$ to be dynamically path controllable is that the matrix $\Psi$ of (3) have rank $n^2$. N-V state that for a system to be dynamically path controllable, "it turns out that the number of instruments must be at least as many as the number of targets, i.e. the Tinbergen rule is exactly carried over to the general dynamic case." (Nyberg and Viotti [1978,

p. 78]). In this section we show that dynamic path controllability is possible when the number of linearly independent instruments is less than the number of linearly independent targets.

The source of N-V's error is a misinterpretation by Aoki of Corollary 2 of his Proposition 1, stating the necessary and sufficient conditions for dynamic path controllability (Aoki [1975, p. 295]). The corollary correctly states that the system (1) is dynamically path controllable only if

$$(10) \qquad n \leq (2 - \frac{1}{n})r.$$

Aoki then interprets this condition erroneously as "only if the number of target variables is less than or equal to the number of instrument variables" (Aoki [1975, p. 295]).

If n=1, the system is indeed only controllable if $r \geq 1$. For n=2, (10) implies $r \geq 4/3$ which, with only integer numbers of controls admissable, requires $r \geq 2$. When n=3, the condition is $r \geq 9/5$ which is satisfied by r=2. In the limit, as $n \to \infty$, the necessary condition is $n \leq 2r$, i.e. the number of instruments must not be less than half the number of targets. This condition is clearly much weaker than the Tinbergen condition for static controllability and does not justify the N-V statement quoted above.

First, we note that the Tinbergen condition is sufficient for dynamic path controllability. Inspection of (3) shows that $\Psi$ has rank $n^2$ if B has rank n. Furthermore if A is the identity matrix, it is clear that B must have rank n for $\Psi$ to have rank $n^2$. The question is, are there weaker conditions that are still sufficient? We have not been able to state a general condition, but it is possible to find numerical examples for which $n > r$ and yet rank $\Psi = n^2$. As the discussion above implies, the lowest value of n for which this search could be successful is three.

Given the result about A equal to the identity matrix, it is clear that an A matrix which fulfills the $\Psi$ condition is not likely to be sparse. This conjecture is economically intuitive: without n instruments, it seems clear that the instruments ought to affect the states not only directly through the B matrix but also indirectly through the A matrix. A pair of A and B matrices, both of full rank, for which rank $\Psi = 9$ is

$$(11) \qquad A = \begin{bmatrix} -3 & -4 & 9 \\ -17 & 2 & 7 \\ -20 & -6 & -4 \end{bmatrix} \qquad B = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$$

The counterexample demonstrates that the mathematical proposition "dynamic path controllability requires the number of linearly independent instruments to equal the number of linearly independent target variables" is incorrect. However, the behavior of the controls required in order to achieve dynamic path controllability with fewer independent instruments than targets may violate unstated physical or economic constraints, even if it does not violate the simple linear structure to which our mathematical path controllability proposition refers. In addition, in a discrete time formulation, both the number of periods until the system must follow the target trajectory and the number of periods for which the system is to remain on the target trajectory further restrict the scope for path controllability. Consider (12).

(12) $x_t = Ax_{t-1} + Bu_t$

$x_t$ is an nx1 vector of state variables, $u_t$ an rx1 vector of controls.

The discrete time model given in (12) is dynamically path controllable if, after attainment of a given point $x_t = x_t^*$, $x_t^*$ given, the model can remain on a given trajectory $x_{t+p} = x_{t+p}^*$, $x_{t+p}^*$ given for p = 0, 1, 2, ..., P-1.

A necessary and sufficient condition for dynamic path controllability is (Uebe [1977]) that the rank of the nP x (τ+P-1)r matrix Ψ' given in (13) has rank nP

(13)

$$\Psi' = \Psi'(A, B, P, \tau) = \begin{bmatrix} A^{\tau-1}B & A^{\tau-2}B \ldots & AB & B & 0 \ldots & 0 \\ A^{\tau}B & A^{\tau-1}B \ldots & A^2B & AB & B \ldots & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot & \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & \cdot & & \cdot \\ A^{\tau-2+P}B & A^{\tau-3+P}B \ldots & A^PB & A^{P-1}B & A^{P-2}B \ldots & B \end{bmatrix}$$

Note that in the discrete time formulation of the problem, time enters in three ways: initial time, t = 0; the start of the control period, t = τ; the end of the control period, t = τ+P-1. Common sense suggests that the sooner one is to get the economy on track, from a given initial position $x_o$, the harder it is to satisfy the dynamic path controllability condition. This result is indeed true. We know that rank $\Psi' \leqq \min(nP, (\tau+P-1)r)$. A necessary condition for dynamic path controllability is therefore that

(14) $P[\frac{n}{r} - 1] + 1 \leqq \tau$

The smaller $\tau$, i.e. the earlier the start of the control period, the harder it is to satisfy (15). In the limit, if we require the economy to get on target immediately ($\tau$=1), it is necessary to have at least as many instruments as controls. It is in this context that the N-V claim is correct. Note also that (12) is less likely to be satisfied the larger P or n and the smaller r.

Dynamic path controllability for the continuous time model is akin to dynamic path controllability for the discrete time model--in the sense that the rank condition on $\Psi$ in (3) is equivalent to the rank condition on $\Psi'$ in (13)--when $P = \tau = n$. In the continuous time model, the controls can be varied continuously; thus the influence of the initial condition $x_o$ can be undone in an arbitrarily short interval even if the system is merely dynamically point controllable. In the discrete time case, it may take many periods to "purge" the influence of the initial state. If the dynamic path controllability condition is satisfied for the continuous time model, the economy can track a given target trajectory of arbitrary length. With the discrete time model, the dynamic path controllability condition becomes harder to satisfy, the longer the control period (the larger P). Even with the discrete time model, however, the condition that the rank of B be equal to n, is not always necessary for dynamic path controllability. By inspection it is clear that, for all $\tau$ and P, equality between the numbers of linearly independent targets and instruments is sufficient for dynamic path controllability. (Tinbergen static controllability can be viewed as the special case $P = \tau = 1$.) Provided the system has sufficient time to get "on track" ($\tau$ is not too small) and provided the control period is not too long (P is not too large), dynamic path controllability may be achieved with fewer independent instruments than targets even with the discrete time model.

Conclusion

The aim of this paper has been to demonstrate the usefulness of dynamic controllability concepts for the theory of ecnomic policy. To make our case we established four propositions: 1) dynamic controllability is a central concept in stabilization policy; 2) the ability to achieve a target state, even if it cannot be maintained, may be of economic interest; 3) dynamic controllability is of special interest for historical models; 4) the conditions for any notion of dynamic controllability are distinct from and weaker than those for Tinbergen static controllability. On the basis of these results, we believe that the two notions of dynamic controllability will play a growing role in the dynamic extensions of the theory of economic policy.

FOOTNOTES

The authors have benefited from discussions with Gregory Chow.

[1]The definition given here is for dynamic point controllability of the state vector x. This is sometimes contrasted with dynamic point controllability of the output vector y, when the complete dynamic system is given by:

(1') $\dot{z} = \alpha z + \beta u$

(1") $y = \gamma z + \delta u$

y is an m- vector of output variables, z an n'-vector of state variables and u and r-vector of controls. In fact, (1) is perfectly general. Use the transformation $\dot{q} = y$. This permits us to rewrite (1') and (1") as

$$\dot{x} = Ax + Bu, \text{ with } x = \begin{bmatrix} z \\ q \end{bmatrix}, \; A = \begin{bmatrix} \alpha & 0 \\ \gamma & 0 \end{bmatrix}, \; B = \begin{bmatrix} \beta \\ \delta \end{bmatrix},$$

i.e. x is an $(n' + m) = n$ vector, A is an $(n' + m) \times (n' + m)$ matrix and B is an $(n' + m) \times r$ matrix. The necessary and sufficient condition for dynamic output point controllability of the system (1'), (1") reduces to the requirement that the rank of the $m \times (r+1)n$ matrix $[\delta, \gamma\beta, \gamma\alpha\beta, \gamma\alpha^2\beta, \ldots, \gamma\alpha^{n-1}\beta]$ be m (Aoki [1976], p. 89.).

[2]See e.g. Preston [1974], Buiter [1975], Gersovitz [1975] and Aoki [1976].

[3]The literature also refers to it as <u>functional reproducibility</u>; Brockett and Mesarovic [1965], Basile and Marro [1971].

[4]See Aoki [1975, 1976]. For the criterion for dynamic path controllability of the output vector y when $y = Cx + Du$; $\dot{x} = Ax + Bu$, see footnote 1.

[5] See e.g. Tinbergen [1955].

[6] Because all trajectories are considered feasible, dynamic controllability may overstate the options available since there may be outside constraints on the path of the u, for instance, inequality constraints.

[7] For proofs see Wonham [1967] and Heymann [1968]. See also Aoki [1973, 1976].

[8] If A is singular, there is at least one zero eigenvalue. (1) is therefore either unstable (there is at least one root with a positive real part) or neutral: all non-zero roots have negative real parts.

[9] This policy rule is not meant to be descriptive. We chose it because it generates the hysteresis property in a particularly transparent manner. A possible interpretation is that monetary authority is prohibited from levying an "inflation tax." If inflation occurs nevertheless, the monetary authority is compelled to compensate holders of money balances.

BIBLIOGRAPHY

[1] Aoki, Masanao [1973], "On Sufficient Conditions for Optimal Stabilization Policies," Review of Economic Studies, 40, January, pp. 131-138.

[2] ----- [1975], "On a Generalization of Tinbergen's Condition in the Theory of Economic Policy to Dynamic Models," Review of Economic Studies, 42, April, pp. 293-296.

[3] ----- [1976], Optimal Control and System Theory in Dynamic Economic Analysis, North Holland, Amsterdam.

[4] Basile, G. and Marro, G. [1971], "On the Perfect Output Controllability of Linear Dynamic Systems," Ricerche di Automatica, 2, pp. 1-10.

[5] Brockett, R.W. and Masarovic, M. [1965], "The Reproducability of Multivariable Systems," Journal of Mathematical Analysis Applied, 11, p. 548-563.

[6] Buiter, Willem H. [1975], Temporary Equilibrium and Long-Run Equilibrium, Unpublished Ph.D. Thesis, Yale University, Ch. 6, "Controllability, Stabilization and Optimization" and Appendix 2, "Stability, Controllability, Stabilization and Optimization."

[7] Gersovitz, Mark [1975], An Essay on Macrodynamics, Unemployment and Inflation, Unpublished Ph.D. Thesis, Yale University.

[8] Heymann, M. [1968], "Comment on Pole Assignment in Multi-Input Controllable Linear Systems," IEEE Trans. Aut. Control, AC-13, Dec., pp. 748-749.

[9] Nyberg, Lars and Viotti, Staffan [1978], "Controllability and the Theory of Economic Policy," Journal of Public Economics, 9, January, pp. 78-81.

[10] Phelps, Edmund S. [1972], Inflation Policy and Unemployment Theory, W.W. Norton, New York.

[11] Preston, A.J. [1974], "A Dynamic Generalization of Tinbergen's Theory of Policy," Review of Economic Studies, 41, January, pp. 65-74.

[12] Tinbergen, Jan [1955], On the Theory of Economic Policy, 2nd ed., North Holland, Amsterdam.

[13] Uebe, G. [1977], "A Note on Aoki's Perfect Controllability of a Linear Macroeconomic Model," Review of Economic Studies, 44, February, pp. 191-192.

[14] Wonham, W.M. [1967], "On Pole Assignment in Multi-Input Controllable Linear Systems," IEEE Trans. Control AAC-12, pp. 660-665.