

THE SCHEDULING OF CONSUMER ACTIVITIES: WORK TRIPS

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## ABSTRACT

How do individuals decide when to carry out certain activities? This question is crucial to a detailed understanding of many important peaking phenomena, such as electricity demand or urban highway congestion. This paper generalizes allocation-of-time models to encompass the scheduling of activities over time. An econometric model is derived, and estimates presented for the problem of a commuter choosing the time-of-day of his work trip.

The econometric results, on data from the San Francisco Bay Area, demonstrate that commuters do indeed perceive a trade-off, in the presence of highly peaked traffic congestion, between travelling under uncongested conditions and travelling at their preferred time of day. Furthermore, this trade-off is affected in the expected way by the worker's official work hours, occupational and family status, work-hour flexibility, and car occupancy.

Finally, the implications of these results for urban transportation forecasting and policy analysis are explored. These models explain the "shifting peak" phenomenon, long noted empirically, by which transportation improvements affect the duration of the peak period more than the degree of congestion. They also provide a way to forecast the quantitative extent of this effect. Furthermore, they permit a revised estimate of the benefits from transportation improvements, taking into account the fact that part of the benefits are in the form of more preferred scheduling of trips rather than decreased travel time.

## I. Introduction

The analysis of time-varying demand phenomena has reached ever-increasing sophistication, most recently spurred by the need for detailed policy guidance in such areas as energy pricing and urban traffic congestion.<sup>1</sup> On the supply side, detailed attention has been paid to costs of capital equipment, the existence of diverse technology, and metering and storage costs. On the demand side, the effects of uncertainty have been analyzed.

The measurement of time-dependent demand functions has received somewhat less attention. Joskow (1976, pp. 203-204) states:

While the total demand for electricity by various classes of consumers has been analyzed of late almost ad nauseam, demands by time of day and season of the year have by necessity not been given very much consideration by econometricians ... Better information about own-price and cross-price elasticities would be helpful for estimating optimal equilibrium prices as well as for determining the revenue yield of any particular set of rates ...

In the area of urban transportation, much effort has been devoted to measuring own- and cross-price elasticities by mode of travel (Kemp, 1973), but not by times of day.

Theoretical work, meanwhile, has generally assumed the existence of arbitrarily defined time periods such as peak and off peak. Analysis of policy changes, even if cross-elasticities are known, can therefore predict changing demand quantities only within these fixed time periods.

For example, consider the effect of adding capacity to a congested highway corridor. By the standard analysis, travel times within a fixed peak period would decrease by an amount depending on the own- and cross-elasticities of demand with respect to service levels during the two periods. Yet empirical

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<sup>1</sup>See, for example, Vickrey (1963), Keeler & Small (1977), the Bell Journal symposium introduced by Joskow (1976), and reference therein.

observation supports a quite different view. Traffic counts on the San Francisco-Oakland Bay Bridge, before and after the opening of a parallel rapid transit line in 1974, showed that this considerable expansion of total capacity resulted in a negligible change in the level of congestion on the bridge, but a substantial decrease in its duration.<sup>1</sup>

This is by no means an aberrant example. The tendency of urban highways to reach a peak level of congestion which is relatively independent of supply and demand conditions has even earned the title of "the fundamental law of traffic congestion," perhaps most carefully and plausibly stated by Downs (1962). The corresponding tendency of peak periods to change in duration has long been known to traffic engineers as the "shifting peak" phenomenon; Joskow (1976, p. 204) notes the same effect for electricity demand.

The neglect of these shifting peaks calls into question the results of important branches of applied microeconomic analysis. Most obviously, predictions may be incorrect as exemplified above. Cost-benefit analyses will then contain inaccuracies, not only from the errors in prediction, but from the omission of benefits or disbenefits in the form of altered schedules. As is demonstrated in this paper, a shift in the timing of an activity involves tangible and measurable welfare effects. Finally, related types of demand analysis, such as mode choice in the transportation example, may contain biases due to incorrectly assuming schedules to be exogenous.

How, then, should these effects be modelled? Fundamentally, the shifting peak phenomenon is due to demand interdependence among points in time, and it might be thought that conventional demand estimation requires only a

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<sup>1</sup>Peat, Marwick, Mitchell & Co. (1978)

sufficiently large number of time intervals to work. However, the sheer number of cross-elasticities to be measured makes this approach difficult to implement. Furthermore, conventional techniques utilize aggregate data, thus preventing the analyst from taking advantage of disaggregate techniques which have been found fruitful in some of the relevant areas of application.

One way out of the dilemma is to model utility as a functional, i.e., as dependent on the continuous time path of consumption over the period. Koenker (1977, 1978) has shown that such a functional, appropriately parametrized, can lead to estimable aggregate demand equations. On the other hand, the time paths of consumption for many activities are inherently discontinuous: an individual will generally not spread his travel, telephone conversation, television viewing, or perhaps even electricity use smoothly over a large number of short time periods, but will instead make one or more "all or nothing" choices. Thus, there is some advantage in an approach which recognizes discreteness in the scheduling decision.

The approach taken here is to model the scheduling of a discrete activity directly at the individual level. The theoretical model is set out in Section II, and is followed by a section which derives and estimates a disaggregate econometric model for the choice of trip schedule by automobile commuters. Finally, using the results of the econometric estimates, an assessment is made of the implications for urban transportation. The finding that work-trip scheduling flexibility is sufficiently great to warrant inclusion in realistic descriptions of congested urban highway systems suggests, I submit, that such phenomena may be of even greater importance for other consumption activities, many of which would appear to take place with considerably more individual freedom as to timing.

## II. Theoretical Model

Ultimately, scheduling choices are part of a grand activity plan which fits a variety of consumption, work, and transportation activities into a schedule (Chapin, 1974; Jones, 1979; Jacobson, 1979). The consumer takes into account preferences (sleeping at night, air conditioning in afternoon, breakfast before work, or whatever) and external constraints (bus schedules, penalties for arriving late for work, times at which another party is reachable by telephone, weather patterns). A theory such as Lancaster's (1966), based specifically on activities, would appear an ideal starting point. However, the choice of an activity schedule to maximize an objective function is a complex problem. Even the ordering of  $n$  activities can in general be done in  $(n!)$  ways, and although mathematicians have devoted some effort to the problem (Graham, 1978), the algorithms proposed have no obvious economic interpretation.

This paper builds instead on models of time allocation pioneered by Becker (1965), by adding scheduling considerations to both the utility function and the constraints. The model postulated has a numeraire good  $x$ , and three types of time: leisure time  $l$ , working time  $h$ , and "consumption" time  $t$  spent on some activity which provides no utility directly, but which is complementary to consumption, leisure, or work.

This "consumption" activity, which also involves a monetary cost  $c$ , must be carried out at a specific time of day,  $s$ . For example, if the activity is making a telephone call from city A to city B,  $s$  could be the time the call is dialed. In the case of electricity demand,  $s$  could be the time at which a clothes dryer is turned on.<sup>1</sup> For work trips,  $s$  could be the time

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<sup>1</sup>Granger et al (1977) have provided an intriguing technique for determining when various electric home appliances are used, employing micro data on 400 Connecticut households.

the trip is begun, the time of arrival at work, or the time at which a particular congested area is entered.

The complexities of scheduling considerations are represented in three ways. First, to allow for preferences among schedule times,  $s$  is included in the utility function. Second, to focus attention on congestion and peak-load pricing phenomena, cost  $c$  and consumption time  $t$  are allowed to depend on  $s$ . Third, to account for limitations imposed by the institutional setting within which employment opportunities are encountered, a constraint is added relating the schedule  $s$  and length  $h$  of the working day to exogenous parameters.

The limitations embodied in the constraint may be flexible or inflexible. For example, if  $s$  is time of arrival at a job with fixed quitting time  $q$  and a policy of docking pay for tardiness (and no other penalty), the constraint would be  $s + h = q$ . Alternatively, the wage rate may effectively depend (through promotions or merit pay increases) on  $s$  and  $h$ :  $w = w(s, h)$ . If lateness penalties are very severe, the constraint is simply  $s = s_0$ , where  $s_0$  is exogenous.

Thus, the consumer is assumed to choose  $x$ ,  $\ell$ ,  $h$ , and  $s$  so as to maximize

$$(1) \quad u = U(x, \ell, h, s)$$

subject to

$$(2) \quad x + c(s) = Y + wh$$

$$(3) \quad \ell + h + t(s) = T$$

$$(4) \quad F(s, h; w) = 0$$

where the exogenous parameters  $Y$ ,  $w$ , and  $T$  are unearned income, wage rate and total time available, respectively.

The Lagrangian for this problem is

$$(5) \quad L = U + \lambda [Y + wh - x - c(s)] + \mu [T - \ell - t(s)] - vF(s, h; w) .$$

Maximizing over  $x$ ,  $\ell$ , and  $h$  (for fixed  $s$ ) yields the time-allocation problem considered by Johnson (1966) except for the additional constraint (4).

The value of leisure time is found to be

$$(6) \quad \mu/\lambda = w + (U_h + vF_h)/U_x ,$$

indicating that if the scheduling constraint is binding, the relation between value of time and wage rate is modified not only by enjoyment of work time as noted by Johnson, but by scheduling considerations as well. An indirect utility functional  $V$  may be defined as the utility achieved at the optimum choice  $(x^*, \ell^*, h^*)$  conditional on  $s$ :

$$(7) \quad V(c(s), t(s), s) = U(x^*(s), \ell^*(s), h^*(s), s) .$$

If  $s$  is continuous and if second-order conditions are satisfied, maximization over  $s$  yields

$$(8) \quad U_s = \lambda c'(s) + \mu t'(s) + vF_s .$$

where the prime denotes differentiation.

Eliminating  $v$  from (6) and (8) and noting that  $U_x = \lambda$ , we obtain

$$(9) \quad \frac{\mu}{\lambda} = \frac{w + U_h/U_x + (F_h/F_s) (U_s/U_x - c')}{1 + (F_h/F_s)t'}$$

This formula displays the complexities introduced into the value of time by scheduling considerations, and can be rewritten in a form which makes clear the marginal trade-off between congestion and other scheduling considerations:

$$(10) \quad c'(s) + (\mu/\lambda)t'(s) = U_s/U_x + (w + U_h/U_x - \mu/\lambda) (-\partial h/\partial s)_F .$$

We see that in choosing schedule time  $s$ , the consumer equates (at the margin) the additional cost plus the value of additional consumption time



incurred, due to a change in schedule, to the value of utility gained both directly and through the additional work hours permitted. With appropriate functional forms assumed for  $(\mu/\lambda)$ ,  $(U_s/U_x)$ ,  $(U_h/U_x)$ , and  $(-\partial h/\partial s)_F$ , equation (10) could serve as an econometric specification.

The difficulties of such a procedure should not be minimized, however. For typical peaking phenomena, both the first and second derivatives of the congestion function  $t(s)$  change sign as  $s$  varies over the peak period, making it difficult to ensure the fulfillment of second-order conditions. If discontinuous peak-load pricing is in effect, some individuals will probably be at corner solutions (e.g., making telephone calls just after the evening rates go into effect). In addition, the slopes  $t'(s)$  may be measured very inaccurately, and  $s$  itself may be subject to considerable rounding error.

An enticing alternative is to view  $s$  as a discrete variable and to specify and estimate equation (7) directly. This is especially attractive if the data on  $s$  are rounded off. Given such data, the discrete specification would appear more plausible a priori, and it has the advantage of being able to accept peaking functions of any shape. It is this approach which forms the basis for the empirical work described in Section III.

### III. Empirical Results: Work Trips

In order to apply discrete choice techniques (McFadden, 1973) to the estimation of equation (7), assume that, for a given consumer  $i$ , indirect utility  $V^i$  contains universal ("strict utility") and idiosyncratic ("stochastic") components:

$$(11) V^i(c^i(s), t^i(s), s) = W(c^i(s), t^i(s), s, \underline{s}^i) + \varepsilon_s^i,$$

where  $\underline{s}^i$  is a vector of observable characteristics of individual  $i$ . The costs and times are superscripted to emphasize that the "menu" available to each consumer need not be the same.\*

The most computationally tractable model for multiple alternatives is the multinomial logit, obtained by assuming the stochastic elements  $\epsilon_s^i$  to be identically and independently distributed with the extreme value distribution. The resulting probabilities are

$$(12) \quad P_s^i = \frac{\exp(W_s^i)}{\sum_r \exp(W_r^i)},$$

where

$$(13) \quad W_s^i = W(c^i(s), t^i(s), s, \underline{s}^i)$$

is the function to be specified and estimated by maximum likelihood techniques.

As is well known, the assumption of mutual independence of the stochastic terms for different alternatives is dubious when those alternatives can be grouped in some natural way. In the present case, it would seem that unobserved stochastic tastes for the scheduling alternatives would be correlated among nearby alternatives. For example, a commuter whose boss is unusually lax about lateness may have a positive stochastic term for all the alternatives which involve arriving late. It is possible, still using the computational algorithms for the logit model, to test for departures from the independence assumption (McFadden, Train, & Tye, 1977; McFadden, 1977); two such tests are described below, in order to assess the severity of the independence

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\* To the extent that such differences result from choices which are not independent of schedule choice, endogeneity bias will result. This is discussed further following the empirical estimates.

assumption in the present case.

### Specification Issues

The problem addressed here is that of scheduling trips to work on a given mode, so as to arrive at the place of work at time  $s$ , when congestion results in a travel-time curve  $t(s)$  facing the individual. Since peak-load pricing is not practiced on most urban transportation systems, the variation of cost with time-of-day can be neglected.<sup>1</sup> Thus, indirect utility depends on  $s$  through the utility function  $U$ , through the travel-time curve  $t(s)$ , and through the work-hours constraint (4).

How should the function  $W$  be empirically specified? The strongest influence on work trip scheduling is, of course, the official work hours. Define schedule delay  $SD(s)$  to be the difference between the chosen time of arrival and the official work start time. Arriving early ( $SD < 0$ ) is likely to involve some time wasted, or at least less productively used, and thereby to decrease utility. Arriving late ( $SD > 0$ ) has, for most workers, more severe repercussions. It is the trade-off between scheduling considerations, as represented by the variable  $SD$ , and travel time  $t$  which is crucial for studying the impact of scheduling behavior on congestion, and upon which the work here is focussed.

The coefficients of this trade-off, reflecting the desires of and

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<sup>1</sup>Automobile running costs vary relatively little with congestion over a wide range of speeds (Keeler and Small, 1977); furthermore, running costs are only part of the costs of auto commuting, especially to downtown areas where parking fees are high. Singapore has recently instituted a peak-period congestion pricing scheme in its central business district; it should prove most interesting to estimate models with cost variables on the data collected following that innovation.

constraints upon the commuter, must depend on such factors as family status, occupation, choice of transport mode, and the employer's policy toward work-hour flexibility. These are the variables which compose the characteristics vector  $\underline{S}$ . Of course, the trade-off depends as well on unobservable traits such as availability of a comfortable place to read the newspaper or to have breakfast, ability to work at home, strength of desire to spend time with the family, necessity to take family members to school or work, and so forth; all of these are subsumed in the stochastic terms  $\epsilon_s^i$ .

Finally, reported work-arrival times obtained from surveys are likely to be rounded off to convenient points such as the nearest quarter-hour. In order to explain them, therefore, it may be necessary to include variables which take on positive values for the most popular rounding points. We denote the effects of such variables by  $R(s)$ .

In summary, it is postulated that the "strict utility" function,  $W(\cdot)$  of equation (13), can be written as follows:

$$(14) \quad W = R(s) + \mu(\underline{S})t(s) + f(SD(s), \underline{S}) .$$

The specification of the functions  $R$ ,  $\mu$ , and  $f$  are dealt with below.

### Data

The sample consists of 527 auto commuters in the San Francisco Bay Area.<sup>1</sup> All report some official work start time, and all report a regular

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<sup>1</sup>A subsample of the "TDFP Pre-BART" sample compiled by the Urban Travel Demand Forecasting Project, Institute of Transportation Studies, University of California, Berkeley, under the direction of Daniel McFadden. The travel time data was calculated using a road network maintained by the Metropolitan Transportation Commission, supplemented by floating-car observations of travel time over the peak periods on major expressways: see Faris, Reid and Small (1976). Some preliminary work on models of trip timing using this sample has been done by S. Cosslett and D. McFadden (McFadden et al., 1977).

time of arrival between  $42\frac{1}{2}$  minutes early and  $17\frac{1}{2}$  minutes late. These times are grouped into twelve five-minute intervals, and each commuter is assumed to select his or her arrival time from one of these twelve alternatives, which are indexed by  $s=1, \dots, 12$ . A number of socioeconomic and transportation-mode variables are also available, including the answer to the question, "How many minutes late can you arrive at work without it mattering very much?"

As the frequency distribution in Table 1 demonstrates, nearly two-thirds of the sample report arriving at work regularly at some time outside the five-minute interval centered around their official work start time; over one-third report having some flexibility for arriving late, and about four percent actually do arrive late. Many of the sample members face a perfectly flat travel-time curve, but a substantial fraction (consisting mainly of those crossing the San Francisco Bay Bridge) face differentials amounting to a substantial fraction of their total commuting time.<sup>1</sup>

The use of discrete five-minute intervals to describe the scheduling possibilities nicely eliminates the problem caused by the survey respondents' tendency to round off their answers to the nearest five minutes. However, preliminary investigation suggested that they also round off to either 10 or 15 minutes. In order to account for this, two "reporting error" variables were defined as in Table 2, footnote a, so that  $R(s)$  in equation (14) is assumed to be given by

$$(15) \quad R(s) = \beta_1 R_{15}(s) + \beta_2 R_{10}(s)$$

where  $\beta_1$  and  $\beta_2$  are coefficients to be estimated.

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<sup>1</sup>Segmenting the sample according to whether or not the Bay Bridge is crossed did not substantially alter the results.

Table 1  
 Distribution of Key Variables  
 San Francisco Bay Area Sample

Flexibility (minutes)	Chosen Schedule Delay			Total	%
	Early (1-8)	On Time (9)	Late (10-12)		
0	239	91	4	334	63.4
1-15	33	42	10	85	16.1
>15	46	54	8	108	20.5
Total	318	187	22	527	
%	60.3	35.5	4.2		100.0

Definitions:

Chosen schedule delay: difference between actual work arrival time and official work start time.

Flexibility: answer to question, "How many minutes late can you arrive at work without it mattering very much?"

## Results

The various models reported in Table 2 all differ in their specification of the (indirect) utility of travel time,  $\mu(S)$ , and of the scheduling considerations,  $f(SD(s), S)$ , in equation (14). Model (1) is a "bare-bones" specification, with  $\mu$  constant and with  $f$  given by

$$(16) \quad f(SD(s), S) = B_4 \cdot SDE + B_5 \cdot SDL + B_6 \cdot D1L,$$

with the variables as defined following Table 2. Allowing separate coefficients on the early- and late-arrival variables SDE and SDL takes into account the quite different disincentives involved; since penalties for late arrival are often strong, one expects to find  $B_5 < B_4 < 0$ . The "late dummy" D1L allows for a discrete jump in utility at the point at which a  $2\frac{1}{2}$ -minute margin of safety for arriving on time is exceeded; one expects  $B_6 < 0$ .

The coefficient estimates for Model (1) verify these expectations. The marginal rate of substitution between minutes of travel time and minutes of early arrival is 0.61. Late arrival is more onerous, with arrival outside the "margin of safety" equivalent to 5.5 minutes of travel time, plus 2.4 minutes for every minute late. This amounts to a penalty of 41.5 minutes of equivalent travel time for arriving 15 minutes late, which apparently explains why only one of the 527 commuters in the sample reports making this choice.<sup>1</sup>

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<sup>1</sup>The models reported in Table 2 were not sensitive to deletion of this individual from the sample, except for model (4a) which contains a more detailed specification of the late-side disutility. It should be mentioned here that these marginal rates of substitution involve congested travel time, since the component of travel time which does not vary according to time-of-day has no effect on these results. It is plausible, though unproven, that travel time is considered more onerous under congested than uncongested conditions; if that is true, the marginal rates of substitution given above are lower than those applicable to uncongested travel time.

Table 2. Work Trip Scheduling Models

Independent Variable <sup>a</sup>	Maximum Likelihood Coefficient Estimates (asymptotic standard errors in parentheses)							
	(1)	(2)	(3)	(4)	(5)	(4a)	(4b)	(4c)
Reporting Error:								
R15	1.09 (0.12)	1.07 (0.12)	0.99 (0.12)	1.05 (0.11)	0.93 (0.11)	1.10 (0.10)	1.06 (0.11)	1.10 (0.10)
R10	0.38 (0.12)	0.38 (0.12)	0.38 (0.13)	0.43 (0.11)	0.33 (0.12)	0.44 (0.12)	0.41 (0.11)	0.40 (0.10)
Travel time:								
TIM	-.106 (.038)	-.038 (.042)	-.142 (.061)	-.147 (.060)	-.056 (.070)	-.143 (.060)	-.134 (.054)	-.093 (.039)
TIM·SGL		-.272 (.107)			-.201 (.117)			
TIM·CP			.097 (.079)	.096 (.080)	.052 (.085)	.097 (.079)	.092 (.078)	
Early arrival:								
SDE	-.065 (.007)	-.063 (.007)	-.072 (.009)	-.076 (.007)	-.067 (.008)	-.083 (.018)	-.065 (.009)	-.067 (.005)
SDE·SGL		-.022 (.012)			-.013 (.012)			
SDE·CP			.022 (.009)	.022 (.009)	.022 (.009)	.022 (.009)	.020 (.009)	
SDE·SDE						-.0002 (.0005)		
Late arrival:								
SDL	-.254 (.030)	-.530 (.184)	-.363 (.076)	-.156 (.029)	-.443 (.184)	-.002 (.093)	-.138 (.031)	-.175 (.029)
SDL·WC		.290 (.184)			.270 (.184)			
SDL·FL			.168 (.080)					
SDLX				-.206 (.080)	-.171 (.081)	-.234 (.088)	-.190 (.079)	-.217 (.081)
SDL·SDL						-.0136 (.0081)		
D1L	-0.58 (0.21)	-1.11 (0.26)	-1.05 (0.24)					
D1L·WC		0.78 (0.20)			0.62 (0.19)			
D1L·FL			1.32 (0.21)					
D2L				-1.14 (0.18)	-1.29 (0.19)	-1.21 (0.20)	-1.19 (0.19)	-1.03 (0.17)
WNEAR							-.311 (.179)	
Sample Size	527	522	453	453	448	453	453	527
Lik.Rat.Ind. <sup>b</sup>	.218	.237	.227	.227	.242	.228	.228	.238
Re-Estimate on Common Sample <sup>f</sup> Lik.Rat.Ind. <sup>b</sup>	.206	.221	.233	.233	.242			
Log-Likelihood <sup>d</sup>	-833.6	-867.3	-853.8	-853.8	-843.7			



## Footnotes to Table 2

<sup>a</sup>Definition of independent variables:

SD = Schedule Delay: actual arrival time minus official work start time, in minutes rounded to nearest 5

RPTR15 =  $\begin{cases} 1 & \text{if SD} = -30, -15, 0, 15 \\ 0 & \text{otherwise} \end{cases}$

RPTR10 =  $\begin{cases} 1 & \text{if SD} = -30, -20, -10, 0, 10 \\ 0 & \text{otherwise} \end{cases}$

TIM = Travel Time in minutes =  $t(s)$

SDE = Min.  $\{-SD, 0\}$

SDL = Min.  $\{SD, 0\}$

D1L =  $\begin{cases} 1 & \text{if SD} \geq 0 \\ 0 & \text{otherwise} \end{cases}$

FLEX = Reported flexibility for arriving late, in minutes

D2L =  $\begin{cases} 1 & \text{if SD} \geq \text{FLEX} \\ 0 & \text{otherwise} \end{cases}$

SGL = dummy for one-person household

CP = dummy for carpool

WC = dummy for white collar

FL = dummy for FLEX > 0

SDLX = Min.  $\{SD - \text{FLEX}, 0\}$

WNEAR = average utility of 4 nearby alternatives, using model(4) coefficients

<sup>b</sup>Likelihood Ratio Index ( $\rho^2$ ) is defined as  $[L(\hat{\beta}) - L(0)] / |L(0)|$  where  $L(\hat{\beta})$  and  $L(0)$  are the log-likelihood with coefficients equal to their estimated values and to zero, respectively. See McFadden (1973). Not comparable between samples of different sizes.

<sup>c</sup>Sample size = 448, log-likelihood with all coefficients zero = - 1113.

<sup>d</sup>With coefficients equal to their estimated values.

We now come to the question of the determinants of scheduling flexibility. It should be possible to find subgroups for which these rates of substitution differ substantially. Although Chapin (1974) has demonstrated the importance of several demographic variables on time budgets, the literature provides little guidance concerning the scheduling decision specifically. In the present study, four variables are used, each of which might in principle interact with travel time or with any of the scheduling variables in equation (16). In order to reduce the number of interactions, I argue below that some of these variables (family status and transportation mode) affect mainly the desirability of early arrival and travel time, whereas others (occupation and work-hour flexibility) affect mainly the work-hour constraints and therefore involve late arrival. Experimentation confirmed that these were the most important interactions.

(a) Family Status is represented by a dummy variable SGL equal to one if the commuter resides in a one-person household. It is hypothesized that, because of fewer family demands, single workers are more flexible in their preferences with regard to early arrival; they may also evaluate travel time differently. Thus, SGL is interacted with SDE and TIM, with a positive sign anticipated in the former case (arriving early is less onerous for single workers). As seen from Models (2) and (5) of Table 2, this expectation is not confirmed, the coefficient of  $SDE \cdot SGL$  being negative and just short of significance at a 5% level. Furthermore, the coefficient on  $TIM \cdot SGL$  is very large, whereas that on TIM drops to a statistically insignificant level, suggesting that commuters living in households with more than one member place very

little value on travel time. The standard errors are sufficiently large to cast some doubt on this, and I conclude only that there are significant differences between single and other workers which could stand further exploration.

(b) Transportation mode is represented by a dummy variable CP equal to one for commuters who regularly carpool. The necessity to match schedules with other riders should make carpoolers more likely to arrive early for work. In addition, travel time is probably less onerous. These hypotheses are confirmed by Models (3), (4), and (5), though in the case of travel time the interaction coefficients are not statistically significant.

(c) Occupation is represented by a dummy variable WC equal to one for white-collar workers. On the hypothesis that white collar workers are less averse to late arrival, WC is interacted with SDL and DLL, with positive signs expected. This is confirmed by Models (2) and (5), in which the interaction coefficients are significant at a 10% level or better (one-tailed test).

(d) Reported work-hour flexibility is available from the question, asked of all respondents, reproduced in Table 1. Like occupational status, the variable FLEX which is the answer to this question should reflect constraints on late arrival. In Model (3), a dummy variable FL, indicating whether or not FLEX is greater than zero, is interacted with the late-arrival variables. The resulting positive coefficients confirm anticipations that late arrival is less onerous for workers who

report some flexibility. Models (4) and (5) employ a more concise specification, in which the dummy variable D2L incorporates a revised "margin-of-safety" hypothesis by picking out those arrival times not within  $2\frac{1}{2}$  minutes of the limit reported for late-side flexibility; and in which the variable SDLX measures the number of minutes beyond that limit. The coefficients of these variables achieve the expected signs with a high degree of statistical significance. To clarify the assumptions behind these alternative specifications, the function  $-(1/\mu)f(\text{SD}(s), \underline{S})$  is plotted for non-carpoolers in Figures 1 and 2, from the results of Models (3) and (4), respectively.

With each of these four proxies, the assumption that only those interactions discussed above are important was tested. In every case the applicable coefficient was statistically insignificant at a 10% level (2-tailed test) and in most cases was smaller than its standard error.

The models presented in Table 2 group these four proxy variables into the socioeconomic indicators used in Models (2) and (5), and the behavioral indicators included in Models (3), (4) and (5). There is reason to suspect that the latter variables are not truly exogenous, since car occupancy and reported work-hour flexibility may both be influenced by the actual time-of-day decision.<sup>1</sup> This raises the possibility, not testable within the present framework, of endogeneity bias in the coefficients of Models (3) through (5). On the other hand, these models, particularly (4), are more appealing in their completeness, conciseness, and plausibility of results.

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<sup>1</sup>For psychological reasons well known to survey researchers, reported values of independent variables may be influenced by the choice itself because respondents try to justify their behavior post hoc. This concern is strengthened by the observation that, among the 193 respondents reporting some flexibility, not one chooses to arrive regularly at a time later than that indicated as being the latest time permitted.

Many tests of additional variables have been carried out.<sup>1</sup> One might hypothesize that each minute of early (late) arrival becomes more onerous the earlier (later) one already is. The results of including quadratic terms in schedule delay (Model 4a) confirm this on the late though not the early side. This result, unlike the others shown, is sensitive to segmentation of the sample and to specification of the rounding-error term  $R(s)$ . Other variables tried and found lacking in explanatory power include: a dummy for the on-time alternative; the ratio of travel time to its minimum value among all twelve alternatives; and travel time multiplied by the dummy D1L. Finally, interactions between the wage rate and other variables gave somewhat ambiguous results suggesting that scheduling considerations may become more important relative to travel time for high-wage workers.

#### Tests of the Logit Model

As discussed earlier, there is reason in the case to doubt the assumption of "independence from irrelevant alternatives" which characterizes the logit model, since the alternatives fall onto a natural scale from the earliest to the latest. Two tests have been designed to detect departures from this assumption.

The first test is based on the observation that, if nearby alternatives are closer substitutes than distant ones, the probability of choosing any particular alternative should be negatively and directly affected (that is, through the numerator as well as the denominator of equation (12)) by the

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<sup>1</sup>Further details are available from the author on request.

desirability of those nearby alternatives. Thus I have computed, for each alternative, the average over several adjacent alternatives of the last two terms in equation (14), using the coefficient estimates of Model (4). The hypothesis to be tested is that the presence of attractive nearby alternatives, as indicated by this variable (WNEAR), reduces the choice probability, causing a negative coefficient. Model 4(b) confirms this expectation, the coefficient being negative and significant at a 5 percent level using a one-tailed test. Although this verifies the anticipated violation of the independence assumption, the coefficients on other variables are not greatly affected, thus providing some evidence that the coefficient estimates are robust to this type of violation.

The second test makes use of a property of "nested logit" models. McFadden (1977) has shown that non-independence within subsets of alternatives can be tested by first estimating the parameters of the model on one or more of the subsets, then forming an "inclusive value"

$$(17) \quad \emptyset_K = \log \sum_{s \in K} \exp(W_s)$$

for each subset  $K$ , and finally estimating the choice among subsets by using a logit model with  $\emptyset$  as independent variable. A coefficient on  $\emptyset$  of less than unity indicates that the stochastic terms  $\epsilon_s^i$  are more closely correlated within than between subgroups. To implement this, the alternatives were grouped into early ( $s=1-8$ ), on-time ( $s=9$ ), and late ( $s=10-12$ ) subsets. As many of the parameters of Model (4) as were identified were then estimated on the subsample choosing among the early alternatives; the remaining parameters were estimated using the late alternatives. The logit estimate of choice among subsets then produced an estimate for the coefficient of  $\emptyset$  of 0.872,

with an asymptotic standard error of 0.078. The departure from unity is of the expected sign, and is of small but statistically significant magnitude (one-tailed test, 5 percent level). Once again, the impression is given that the true choice model does depart from the independence assumption, but not enough to cast doubt on the approximate validity of the logit model.

#### Summary of Results

Some marginal rates of substitution implied by the models described in this section are compared in Table 3 for selected population subgroups. A reasonable characterization of these results is that, on average, urban commuters will shift their schedules by one to two minutes toward the early side, or by one-fourth to one-half minute toward the late side, in order to save a minute of travel time. Furthermore, there is considerable variation depending on family status, occupation, transportation mode, and employer's policy toward work-hour flexibility. The import of these findings is the subject of the next section.

#### IV. Implications

It was suggested in the introduction that scheduling considerations have widespread ramifications. With the empirical results of Section III, we can better assess their importance, and indicate the kinds of analysis required to fully incorporate scheduling behavior into other areas of economics.

Table 3  
Marginal Rates of Substitution

Characteris- tic	Equa- tion	Minutes of Travel Time equivalent to:			
		one minute early arrival	one minute late arrival	one minute late arrival outside flex. range	arrival beyond flex. range
		$-\left(\frac{\partial \text{TIM}}{\partial \text{SDE}}\right)$	$-\left(\frac{\partial \text{TIM}}{\partial \text{SDL}}\right)$	$-\left(\frac{\partial \text{TIM}}{\partial \text{SDL}} + \frac{\partial \text{TIM}}{\partial \text{SDLX}}\right)$	$-\left(\frac{\partial \text{TIM}}{\partial \text{D2L}} + \frac{\partial \text{TIM}}{\partial \text{D1L}}\right)$
All	(4c)	.72	1.9	4.2	11.1
Drive alone	(4)	.52	1.1	2.5	7.8
Carpool	(4)	1.06	3.1	7.1	22.4
Drive alone, household with two or more members:					
Blue collar	(5)	1.20	7.9	11.0	23.0
White collar	(5)	1.20	3.1	6.1	12.0
Drive alone, household with one member:					
Blue collar	(5)	0.31	1.7	2.4	5.0
White collar	(5)	0.31	0.7	1.3	2.6



### Value of Time

One implication of scheduling considerations is that, when a consumption activity is subject to peaking, the value of a time saving depends on when it occurs. For example, consider a work trip consisting of a "line-haul" portion on a congested freeway with travel time curve  $t(s)$ , plus "access" and "egress" portions involving fixed amounts of travel time. It can be shown that the value of a minute of time saved on the "egress" portion differs from that on the "access" portion by a factor  $(1-t')$ . This is because the access-time saving can be taken directly in increased leisure, and is therefore evaluated at the goods-leisure tradeoff, whereas a unit reduction of egress time permits more or less than one unit of additional leisure, depending on the sign of  $t'$ .

A perusal of the San Francisco Bay area data used in Section III reveals that many commuters face travel-time slopes of absolute value  $|t'| = 0.25$  (minutes travel time per minute time-of-day), and it is not uncommon to find slopes as large as 0.5. Thus, this phenomenon could exert a significant influence on measurements of value of time, and may account for some of the discrepancies in the literature on that subject.

### Transportation Mode Choice

As mentioned earlier, models of mode choice assume that the travel times for the various modes are exogenous. Faris, Reid, and Small (1976) have investigated the effects on mode choice models of imprecise representation of the variation of travel time with time-of-day; their findings suggest that total neglect of time-of-day variation produces substantial bias, but that rough approximations work quite well. One might expect, then, that treating travel time as endogenous, due to the possibilities of altering

schedules, would have a rather small effect on the results of mode choice models. A natural way to investigate this question would be through simultaneous models of mode and schedule choice.

### Forecasting

The most important result of the empirical work presented here is that the marginal rates of substitution between travel time and schedule delay for many individuals, particularly for single white-collar workers, are comparable in size to frequently-encountered values of  $|t'(s)|$ , the slope of the travel time curve. It is therefore likely that many commuters are currently travelling at other than their preferred times of day in order to avoid traffic congestion. This suggests that the concerns expressed at the beginning of this paper are indeed of quantitative significance: a sharp reduction in congestion can be expected to induce considerable shifting of travel schedules. This affects both the resulting equilibrium and the benefits from any proposed policy. The computation of these effects requires combining a scheduling model, such as has been presented in this paper, with a simulation model of considerable sophistication for the transportation facility.

### V. Conclusion

This paper has demonstrated that the scheduling of activities by consumers can be explicitly modelled in a theoretically satisfactory and empirically productive way. Even in the case of urban work trips, probably one of the most tightly constrained of everyday activities, schedule shifting has been found to be of quantitative importance for the understanding of urban transportation systems. Considerable effort will be required to fully assess the

implications of this type of behavior for such important areas of transportation analysis as demand studies, value-of-time measurement, policy simulation, and cost-benefit analysis. Meanwhile, it seems likely that this approach can be productively applied to other goods subject to peaked demands, including electricity, telephone service, computer services, and products with seasonal demands. In short, it has been argued here that much of economic analysis could be profitably modified to include scheduling behavior.

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Figure 1

Disutility of Schedule Delay: Model (3)

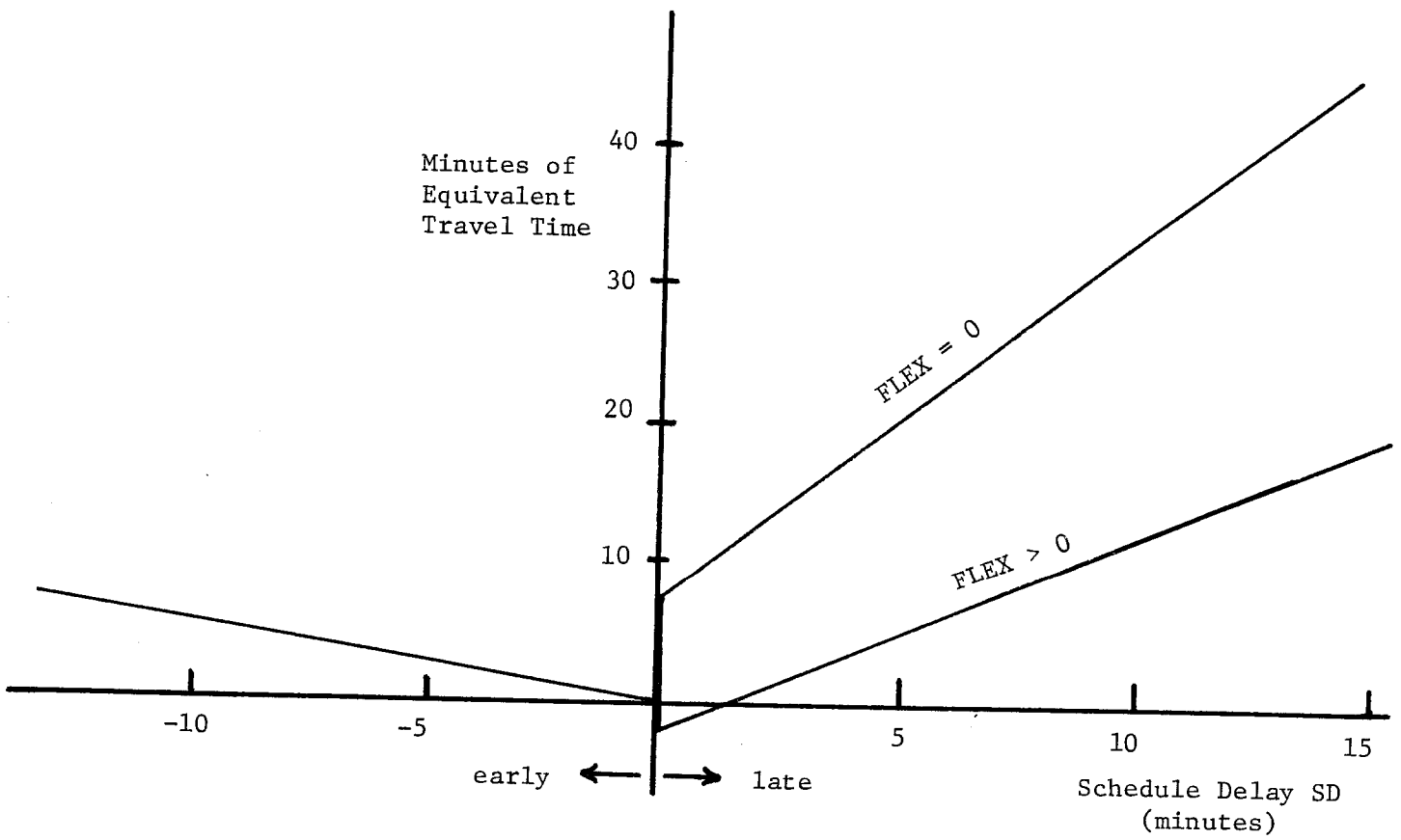


Figure 2

Disutility of Schedule Delay: Model (4)

