MONEY, INFLATION AND ECONOMIC GROWTH: CONDITIONS

FOR SUPERNEUTRALITY IN A GENERAL OPTIMIZING FRAMEWORK

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This paper addresses an issue that has dominated the literature on growth in a monetary economy -- the effect of the rate of monetary expansion on steady-state consumption per capita and income distribution. This issue divides naturally into two questions. In the first place, does money have any real effects at all? This question is usually phrased in terms of the superneutrality[1] of money. Secondly, if money is not superneutral, are its effects on real variables nonetheless predictable?

The early contributions to the literature,[2] based on descriptive (non-optimizing) analysis, are unified in their rejection of superneutrality. Their rejection stems from the argument that money and capital are substitutes in the eyes of portfolio holders. A change in the rate of monetary expansion changes the rate of inflation and thus the rate of return on real balances. This sets income and substitution effects into motion which alter the real equilibrium of the system. These contributions are less unified in their conclusions on the predictability of these real effects: the source of the ambiguity being the relative sizes of the income and substitution effects.[3]

These analyses have been justifiably criticized on the grounds that the portfolio behaviour they represent is not, in general, derivable from rational utility-maximizing foundations. An alternative approach, based on explicit optimization, was first formulated by Sidrauski (1967a). His typical agent is an "immortal family" whose size grows at a constant rate. The family maximizes the present value of its intertemporal stream of utility per head where utility is an increasing function of consumption and real balances. In this model, the steady-state capital intensity converges to the modified golden rule level and is invariant to changes in the rate of monetary expansion.

In contradicting such a widely accepted result, Sidrauski re-opened interest in an issue of central importance to monetary theory and policy.[4] The restrictive nature of his assumptions, however, led several economists to question the generality of his result. In their survey of recent developments in monetary theory, Barro and Fischer (1976) suggest that super-neutrality may be dependent upon the assumption of an infinite-lived household with a fixed discount rate. Their speculation gains support from Drazen (1976) who shows that superneutrality is not, in general, a property of models with money and capital assets in which individuals have finite horizons. Dornbusch and Frenkel (1973), on the other hand, suggest by counter-example that superneutrality is a consequence of the assumption that the yield on capital is independent of the real quantity of money. None of these authors indicate whether they are concerned with necessary or sufficient conditions, nor do they distinguish between conditions for superneutrality and non-superneutrality.

The optimal control framework in which Sidrauski's model is cast does not lend itself readily to alternative assumptions. This makes it difficult to identify which of his assumptions are necessary and/or sufficient for superneutrality. In this paper, I approach the problem within the framework of the overlapping-generations model. Inclusion of a bequest motive enables us to replicate Sidrauski's analysis while retaining the flexibility to relax each of his assumptions.

The overlapping-generations equivalent of Sidrauski's immortal-household model is presented in Section I. In this section I show that, of the assumptions that he makes, the following five are sufficient for super-neutrality:

(i) Each individual has an infinite optimization horizon.

(ii)   Each individual has an intertemporally additively separable
       utility function with a constant discount rate.

(iii)  All individuals are identical.

(iv)   Money is an argument in the utility function.

(v)    The private[5] rate of return on capital depends upon the capital
       intensity only.


While these conditions are sufficient for superneutrality in Sidrauski's
model, they need not be necessary.  Of particular interest is the possibility
that plausible alternative formulations of the optimization problem are
capable of generating superneutrality under assumptions less restrictive
than (i)-(v) above.  This issue is taken up in Section II.  The model is
extended to include a wider range of assets, and an alternative treatment
of money.  These extensions suggest that of the above assumptions, only (v)
and a weaker version of (iv), namely, that the private rate of return on
money depends upon the level of real balances held, need to be regarded as
necessary conditions for superneutrality in the more general class of opti-
mizing models.  By implication, the absence of either one will be suffi-
cient for non-superneutrality.  The case for superneutrality is not strength-
ened, however, because the additional assumptions needed under these exten-
sions are as restrictive as the ones they replace.  Section III makes some
brief comments on non-superneutrality and the predictability of the rela-
tionship between the rate of monetary expansion and the steady-state capital
intensity.

## I. SUFFICIENT CONDITIONS FOR SUPERNEUTRALITY IN AN OVERLAPPING-GENERATIONS EQUIVALENT OF SIDRAUSKI'S MODEL

The purpose of this section is to isolate the key assumptions in Sidrauski's superneutrality conclusion by replicating his result in an overlapping-generations framework. The basic model is in the tradition of the two-period Samuelson (1958)-Diamond (1965) consumption-loan model with production. Some of the implications of introducing money to this model have been explored by Drazen (1976) and Starrett (1978).

### I(a) The Structure of the Model

The economy consists of overlapping generations of identical individuals,[6] each of whom lives for two periods (denoted by the superscripts 1 and 2). Saving takes the form of accumulating real money balances or capital. This allows individuals to allocate consumption intertemporally given the constraint that they work in the first period and retire in the second. Each individual cares about the welfare of his descendant and can influence the descendant's behaviour through bequests. Firms produce a single, non-depreciating[7] output according to a constant-returns-to-scale production function with labor and capital as inputs. The economy is fully decentralized and all markets are competitive.

In each period $t$, the government has on issue a stock of nominal money balances, $M_t$. It expands the stock according to the rule

$$M_t = (1 + \theta)M_{t-1}$$

where $\theta$ is the chosen rate of monetary expansion. I will assume that the new issue of money in each period, $\theta M_{t-1}$, is paid as a transfer to the young generation.[8] Real per capita money balances are defined as

$m_t \equiv M_t / P_t L_t$ where $P_t$ is the absolute price level, and $L_t$ is the labor force.

It will be useful at this point to list the main assumptions made by Sidrauski in his analysis of superneutrality.

(i)   There are a finite number of infinite-lived individuals.

(ii)   The individual's utility function is additively separable with a constant discount rate.

(iii)   All individuals are identical.

(iv)   Money yields utility directly.

(v)   The private rate of return on capital depends upon the capital intensity only.

(vi)   The labor supply is perfectly inelastic.

(vii)   The individual knows the production technology.

(viii)   The individual has adaptive expectations about inflation.

## I(b) The Individual's Choice Problem

The individual's concern for his descendant is expressed through a bequest motive. This assumption replaces (i) above and is modeled by including the maximum utility attainable by the descendant, $U_{t+1}^*$ , as an argument in the individual's utility function. His utility function will thus be defined over first- and second-period consumption, $c^1$ and $c^2$ respectively, real balances, $s$ , and $U_{t+1}^*$ . The counterpart of (ii) is that the utility function is additively separable with respect to the descendant's utility and has a constant interpersonal discount rate, $\delta$ .[9]

Using (iv), the individual's utility function can be written as

(1) $$U_t = V(c_t^1, c_t^2, s_t) + U_{t+1}^*/(1 + \delta)$$

where $V(\cdot)$ is assumed to be strictly quasi concave, and to satisfy the Inada conditions. Henceforth, I will drop the time subscript whenever there is no ambiguity.

In the first period of his life, the individual supplies, inelastically, one unit of labor. He receives as income, a real wage, $w$, and a monetary transfer payment from the government. Let $h$ denote the real value of transfers.[10] The individual divides his total income between consumption, purchases of capital, and holdings of real balances. Second-period consumption and bequests are financed from savings and bequests received from his forebearer, where rates of return are assumed to be perfectly foreseen.[11] Thus, he faces the following budget constraint:

(2) $$c^1 + a + s \leq v$$

$$c^2 + (1 + n)b_t \leq (1 + r_{t+1}^k)(a + b_{t-1}) + (1 + r_{t+1}^m)s$$

where $v = w + h$ is total real first-period income, $a$ is his holdings of capital, $(1 + r_{t+1}^k)$ is the (perfectly foreseen) real market yield on capital, $(1 + r_{t+1}^m)$ is the (perfectly foreseen) real market yield on real balances, $b_t$ is the per capita bequest that he leaves to his descendant, and $b_{t-1}$ is the bequest that he receives from his forebearer. The analysis is simplified greatly, without altering any of the results, if we assume that bequests are made entirely in the form of capital.

The first-order conditions for a maximum of (1) subject to (2) are[12]

(3) $$V_1/V_2 \geq (1 + r_{t+1}^k) \quad , \quad \text{if} > \text{ then } a = 0$$

(4) $\qquad V_1/V_2 = (1 + r^m_{t+1}) + V_3/V_2$

(5) $\qquad (1 + n)V_2 \gtreqqless \partial u_t/\partial b_t \qquad\qquad$ , if $>$ then $b = 0$

where $V_{i,t+j}$ is the general notation for the partial derivative of $V(\cdot)$ with respect to its $i^{th}$ argument for a member of generation $t+j$ .

The interpretation of these conditions is straightforward. Accumulation of money and capital are the two available mechanisms through which the individual can transform first-period consumption into second-period consumption. Each asset will each be accumulated until the marginal rate of substituting present for future consumption (that is $V_1/V_2$ ) is equal to the marginal rate of transforming present into future consumption. The marginal rate of transformation under money includes the utility yield as well as the market yield. A bequest involves a sacrifice of second-period consumption. Equation (5) states that the utility from giving a bequest must equal the consumption utility foregone (with the units adjusted for the rate of population growth).

A unit of bequests provides the descendant with $(1 + r^k_{t+2})\,db_t$ of extra endowment in the second period of his life. He can consume it as $c^2_{t+1}$ , consume it as $c^1_{t+1}$ (provided that he anticipates the bequest), leave it as a bequest, $b_{t+1}$ , to his descendant, or some combination of the three. If the descendant is at a utility maximum then he will be indifferent between any of these actions that are already at non-zero levels. As a result, we can use his budget constraint to measure his utility from a bequest as

$$\partial U^*_{t+1}/\partial b_t = (1 + r^k_{t+2})V_{2,t+1}$$

This enables us to express (5) in the convenient form

$(5^1)$
$$1 + r^k_{t+2} \leq (1 + n)(1 + \delta)v_{2,t}/v_{2,t+1}$$

## I(c) Equilibrium in the Aggregate Economy

The aggregate economy is assumed to consist of overlapping genera-tions of identical individuals. In each period there are two generations living: $L_t$ young individuals born at the beginning of period $t$, and $L_{t-1}$ old individuals born at the beginning of period $t-1$. Population grows at the constant rate $n$, so that

$$L_t = L_0 (1 + n)^t$$

Technology is described by a linear homogenous production function in capital and labor. This can be written in per capita terms as[13]

$$y_t = f(k_t) \qquad f' > 0, \ f'' < 0, \ f(0) = 0$$

where $y$ is per capita output, $k$ is the capital intensity, and $f(\cdot)$ satisfies the Inada conditions. Markets are assumed to be competitive so that each factor of production receives the value of its marginal product

(6)
$$w_t = f(k_t) - k_t f'(k_t) \qquad \text{and}$$

(7)
$$r^k_t = f'(k_t)$$

Under the assumption that capital accumulated in one period becomes available for production in the next period, the general equilibrium of the system in period $t$ is described by the following conditions for equilibrium in the capital and money markets

(8)
$$f'([a(v_t, \ r^m_{t+1}, \ r^k_{t+1}) + b(v_t, \ r^m_{t+1}, \ r^k_{t+1})]/(1 + n)) = r^k_{t+1}$$

(9)
$$s(v_t, r^m_{t+1}, r^k_{t+1}) = m_t$$

where $a(\cdot)$ , $b(\cdot)$ and $s(\cdot)$ are the demand functions found by solving (3), (4) and (5') and $r^m_{t+1} = - (P_{t+1} - P_t)/P_{t+1}$ . These market equilibrium conditions solve for $r^k_{t+1}$ and $P_t$ , which in turn generate values for $k_{t+1}$ and $m_t$ . The equilibrium path of the economy can then be described by a difference equation system in either $r$ and $P$ , or $k$ and $m$ .

## I(d) The Steady-state Properties of the Economy

The question of superneutrality involves the comparative dynamics properties of the steady state of the system described above. In the steady state, the following relationships hold:

(10)
$$v(k,m,\theta) = w(k) + h(\theta,m)$$

(11)
$$h(\theta,m) = \theta m/(1 + \theta)$$

(12)
$$w(k) = f(k) - kf'(k)$$

(13)
$$1 + r^k(k) = 1 + f'(k)$$

(14)
$$1 + r^m(\theta) = (1 + n)/(1 + \theta)$$

where (14) is derived from the steady-state requirement that real balances per head be constant. The steady-state market equilibrium conditions are

(15)
$$a + b = (1 + n)k$$

(16)
$$s = m$$

Substituting equations (10)-(16) into the budget constraint (2) defines the steady-state consumption levels.

(17)　　$c^1(k,m,b,\theta) = w(k) - (1 + n)k + b - m/((1 + \theta)$

$c^2(k,m,b,\theta) = (1 + n)(1 + r^k(k))k - (1 + n)b + (1 + n)m/(1 + \theta)$

The steady-state properties of the system are contained in the steady-state marginal conditions obtained by substituting (10)-(17) into (3)-(5).

(18)　　　　　$\dfrac{\hat{V}_1(k,m,b,\theta\ )}{\hat{V}_2(k,m,b,\theta)} = 1 + r^k(k)$

(19)　　　　　$\dfrac{\hat{V}_1(k,m,b,\theta)}{\hat{V}_2(k,m,b,\theta)} = \dfrac{(1 + n)}{(1 + \theta)} + \dfrac{\hat{V}_3(k,m,b,\theta)}{\hat{V}_2(k,m,b,\theta)}$

(20)　　　　　$(1 + n)(1 + \delta) \geq 1 + r^k(k)$

where the equality in (18) follows from the restrictions on $f(\cdot)$ , and $\hat{V}_i(k,m,b,\theta) \equiv V_i(c^1(k,m,b,\theta)\ ,\ c^2(k,m,b,\theta),m)$  for $i = 1$ , 2 and 3.

To replicate Sidrauski's model, we need to assume an interior solution with respect to bequests so that (20) holds with equality. Given this assumption, (20) determines the capital intensity at the modified golden rule level. Equations (18) and (19) solve for steady-state real balances and bequests, given the golden rule capital intensity and the monetary growth rate. In this system, money is superneutral.

We can describe the generation of superneutrality in the following way. Steady-state equilibrium requires equality between the marginal rate of substituting $c^2$ for $c^1$ and the marginal rate of transforming $c^1$ into $c^2$ under each of the three transformation schemes: capital, money, and bequests.[15] One of these transformation rates -- that on capital -- is a function of the steady-state capital intensity

only. If either the marginal rate of substitution or one of the other transformation rates is independent of steady-state real balances, bequests, and the monetary expansion rate, superneutrality will result. The monetary transformation rate is ruled out as a possibility because $r^m$ is necessarily dependent on $\theta$.[16]

## I(e) Superneutrality

The superneutrality conditions are met in Sidrauski's model because the marginal rate of substitution is an exogenous steady-state constant. In the overlapping-generations model above, superneutrality follows from the exogenous constancy of the steady-state marginal rate of transforming $c^1$ into $c^2$ through bequests. The latter can be interpreted as the finite-lived analogue of the former. In the first place,(5') suggests that the transformation rate under bequests can also be interpreted as a marginal rate of substitution -- an intergenerational substitution rate of $c^2_{t+1}$ for $c^2_t$. From the optimality conditions, this transformation rate can also be translated into a substitution rate of $c^1_{t+1}$ for $c^1_t$. Secondly, in the infinite-lived model, steady-state constancy of the capital intensity implies constancy of per capita consumption for each individual. In the finite-lived model, constancy of the capital intensity implies constancy of the life-time consumption pattern from one generation to the next. Stationarity between periods with infinite lives is equivalent to stationarity between generations with finite lives. Thus, assumptions about intertemporal substitutability in the former have a natural parallel in assumptions about intergenerational substitutability in the latter. In particular, the same set of assumptions will be required for each of the substitution rates to be an exogenous constant in

the steady state. These assumptions are contained in the following proposition.

PROPOSITION: In the model described by (18)-(20), the following conditions are sufficient for neutrality:

(i)   Individuals have an (effectively) infinite horizon.

(ii)  Each individual has an additively separable utility function with a constant discount rate.[17]

(iii). All individuals are identical.

(iv)  Money is an argument in the utility function.

(v)   The private rate of return on capital depends upon the capital intensity only.

To prove this proposition consider relaxing these assumptions one at a time. Relaxing (i) is equivalent to assuming that the individual is at a corner solution with respect to bequests. In this case, (18) and (19) solve for $k$ and $m$, given $\theta$. As both equations involve all three variables, non-superneutrality will result. If we relax (ii), we must allow utility functions of the general form

$$U_t = V(c^1, c^2, s, U^*_{t+1})$$

In this case, (20) becomes

(20′)                         $(1 + n)\hat{V}_4(k, m, b, \theta) \geq 1 + r^k(k)$

The marginal rate of transformation under bequests is no longer independent of $m$, $b$ and $\theta$ and non-superneutrality will follow. If

assumption (iii) is relaxed, discount rates may vary between individuals. This implies that some individuals are necessarily at a zero-bequest corner solution. Changing the rate of monetary expansion may change the marginal individual who determines the corner. If assumption (iv) is relaxed, the transformation rate under money is just the market yield, $1 + r^m$, which is also a steady-state constant, given $\theta$. In this situation, unless $(1 + \delta)(1 + \theta) = 1$, either bequests or money will be at a corner solution. If money is at a corner solution then it is superneutral, but in a trivial sense; the model cannot be regarded as a plausible description of monetary behaviour if no-one holds money. If bequests are at a corner solution then the infinite horizon is lost and money is non-superneutral. The inclusion of money in the utility function allows both an infinite horizon and positive money holdings to co-exist in the steady state. Finally, if (v) does not hold, the constancy of the transformation rate under bequests is no longer sufficient for superneutrality. Assumption (v) excludes the following: long-run uncertainty about $r^k$, money in the production function, capital in the utility function, and production functions other than constant-returns-to-scale.[18]

## II. NECESSARY CONDITIONS FOR SUPERNEUTRALITY IN A GENERAL OPTIMIZING FRAMEWORK

In the preceding section, I attempted to isolate the key sources of superneutrality in Sidrauski's model. The factors suggested by Barro and Fischer, and Dornbusch and Frenkel were shown to be relevant but not sufficient by themselves for superneutrality. Their suggestions can be viewed more as attempts to identify some of the sufficient conditions for non-superneutrality. In this section, I will consider altering the specification

of the model in order to determine which, if any, of the five assumptions listed above are necessary for superneutrality in the general context of optimizing models.

A logical starting point in the search for general necessary conditions for superneutrality is with the first-order conditions for the individual to be at a utility maximum, (18)-(20). In a multi-period[19], multi-asset model, these marginal conditions will require equality between all marginal rates of substitution and transformation[20] of comparable time dimensions. The transformation rates will include market and non-market yields as perceived by the individual. We can write the steady-state representation of these conditions in the convenient form

$$(21) \qquad \qquad _\tau R^c_t(k,m,\dots,\theta) \geq \; _\tau R^a_t(k,m,\dots,\theta)$$

where $_\tau R^c_t$ is the intertemporal marginal rate of substitution, as perceived at time $t$, on any feasible commodity pair $c = \{c^\alpha_t, c^\beta_t\}$, where the commodities $c^\alpha$ and $c^\beta$ are separated in time by $\tau = \alpha - \beta$, and $_\tau R^a_t$ is the marginal rate of transforming $c^\alpha$ into $c^\beta$ by holding the asset, $a$, for the period $\tau$. The other elements in $R^c$ and $R^a$ will be the steady-state values of the other assets in the system.

In a general optimizing model, superneutrality requires the following three conditions:

(a) One of the steady-state substitution or transformation rates, $_\tau R^i_t$, must have the form $_\tau R^i_t(k,m,\dots,\theta) = \; _\tau R^i_t(k)$

(b) Another of the substitution or transformation rates, $_\tau R^j_t$, must have either one of the two forms $_\tau R^j_t(k,m,\dots,\theta) = \; _\tau R^j_t(k)$ or $_\tau R^j_t(k,m,\dots,\theta) = \; _\tau \bar{R}^j_t$, where $_\tau \bar{R}^j_t$ is a constant.

(c) The assets (or commodities), $i$ and $j$, as well as money must be held (consumed) in positive quantities in the steady state.

These conditions leave open a number of avenues for contriving a superneutrality result.[21] Common sense dictates that we restrict ourselves to economically plausible cases. There appear to be two extensions of Sidrauski's model that are worthy of investigation: expanding the range of assets, and specifying the role of money in alternate ways.

Consider the model of Section I with assumption (v) in force. Suppose that we introduce a new asset, $\hat{a}$. Provided that $\hat{a}$ does not enter the utility function (or affect marginal utilities indirectly through uncertainty), neutrality will follow if the real market yield on $\hat{a}$ is fixed. An external asset, such as foreign capital, could serve this function. The rate of return on the foreign capital would need to be determined entirely in the rest of the world. It would also have to be free of any long-run uncertainty about either the productive process or the exchange rate.[22] Superneutrality in these instances will be independent of assumptions (i), (ii) and (iii). Assumption (iv), (or the weaker version that follows below), is still needed if both money and the fixed-yield asset are to be held in private portfolios.[23]

One of the more controversial assumptions in Sidrauski's analysis is his treatment of money as yielding utility directly. The fact that this assumption is an essential link in his superneutrality argument is all the more reason to investigate alternative specifications of monetary behaviour.

There are two main schools of thought on how to model the roles of money. The neoclassical approach[24] includes money directly into the

utility function on the grounds that money yields a flow of non-pecuniary transactions and liquidity services. The alternative inventory-theoretic approach[25] attempts to model these transactions costs explicitly. The importance of including money in the utility function in Sidrauski's model is that it provides a link between the private yield on money and the quantity of money held, thus ensuring that money is held as an asset. The question is whether or not the inventory-theoretic approach also provides this link. The answer is "yes" although the relationship is considerably more complex. The essence of the inventory approach can be stated quite simply. For a given transactions technology, the individual will hold the quantities of money, earning asset, and commodity inventories that minimize the cost of achieving a desired pattern of consumption within each period. A change in the monetary expansion rate changes the structure of transactions costs which changes both the cost-minimizing level of real balances and the implicit private yield on those balances ($r^m$ plus transactions costs saved). For reasonable transactions technologies, money will always be held in positive quantities. Thus, we can replace assumption (iv) by the weaker assumption that, over some range of money holdings, the private real rate of return on real balances varies with the quantity held. This could result from including money in the utility function or from transactions costs.

### III. NON-SUPERNEUTRALITY: THE RELATIONSHIP BETWEEN THE CAPITAL INTENSITY AND THE RATE OF MONETARY EXPANSION

The principal conclusion of the preceding two sections is that superneutrality requires excessively stringent restrictions on behaviour. If we are prepared to relax some of these assumptions and consider

non-superneutrality as the likely state of the world, can we say anything about the predictability of the real effects of changing the rate of monetary expansion?

The most general analysis of the relationship between the capital intensity and the monetary expansion rate in an explicit optimizing framework is that by Dornbusch and Frenkel. They use Sidrauski's model, in which the steady-state marginal rate of substitution is fixed, and generate non-superneutrality by arbitrarily including capital in the utility function and money in the production function. The latter of these violates assumption (v) so that money is non-superneutral. With the marginal rate of substitution fixed, the solution for $dk/d\theta$ is a mixture of the utility and production effects of money and capital on their private rates of return. Given the arbitrary way in which they treat money and capital, it is hardly surprising when they arrive at the superficially paradoxical conclusion that increasing the rate of monetary expansion decreases the capital stock.[26]

The analysis in Sections I and II suggests that Dornbusch and Frenkel have relaxed probably the least objectionable of the five assumptions needed for superneutrality in Sidrauski's model. Non-superneutrality is more likely to result from failure of one of the assumptions (i)-(iii) needed for the marginal rate of substitution to be an exogenous steady-state constant. With the marginal rate of substitution endogenous in the long run $dk/d\theta$ is ambiguous regardless of the roles assigned to money. Further, the ambiguity is not easily resolved by arbitrary assumptions about own- and cross-derivaties. The use of a well-specified optimization framework does not help us to untangle the general-equilibrium relationship between the rate of monetary expansion and the steady-state capital intensity.

## IV. CONCLUSION

The main purpose of this paper has been to establish that the long-standing superneutrality result due to Sidrauski is dependent on an unrealistically restrictive set of assumptions. Using the overlapping-generations framework with money, capital and bequests I showed that superneutrality requires:

(i)   An infinite optimization horizon.

(ii)  An additively separable utility function with a constant (interpersonal) discount rate.

(iii) Identical individuals;

(iv)  A utility yield on money; and

(v)   A private rate of return on capital that depends only on the capital intensity.

If a transactions cost approach to monetary behaviour is followed, the assumption that money yields utility directly can be replaced by the weaker assumption (iv) that the private yield on money depends on the quantity held. In a multi-asset setting superneutrality may result if assumptions (v) and either (iv) or (iv') hold, and there exists an asset on which the real rate of return is either exogenously fixed or depends on the capital intensity. This framework reinforces the argument that the relationship between the rate of monetary expansion and the capital intensity is ambiguous without specific numerical knowledge of the parameters of the economy.

## FOOTNOTES

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(1) In the general class of models considered by the literature, money is always neutral in the sense that a once-for-all change in the level of the money stock has no real effects. This concept of neutrality is distinguished from superneutrality which concerns the relationship between the real variables of the system and the rate of growth of the nominal money stock.

(2) See, for example, Tobin (1965), Johnson (1967), Sidrauski (1967b), and Levhari and Patinkin (1968).

(3) The importance and interpretation of this effect is brought out clearly by Dornbusch and Frenkel (1973).

(4) For example, superneutrality is implicit in the modern versions of the natural rate hypothesis as put forward by Friedman (1968). See the subsequent work by Sargent (1973) and the papers by Lucas that he refers to.

(5) The term "private" is used to reflect all sources of yield perceived by the individual including the market yield, utility yield and so on.

(6) The basic consumption unit is the household but as the analysis does not distinguish between the household and its members, I will refer to it as though it consisted of a single individual.

(7)  Sidrauski includes a positive depreciation rate.  For the purpose of examining superneutrality, depreciation is an unnecessary complication that does not influence any of the results, and so is ignored.

(8)  All results in this chapter are unaltered by distributing the transfer to the old generation or some combination of the two generations.

(9)  An intuitive justification of this assertion will be given in Section I(e) below.

(10)  We rule out the possibility that the individual regards  h  as an interest payment on his money holdings.

(11)  The use of perfect foresight instead of Sidrauski's Assumptions (vii) and (viii) do not alter the superneutrality properties of the model.

(12)  The assumptions about  $V(\cdot)$  ensure that  $c^1$ ,  $c^2$ , and  s  are strictly positive and that the budget constraints hold with equality. This also ensures that equation (4) below holds with strict equality.

(13)  Throughout the paper, lower case letters will be used to denote per capita quantities.

(14)  Note that the definition of superneutrality as the relationship between  k  and  $\theta$  is quite narrow.  Monetary growth still affects the division of life-time consumption between the two periods.

(15)  Bequests are simply a disguised means of reallocating life-time consumption -- see equation (17).

(16)  Given the assumption that interest is not paid on money balances.

(17)  The reader is reminded that additive separability in Sidrauski's model relates to interpersonal consumption in the overlapping-generations model.

(18) As pointed out by Brock (1976), if the labor supply is chosen endogenously then assumption (v) requires linear homogeneity of the production function for superneutrality.

(19) Multi-period is defined here to mean that the individual lives for more than two periods.

(20) Transformation rates on assets not held by the individual will be less than the corresponding substitution rates.

(21) For example, suppose that assumption (v) holds and the utility function (1) is additively separable and linear in $c^1$ and $c^2$. In this case, the marginal rate of substitution, $v_1/v_2$, is a constant. This utility function, however, is not quasi-concave and does not present a convincing argument for superneutrality. I am grateful to Leo Simon for pointing out this particular example.

(22) The economy in this instance would be a perfect small country case and would behave according to the rules of the monetary approach to the balance of payments. For example, see Johnson (1973b).

(23) Another asset that could generate superneutrality is an indexed bond issued by the government and offering a fixed yield. In this model, in which money only performs an explicit asset function, indexed bonds are essentially equivalent to paying interest on money and so contravene the ground rules of the superneutrality debate.

(24) For example, see Friedman (1969), Johnson (1973a), Samuelson (1968), and Patinkin (1965).

(25) This approach follows Baumol (1952) and Tobin (1956).

(26) This conclusion requires own-derivates to be negative, cross-derivatives to be positive, and own-effects to dominate cross-effects.

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