THE ESTIMATION OF TOTAL INVESTABLE RESOURCES

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Abstract

The problem of estimating the total amount of resources available for government investment appears at first glance to be a standard econometric forecasting problem, but it turns out to require the method of optimal control for solution.

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1. Introduction

This note attempts to answer the question, "what is the total amount that a central government can invest in order to satisfy the goal of full utilization of the nation's resources?" This question was raised to me while I was consultant to the Economic Planning Council of the Republic of China in Taiwan, in July, 1978, but the answer appears to be of more general applicability. Before presenting the answer, I should emphasize that there is a second, and perhaps more important question, that of the most efficient allocation of the total economic resources among alternative uses. This question was also raised by the government officials in Taiwan and was answered in a report (in Chinese) entitled Economic Planning and the Efficient Utilization of Resources coauthored by other consultants (whose names are listed in the first footnote) and myself, but it is not treated here.

2. Definition of Investable Resources

To obtain an estimate of total investable resources I, one may begin with the following definition:

$$I \equiv GDP^* - C_p - C_g + Im - Ex \tag{1}$$

$$= (GDP_{-1} - C_p - C_g) + (Im - Ex) + (GDP^* - GDP_{-1})$$

where GDP* denotes potential gross domestic product, C_p and C_g denote private and government consumption, and Im and Ex denote imports and exports, all in real terms. The three terms in parentheses on the second line of (1) measure, respectively, total private and government savings out of the total output from the last period, import surplus and the potential growth of gross domestic product. These are the three sources of the total investable resources available in the economy for private and government investments. One might attempt to estimate the items on the right-hand of (1) in order to estimate the total investable resources I. This would require an estimate of potential GDP, which can be obtained by using an aggregate production function relating potential output to total labor force, total capital and technological trend, for example. It would also require estimates of total consumptions by the private and government sectors (the latter being a policy variable subject to government control), import surplus, and private investment I_p. I_p will be subtracted from I to obtain the amount of investable resources I_g available to the government. These estimates can be obtained from a suitable econometric model of the economy which includes the above variables (excepting C_g) as endogenous variables.

In order to estimate the above endogenous variables from an econometric model, the values of certain exogenous and policy variables including government consumption and money supply have to be given. Using the projected values of these exogenous and policy variables, one can solve the econometric model to obtain estimates of the endogenous variables required in our estimation of total investable resources.

3. The Matching of Instruments and Targets

When we attempt to use an econometric model to estimate the required endogenous variables, our solution is subject to the restriction that the value of one endogenous variable, namely real GDP, has to equal its potential value estimated above. On the other hand, one of the policy variables, namely government investment expenditures, is yet to be determined. This situation should present no difficulty for solution. Following the well-known approach of J. Tinbergen to economic policy, we can fix the value of one target variable (GDP) and find the value of the instrument (government investment expenditures) which would achieve the target, given the values of the other exogenous variables in the model. In other words, we exchange the roles of one dependent variable (GDP) and one exogenous variable (government investment) and solve the system of simultaneous econometric equations for the latter variable, together with other dependent variables, given the target value of the former variable, together with the projected values of the other exogenous and policy variables.

From this solution, we obtain not only the total investable resources I but its decomposition into private investment I_p (a dependent variable in the model) and government investment I_g. We can rewrite (1) as

$$GDP = GDP^* = I_p + I_g + C_p + C_g + Ex - Im . \tag{2}$$

(2) appears to be an ex post GDP identity. However, it should be interpreted as an ex ante relationship between the estimates of the included endogenous and exogenous variables. It shows how potential GDP shall be distributed among its various uses, and is not simply an ex post accounting identity.

At first glance, we might appear to have obtained a satisfactory solution to the problem of estimating the government's total investable resources. A moment's reflection shows that this is not the case, because our solution might entail a high rate of inflation. If one finds the rate of inflation obtained from solving the econometric model in the way described to be other than desired, one can try to fix the inflation rate as another target variable and solve for the value of the required money supply (a natural instrument to control inflation). Here one exchanges the roles of a second pair of dependent variable (inflation rate) and exogenous variable (money supply). In terms of Tinbergen's approach to economic policy, we are using government investment and money supply as two instruments for the two targets, real GDP and the inflation rate. Computationally, this problem could be, but need not be, solved by using an (approximately) optimal control algorithm of Chow (1975, pp. 280-285) which aims at minimizing the expected value of a quadratic loss function subject to the constraint of a nonlinear econometric model. In this problem, the loss function includes the terms $(GDP-GDP^*)^2$ and $(\text{Inflation Rate-Target Rate})^2$. The two instruments or control variable are as mentioned above, the values of other exogenous and control variables being treated as given.

4. The Need for Optimal Control

At this stage, one might be tempted to conclude that we have solved our problem of finding an aggregate level of government investment to achieve full utilization of resources without inflation. Again, there are problems with our solution. First, there may be other objectives not yet allowed for. For example, the solution may entail an undesirable amount of trade surplus or deficit. To prevent this and possibly other undesirable results from occuring in the solution, one can include other target variables in the objective function and apply the method of optimal control to find a satisfactory solution. Not the least is the problem of possible severe fluctuations in some instruments, which can also be treated under the framework of optimal control by including the fluctuating instruments in the loss function with assigned target paths for their levels and/or rates of change. The consideration of other objectives necessitates the use of optimal control in order to minimize the expected value of a loss function subject to the constraint of a nonlinear stochastic model.

Second, the dynamic nature of our problem should be emphasized. Government investment and other policy variables in period t will have delayed effects on future GDP, inflation rate and balance of payment. These delayed effects have to be taken into account in the planning of current government investment. Therefore, a multiperiod optimization problem as it is formulated in the optimal control framework has to be solved in order to achieve a desirable time path for government investment expenditures.

There are at least four advantages in using the method of optimal control to determine the level of government investment, as compared with the simple method of finding the values of the instruments to match the target values of an equal number of target variables. First, the assumed target values for GDP and inflation rate might be too conservative. Using the optimal control approach, one can start with more ambitious target values and find a solution which will, through optimization, come as close to these target values as possible (close in the sense of minimizing a weighted sum of squared deviations of the variables from their target values). Second, as we have mentioned, more target variables can be incorporated in the objective function than the number of instruments. The target variables may include some instruments themselves. Third, the determination of government investment and other policy variables is achieved in a dynamic setting, with due allowance for the delayed effects of policy action on future economic welfare. Fourth, uncertainty in the econometric model can be incorporated in the solution, as described for example in Chow (1975, Chapter 10).

To recapitulate, this note began with an apparently straight-forward econometric problem of estimating total investable resources defined by equation (1). It then turned into a problem of finding the value of governemnt investment to achieve potential GDP using an econometric model in a static, one-instrument, one-target context. As the solution was being worked out, one was soon confronted with the additional problem of controlling inflation by a suitable adjustment in the supply of money. This led naturally to the problem of multiple targets and instruments. When the former outnumbers the latter because of other objectives than full employment without inflation, and when dynamic considerations are taken into account, not to speak of the difficulties created by uncertainty in the econometric model, our problem becomes one of optimal control. From the viewpoint of optimal control, to estimate a government's total investable resources is to find an optimal path for this variable which, together with the optimal paths of other policy variables, will help achieve as much as possible the objectives of full resource utilization, price stability, balance of payments equilibrium and reasonable behavior of the instruments themselves.

It is hoped that this note will contribute, in a modest way, to solving an important practical problem of economic planning and to clarifying the potential usefulness as well as the conceptual basis of the method of optimal control.

Reference

Chow, G. C. Analysis and Control of Dynamic Economic Systems. New York: John Wiley & Sons, 1975.