

The Interest Arbitrage Equation
as a Multiple Input Transfer Function

by

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The wide day-to-day fluctuations in exchange rates since the advent of a system of floating rates in 1971 resemble movements in the prices of other financial assets, such as stock prices. This observation has led economists into taking another look at the asset market theory of exchange rate determination as originally presented by David Hume.¹ The essence of this approach is in viewing national money as a financial asset whose supply is fixed at any instant in time. Exchange rates are the relative prices of these monies. The short run equilibrium exchange rates are those rates, determined with other equilibrium asset prices, such that the existing stocks of domestic and international assets are willingly held. Thus, exchange rate determination in the short run is a phenomenon based on stock equilibrium.

Several points should be stressed. First, domestic and foreign securities are viewed in this theory as assets to be considered among other assets in a portfolio selection. Clearly, the rates of return of other assets which are substitutes or compliments to national securities in the portfolio should cause changes in the demand for the monies. Inasmuch as the stocks of these assets are fixed every instant, these demand changes translate directly into changes in exchange rates.

The second point is that the expectations of both exchange rates and the prices of other assets are arguments in the demand for assets. This is simply the capital gain aspect in investment. Considerations of the future rates of financial assets play a large role in determining the present rates of these assets.

It is well accepted that changes in expectations play a significant role in accounting for day-to-day fluctuations in stock prices. When one views the exchange rate as the asset price of the stock of national monies, the daily fluctuations which exchange rates exhibit should come as no surprise. Either changes or expected changes in the prices of other financial assets are sufficient to prompt changes in the exchange rate.

The goal of this paper is to investigate the time series properties of exchange rates in light of the asset market theory. This study departs from previous Box-Jenkins time series studies of exchange rates in that a formal multiple input transfer function (MITF) is developed and estimated. The model itself is based on a simple asset market view of exchange rate determination.

Investigations of the time series properties of exchange rates have been reported by Levich (1977), Giddy and Dufey (1975), and Cornell and Dietrich (1978).² Levich studied the time series properties of weekly percentage changes in spot rates over the period from 1962 until 1975. Giddy and Dufey examined daily exchange rate data for three countries during two floating periods: post World War I and the early 1970's up to 1974. Using daily data from 1973 until 1975, Cornell and Dietrich examined the efficiency of foreign exchange markets in six currencies relative to the U.S. dollar. However, their estimation procedure entertained lags up to only eight days, although, as in the other two studies cited, univariate time series models were developed and estimated. Yet another study using daily rates from 1973 to 1975 for nine countries was reported by Dooley and Shafer (1976). Formal time series models, however, were not estimated.

The analysis contained in this paper contributes to the previous work of these authors in using univariate time series models for daily exchange rates during the years 1975 and 1976. More importantly, this study advances

from univariate analysis to develop and estimate a multiple input transfer function to analyze exchange rate movements.³

The thrust of the paper is to study the interrelationship through time among the spot exchange rate, the forward rate, and the domestic and foreign interest rates. This is an essentially empirical investigation on the asset market determination of exchange rates in the very, very short run. Daily exchange rate data and interest rate data are examined to test whether previous fluctuations in interest rates and the forward rate have a significant effect on the current spot rate.

Section I develops the model of exchange rate determination in the asset market on a day-to-day basis. A simple extension of the interest arbitrage equation is presented which emphasizes the role of a fixed supply of arbitrage funds available on any given day. The role that time series techniques will play in the empirical work is indicated. Section II presents the empirical representation of the individual variables as stochastic processes of autoregressive, moving average (ARIMA) form. Examination of the ARIMA structure for each variable is done in detail and the random walk hypothesis for daily exchange rates is tested. Likelihood ratio tests are employed to select the "best" ARIMA models. Section III estimates the multiple input transfer function (MITF) which dynamically relates the spot exchange rate to the forward exchange rate, the domestic interest rate, and the foreign interest rate. The impulse response patterns are derived. A discussion of causality and an examination of predictive performance of the model is included. Section IV contains concluding remarks.

SECTION I

The asset market view of exchange rate determination sheds a new light on the familiar interest arbitrage equation. Traditionally, the arbitrage equation has been viewed as a zero profit condition for arbitragers in the foreign exchange market. It is the equilibrium condition relating the interest differential to the forward premium. Once it is recognized that the exchange rate is determined in the financial markets, the arbitrage equation ceases to be merely a condition fulfilled in equilibrium, but becomes a channel through which the exchange rate is actually established. This can be seen simply as follows.

Assume that there is perfect capital mobility between countries, that the financial assets of the home country are perfect substitutes with those of the rest of the world, and that coupon rates are invariable over the duration of the bonds. Given perfect capital markets and the absence of transaction costs, the familiar arbitrage equation follows:

$$\frac{(1+r_t^d)}{(1+r_t^*)} = \frac{e_t^f}{e_t^s} \quad (1)$$

where r_t^d is the domestic interest rate, r_t^* is the foreign interest rate, and e_t^s and e_t^f are the spot and forward exchange rates respectively, expressed as units of home currency per unit of foreign currency. All variables are evaluated at time t . Henceforth the subscript t will be omitted where confusion does not result. When condition (1) is not met, the bond market is in disequilibrium as portfolio-adjusters try to move into the asset with the higher rate of return. Arbitrage profits are earned until the rates of return are equalized.

The model we present relaxes the foregoing assumption of perfect capital

markets in order to better view exchange rate determination in a daily framework. In particular, we will assume that funds available for investment in the bond markets (and in arbitrage opportunities) are fixed over the course of a day or two. This certainly seems to be a realistic assumption.⁴

Following Dornbusch (1976), assume a money demand equation in the form

$$M^d = Pm(r^d, u)y \quad (2)$$

where prices, P , and real income, y , are held fixed in the very short run, and u is a stochastic disturbance term. The demand for money is negatively related to the opportunity cost of holding money.

Assume that domestic and foreign assets are perfect substitutes, that there is perfect capital mobility, and that there are no transactions costs. However, available investment funds are fixed in the very short run. This leads to a modified version of the arbitrage equation:

$$\frac{1 + r_t^d}{1 + r_t^*} = \frac{e_t^f}{e_t^s} - \alpha \quad (3)$$

The usual approximation to (3) is given by:

$$r_t^d = r_t^* + \frac{e_t^f - e_t^s}{e_t^s} - \alpha \quad (3')$$

In (3) and (3') α is the "arbitrage opportunity factor." This concept needs amplification.

Consider the investor with a fixed amount of funds available to him in the short run (a few days, say). He allocates these funds between holding money and holding foreign and domestic securities. Assume that these

securities are identical in all respects (risk class, maturity, etc.) except for the currency of their denomination. In choosing his mix of domestic and foreign bonds, the investor considers not only the rates of return of the foreign and domestic securities, but also the position he will be in to take advantage of future arbitrage opportunities. An example will clarify things.

Suppose in day 1 that there is no arbitrage opportunity, i.e.,

$r_t^d = r_t^* + (e_t^f - e_t^s)/e_t^s$, and the investor sinks all his funds in domestic securities. If the foreign interest rate rises at $t+h$, where h is small, the investor is in a position to capture extra gains from an arbitrage operation. This will be the case if $r_t^d < r_{t+h}^* + \frac{(e_{t+h}^f - e_{t+h}^s)}{e_{t+h}^s}$ where

$$\frac{e_t^f - e_t^s}{e_t^s} = \frac{e_{t+h}^f - e_{t+h}^s}{e_{t+h}^s} .$$

He will simultaneously sell his domestic bonds,

buy foreign exchange at the spot rate, buy foreign securities at the new interest rate, and buy the domestic currency forward, thereby covering his transactions. In this process, he will earn a total rate of return of $r_{t+h}^* + (e_t^f - e_t^s)/e_t^s$, and will increase his returns by $r_{t+h}^* + (e_t^f - e_t^s)/e_t^s - r_t^d$ compared to his original position in the absence of the arbitrage transactions.

If, however, the investor had sunk all his funds in foreign securities at time t and the foreign interest rate rose at time $t+h$ the investor would be in no position to transact an arbitrage operation. It is true that $r_t^d < r_t^* + (e_t^f - e_t^s)/e_t^s$, but the investor is already in foreign currency and has no uninvested funds in the portfolio. Since all his funds are already in foreign exchange, he must watch this profit opportunity go by. He can not sell his foreign bonds and buy new foreign bonds with the higher interest rate since the capital loss he would incur by selling his bonds would just offset the increased interest rate gains on the new bonds he would buy.

Since his access to investible funds is limited and these funds are already in foreign securities, he has no feasible arbitrage opportunity. He will keep the foreign securities which he originally purchased and his total return will be $r_t^* + \frac{e_t^f - e_t^s}{e_t^s}$ at maturity. This return is less than that

which he would have earned if he had been in domestic securities originally and thus able to arbitrage into foreign ones when r rose. The return in this case is $r_{t+h}^* + \frac{e_t^f - e_t^s}{e_t^s}$.

The foregoing argument suggests that with a finite supply of funds and expectations of future arbitrage opportunities, the relative demand for domestic securities vis-à-vis foreign securities will be a function of $r^d - r^* - (e^f - e^s)/e^s$, and also the expectations of the size and direction of future arbitrage opportunities. This follows from the recognition that in addition to offering current rates of return, bond holdings offer a position from which it is possible to capture additional arbitrage profits. The relative prices (and rates of return) of bonds should reflect this dimension. One should not expect the domestic security (say) to offer the same current rate of return as the foreign security if, in addition, the domestic security also offers a superior position from which expected arbitrage profits can be captured. The extra attractiveness of the domestic security should bring about an increased demand for it, and a higher price, and therefore, a lower current rate of return as compared with the foreign security. This discount factor is captured in the variable α in equation (3).⁵

It is clear from the foregoing that α is based on expected future arbitrage opportunities. In order to make equation (3) operational for empirical investigation it is necessary to specify more explicitly the expectations formation process. In so doing, we will formalize the portfolio decision

(and thereby, the exchange rate determination) in the very short run, where funds are fixed and future arbitrage opportunities are relevant considerations.

Assume that individuals have a quadratic utility function in wealth, W , of the simple form

$$U = aW - bW^2 ,$$

and that they are expected-utility-maximizers in the sense implied by von Neumann and Morgenstern. It is easy to show that this gives rise to mean-variance considerations in the design of their optimal portfolios.

To proceed further, we need to develop the concept of the net arbitrage return gained from a future arbitrage transaction. R_{t+h} will denote the arbitrage rate of return which comes from moving funds out of domestic securities into foreign ones; R'_{t+h} denotes the arbitrage profit or rate of return which results from the reverse operation. They are given by the following expressions:

$$R_{t+h} = r_{t+h}^* + \frac{e_{t+h}^f - e_{t+h}^s}{e_{t+h}^s} - r_t^d$$

$$R'_{t+h} = r_{t+h}^d - \frac{e_{t+h}^f - e_{t+h}^s}{e_{t+h}^s} - r_t^*$$

Both R_{t+h} and R'_{t+h} are actually vectors of opportunities expected on subsequent days during which the supply of funds is fixed. Thus $f(R_{t+h})$ is the joint distribution $f(R_{t+1}, R_{t+2}, \dots, R_{t+T})$, whereas $g(R'_{t+h})$ denotes the joint distribution $g(R'_{t+1}, R'_{t+2}, \dots, R'_{t+T})$. The formulation of R_{t+h} and R'_{t+h} abstracts from the complication and views the future as some aggregation of expectations for future time periods, occurring, however, at time $t+h$, where h is considered small. This assumption yields considerable simplification and clarifies the main point we are making.⁶ The

import of the notation R_{t+h} and R'_{t+h} is that shortly after time t , one learns of the realization for the next period. Thus one can act as if r and r^* were rates of interest covering the entire period and not just part of it. The portfolio composed of domestic and foreign securities has the following expected return:

$$\begin{aligned} \hat{P} \equiv \varepsilon(P) = & \lambda \left[r_t^d F(0) + \int_0^\infty \left(r_t^d + R_{t+h} \right) f \left(R_{t+h} \right) dR_{t+h} \right] \\ & + (1-\lambda) \left[\left[\frac{e_t^f - e_t^s}{e_t} + r_t^* \right] G(0) \right. \\ & \left. + \int_0^\infty \left[\frac{e_t^f - e_t^s}{e_t} + r_t^* + R'_{t+h} \right] g \left(R'_{t+h} \right) dR'_{t+h} \right] \quad (4) \end{aligned}$$

where $F(x)$ is the cumulative distribution of $f(R_{t+h})$ to the value x , $G(x)$ is the cumulative distribution of $g(R'_{t+h})$ to the value x , and λ is the proportion of wealth invested in domestic securities.

In order to understand the terms of the first set of brackets, consider the situation facing the investor. In assuming a position in domestic bonds he realizes that he might not keep those securities until their maturation. If he does keep them his assured return will be r^d . If sometime in the future, however, there exists an attractive arbitrage opportunity which involves selling domestic securities and buying German ones, his return will be greater than r^d .⁷ The term inside the bracket, then, is the expected rate of return he will receive from taking a position in U.S. bonds.⁸ $F(0)$ is the probability that no future arbitrage opportunities will arise calling for the movement of funds from domestic to foreign securities. Since λ is the percentage of wealth already in domestic securities, these funds are

not able to respond to the opportunity. They will remain in domestic securities and earn r^d . The first term in the brackets, then, is the return from remaining in domestic securities weighed by the probability that this event will occur. The second term in the first set of brackets is the expected rate of return a current position in domestic securities offers, given that this position will be used to conduct an arbitrage transaction into foreign securities.

Hence the eventual returns from holding domestic bonds today will come from one or two sources: holding the bonds to maturity or using them for arbitrage transactions into foreign securities. The first set of brackets captures the expected return of these two uncertain returns.

The second set of brackets is explained in exactly the same way. The first term is the return from holding foreign securities to maturity multiplied by the probability that they will be so held. The second term is the expected return yielded by foreign securities in their role of enabling arbitrage operations to run from foreign securities into domestic ones.

Rewriting equation (4) yields

$$\hat{P} = \lambda Z^D + (1-\lambda) Z^F \quad (5)$$

$$\text{where } Z^D \equiv F(0) r_t^d + \int_0^{\infty} \left(r_t^d + R_{t+h} \right) f \left(R_{t+h} \right) dR_{t+h}$$

$$\text{and } Z^F \equiv G(0) \left(r_t^* + \frac{e_t^f - e_t^s}{e_t^s} \right) + \int_0^{\infty} \left(r_t^* + \frac{e_t^f - e_t^s}{e_t^s} + R'_{t+h} \right) g \left(R'_{t+h} \right) dR'_{t+h}$$

It is clear from equation (5) that the portfolio selection between establishing positions in U.S. securities and foreign securities is precisely the portfolio selection between two assets with uncertain rates of

return. It should be noted that although the returns are uncertain, each position offers a minimum assured rate of return r_t^d , and $r_t^* + \frac{e_t^f - e_t^s}{e_t^s}$. The returns are uncertain in the upward direction, since additional arbitrage profits could be earned. From equation (5) it follows that

$$\lambda = \frac{\hat{P} - Z^F}{Z^D - Z^F} \quad (6)$$

A look at a specific case will help clarify things. Suppose that $F(0) = 1$ in equation (4). This implies

$$\hat{P} = \lambda r_t^d + (1-\lambda)Z^F \quad (7)$$

The portfolio decision simplifies to the choice between an asset with a certain rate of return, a position in domestic securities, and a risky asset, a position in foreign securities. The position in domestic assets has a certain rate of return since the future arbitrage opportunities will come in the direction of moving from foreign asset into domestic assets with probability 1. This is the implication of $F(0) = 1$.

The following proposition follows directly in this case:

$$\text{if } F(0) = 1, \text{ then } r_t^d > r_t^* + \frac{e_t^f - e_t^s}{e_t^s} \quad (8)$$

The proof of this proposition is simple. Suppose $r_t^d = r_t^* + \frac{e_t^f - e_t^s}{e_t^s}$. Since investment in the foreign assets offers a minimum return of $r_t^* + \frac{e_t^f - e_t^s}{e_t^s} = r_t^d$ and a chance with a probability one of attaining a higher return through future arbitrage transactions, everyone will invest in foreign securities and so no one will hold domestic securities whose return is fixed at r_t^d . This cannot be an equilibrium since there would be excess supply of domestic bonds. Its rate of return vis-à-vis the foreign rate must rise

until the certain return of r_t^d is as attractive as the lower return $r_t^* + \frac{e_t^f - e_t^s}{e_t^s}$ plus the arbitrage opportunities foreseen by being in foreign securities. Hence $r_t^d > r_t^* + \frac{e_t^f - e_t^s}{e_t^s}$ or, equivalently $r_t^d = r_t^* + \frac{e_t^f - e_t^s}{e_t^s} - \alpha$, where α is negative in this case. The value of α depends on expectations of future arbitrage opportunities, as the above example highlights. The nature of these expectations is captured in the subjective density functions held for R_{t+h} and R'_{t+h} . Hence,

$$\alpha = b(f(R_{t+h}), g(R'_{t+h})) \quad (9)$$

It is interesting to note that the expectation of future arbitrage profits can create an arbitrage situation that is not exploited. The reason is simply that funds are fixed in the very short run. Once they are invested in particular securities, they are useless for later arbitrage opportunities requiring a movement into those securities. If there exist strong expectations of arbitrage opportunities in the near future, and if these expectations are widely held, a small current arbitrage situation will be created as investors position themselves for the big opportunity they believe is forthcoming. This small arbitrage opportunity goes unexploited since it reflects investors' expectations of larger gains in the near future. Clearly, this phenomenon hinges on there being fixed funds available over a horizon of a day or two.

Substituting equation (9) into equation (3') and with some manipulation we have

$$\ln e_t^s = \ln e_t^f + \ln (1+r_t^*) - \ln (1+r_t^d) - b(f(R_{t+h}), R'_{t+h}) \quad (10)$$

This formulation sheds light on the asset market determination of the exchange rate. Given that the domestic interest rate is determined in the

money markets (see equation (2)), that r^* is determined exogenously by the rest of the world, and that e^f is fixed in the very short run then equation (10) determines the spot exchange rate.⁹ Given r^* , r^d , e^f , and expectations, the spot exchange rate is determined so as to yield equilibrium in the bond market.

The last step to make this theory operational and to set the stage for empirical investigation is to specify the variables determining the frequency distribution for expected arbitrage opportunities. This is the task to which we now turn.

Consider R_{t+h} and R'_{t+h} . In this analysis, money supplies in the short run are assumed to be held constant and changes in r_t^* and r_t^d are assumed to result from stochastic disturbances in the money demand equations. Similarly, expectations for future exchange rates are assumed stochastic and move in response to exogenous shocks in the form of white noise. Hence, components of R_{t+h} and R'_{t+h} are seen as stochastic processes evolving through time. In seeking a distribution for R_{t+h} and R'_{t+h} , we are actually looking for the joint probability function

$$k(R_0, R'_0, R_1, R'_1, \dots, R_t, R'_t, R_{t+h}, R'_{t+h})$$

from which we can calculate the conditional probability functions

$f(R_{t+h} | R_0, R_1, \dots, R_t)$ and $g(R'_{t+h} | R'_0, R'_1, \dots, R'_t)$. The time series is viewed as one realization from the distribution produced by the underlying probability mechanism.

In practice, the joint probability distribution is never developed. It is easier to use the information in the time series to infer the mechanism generating the data, and from this derive the conditional distribution for future values of the time series. Since the conditional distribution of

R_{t+h} and R'_{t+h} depend on the underlying mechanism and this mechanism is a function of the current and past values of r^d , r^* , e^f and e^s , we write the following expressions for $f(R_{t+h})$ and $g(R'_{t+h})$:

$$\begin{aligned} f(R_{t+h}) &= \gamma(\tilde{r}^d, \tilde{r}^*, \tilde{e}^f, \tilde{e}^s) \\ g(R'_{t+h}) &= \delta(\tilde{r}^d, \tilde{r}^*, \tilde{e}^f, \tilde{e}^s) \end{aligned} \quad (11)$$

The tilde over a variable denotes its current and past values.

Combining equation (11) with equation (10) yields the following equation:

$$\ln e_t^s = V_1(B) \ln e_t^f + V_2(B) \ln(1+r_t^d) + V_3(B) \ln(1+r_t^*) + u_t \quad (12)$$

where $V_i(B)$ is a polynomial in the lag operator B , which acts on t , i.e., $V_i(B) = (V_i + V_{i1}B + V_{i2}B^2 + V_{i3}B^3 + \dots)$ and $B^m X_t = X_{t-m}$. Note that this is the same general form of the arbitrage equation (1) except that there are lag structures and coefficient weights for each of the independent variables.

Equation (12) is the equilibrium equation for the determination of the spot exchange rates in the asset markets in the very short run. In its general form it suggests that lagged values of interest rates and the forward exchange rates may exert some influence on the current spot rate. This need not be inferred. If all of the underlying probability mechanisms of the variables generate random walks then their lagged values will yield no information for future values. Equations (11) then simplify to

$$\begin{aligned} f(R_{t+h}) &= \gamma(r^d, r^*, e^f, e^s) \\ g(R'_{t+h}) &= \delta(r^d, r^*, e^f, e^s) \end{aligned} ,$$

and equation (12) reduces to

$$\ln e_t^s = V_1 \ln e_t^f + V_2 \ln (1+r_t^d) + V_3 (1+r_t^*) + U_t .$$

Whether such a simplification is justified is an empirical question which we investigate.

A time series approach for investigating equation (12) seems appropriate for two reasons. First, much useful information is contained in an individual series which would otherwise be neglected if one considered only contemporaneous relationships with a few lags thrown in. Perhaps more importantly, time series analysis takes specific note of autocorrelation among the errors u_t . It makes no a priori assumption that u_t is a white noise process. Neglect of autocorrelation among error terms leads to inefficient estimators and, as is well known, in the case of lagged dependent variables, inconsistent estimates. In attempting to incorporate past information available in the series and in filtering the possibly autocorrelated error series we can obtain efficient estimates and optimal linear predictors.

Having developed a simple model of exchange rate determination which suggests that lagged values of interest rates and the exchange rates may systematically influence daily spot rates, we turn to investigate this issue empirically.

SECTION II

The general ARIMA structure for a time series attempts to incorporate in parsimonious fashion as much information as is contained in the series itself. It is of the form

$$\phi(B) (1-B)^d y_t = \theta(B) a_t \quad (13)$$

where a_t is white noise and

$$\phi(B) = 1 - \phi_1 B - \phi_2 B^2 - \dots - \phi_p B^p$$

$$\theta(B) = 1 - \theta_1 B - \theta_2 B^2 - \dots - \theta_q B^q$$

The original series y_t is either covariance stationary or it is assumed that it can be suitably differenced $z_t = (1-B)^d y_t$ and/or transformed so that z_t is covariance stationary. An implication of covariance stationarity is that $\phi(B)$ has all zeroes outside the unit circle. As an identification restriction we also require that the process is invertible, i.e., that $\theta(B)$ has all zeroes outside the unit circle. The first stage in specifying an ARIMA structure is to deduce the order of the autoregressive parameters, p , the required degree of differencing for stationarity, d , and the order of the moving average parameter, q . This is done by analysis of the autocorrelation patterns, the partial autocorrelations, and the inverse autocorrelations.

Daily data extending for a two year period from January 1, 1975 to December 31, 1976 were employed in the study. Data for the U.S.-German exchange rates and the interest rates in each of the two countries were used. The interest rates were the ninety day U.S. treasury bill rate (r^d) and the ninety day German inter-bank rate (r^f); exchange rates were the spot (${}_t e_t^S$) and ninety day forward (${}_t e_{t+90}^F$) U.S.-German exchange rates.

We checked the autocorrelation patterns for the thirty and sixty day forward rates and they revealed the same pattern present in the ninety day rates. Thus we expect that should someone conduct a similar analysis to ours using thirty or sixty day time series they would obtain the same results we did. However, we are willing to be surprised.

The 1975-76 interval was chosen because it was relatively free of the structural changes and disturbances of earlier periods. The abandonment of the Bretton Woods system in 1971 and the world monetary disturbances following the formation of the OPEC cartel in the fall of 1973 made these earlier periods unsatisfactory for analysis. No such abrupt shocks occurred in 1975-76 and so one can better study the typical workings of the exchange market under fairly constant structural conditions.

The ARIMA identification¹⁰ was done for each of these data series. Examples of the autocorrelation and partial autocorrelation patterns are shown in Figures 1 and 2. Lags are given along the horizontal axis while the estimated autocorrelations or partial autocorrelations are on the vertical axes. The dotted lines indicate $2\hat{\sigma}$ bands for the variation in the estimated (partial) autocorrelations. The models for the spot and ninety day forward rates exhibited long lags. The autocorrelations were significant at lags out through 30 and exhibited some signs of weekly seasonality after a delay of about a week. To test to see whether we were dealing with autocovariance stationary series we split the sample in half and separately estimated the autocorrelation structure for 1975 by itself and for 1976 by itself. The autocorrelation patterns were remarkably robust in each of the period subsets. While the long lags might seem high compared with those found in other analyses of time series, it is to be remembered that we are dealing with a pure moving average process for daily data. Our findings indicate that disturbances in the foreign exchange market take up to seven

weeks to work their way through the system.

As mentioned earlier, for identification purposes it is required that $\theta(B)$ be invertible, i.e., have all zeroes outside the unit circle. In the actual estimation process described below we had some difficulty in making sure that this condition was fulfilled for the spot and forward rate ARIMA processes. Though $\theta(B)$ does indeed have all zeroes outside the unit circle for all the models we present (even those which we ultimately reject), some of the roots are close to one. Analytically it is easy to see why, given significant autocorrelations at long lags, this must be the case. For a pure moving average process of order q the covariance generating function is given by:

$$\gamma(B) = \sigma_a^2 \theta(B) \theta(B^{-1})$$

Let $\theta(B)$ be expressed in multiplicative form:

$$\theta(B) = \prod_{i=1}^q (1 - \theta_i^* B)$$

For the invertibility of $\theta(B)$ it is required that $|\theta_i^*| < 1$ for $i=1, \dots, q$.

Then $\gamma(B)$ may be written as:

$$\gamma(B) = \sigma_a^2 \prod_{i=1}^q (1 - \theta_i^* B) \prod_{i=1}^q (1 - \theta_i^* B^{-1})$$

It is now easy to see that the autocovariance at lag q is

$$\gamma(q) = \sigma_a^2 \prod_{i=1}^q \theta_i^*$$

The population autocorrelation at lag q is then given by

$$\rho(q) = \frac{\gamma(q)}{\gamma(0)} = \frac{\sigma_a^2 \prod_{i=1}^q \theta_i^*}{\sigma_a^2 \left[1 + \sum_{i=1}^q (\theta_i^*)^2 + g(\theta_1^*, \dots, \theta_q^*) \right]} = \frac{\prod_{i=1}^q \theta_i^*}{1 + \sum_{i=1}^q (\theta_i^*)^2 + g(\theta_1^*, \dots, \theta_q^*)}$$

where $g(\) \geq 0$.

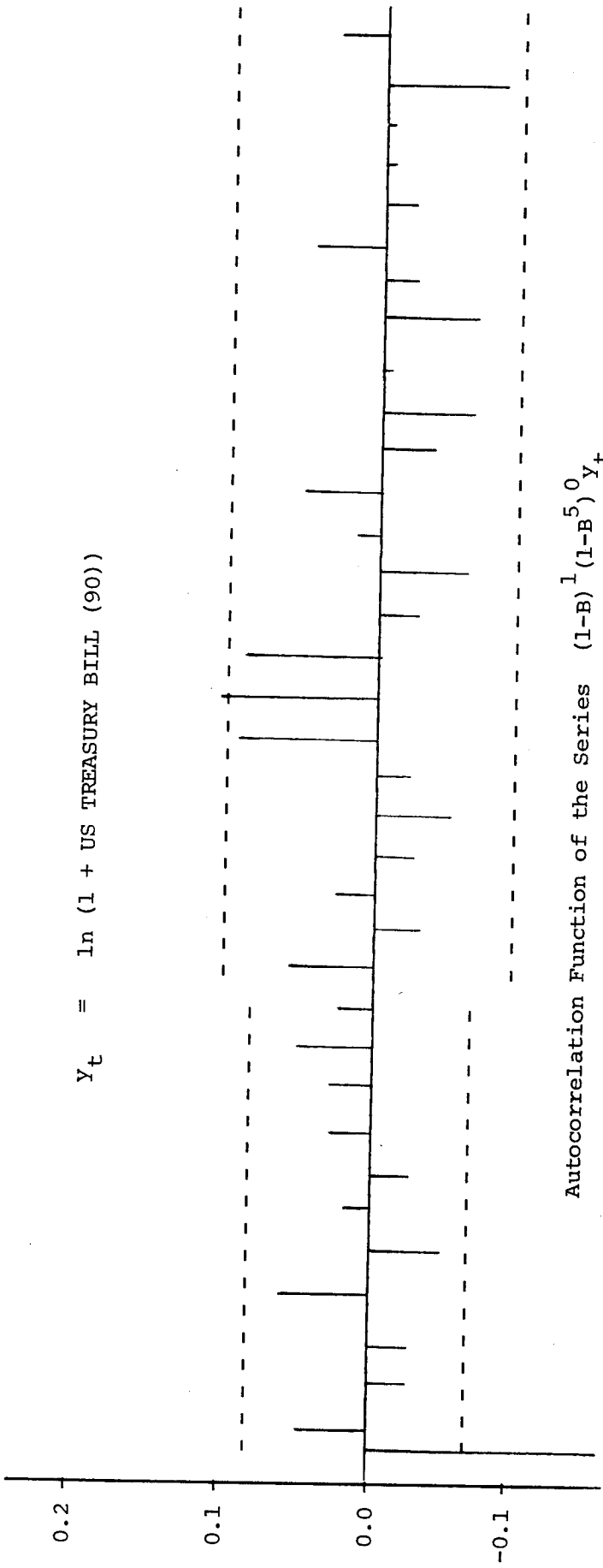
(14)

Suppose $q = 20$ and the estimated autocorrelation at lag 20 is $r_{20}^2 \approx .10$ (and therefore we expect $\rho(20) \approx .10$). Assume further that the denominator in (14) is small; a large denominator would make the following point even more telling. Even if all θ_i^* were equal to .89, $\rho(20)$ would only be .097. Naturally if one of the θ_i^* is significantly smaller than .89 this will push other roots even closer to one. Thus the price we pay for using daily data and thereby incurring long lags is having the roots of $\theta(B)$ close to one.

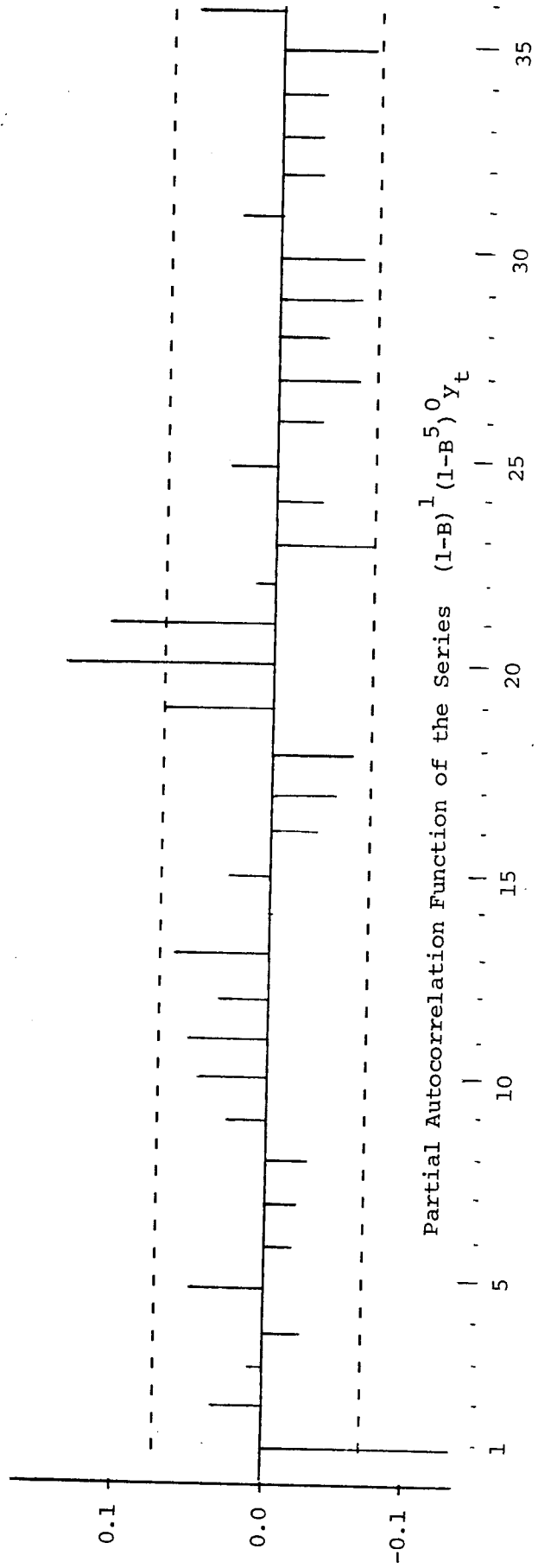
Once tentative models were identified for each series, parameters were estimated by iterative minimization of the sum of squared residuals, $\sum_{t=1}^T a_t^2$. Such a procedure yields maximum likelihood estimates under the assumption that a_t follows a Gaussian white noise process. Two diagnostic checks were applied to check the adequacy of the models. The Box-Pierce statistic¹¹ was used in a test to determine whether the estimated residuals, when taken as a group, are white noise. The second diagnostic check, the Kolmogorov-Smirnov goodness-of-fit test, checks for the presence of periodic non-randomness in the estimated noise process. Additional measures such as overfitting and testing the stability of estimates in different time periods were also carried out. Models which failed either the Box-Pierce chi-squared test or the Kolmogorov-Smirnov tests were rejected.

Table 1 gives a listing of the random walk model for each series plus all those models which passed both the Box-Pierce test and the Kolmogorov-Smirnov test. In some trial runs it was discovered that the constant term was never significant. Hence it was usually suppressed in subsequent estimations. In our notation a random walk is expressed as $(0,1,0)$, i.e., zero order autoregressive and moving average process plus a first difference to induce stationarity. The random walk model is included for each of the series because it provides a benchmark, though it should be noted that the

$$Y_t = \ln(1 + \text{US TREASURY BILL (90)})$$



Autocorrelation Function of the Series $(1-B)^{-1}(1-B^5)^0 Y_t$



Partial Autocorrelation Function of the Series $(1-B)^{-1}(1-B^5)^0 Y_t$

FIGURE 1

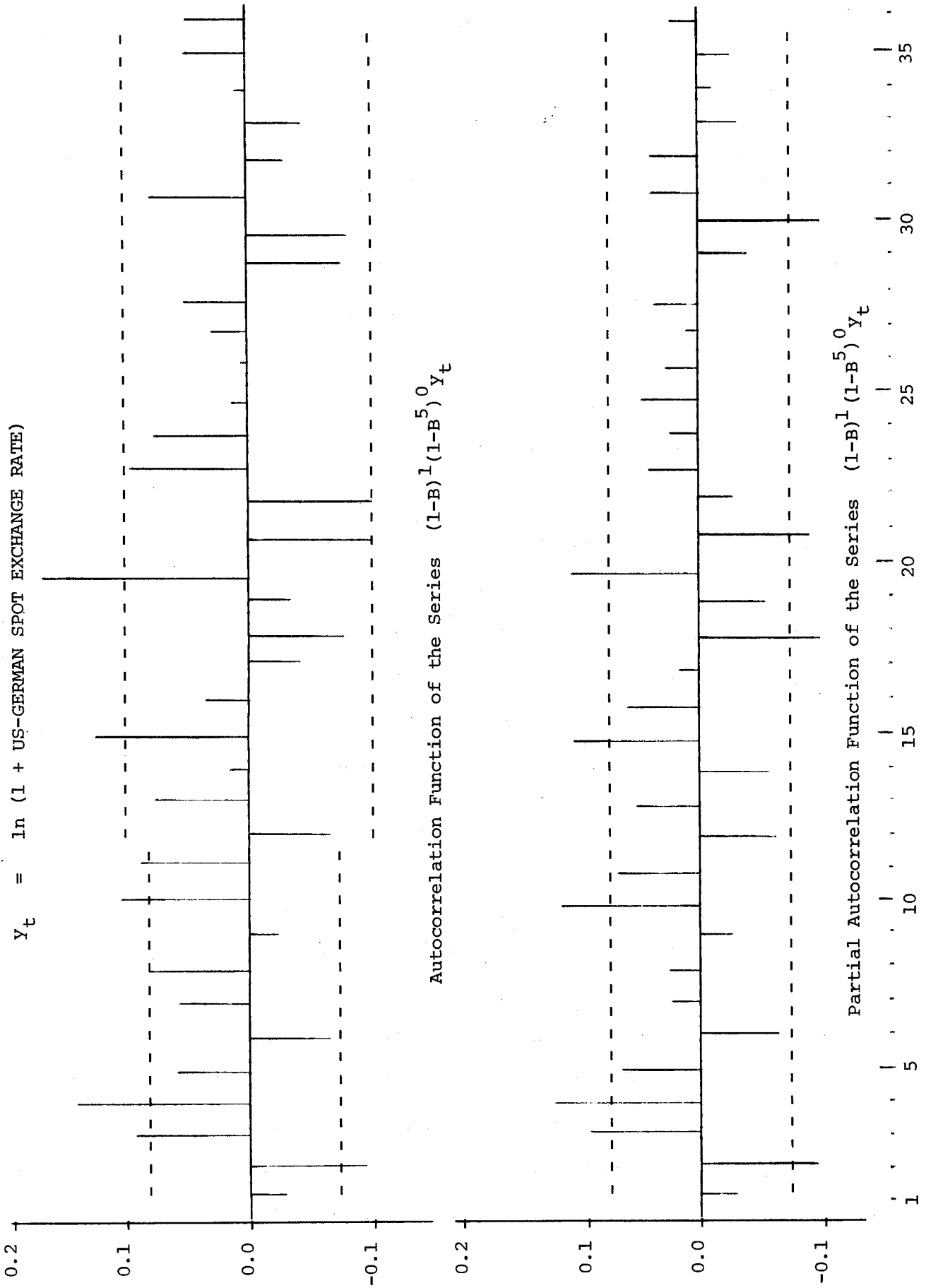


FIGURE 2

random walk never left white noise residuals. For the random walk model all four series failed the Kolmogorov-Smirnov goodness-of-fit test. The U.S. treasury bill rate did reasonably well on the Box-Pierce test; however, the other three failed decisively. The failure of the random walk model to adequately depict exchange rate movements is consistent with the results in the studies mentioned above by Levich, Dooley and Shafer, and Giddy and Dufey. In each of these studies, time series analysis suggested departures from the random walk model for the exchange rates of most of the countries under study.¹²

When there were a plurality of models which passed both tests and if the models were nested, the likelihood ratio (or the posterior odds ratio) tests¹³ were carried out.

These tests are carried out pairwise on two competing models. Consider two distinct models used to explain a single, dependent variable. Assume that we have generally diffuse prior information about the models. One way of choosing between them would be to select the one with the "higher" posterior probability, i.e., the probability which results after the prior probability density function has been transformed by the likelihood function. The term "higher" can be made operational by working with ratios of probability functions.

$$\lambda^* = \frac{L(y|H_0)p(H_0)}{L(y|H_1)p(H_1)}$$

Then the ratio of the prior density under the null hypothesis to the prior density under the alternate hypothesis is transformed (multiplied) by the ratio of the likelihood functions under the respective hypotheses. This yields λ^* , the posterior odds ratio or, under diffuse prior information, the likelihood ratio. If λ^* is high, H_0 is very probable. Only when

λ^* is close to zero would one be inclined to reject H_0 . For large samples $-2\ln\lambda^*$ has the χ^2 distribution with degrees of freedom given by the difference between the number of parameters to be estimated under H_1 and the number under H_0 . It is to be noted that the test is appropriate only when the model under H_0 is nested in that under H_1 .

Table 2 presents the results of these tests. The asterisk in Table 1 indicates the model which was finally selected for each series. It will be noted that both the spot and the ninety-day forward exchange rates always involve models with fifteen parameters. Many other models with autoregressive terms and lower order moving averages were attempted. None however (except those listed) passed our double test. Indeed it would be desirable to overfit the model with more than fifteen parameters and then conduct a likelihood ratio test. However, the estimation program which we used allowed a maximum of fifteen parameters to be estimated. Selection in the case of nonnested hypotheses was made on the basis of the lowest sum of squared residuals.

Aside from the long moving average processes characterizing the two exchange rates and to a certain extent the German interbank interest rate, it is noteworthy that only one of the series can be satisfactorily modelled by using an autoregressive process. Even then, the (1,1,1) process for the U.S. treasury bill rate is rejected in favor of a pure moving average process.

Having selected a "most likely" ARIMA model for each series, the model was then used to forecast data. These forecasts were made for time periods within and outside the sample period used for parameter estimation. As Box-Jenkins (1976, pp. 127-128) points out the minimum mean square error forecast of a data series, z , at origin in period t , for lead time ℓ , is the conditional expectation of $z_{t+\ell}$ made at time t . Thus, if we represent

TABLE 1

| MODEL | RSS (residual sum of squares) | DF | CONSTANT | PARAMETERS |
|-----------|-------------------------------|-----|--|---|
| (0,1,0) | $.1005014 \times 10^{-2}$ | 520 | $-.54118 \times 10^{-4}$ ($.6090 \times 10^{-4}$) | $\ln(1+GERINTBK(90))$ |
| (0,1,10)* | $.95617 \times 10^{-3}$ | 511 | S (suppressed) | MA1 .0943 (.0437) MA2 .0134 (.0438) MA3 .0791 (.0438) MA4 -.0319 (.0438) MA5 -.1219 (.0437) MA6 -.0883 (.0437) MA7 .0459 (.0438) MA8 .0543 (.0437) MA9 .0725 (.0436) MA10 -.0654 (.0436) |
| (0,1,12) | $.95306 \times 10^{-3}$ | 509 | $-.5346 \times 10^{-4}$ ($.5276 \times 10^{-4}$) | MA1 .0978 (.0438) MA2 .0152 (.0440) MA3 .0753 (.0440) MA4 -.0368 (.0440) MA5 -.1165 (.0443) MA6 -.0726 (.0443) MA7 .0530 (.0442) MA8 .0443 (.0440) MA9 .0583 (.0439) MA10 -.0741 (.0438) MA11 .0333 (.0439) MA12 .0467 (.0439) |

| MODEL | RSS | DF | CONSTANT | PARAMETERS |
|------------------------|----------------------------|-----|--|--|
| | | | $\ln(1+USTB(90))$ | |
| (0,1,0) | .429978 x 10 ⁻³ | 520 | -.4770 x 10 ⁻⁴ (.3984 x 10 ⁻⁴) | |
| (0,1,1)* | .42408 x 10 ⁻³ | 520 | S | MA1 .12394 (.04340) |
| (1,1,1) | .423177 x 10 ⁻³ | 519 | S | AR1 -.32552 (.3056) MA1 -.19533 (.3160) |
| (0,1,0) | .83406 x 10 ⁻² | 520 | .48034 x 10 ⁻⁴ (.1755 x 10 ⁻³) | |
| (0,1,15) x (0,0,4)* | .73533 x 10 ⁻² | 506 | S | MA1 .0156 (.0424) MA2 .0669 (.0430) MA3 -.0975 (.0417) MA4 -.1493 (.0424) MA5 S MA6 .1105 (.0425) MA7 S MA8 -.1356 (.0424) MA9 S MA10 -.1335 (.0686) MA11 -.1068 (.0420) MA12 .0779 (.0428) MA13 -.0615 (.0416) MA14 S MA15 .2329 (.0651) SEMA1 -.0035 (.0445) SEMA2 .0243 (.0690) SEMA3 -.3179 (.0666) SEMA4 -.1290 (.0458) |

MODEL RSS DF CONSTANT PARAMETERS

| | | | | | | | | | | | | | | | | | | |
|-----------------------|---------------------------|-----|---|------------------------------|---|-------------------------------|------------------------------|--|------------------------------|------------------------------|------------------------------|-------------------------------|------------------------------|-------------------------------|------------------------------|------------------------------|------------------------------|-------------------------------|
| (0,1,15) x (0,0,5) | .73537 x 10 ⁻² | 506 | S | MA1 S .0672 (.0430) | MA2 MA3 MA4 MA5 MA6 MA7 MA8 MA9 MA10 MA11 MA12 MA13 MA14 MA15 SEMA1 SEMA2 SEMA3 SEMA4 SEMA5 | MA1 S .10876 (.0424) | MA2 S .0672 (.0430) | MA3 MA4 MA5 MA6 MA7 MA8 MA9 MA10 MA11 MA12 MA13 MA14 MA15 SEMA1 SEMA2 SEMA3 SEMA4 SEMA5 | MA1 S .1484 (.0422) | MA2 S .0979 (.0416) | MA3 S .1350 (.0421) | MA4 S .1555 (.1042) | MA5 S .2377 (.0661) | MA6 S .15144 (.1117) | MA7 S .2473 (.1020) | MA8 S .0186 (.0714) | MA9 S .0223 (.0710) | MA10 S .0205 (.0755) |
| (0,1,15) x (0,1,6) | .73867 x 10 ⁻² | 506 | S | MA1 S .0980 (.0419) | MA2 S .0630 (.0417) | MA1 S .1121 (.0413) | MA2 S .0630 (.0417) | MA3 MA4 MA5 MA6 MA7 MA8 MA9 MA10 MA11 MA12 MA13 MA14 MA15 SEMA1 SEMA2 SEMA3 SEMA4 SEMA5 | MA1 S .1528 (.0421) | MA2 S .0970 (.0415) | MA3 S .1242 (.0421) | MA4 S .15144 (.1117) | MA5 S .2473 (.1020) | MA6 S .0186 (.0714) | MA7 S .0223 (.0710) | MA8 S .0205 (.0755) | MA9 S .0223 (.0710) | MA10 S .0205 (.0755) |

| MODEL | RSS | DF | CONSTANT | PARAMETERS |
|-------------------------|---------------------------|-----|---|---|
| (0,1,0) | .92200 x 10 ⁻² | 520 | .3804 x 10 ⁻⁴ (.1843 x 10 ⁻³) | MA1 .0101 (.0441) |
| (0,1,15) x (0,0,4) * | .83518 x 10 ⁻² | 506 | S | MA2 .1169 (.0439) MA3 -.0788 (.0436) MA4 -.1425 (.0436) MA5 S MA6 .0638 (.0438) MA7 S MA8 -.0918 (.0436) MA9 S MA10 -.1440 (.1164) |
| (0,1,15) x (0,0,5) | .83525 x 10 ⁻² | 506 | S | MA11 -.0989 (.0434) SEMA1 .00026 (.0446) SEMA2 .0754 (.1174) SEMA3 -.1627 (.09822) SEMA4 -.1262 (.0511) MA12 .0836 (.0445) MA13 -.0700 (.0437) MA14 S MA15 .0774 (.0952) MA16 S MA17 .1196 (.0437) MA18 -.0800 (.0430) MA19 S MA20 -.0892 (.0431) MA21 S MA22 .1035 (.0420) SEMA5 -.00196 (.0444) SEMA6 .1106 (.1299) SEMA7 -.2120 (.0933) SEMA8 -.1300 (.0551) SEMA9 -.1300 (.0917) SEMA10 -.0360 (.0673) |

ln (US-German exchange rate (90))

| MODEL | RSS | DF | CONSTANT | PARAMETER |
|-----------------------|----------------------------|-----|----------|--|
| (0,1,15) x (0,0,6) | .845717 x 10 ⁻² | 506 | S | MA1 S MA2 S MA3 -.0767 (.0438) MA4 -.1430 (.0438) MA5 S MA6 .0373 (.0437) MA7 S MA8 -.0812 (.0445) MA9 S MA10 -.1547 (.1844) MA11 -.0989 (.0434) MA12 .0704 (.0435) MA13 -.0802 (.0450) MA14 S MA15 0.0535 (.1430) SEMA1 -.0106 (.0443) SEMA2 .0770 (.1890) SEMA3 -.0418 (.1450) SEMA4 -.1438 (.0620) SEMA5 -.0259 (.0540) SEMA6 .0248 (.0534) |

Explanation to Table 1

(p,d,q) x (P,D,Q) refers to an ARIMA model with d simple differences, an autoregressive polynomial of order p and a moving average polynomial of order q. The second set of parentheses indicates seasonal parameters: seasonal AR or order P, seasonal differencing of order D and seasonal MA of order Q. For our daily data, the relevant seasonal interval is 5 days, a business week. The numbers in parentheses under the parameter estimates are the standard error of the parameters under the linear hypothesis. The asterisk indicates the model which was finally selected for each series on the basis of the likelihood ratio tests.

GERINTBK(90) ≡ German inter-bank rate for 90 day loans
 USTB(90) ≡ rate of interest on 90 day U.S. Treasury bills

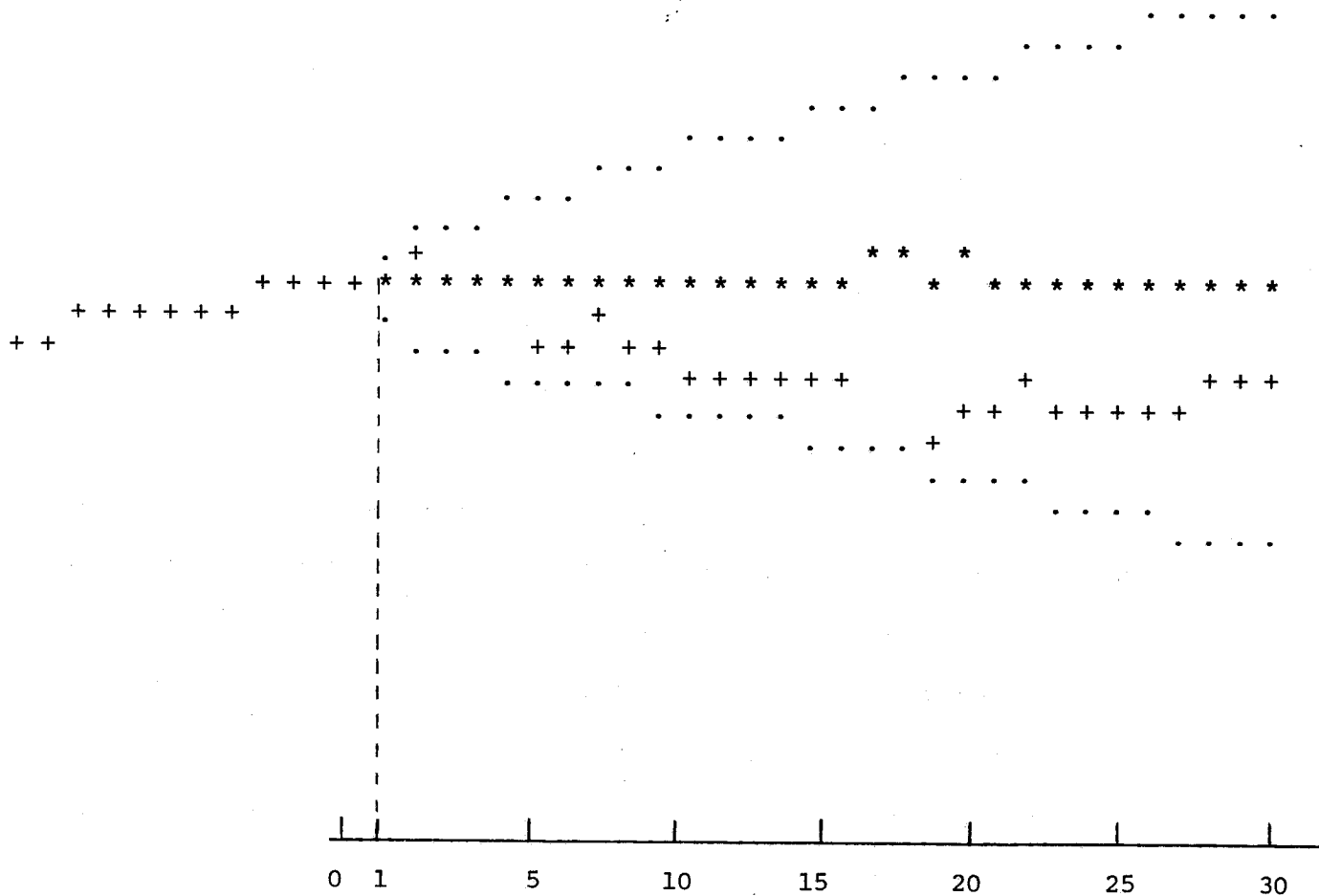
TABLE 2

| | $\lambda = \frac{L(X H_1)}{L(X H_0)}$ | $2\ln\lambda$ | r | Critical points for χ^2 | |
|--------------------------------|---------------------------------------|---------------|-----|------------------------------|--------|
| | $\frac{\ln(1+GERINTBK(90))}{\alpha}$ | | | | |
| $H_0(0,1,0)$ vs $H_1(0,1,10)$ | 414900.25 | 25.87 | 10 | 18.307 | 15.987 |
| $H_0(0,1,0)$ vs $H_1(0,1,12)$ | 1038245. | 27.71 | 12 | 21.026 | 18.549 |
| $H_0(0,1,10)$ vs $H_1(0,1,12)$ | 2.5024 | 1.8345 | 2 | 5.991 | 4.605 |
| | | | | .05 | .10 |
| | | | | | .25 |
| | | | | | 12.549 |
| | | | | | 14.845 |
| | | | | | 2.773 |
| | $\frac{\ln(1+USTB(90))}{\alpha}$ | | | | |
| $H_0(0,1,0)$ vs $H_1(0,1,1)$ | 36.686 | 7.2048 | 1 | 7.879 | 6.635 |
| $H_0(0,1,0)$ vs $H_1(1,1,1)$ | 64.1796 | 8.3234 | 2 | 10.597 | 9.210 |
| $H_0(0,1,1)$ vs $H_1(1,1,1)$ | 1.7494 | 1.1186 | 1 | 7.879 | 6.635 |
| | | | | .05 | .10 |
| | | | | | .25 |
| | | | | | 5.024 |
| | | | | | 7.378 |
| | | | | | 5.024 |

r = degrees of freedom

DATA LISTING FOR FIGURE 3

| Origin | Observation | Lower Conf. Limit | Forecast | Upper Conf. Limit | Actual | Error |
|--------|-------------|----------------------|--------------|----------------------|--------------|---------------|
| | 523 | 0.421298E 00 | 0.424456E 00 | 0.427632E 00 | 0.426700E 00 | 0.224382E-02 |
| | 524 | 0.420388E 00 | 0.424818E 00 | 0.429282E 00 | 0.427100E 00 | 0.228196E-02 |
| | 525 | 0.419033E000 | 0.424316E 00 | 0.429647E 00 | 0.425900E 00 | 0.158441E-02 |
| | 526 | 0.418110E 00 | 0.424280E 00 | 0.430517E 00 | 0.425800E 00 | 0.151992E-02 |
| | 527 | 0.417477E 00 | 0.424653E 00 | 0.431921E 00 | 0.423500E 00 | -0.115335E-02 |
| | 528 | 0.417005E 00 | 0.425070E 00 | 0.433250E 00 | 0.419400E 00 | -0.567025E-02 |
| | 529 | 0.416633E 00 | 0.425359E 00 | 0.434220E 00 | 0.419200E 00 | -0.615925E-02 |
| | 530 | 0.416035E 00 | 0.425369E 00 | 0.434857E 00 | 0.421000E 00 | -0.436902E-02 |
| | 531 | 0.415416E 00 | 0.425474E 00 | 0.435710E 00 | 0.419500E 00 | -0.597376E-02 |
| | 532 | 0.414702E 00 | 0.425430E 00 | 0.436362E 00 | 0.418100E 00 | -0.733000E-02 |
| | 533 | 0.414139E 00 | 0.425620E 00 | 0.437334E 00 | 0.417700E 00 | -0.791961E-02 |
| | 534 | 0.413651E 00 | 0.425957E 00 | 0.438532E 00 | 0.416400E 00 | -0.955701E-02 |
| | 535 | 0.412784E 00 | 0.425770E 00 | 0.439057E 00 | 0.415400E 00 | -0.103705E-01 |
| | 536 | 0.412214E 00 | 0.425913E 00 | 0.439946E 00 | 0.415200E 00 | -0.107132E-01 |
| | 537 | 0.411563E 00 | 0.425931E 00 | 0.440668E 00 | 0.416800E 00 | -0.913119E-02 |
| | 538 | 0.410929E 00 | 0.426018E 00 | 0.441512E 00 | 0.415700E 00 | -0.103179E-01 |
| | 539 | 0.410590E 00 | 0.426374E 00 | 0.442601E 00 | 0.411100E 00 | -0.152736E-01 |
| | 540 | 0.409782E 00 | 0.426192E 00 | 0.443083E 00 | 0.410900E 00 | -0.152921E-01 |
| | 541 | 0.409038E 00 | 0.426076E 00 | 0.443633E 00 | 0.412100E 00 | -0.139760E-01 |
| | 542 | 0.408438E 00 | 0.426125E 00 | 0.444373E 00 | 0.414100E 00 | -0.120251E-01 |
| | 543 | 0.407615E 00 | 0.426026E 00 | 0.445044E 00 | 0.414100E 00 | -0.119258E-01 |
| | 544 | 0.406999E 00 | 0.426078E 00 | 0.445811E 00 | 0.415900E 00 | -0.101781E-01 |
| | 545 | 0.406355E 00 | 0.426071E 00 | 0.446485E 00 | 0.412700E 00 | -0.133705E-01 |
| | 546 | 0.405628E 00 | 0.425997E 00 | 0.447114E 00 | 0.413100E 00 | -0.128973E-01 |
| | 547 | 0.405017E 00 | 0.426040E 00 | 0.447858E 00 | 0.413300E 00 | -0.127396E-01 |
| | 548 | 0.404276E 00 | 0.425960E 00 | 0.448492E 00 | 0.412600E 00 | -0.133597E-01 |
| | 549 | 0.403524E 00 | 0.425862E 00 | 0.449102E 00 | 0.414500E 00 | -0.113621E-01 |
| | 550 | 0.402892E 00 | 0.425851E 00 | 0.449764E 00 | 0.415100E 00 | -0.107512E-01 |
| | 551 | 0.402282E 00 | 0.425872E 00 | 0.450471E 00 | 0.415800E 00 | -0.100724E-01 |
| | 552 | 0.401755E 00 | 0.425962E 00 | 0.451233E 00 | 0.415800E 00 | -0.101625E-01 |
| | 553 | 0.401194E 00 | 0.425960E 00 | 0.451840E 00 | 0.417600E 00 | -0.836045E-02 |



* = forecast

+ = actual

. = upper and lower confidence limits

o = origin of forecast, December 31, 1976

In cases where the forecast (*) is nearly equal to the actual (+), the * is printed.

FIGURE 3

the ARIMA process in its infinite moving average form, we have

$$z_{t+l} = a_{t+l} + \psi_1 a_{t+l-1} + \psi_2 a_{t+l-2} + \dots + \psi_l a_t + \psi_{l+1} a_{t-1} + \dots \quad (15)$$

The forecast \hat{z}_{t+l} for z_{t+l} is simply

$$z_{t+l} = \psi_l a_t + \psi_{l+1} a_{t-1} + \dots \quad (16)$$

since $E(a_{t+1}) = 0$ for $l \geq 1$. The forecasts made in this manner are unbiased.

Daily forecasts for various horizons were made from 41 different origins, 24 within the sample period and 17 outside of it. As an example, Figure 3 and its accompanying table show the forecasts of the spot exchange rate for the thirty days following the end of our sample period. Figure 3 presents the actual data, the forecasted value and the confidence limits for the spot series. In practice these forecasts could be updated daily as new forecast errors in the ARIMA process become available.

SECTION III

While an individual time series y_t can be parsimoniously represented by an ARIMA process, it is often the case that y_t is correlated with other time series x_t which may be helpful in predicting more accurately the future values $y_{t+\ell}$, $\ell = 1, \dots, N$. Of course, correlation does not imply causality nor does causality imply correlation. It is essential to have a clear criterion for establishing whether the real world, i.e., the data, reveals causal relationships. Under the perhaps not so obvious assumption that the future does not cause the past, Granger and Newbold [1976, pp. 224-226] propose the following criterion, which we adopt, to establish whether series x_t causes series y_t . Let $P(y_t | \Omega_{t-1})$ denote the conditional probability of y_t given Ω_{t-1} , where Ω_{t-1} denotes the universe of information available at time $t-1$. If

$$P(y_t | \Omega_{t-1}) = P(y_t | \Omega_{t-1} - x_{t-1}) \quad (17)$$

then x_t does not cause y_t , where $\Omega_{t-1} - x_{t-1}$ represents the universe of information at time $t-1$ but excluding the past realizations of the x series. Instantaneous causality exists if

$$P(y_t | \Omega_t - y_t) > P(y_t | \Omega_t - y_t - x_t) \quad (18)$$

Feedback is present if x causes y (non-instantaneously) and y causes x (non-instantaneously). Of course such definitions are non-operational since we rarely have enough information to estimate a conditional distribution function. In practice we restrict ourselves to looking at the mean of the forecast and its variance. The variance of the forecast will in turn be influenced by the cross-correlation functions $\gamma_{yx}(k)$ where k

indicates the lag between the series.

As a first approximation to a determination of the spot rate we consider a more general version of equation (12). This is a single output, multiple input transfer function:

$$y_t = \sum_{i=-\infty}^{\infty} v_{1i} B^i x_{1t} + \sum_{i=-\infty}^{\infty} v_{2i} B^i x_{2t} + \sum_{i=-\infty}^{\infty} v_{3i} B^i x_{3t} + \varepsilon_t \quad (19)$$

where

- y_t = natural log of the spot exchange rate
- x_{1t} = natural log of the ninety day forward exchange rate
- x_{2t} = natural log of 1 plus the three month German interbank rate
- x_{3t} = natural log of 1 plus the three month U.S. treasury bill rate
- ε_t = a zero-mean stationary process

Our first task is to see what, if any, causality relationship characterizes the various series. By using the ARIMA filter for an input variable, we can prewhiten the input. Using the input filter, we can prewhiten the output and then study the cross-correlation between the prewhitened input and output. Such a procedure preserves the causality relationship between the two variables. This exercise showed only meagre evidence of any correlation at all. In particular, as evidenced by insignificant cross correlations when the output leads the input, there seems no reason to believe that there is any feedback relationship between the spot exchange rate and the other three variables. The cross correlation between the spot rate and the ninety day forward exchange rate evidences very strong instantaneous causality as well as a pronounced lag relationship (spot following the forward rate) at twelve days. The fact that there is instantaneous causality should come as no surprise since in Section II we indicated that our transfer function would represent a first approximation inasmuch as it combines a two-equation

simultaneous system into a single equation. We are aware of the difficulty of simultaneous equations bias. However neither the theoretical framework nor the required data are available for the joint estimation procedure which would eliminate the bias.

That there was little evidence for lag relationships between the spot rate on the one hand and the two interest rates on the other was indeed surprising. We decided to pursue further the possibility of causal relationships among these variables as well as between the spot and forward exchange rates. Allowing for the possibility of significant lag relationships but excluding feedback relationships we can write a more parsimonious version of (19):

$$\begin{aligned}
 y_t &= \sum_{i=0}^{K_1} v_{1i} B^i x_{1t} + \sum_{i=0}^{K_2} v_{2i} B^i x_{2t} + \sum_{i=0}^{K_3} v_{3i} B^i x_{3t} + \epsilon'_t \\
 &= v_1(B)x_{1t} + v_2(B)x_{2t} + v_3(B)x_{3t} + \epsilon'_t \quad (20)
 \end{aligned}$$

where $v_j(B)$ are polynomials in B which operate on t . The order of the $v_j(B)$ may be quite high since both the forward exchange rate and the German interbank rates had high order ARIMA representations. Although it is not assumed that the error process is white noise, nevertheless consistent estimates of the v_{ji} can be obtained by ordinary least squares regression. Initially we regressed the spot exchange rates on the forward rates, the German interbank rates and the U.S. treasury bill rates, each of these lagged from zero to forty. Since the t-statistics on some of the v_{ji} 's were still significant at or near forty, the lag structure was increased from zero to forty-six for each of the input variables.

Given estimates for the $v_j(B)$ functions, a yet more parsimonious representation was sought by means of the ratio of two polynomials:

$$v_j(B) \approx \frac{\omega(B)}{\delta(B)}$$

where ω and δ are polynomials in B of order r and s respectively. Orders of ω and δ , as well as those individual polynomial coefficients to be suppressed, are chosen so that they reproduce in a general fashion the pattern of the v_{ji} .

To actually estimate the values of the ω and δ coefficients, one must model the error process. Inspection of the autocorrelation and partial autocorrelation functions for the residuals from the ordinary least squares regression indicates that the residuals follow a MA(2) process:

$$\varepsilon'_t = (1 - \theta_1 B - \theta_2 B^2) a_t$$

where a_t is a white noise process. If we further assume that a_t is Gaussian, then minimizing the sum of these squared residuals, conditional on starting values $y_0, x_{1,0}, x_{2,0}, x_{3,0}$ and a_0 , gives a good approximation to the maximum likelihood estimates.

Several different models were fitted and the generic results (excluding parameter values) are given in Table 3. ω^j refers to the numerator polynomial in B of input j whereas δ^j refers to the denominator polynomial of j . The numbers in parentheses indicate the polynomial coefficients which are actually estimated; all other coefficients of that polynomial are constrained to be zero. The exception is that $\delta^j(0) = 1$ for all j . Thus $\omega^3(0,6,18)$ indicates the numerator polynomial for the U.S. treasury bill rate (input 3). It has the form

$$\omega^3(0,6,18) = \omega_0^3 - \omega_6^3 B^6 - \omega_{18}^3 B^{18}$$

The order of the delay impulse operator is given by b . If, for example, $b = 3$, then the indicated input, input 3 in our case, is constrained to have no effect on the output until three periods have elapsed. All of the models,

TABLE 3

| MODEL | DESCRIPTION | SSR | F | R ² |
|-------|--|-------------------------|-----------------|----------------|
| 1 | $\omega^1(0,2)$ $\omega^2(0,2,8,18)\delta^2(1,2)$ $\omega^3(0,3,15)\delta^2(1,2)b = 3$ | $.96673 \times 10^{-3}$ | F(15,488) = 245 | .879 |
| 2 | $\omega^1(0,2)$ $\omega^2(0,2,8,18)\delta^2(1,2)$ $\omega^3(0,6,18)\delta^3(1,2)b = 0$ | $.96534 \times 10^{-3}$ | F(15,488) = 245 | .879 |
| 3 | $\bar{c}, \omega^1(0,2)$ $\omega^2(0,2,18)\delta^2(1,2)$ $\omega^3(0,3)\delta^3(1,2)b = 3$ | $.9995 \times 10^{-3}$ | F(14,488) = 252 | .875 |
| 4 | $\omega^1(0,2)$ $\omega^2(0,2,8,18)\delta^2(1)$ $\omega^3(0,3)\delta^3(1,2)b = 3$ | $.10005 \times 10^{-2}$ | F(13,490) = 273 | .875 |
| 5 | $\omega^1(0,2)$ $\omega^2(0,2,8,18)\delta^2(1)$ $\omega^3(0,6)\delta^3(1,2)b = 0$ | $.96961 \times 10^{-3}$ | F(13,490) = 282 | .879 |

| MODEL | DESCRIPTION | SSR | F | R ² |
|-------|--|---------------------------|-----------------|----------------|
| 6 | $\bar{c}, \omega^1(0,2)$ $\omega^2(0,2,8,18)\delta^2(1,2)$ $\omega^3(0,3)\delta^3(1,2)b = 3$ | .10004 x 10 ⁻² | F(13,489) = 272 | .875 |
| 7 | $\omega^1(0,2)$ $\omega^2(0,8,18)\delta^2(1,2)$ $\omega^3(0,6,18)\delta^3(1,2)b = 0$ | .96542 x 10 ⁻³ | F(14,489) = 263 | .879 |
| 8 | $\omega^1(0,2)$ $\omega^2(0,2,8,18)$ $\omega^3(0,6,18)\delta^3(1,2)b = 0$ | .96723 x 10 ⁻³ | F(13,490) = 283 | .879 |
| 9 | $\omega^1(0,2)$ | .10970 x 10 ⁻² | F(4,515) = 850 | .867 |
| 10 | $\omega^2(0,8,18)\delta^2(1,2)$ | .76490 x 10 ⁻² | F(7,496) = 5.42 | .058 |

For each model the noise process was $\epsilon_t = (1-\theta_1 B - \theta_2 B^2)a_t$. \bar{c} means a constant was estimated.

except 9 and 10, performed well on the diagnostic checks, which are referred to in detail below.¹⁴ To help facilitate the choice of models, likelihood ratio tests were conducted in those cases where we were dealing with nested hypotheses. These results are given in Table 4.

The OLSQ regression referred to above indicated that the U.S. treasury bill may act on the spot exchange rate with a lag of about three days. A comparison of the SSR in model 1 with that in model 2 indicates there is a slight preference for model 2, i.e., the one in which the U.S. treasury bill impacts instantaneously on the spot exchange rate. Thus, most of the subsequent models do not constrain the initial impact of the U.S. treasury bill rate to zero.¹⁵ Table 4 indicates that model 4 (without a constant) cannot be rejected in favor of model 6 (same as 4 but including a constant). In addition, in other trial runs where the constant was included, its t-statistic was never significant. Hence the constant was suppressed in order to allow efficient choice of the maximum number of fifteen parameters to be estimated. Table 4 indicates that there are a number of models at our disposal among which we cannot choose a "best one." Happily the likelihood ratio test does show that if we include only one of the inputs, e.g., the forward exchange rate, then that model is rejected in favor of a model which includes all of the inputs.

On balance model 2 and model 7 seem attractive. Since a likelihood ratio test indicates that, under the assumption that model 7 is the correct model, it cannot be rejected in favor of model 2, model 7 was chosen for further analysis.

Before presenting the parameter estimates for model 7 we turn to some of the diagnostic checks, which offer us further assurance that our model is approximately correct. The residuals of the nonlinear estimation procedure produce no autocorrelation at any lags. This is the case both when individual lags are considered and when one groups the autocorrelations over lags from 1 to 60. This indicates that there is practically no information in the residuals which could be

TABLE 4

| λ | $2\ln\lambda$ | r | Critical values of χ^2 for $\alpha =$ | | |
|-----------------------|----------------------------|-----|--|---------|---------|
| | | | .05 | .10 | .25 |
| H_0^4 vs H_1^6 | 1.02633 | 1 | 3.84146 | 2.70554 | 1.3233 |
| H_0^5 vs H_1^2 | 3.15039 | 2 | 5.99147 | 4.60517 | 2.77259 |
| H_0^6 vs H_1^3 | 1.26366 | 1 | 3.84146 | 2.70554 | 1.3233 |
| H_0^8 vs H_1^2 | 1.66287 | 2 | 5.99147 | 4.60517 | 2.77259 |
| H_0^7 vs H_1^2 | 1.0217797 | 1 | 3.84146 | 2.70554 | 1.3233 |
| H_0^9 vs H_1^7 | 2.6760077×10^{14} | 10 | 18.3070 | 15.9871 | 12.5489 |
| H_0^{10} vs H_1^7 | greater than 10^{100} | 9 | 16.9190 | 14.6837 | 11.3887 |

useful in providing a better predictor for the output variable.

Other important signs that both the noise process and the transfer function are correctly modelled are to be found in the cross-correlation functions between the residuals and the input variables. If there is residual autocorrelation and cross correlation with the input, then this is evidence that the transfer function has been modelled incorrectly. If there is residual autocorrelation but no cross correlation, this points to a misspecification of the noise process.¹⁶ Our results indicate no such problems.

When one inspects the cross correlations between the estimated residuals and the three input series, the results are very satisfactory. Both in the case of the residuals leading the input series and when the residuals lag the input series, the Box-Pierce statistics are good and the individual cross correlations are practically never significant up to sixty leads and lags. Only in the case of the U.S. treasury bill rate is there some indication that at short leads (because the cross correlations between \hat{a}_{t+k} and x_{3t} for $k = 4$ and $k = 6$ are barely significant) there may be some slight misspecification in the model. The other input series behave in exemplary fashion.

Model 7, about which we can be reasonably confident, has the following parameter values:

$$\begin{aligned}
 \ln \ell_t = & \frac{(.91433 + .0776B^2)}{(.01402) \quad (.01403)} \ln_{t+90} \\
 & + \frac{(.14967 + .04387B^8 + .068976B^{18})}{(.04144) \quad (.04191) \quad (.03963)} \ln(1+GRINTBK(90)) \\
 & \quad 1 + \frac{.23570B}{(.24134)} + \frac{.41304B^2}{(.24351)} \\
 & + \frac{(-.11593 + .04998B^6 + .01701B^{18})}{(.02559) \quad (.02470) \quad (.01345)} \ln(1+USTB(90)) \\
 & \quad 1 - \frac{.07786B}{(.1343)} - \frac{.78939B^2}{(.12603)} \\
 & + (1 - \frac{.39692B}{(.03990)} - \frac{.50351B^2}{(.03947)}) a_t \tag{21}
 \end{aligned}$$

Though the model is much more complicated than the simple interest arbitrage equation (3'), it nevertheless retains some of its characteristics. The coefficient of $\ln \ell_{t+90}$ is fairly close to 1, though the 95% confidence interval, based on the linear hypothesis, does not include 1. The signs on the coefficient of the unlagged German interbank rate and the unlagged U.S. treasury bill rate are respectively positive and negative, which is what one would expect from the simple story. Though the absolute values of the coefficients are not close to 1, they are about equal in magnitude and the absolute values of the 95% confidence intervals overlap.

Dividing ω^j by δ^j one obtains the $v_{ji} B^i$ and gets a feel for the way in which the inputs drive the output. The forward exchange rate expends all its influence after just two periods. The weights on the German interbank rate begin positive for the unlagged operator and then decline in magnitude and alternate in sign with two negative, two positive, two negative, etc. They are of significant size out to about lag 25. The weights on the U.S. treasury

bill rate are all negative and roughly declining out to lag 18, at which point they take a jump and then continue to decline again, but now alternatively positive and negative. They remain significant out to approximately 35 lags.

The steady state gain $g_j = \sum_{i=0}^{\infty} v_{ji}$ for each of the input series is perhaps more properly considered the analog to the coefficient of the simple interest arbitrage equation. Since $g_1 = .992$ the total effect over time of the forward rate is that predicted by (3'). On the other hand, $g_2 = .076$ and $g_3 = -.369$ and these bear little resemblance to 1 and -1 in (3').¹⁷

All this indicates that values of the input series going back as far as seven weeks provide useful information about the present value of the spot exchange rate. It is not surprising that the impulse (v) weights are significant at long lags for the interest rates but only at short lags for the forward exchange rate. It will be remembered that the ARIMA processes for the spot and forward exchange rates were very similar. Thus the forward rate would not be expected to contribute much information at long lags to the determination of the spot rate. On the other hand the spot ARIMA process was such that information on the a_t up to thirty periods in the past was useful in determining the present spot rate. Apparently both interest rates are sufficiently different from, yet related to, the spot rate so that interest rates at rather long lags offer useful information about the present spot exchange rate.

A good measure of the multiple input transfer function's predictive power has been suggested by Pierce (1975). He proposes that we look at a modified R^2 . It is modified because we are not really interested in its explanatory power for current values of the output variable, but rather for future values. More importantly we know that the output y_t may already

contain considerable information in its own history which can be captured in a suitable ARIMA process. The interesting question is: what can the inputs x_{it} explain of y_t which is not already explained by y_t 's own ARIMA process? Put another way; how much of the variance of the innovation process of y_t can the multiple input transfer function eliminate? The proper measure of this is

$$R_+^2 = \frac{v_1(1) - v_2(1)}{v_1(1)} \quad (22)$$

where $v_1(1)$ is the one step ahead forecast variance of the single variable ARIMA process and $v_2(1)$ is the one step ahead forecast variance of the multiple input transfer function. Since the one step ahead forecast variance is simply the variance of the innovation process R_+^2 is easily calculated in our case to be .864. Thus the multiple input transfer function has a residual variance substantially smaller than that of the univariate ARIMA process.

SECTION IV

When uncertainty with respect to future arbitrage opportunities was introduced, this led to a modified version (10) of the interest arbitrage equation. The conditional distribution of future arbitrage opportunities then suggested a version (12) for the arbitrage equation which had the form of a multiple input transfer function with possibly long lags on the input variables. However, before the MITF was estimated, each series was modelled by a suitable ARIMA process. While the individual series had a moderate amount of information which was recoverable from their past history, the multiple input transfer function had a residual variance substantially smaller than that of the univariate ARIMA process. All of the individual series were characterized by MA processes. The exchange rate series had long lags, whereas the interest rates were rather different from each other. The German interbank series had lags of intermediate lengths while the U.S. treasury bill rate only required a first order moving average process.

Out of a plethora of multiple input transfer models we were able to choose one with a fair degree of certainty. It has perhaps a few too many parameters, but, since our interest was in forecasting, it was decided that a few extra parameters could do no harm. As one step in checking out the predictive power of the model, we employed Pierce's predictive R_+^2 and found that the model performed well.

Further forecasts over multi-period horizons remain to be carried out. For such work the ARIMA structures would be useful since they provide forecasts for the input variables as a first step toward obtaining unconditional forecasts of the output series. Then it would be appropriate to forecast, via the MITF, future spot exchange rates and compare them with the present forward exchange rate in an effort to determine whether the forward rate is a "rational" forward rate, i.e., a forward rate based on all the information in the system. We leave such analysis to further work.

FOOTNOTES

¹ See Hume (1752).

² For a succinct survey of recent time series studies of exchange rates see Levich (1977).

³ This is the first study we are aware of that employs a MITF in exchange rate analysis.

⁴ Branson (1969) and Einzig (1961) cite instances where departures from the strict interest arbitrage equation can be explained by supply elasticities. The basic concept is that there exists a point after which additional arbitrage funds are available only at increasing marginal cost. The marginal return from arbitrage transactions might be less than the marginal cost of acquiring the necessary funds.

In this paper we emphasize another implication of a less than perfectly elastic supply of arbitrage funds. Given that funds are in limited supply, arbitrage opportunities might be deliberately passed up on a given day due to portfolio considerations concerning the expectations of future arbitrage opportunities and the variance of these expectations.

⁵ It might be argued that equation (1) still holds if securities are considered identical in all respects, including the position they offer for future arbitrage opportunities. This definition of perfect substitutes does not appear to be very useful. We continue to use equation (3) in the analysis.

⁶ A note on the additional complications of the expanded model is worth-

while. The explicit treatment of the future involves looking at the path in which information is acquired. When considering his investment decisions today, the investor looks at the position he will be in for arbitrage tomorrow, the day after, the day after that, etc. He has expectations for these arbitrage opportunities for each day in the future.

When calculating the expected return from taking his positions today, he must include what his reactions to future expected events will be. An example illustrates this.

Assume the investor puts λ of his funds in domestic securities and he has conditional probability distributions for R_{t+1} , R_{t+2} , R_{t+3} entailing movements into foreign securities. Now suppose that the next day arrives and R_{t+1} materializes as expected. The investor will not put all of λ into the arbitrage opportunity since that would lock him out of the opportunities for R_{t+2} and R_{t+3} . He has the choice of a certain arbitrage return against uncertain future returns. It is the portfolio selection problem at the second level, and as such would involve dynamic programming.

⁷ We use the word "attractive" since, given a limited supply of funds, the investor might pass up some initial arbitrage opportunities if he has an expectation that a particularly profitable one will soon be available.

⁸ It is again stressed that the asset the investor is purchasing is his position in bonds. This position may yield the bond's own rate of return or may be used to capture the higher rates of return from arbitrage activities.

⁹ We are aware that there is risk of simultaneous equation bias in assuming that only e^s (and not e^f and e^s) will adjust to clear the bond market. This will be emphasized again in the text below.

¹⁰ Computing was done by programs developed by or under the direction of Charles Nelson. We gratefully acknowledge his work. For a good discussion of ARIMA techniques see Nelson (1973).

¹¹ For a more thorough treatment of the Box-Pierce statistic and the Kolmogorov-Smirnov test see Box and Jenkins (1976), 289-298.

¹² For example, Levich investigated the percentage change in exchange rates for nine industrial countries and found strong evidence for the random walk model for only two countries, Italy and Switzerland.

¹³ These tests were developed by Zellner (1971), 291-318. In our calculations the simplified expression $= (S_0^2/S_1^2)^{-T/2}$ is used. This assumes diffuse prior information as to the relative suitability of the two models, a large number of observations, and a symmetric loss function. Zellner and Palm (1974) also contains a useful discussion of this technique as well as applications.

¹⁴ All the models calculated were both stationary and noninvertible, i.e., all the roots of ω^j and δ^j lay outside the unit circle. As it turns out stationarity is a necessary assumption in a MITF though invertibility is not.

¹⁵ Whether the U.S. treasury bill acts instantaneously or with a lag has important consequences for the predictive R^2 , to be discussed below. Pierce (1957) has shown that if a MITF is noninvertible, and this is always the case if $b > 0$, there is additional predictive punch which arises from this noninvertibility.

¹⁶ This is a trivial generalization of the analysis in Box and Jenkins (1976), 392-393.

¹⁷ The computer program to estimate the MITF does allow one to constrain the gain to a certain value. This would, however, adversely affect the predictive power of the estimated equation.

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