MARGINAL CONSUMERS AND

NEOCLASSICAL DEMAND THEORY

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1. INTRODUCTION

Ask any economist to explain the elasticity of demand for a product, and it is likely he will speak not only of income and substitution effects, but also of "marginal consumers" who have reservation prices in a neighborhood of the existing price. Nevertheless, most textbook aggregate demand analysis at the advanced level ignores "marginal consumers" when describing price induced changes in the demand for a single commodity. This is because, with a finite number of consumers, an infinitesimal change in price will typically not change the number of consumers with positive demand for a commodity; thus, all changes in demand are properly explained by summing the neo-classical income and substitution effects. In order to have the effect of "marginal consumers" have force, it is necessary to model demand as being generated by a continuum of agents; e.g., one for each point a in the interval [0,1]. Then, an infinitesimal change in price will typically influence the number of consumers who purchase the commodity.[1]

1/ Of course, the idealization of a continuum of agents has always been assumed (explicitly or implicitly) whenever the slope of a continuous demand function has been related to the number of consumers who enter (or leave) a market in response to a price change.

Our purpose here is to indicate how the neoclassical theory of demand can be extended so that marginal consumers play a significant role in the determination of the slope of an aggregate demand function. Theories of consumers who pick one unit of one type of a differentiated commodity (see; e.g. Rosen (1974) and McFadden (1977)) as well as the neoclassical theory obtain as special cases. Furthermore, the framework is suitable for the analysis of empirical situations in which a discrete change in price leads some consumers to switch brand (or mode) and others to change the magnitude of their consumption.[2] The analysis leads to a decomposition of price induced demand changes into three effects.

(S) (The Aggregate Substitution Effect) With real income held constant, an increase in the price of commodity γ will cause each consumer of γ to substitute away from that commodity.

(I) (The Aggregate Income Effect) An increase in the price of a commodity purchased by agent <u>a</u> lowers the amount of real income available to <u>a</u>, and lower real income "normally" results in a reduced consumption of each commodity.

(C) (The Change of Commodity Effect) A small increase in the price of a commodity leads some consumers to switch consumption from the commodity whose price was raised to a similar commodity.

2/ Houthakker's (1952) pioneering analysis is confined to a single consumer; also he does not consider the case of a change in the price of a single differentiated commodity. For the econometrics of inter-related discrete and continuous choice, see J. Heckman (1978) and Richard Westin (1974).

It is of course possible for the sum of the aggregate income and price effects to be positive. A principal use of the neoclassical theory is to indicate this possibility for an individual consumer.[3] But the Aggregate Change of Commodity Effect is Always Negative. As an application of the framework introduced here, we show that even if individual demand functions are upward sloping, the unambiguous change of commodity effect will guarantee that market demand for a commodity must slope downwards whenever there are differentiated commodities which are sufficiently close to the commodity in question.

2. INGREDIENTS OF THE MODEL

The specification of the model is based on a situation in which a large number of consumers choose among alternative brands of a highly divisible commodity; e.g., gasoline or scotch whisky. Some instances of job choice and transportation mode choice are also accommodated.

Consider an economy with an infinite number of consumers. There is a numeraire commodity x and a finite number of differentiated commodities. All products are completely divisible and the standard neoclassical figure applies nicely to explain how much of each commodity an individual chooses to consume. Let a price vector be given. Even though there are many commodities, almost all consumers choose a positive amount of only one of the differentiated commodities.[4] For each

3/ Some have declared that this is the only use of the neoclassical theory. Certainly, the major message in intermediate textbook treatments is that the effect of a price change can be decomposed, and as a result of the ambiguous income effect, it is possible for demand and price to rise together.

4/ This is not necessarily due to indivisibilities, which would specialize the analysis to a logit-type framework, but rather because of the form of consumers' preferences.

consumer, a significant (non-infinitesimal) increase in the price of the differentiated commodity of his choice, call it γ, will have one of two possible effects. Either the consumer will continue to purchase γ and the standard income and substitution effects concerning x and γ apply, or the consumer will leave the market for γ. In terms of infinitesimal analysis, we can compute for each consumer who is choosing γ the neoclassical income and substitution effects; the aggregate effects are obtained by integration. In addition, we can compute the rate of change in demand associated with the rate at which consumers leave (enter) the market for γ and the amount which was demanded by each of those consumers. Thus, the effect of a price change is conveniently decomposed into an aggregate neo-classical income effect, an aggregate neo-classical substitution effect, and an aggregate change of commodity effect. For example: the price of "Pepsi" rises slightly; people who choose to drink "Pepsi" both before and after the price change exhibit neo-classical income and substitution effects, which we aggregate in the usual manner. Also, a small group of "marginal consumers" switch to "Coke" or "R.C.", and their action is aggregated as well. The theory is more definite because a rise in the price of "Pepsi" does not attract new "Pepsi" consumers.

Commodity zero has its price fixed at unity; amounts of commodity zero are indicated by x, x', etc. In addition, there are a finite number of differentiated commodities which are indexed by a finite subset of the closed unit interval. For $\gamma \epsilon [0,1]$, the real number $y(\gamma)$ denotes an amount of the commodity of type γ. Consumers will also be indexed by the closed unit interval. Suppose there are two differentiated commodities: γ_1 and γ_2. Then, a generic commodity bundle is represented by a triple of real numbers $(x, y(\gamma_1), y(\gamma_2))$. In this case, preferences are naturally represented with the aid of figure 1. For each level of numeraire commodity x, the indifference curves relating γ_1 and γ_2 are straight lines, which reflect the fact that one unit of γ_1 and β units of γ_2 are viewed as "perfect substitutes" for some choice of β. (The choice of β depends in general on the level of x, γ_1 and γ_2 consumption. Also, for any value of $y(\gamma_1)$ (respectively $y(\gamma_2)$), the indifference curves relating x and $y(\gamma_1)$ (respectively $y(\gamma_2)$) are strictly convex.[5] In particular, the possibility that the numeraire, γ_1, or γ_2 are Giffen goods is considered. Since the differentiated commodities are perfect substitutes (up to a scaling), a consumer will "normally" buy only one of the differentiated commodities. (However, for each consumer there are prices which make him indifferent among several differentiated commodities.)

A point (x, y, γ) in figure 2 represents the level of numeraire commodity x, and a level y of a particular differentiated commodity γ. Given prices $\pi(\gamma)$ for each

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5/ If $p = (x, y_1, y_2)$ is indifferent to $p' = (x', y_1', y_2')$ and $0 < t < 1$, we require $tp + (1-t)p'$ is preferred or indifferent to p accordingly as $x \neq x'$ or $x = x'$.

of the differentiated commodities γ, and income R, the point (x,y,γ) is affordable if $x + \pi(\gamma)y \leq R$. We have denoted the affordable set by B. The indifference curves c_1, c_2 and c_3 are drawn for three different values of γ, and correspond to the same level of utility. The consumer whose preferences are illustrated is indifferent among bundles composed of points from these curves. As we have noted, it is possible for more than one choice in B to be maximal.

Consistent with the above explanation, the utility maximizing action of each consumer contains at least one bundle in which only one commodity other than numeraire is being consumed in a positive amount. Thus, we assume without loss of generality that consumers' preferences are represented by the function U (with generic argument (x,y,γ,a)) together with a function ρ which specifies the distribution of a. For each agent a, $U(\cdot,\cdot,\cdot,a)$ represents a's preferences. For each a and value of U, an indifference surface is defined.[6] We are interested in situations in which only a finite number of commodities are available; thus, only a finite number of the γ_i

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6/ Unless stated to the contrary, we maintain the hypothesis that U and ρ are twice continuously differentiable in their arguments.

have prices. The notation (p_i,γ_i), $i = 1,2,\ldots,n$ will refer to a situation in which commodities $\gamma_1 < \gamma_2 < \ldots < \gamma_n$ are available at prices $p_1,p_2\ldots,p_n$.[7] The budget set of the consumer with income R who faces (p_i,γ_i), $i = 1,2,\ldots,n$ consists of affordable points (x,y,γ) such that $\gamma\varepsilon\{\gamma_1,\gamma_2,\ldots,\gamma_n\}$; in the second figure this is $B_1 \cup B_2 \cup B_3$.

A simple example serves to illustrate the theory. Let $U(x,y,\gamma,a) = xy - (\gamma-a)^2$, and assume that a is distributed uniformly over the unit interval with density one. Let commodities $\{i/4\}$, $i = 1,2,3$ be available and let the income of each consumer be fixed at 1. We consider the market for commodity 1/2 when the price of commodities 1/4 and 3/4 are fixed at one. The marginal consumer (for $a > 1/2$) depends on the price of commodity 1/2, and is identified by the condition that the indirect utility of commodity 1/2 at price p is the same as the indirect utility of 3/4 at price 1. This yields the equation $1/4p-((1/2)-a)^2 = 1/4-((3/4)-a)^2$. Starting at $p = 1$, we can easily evaluate the aggregate price, income and change of commodity effects of a change in the price of commodity $\gamma = 1/2$. For consumers to the right of $a = 1/2$, the total effect is simply

$$\int_{1/2}^{a(1)=5/8} \frac{d\xi^a(p)}{dp}\rho(a)da + [\xi^a(1)\rho(5/8)\frac{da(1)}{dp}],$$

7/ Throughout we limit attention to "situations" which have the characteristic that the demand for each commodity is positive.

where ξ^a is the demand for commodity 1/2 by agent a. Decomposing the first term into the neoclassical substitution and income effects and substituting $\xi^a=1/2p, \rho=1$, and $\frac{da}{dp}=-1/2p^2$ yields

$$\int_{1/2}^{5/8} -(1/2p^2)da + \int_{1/2}^{5/8} 0da - (1/4p^3).$$

For p = 1, the aggregate substitution, income and commodity change effects (for a > 1/2) are thus -1/16, 0, and -1/4 respectively. By symmetry, the effects for a unrestricted are double these.

We proceed to a general case.

Consumers : $U = U(x,y,\gamma,a)$

$R = R(a)$, and

a is distributed over [0,1] with density $\rho > 0$.

Let $V(\gamma_j,p,a)$ be the indirect utility function for consumer a when commodity γ_j has price p, and the consumer is constrained to purchase only γ_j and the undifferentiated commodity, x. Given (p_i,γ_i) $i = 1,2,\ldots,n$, consumer a purchases good γ_j only if $V(\gamma_j,p_j,a) \geq \max_{i \neq j} V(\gamma_i,p_i,a)$. With all other prices fixed, if p_j is increased, $V(\gamma_j,p_j,a)$ is decreased for all a,[8] while $V(\gamma_i,p_i,a)$ is unchanged for all $i \neq j$. Thus the change of commodity effect is negative: an increase in price leads some consumers to switch away from the commodity, but no consumers switch to the commodity.

8/ The partial derivative $V_p(\gamma_j,p_j,a)$ is nonpositive, and we make the regularity assumption that it is strictly negative.

We will study the demand for $\bar{\gamma}\varepsilon(0,1)$. Independent of which commodities are available, prices are specified by a positive valued function π defined on $[0,1]$. For each $\gamma\varepsilon[0,1]$, $\pi(\gamma)$ denotes the price of a unit of commodity γ when it is available; e.g., if $\gamma_1,\gamma_2,(\gamma_3=\bar{\gamma}),\gamma_4$ are available, then their prices are $\pi(\gamma_1)$, $\pi(\gamma_2)$, $\pi(\gamma_3)$, and $\pi(\gamma_4)$. The prices (and availability) of all goods other than $\bar{\gamma}$ are fixed throughout the analysis. We restrict attention to price functions π which have the characteristic that if all commodities are available, then the density of demand at each γ is positive. Furthermore, we assume that consumers are indexed so that if all commodities are available and if prices are given by π, then consumer a chooses commodity a.

Assume (for simplicity only) that the set of consumers purchasing $\bar{\gamma}$ form an interval.[9] We can there unambigously define the marginal consumers "to the left" and "to the right" of $\bar{\gamma}$ as functions of the price p, of $\bar{\gamma}$, $f^{\ell}(p)$ and $f^{r}(p)$ respectively. By our assumptions about π and the available commodities, $f^{r}(p)$ is defined implicitly by $V(\bar{\gamma},p,f^{r}(p))=V(\gamma',\pi(\gamma'),f^{r}(p))$ where γ' is the closest available commodity to the right of $\bar{\gamma}$. Thus

$$(*)\quad \frac{df^{r}(p)}{dp} = \frac{V_p(\bar{\gamma},p,f^{r}(p))}{V_a(\gamma',\pi(\gamma'),f^{r}(p)) - V_a(\bar{\gamma},p,f^{r}(p))} < 0.$$

Similarly, for $f^{\ell}(p)$,

$$\frac{df^{\ell}(p)}{dp} = \frac{V_p(\bar{\gamma},p,f^{\ell}(p))}{V_a(\gamma'',\pi(\gamma''),f^{\ell}(p)) - V_a(\bar{\gamma},p,f^{\ell}(p))} > 0$$

where γ'' is the closest available commodity to the left of $\bar{\gamma}$.

9/ In general, this set will be a union of intervals; however, if the mesh of available commodities is sufficiently fine, and if preferences are decent, the assumption will be satisfied.

Aggregate demand for $\bar{\gamma}$ as a function of p is defined by

$$\Phi(p) = \int_{f^{\ell}(p)}^{f^{r}(p)} Y^{\bar{\gamma},a}(p,R(a))\rho(a)da,$$

where $Y^{\bar{\gamma},a}$ is consumer a's demand for commodity $\bar{\gamma}$ as a function of the price, p, of $\bar{\gamma}$, and a's wealth ($Y^{\bar{\gamma},a}$ is the maximizer of $U([R(a)-py],y,\bar{\gamma},a)$). Application of Leibnitz rule and the normal decomposition of consumer demand into substitution and income effects yields the principal decomposition

$$(**)\quad \frac{d\Phi(p)}{dp} = -Y^{f^{\ell}(p)}(p,R(f^{\ell}(p))\rho(f^{\ell}(p))\frac{df^{\ell}(p)}{dp}$$

$$+ Y^{f^{r}(p)}(p,R(f^{r}(p))\rho(f^{r}(p))\frac{df^{r}(p)}{dp} \qquad (C)$$

$$+\int_{f^{\ell}(p)}^{f^{r}(p)} [U(\bar{\gamma})]Y_{p}^{a}(p)\rho(a)da \qquad (S)$$

$$+\int_{f^{\ell}(p)}^{f^{r}(p)} Y^{a}(p,R(a))Y_{R}^{a}(p,R(a))\rho(a)da \qquad (I)\ .$$

In the expression (S), $^{[\cdot]}Y^{a}(\cdot)$ is a compensated demand function for agent a and depends on the price of $\bar{\gamma}$ (which appears in round brackets) and a utility level, $U(\bar{\gamma})=U([R(a)-pY^{\bar{\gamma},a}],Y^{\bar{\gamma},a},\bar{\gamma},a)$ (which appears in square brackets). The terms (C), (S), and (I) are respectively the aggregate change of commodity effect, the aggregate substitution effect, and the aggregate income effect. By the neoclassical theory, (S) is negative and (I) can have either sign; their sum can be either positive or negative. Since both $\frac{df^{r}(p)}{dp}$ and $-\frac{df^{\ell}(p)}{dp}$ are negative, the change of commodity effect is negative.

3. AN APPLICATION

As an application of the framework introduced here, we show that even if individual demand functions are upward sloping, the unambiguous change of commodity effect will guarantee that market demand for a commodity must slope downwards whenever there are differentiated commodities which are sufficiently close to the commodity in question.[10] This claim should come as no surprise. Although we could not find such a result in formal demand theory, arguments which relate to the assertion appear throughout the literature. The current framework, with a continuum of consumers and the possibility of an increasingly large number of commodities, is sufficiently rich to accommodate a precise analysis.

Theorem: Let U,R, and $\rho > 0$ define the distribution of agents characteristics, let π be a continuously differentiable function, and let $\gamma'' < \bar{\gamma} < \gamma'$ be available commodities. For every $N > 0$, there exists $\varepsilon > 0$ such that $\frac{d\Phi(p(\bar{\gamma}))}{dp} < -N$ whenever $|\gamma' - \gamma''| < \varepsilon$, where the derivative is evaluated with the available commodities priced according to π.

10/ In fact we demonstrate the stronger result that $d\Phi/dp$ approaches negative infinity as closer substitutes are made available. We note that for location models, it is not natural to assume that U is differentiable in γ (e.g., $U(x,y,\gamma,a) = xy - |\gamma-a|$ should be included). In this case, market demand slopes downward when there are close substitutes, but lim $d\Phi/dp$ is bounded.

The proof follows from (**). As ε converges to zero, $f^r(p) - f^\ell(p) < \gamma' - \gamma'' < \varepsilon$ converges to zero, and the aggregate substitution and income effects become arbitrarily small. Given our assumptions, from (*), we note that the Theorem is true if $V_a(\gamma',\pi(\gamma'),f^r(p)) - V_a(\bar{\gamma},\pi(\bar{\gamma}),f^r(p))$ converges to zero. We know $V(\gamma,\pi(\gamma),a) = U([R(a)-\pi(\gamma)Y^{\gamma,a}],Y^{\gamma,a},\gamma,a)$ so $V_a(\gamma,\pi(\gamma),a) = (R'(a)=\pi(\gamma)Y_a^{\gamma,a})U_x + (Y_a^{\gamma,a})U_y + U_a$, and, by the first order condition for utility maximization, this is equal to $(R'(a))U_x + U_a$. By our differentiability assumptions, as $\gamma' - \bar{\gamma}$ converges to zero, the partial derivatives of $U[(R(a) - \pi(\gamma')Y^{\gamma',a}],Y^{\gamma',a},\gamma',a)$ converge to the partial derivatives of $U[(R(a) - \pi(\bar{\gamma})Y^{\bar{\gamma},a}],Y^{\bar{\gamma},a},\bar{\gamma},a)$, and so the denominator of $\frac{df^r(p)}{dp}$ converges to zero. (Similar reasoning shows $-\frac{df^\ell(p)}{dp}$ also converges to $-\infty$.)

4. CONCLUSION

We have presented a framework, designed to conform to standard intuition, in which price induced changes in aggregate demand are decomposed into income, substitution, and change of commodity effects. Clearly, any aggregate demand theory which ignores either marginal consumers or the neoclassical income and substitution effects invites misspecified application. The present decomposition will hopefully help to reduce misspecification and lead to sharper analysis.

5. FIGURES

For two differentiated commodities, a pair of curves (c_1, c_2) defines an indifference surface in Figure 1. The same two curves in Figure 2, c_1 in the B_1 plane and c_2 in the B_2 plane, define the same indifference surface. The latter diagram is used because it is adequate for defining preferences when there are three or more differentiated commodities (the triples c_1, c_2, c_3 and c'_1, c'_2, c'_3 each define a three dimensional indifference surface in four dimensional Euclidean space.) In Figure 2, the line GF indicates how B_2 would change with an increase in the price of γ_2. The consumer with the indicated preferences would purchase γ_2 both before and after the price change, and his consumption change can be decomposed into income and substitution effects in the B_2 plane. On the other hand, the consumer who took γ_2 at $\pi(\gamma_1), \pi(\gamma_2), \pi(\gamma_3)$, but could have received close to the same level of substitution from choosing γ_1, would switch to γ_1 as a result of the price change. For such a consumer c_1 would lie above the boundary of B_1 and c'_1 would cut the boundary of B_1.

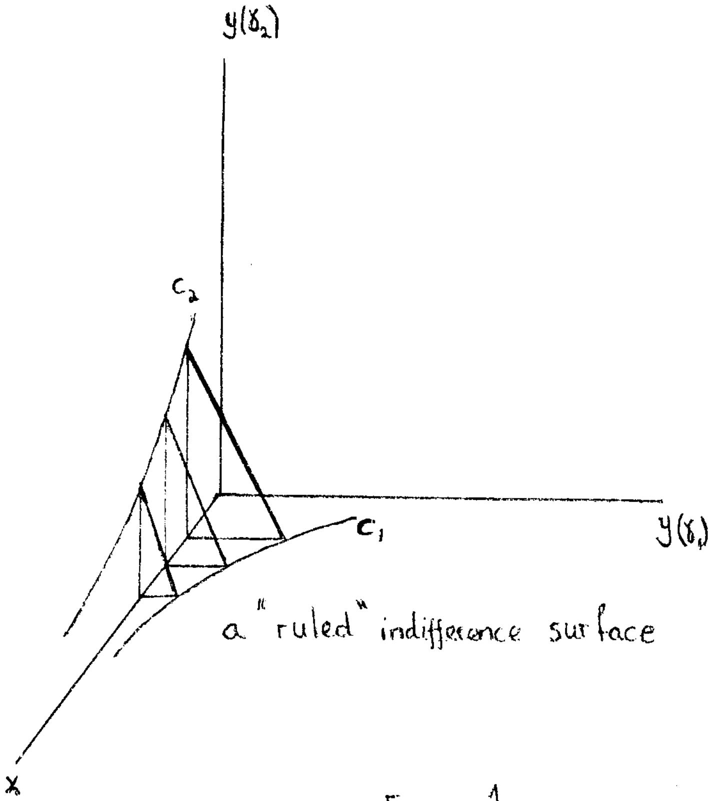

Figure 1

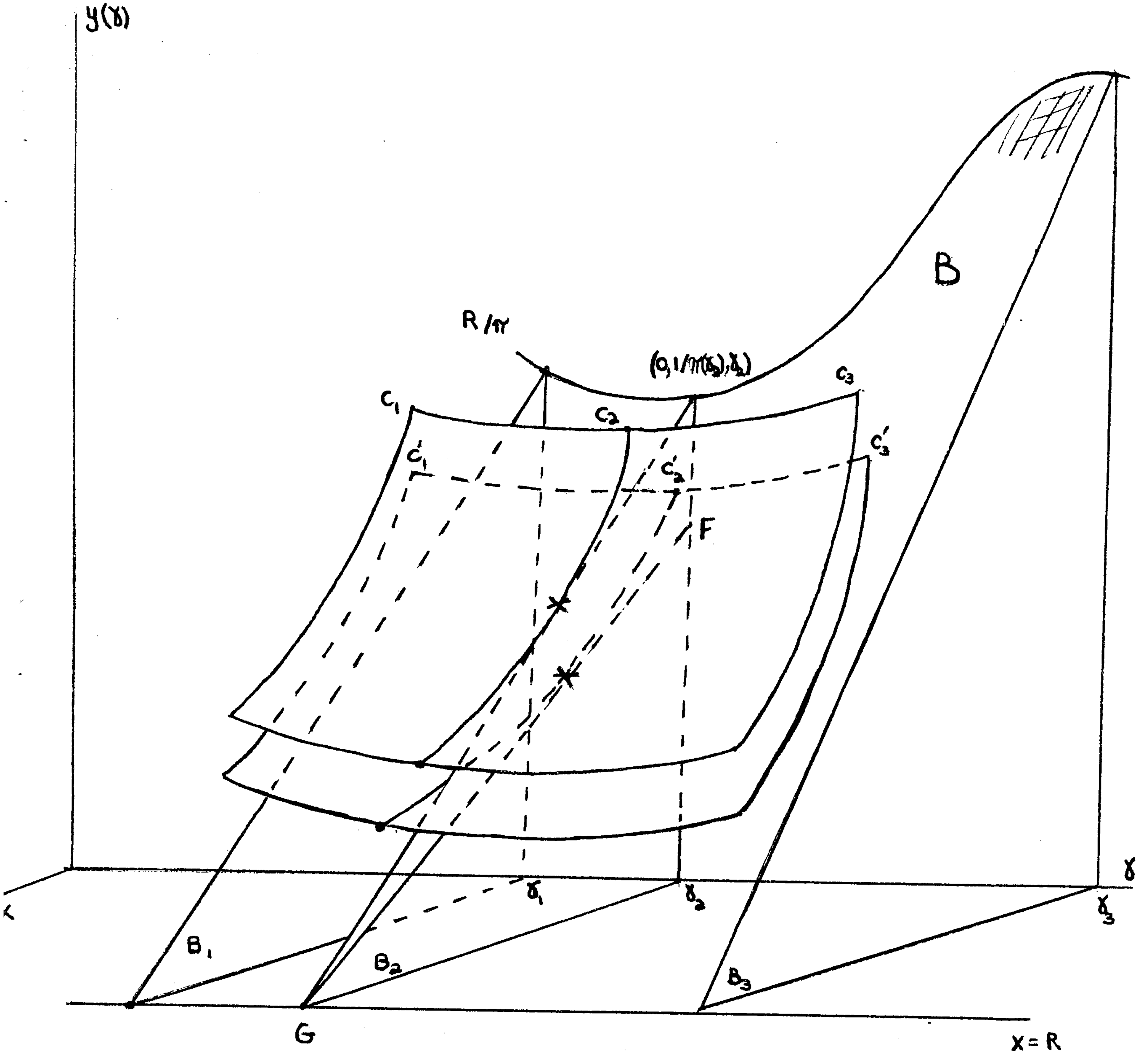

Figure 2

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