

INVOLUNTARY UNEMPLOYMENT
AND THE CONSUMER'S BEHAVIOR

Henri Sneessens*

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Econometric Research Program
Princeton University
207 Dickinson Hall
Princeton, New Jersey

I. INTRODUCTION

Much effort has been devoted in the recent years to the development of rationing (or "disequilibrium") models. In this context, it is emphasized that a constraint on a particular market affects the behavior of the economic agent on other markets. A typical example of this interaction is given by the aggregate consumption function, rewritten in the linearized form:

$$CO = CO^W + s (\overline{HWO} - HWO^W) \quad (1.1)$$

where CO , CO^W are respectively the effective and Walrasian (or notional) aggregate demand for consumption goods, at constant prices¹.

\overline{HWO} , HWO^W are respectively the actual (constrained) and desired level of aggregate labor income at constant prices, satisfying $\overline{HWO} \leq HWO^W$.

s is the "spill-over" coefficient, defined by $s = \partial CO / \partial \overline{HWO}$.

The purpose of this paper is to measure both the short-run and the long-run value of the spill-over coefficient s . Many attempts have been made in the past, yet none of them has been able to make the distinction between the behavior of employed and unemployed consumers or to give a correct appraisal of dynamic phenomena. The specification adopted for the consumption function is usually very much like the traditional static

$$CO = m_0 + m_1 YDO \quad (1.2)$$

where YDO denotes aggregate disposable income at constant prices, which imposes that employed and unemployed workers behave in the same way. When terms in the unemployment rate are added, their coefficients usually appear not

significant or unacceptable. An overview of some of those researches is given in the following section. Their common characteristic is that they do not take full advantage of what we know from choice theory. Using a particular utility function including labor as an argument should enable us to specify from the start the behavioral differences between constrained and unconstrained consumers. It is the approach adopted in this paper, where use is made of the Stone-Geary utility function. This choice is justified by the very simple and powerful econometric formulation obtained by Lluch (1973) as a result of the intertemporal optimization of this type of utility function. Lluch's "Extended Linear Expenditure System" is presented in Section III. Section IV uses the same framework to derive aggregate static consumption demand and labor supply functions, taking account of the constraints observed on the labor market. The Keynesian linear consumption function obtains as a special case. In Section V, I further differentiate employed and unemployed workers, allowing for an effect of past constraints as well as of current ones. The rationale is that a worker who has been unable to find a job for two years will have a "permanent" constraint different from the one expected by a worker experiencing his first year of unemployment, and accordingly will also have a different consumption behavior. This effect is introduced through a simple lag polynomial function, under two alternative hypotheses. The empirical results reported in Section VI show how sensitive the estimates of the spill-over effect are to the precise dynamic specification adopted. The estimation has been performed on Belgian annual data, for the period 1954-1976. Taking account of the unemployment benefits granted by that country, the most reliable estimate of the long run (total) spill-over coefficient seems to be .7689, only half of which is realized during the first year, the rest needing a long time to occur. This figure is to be compared with the one obtained from the traditional linear consumption function,

.52 for both the short-run and the long-run. The precise meaning of those values is detailed in the same Section VI. Empirical estimates of the labor supply function are given as a by-product. Section VII concludes with a few remarks.

One remark is in order before turning to Section I. Throughout the paper, I shall maintain the following three assumptions:

- A1. The effective demand for consumption goods and the effective supply of labor are of the Clower type².
- A2. Every consumer believes his Walrasian demand for consumption goods, if it happened to be expressed on the market, would certainly be satisfied.
- A3. The effective demand for consumption goods is never rationed.

Without going into details, let us simply say that these assumptions avoid any ambiguity about the effective demand concept and ensure that C_0 is actually observed. Without them, both the specification and the estimation of the model developed in Sections IV - V would become much more complicated. A secondary consequence is that the aggregate effective labor supply, will always take the simple form:

$$H^W O = H^W O^W$$

II. THEORY AND ESTIMATION OF THE UNEMPLOYMENT EFFECT ON CONSUMPTION

There are two possible justifications for an individual consumption function of the form (1.2). Both will be briefly described in this section in order to examine how previous empirical studies fit in that framework.

More details will be provided in Sections III - IV where the same approach is used to specify a new consumption function. Clower (1965) elaborated the first justification, which Keynes was likely to have in mind when writing the "General Theory." Consider a representative household i and assume its behavior can be simulated by the maximization of the utility functional

$$\int_{\tau} e^{-\rho\tau} u_i(CO_{i\tau}, HL_{i\tau}) d\tau \quad (2.1)$$

under the budget constraint

$$\dot{A}_{i\tau} = R \cdot A_{i\tau} + W \cdot (HT - HL_{i\tau}) - P \cdot CO_{i\tau}$$

The following notation has been used:

- $A_{i\tau}$ - non-human wealth owned by individual i at time τ
- $\dot{A}_{i\tau}$ - time derivative of $A_{i\tau}$
- R - rate of interest on non-human wealth
- P - price of consumption goods
- W - nominal wage rate
- CO_i - real consumption
- HL_i - hours of leisure
- HT - total number of hours available for leisure and work, such that $HW_i = HT - HL_i$ is the number of working hours.

This program defines the effective consumption demand and labor supply as equal to their Walrasian (or notional) counterparts:

$$CO_i = CO_i^W, \quad HW_i = HW_i^W$$

where CO_i^W and HW_i^W are functions of prices and unearned income only.

Keynes, however, introduced total disposable income as an argument in the demand for consumption goods, considering that the household is actually

constrained on the labor market and is only able to sell a quantity of labor \overline{HW}_i smaller than the desired quantity HW_i . Under this assumption, the program of the household is to maximize the constrained utility functional:

$$\int_{\tau} e^{-\rho\tau} u_i(CO_{i\tau}) d\tau$$

subject to

$$\dot{A}_{i\tau} = R A_{i\tau} + W \overline{HW}_{i\tau} + r W (HW_{i\tau}^W - \overline{HW}_{i\tau}) - P CO_{i\tau}$$

where r is the rate of unemployment compensation³. This program defines the effective (linearized) demand as notional demand plus a term accounting for the constraint:

$$CO_i = CO_i^W + s (\overline{HWO}_i^P - HWO_i^W)$$

where \overline{HWO}_i^P is the perceived "permanent" constraint on labor income. Under a specific hypothesis on the form of u_i , it can be rewritten as a linear function of total permanent income:

$$CO_i = m_0 + m_1 \overline{YDO}_i^P \tag{2.2}$$

where the main component of \overline{YDO}_i^P is exogenous, as \overline{YDO}_i^P is defined by:

$$\overline{YDO}_i^P = R(A_i/P) + \overline{HWO}_i^P + r (HWO_i^W - \overline{HWO}_i^P)$$

I shall call this function the Keynesian individual consumption function.

It follows that the long-run spill-over coefficient corresponds to the Keynesian marginal propensity to consume corrected for the impact of unemployment benefits:

$$s_i = \partial CO_i / \partial \overline{HWO}_i^P = m_1 (1 - r)$$

Yet an individual consumption function of the form (1.2) can be given an

alternative interpretation. Provided (2.1) takes the additive form

$$u_i (CO_{iT}, HL_{iT}) = u_{1i} (CO_{iT}) + u_{2i} (HL_{iT}),$$

decisions on consumption and leisure will be separable. This means that the notional demand for the consumption goods can also be viewed as a function of total permanent income:

$$CO_i^W = m_0^* + m_1^* YDO_i^W \quad (2.3)$$

In this expression, unlike in the Keynesian one, permanent income is an endogenous variable defined by

$$YDO_i^W = R (A_i/P) + HWO_i^W$$

m_1^* can be called a marginal propensity to consume only by analogy with m_1 , as the source of variation in labor income will now be price or wage changes only. Moreover, as there is no constraint on the labor market, there is also no spill-over effect.

If we consider two classes of individuals only and accept the proposition that employed workers perceive no constraint on the labor market while unemployed workers do of course, the aggregate consumption function should then be a weighted average of both types of equations (2.2) - (2.3). The difficulty is usually escaped by taking

$$m_0 = m_0^*, \quad m_1 = m_1^*$$

Consumption becomes a linear function of constrained permanent aggregate income⁴:

$$CO = m_0 NA + m_1 \overline{YDO}^P$$

where $NA =$ available labor force

$$\overline{YDO}^P = R \cdot (A/P) + \overline{HWO}^P + r (\overline{HWO}^W - \overline{HWO}^P)$$

\overline{HWO}^P = aggregate constraint on permanent labor income.

A further simplification is to approximate permanent income by current income:

$$\overline{YDO}_t = RAO_t + \overline{HWO}_t + UBO_t$$

where RAO_t = current (as opposed to permanent) property income

\overline{HWO}_t = current wage bill

UBO_t = current unemployment benefits .

A typical empirical result is obtained from Belgian data:

$$CO_t = 24.39 NA_t + .74 YDO_t \quad (2.4)$$

With an average rate of unemployment compensation r equal to .29, this gives an average spill-over coefficient of .52; no distinction is possible between long-run and short-run spill-overs. ANDO-MODIGLIANI (1963) tried to obtain a better approximation to permanent income by using data on wealth and adding the effect of labor income expectations:

$$CO_t = \alpha_1 \overline{HWO}_t + \alpha_2 \overline{HWO}_t^e + \alpha_3 A_t$$

They assume that expectations of employed workers are almost equal to their current income \overline{HWO}_t / E_t , where E_t is the number of persons engaged in production. With respect to unemployed persons, however, "it seems reasonable to suppose that some of them would expect their current unemployment status to continue for some time and, possibly to recur." Accordingly, their expectations will be approximated by $k \overline{HWO}_t / E_t$, with $0 \leq k \leq 1$. The resulting equation is:

$$CO_t = \alpha'_1 \overline{HWO}_t + \alpha'_2 \frac{NA_t}{E_t} \overline{HWO}_t + \alpha_3 A_t$$

This is a crude way to introduce expectations and coefficient α'_2 never appeared to be significant. Much more recently, VANKEERBERGHEN (1978) also tried to introduce a specific effect of unemployment in the consumption function. She used the following specification:

$$CO_t = m_0^* E_t + m_0 (NA_t - E_t) + m_1^* (\overline{HWO}_t + RAO_t) + m_1 UBO_t$$

where RAO= current (as opposed to permanent) unearned income, in constant prices

UBO = unemployment benefits, in constant prices.

This formulation assumes that employed and unemployed persons have the same marginal propensity to consume out of unearned income, but different propensities to consume out of labor income or its substitute. Using:

$$UR_t = \frac{NA_t - E_t}{NA_t} = \frac{HWO_t - \overline{HWO}_t}{HWO_t} = \frac{HWO_t^W - \overline{HWO}_t}{HWO_t^W}$$

(which implicitly postulates that all consumers have the same labor supply), the equation can be recast in the form:

$$CO_t = m_0^* NA_t + (m_0 - m_0^*) NA_t \cdot UR_t + m_1^* (\overline{HWO}_t + RAO_t) + m_1 UBO_t$$

Empirical results obtained from Belgian postwar data are⁵:

$$CO_t = 30.55 NA_t - 90.90 NA_t \cdot UR_t + .71 (\overline{HWO}_t + RAO_t) + 2.51 UBO_t \quad (2.5)$$

(52.23) (.013) (.79)

which implies $m_0 < m_0^*$
 $m_1 > m_1^*$

The corresponding spill-over effect is computed by:

$$s_t = \frac{\partial CO_t}{\partial \overline{HWO}_t}$$

$$= 90.90 \left(\frac{HWO_t^W}{NA_t} \right)^{-1} + .71 - 2.51 r_t$$

Its average value is .57. While these results suggest a strong specific effect of unemployment, it seems dubious that the coefficients can be interpreted as behavioral parameters. The high marginal propensity to consume of unemployed workers implies that, should the level of real wages and unemployment benefits continue to rise as they did in the past, the spill-over effect would quickly become negative even for reasonable values of r . An increase in the unemployment rate would then result in increased consumption. Moreover the consumption function has a static form, so there is no way to measure the intertemporal effect of unemployment. DAVIDSON et al., (1978) proceeded differently. They first specified a dynamic consumption function, thereafter adding a term in UR_t . They start from the simple long-run behavioral relationship:

$$CO = m_1 YDO \approx YDO$$

which they dynamize as:

$$\ln CO_t = \Lambda(L) \ln YDO_t$$

$$\text{with } \Lambda(L) = \Lambda_2(L) / \Lambda_1(L)$$

$$\Lambda_1(L) = 1 - \lambda_1 L$$

$$\Lambda_2(L) = \lambda_2 + \lambda_3 L + \lambda_4 L^2$$

and L the lag operator. If the λ_i are constrained to sum to unity, we derive after some manipulations:

$$\Delta_1 \ln CO_t = (\lambda_2 - \lambda_4) \Delta_1 \ln YDO_t + \lambda_4 \Delta_1^2 \ln YDO_t + (1 - \lambda_1) \ln \frac{YDO_{t-1}}{CO_{t-1}}$$

where $\Delta_1 = (1 - L)$ is the first difference operator. On a steady-state growth path characterized by $\Delta_1 \ln CO_t = \Delta_1 \ln YDO_t = g$, we get

$$CO_{t-1} = K \cdot YDO_{t-1}$$

where $K = \exp(-(1 - \lambda_2 + \lambda_4)g / (1 - \lambda_1))$ is a decreasing function of g . K is not to be interpreted as a behavioral parameter in the usual sense, as it simply reflects that consumers do not react instantaneously to changing real income. The impact elasticity is λ_2 only, changing to $((1 + \lambda_1)\lambda_2 + \lambda_3)$ after one period. The remaining adjustment will take place more or less slowly, depending on the value of λ_1 : the bigger λ_1 , the slower the adjustment will be. Davidson et al, estimated their equation using British quarterly data. As the estimated relation turned out to give bad forecasts, they tried several additional regressors. Attempts with the unemployment rate did not give significant results. Their final result including only the inflation rate was:

$$\begin{aligned} \Delta_4 \ln CO_t = & .47 \Delta_4 \ln YDO_t - .21 \Delta_1 \Delta_4 \ln YDO_t + .10 \ln(YDO_{t-4}/CO_{t-4}) \\ & - .13 \Delta_4 \ln P_t - .28 \Delta_1 \Delta_4 \ln P_t \end{aligned} \quad (2.6)$$

where use is made of $\Delta_4 = (1-L^4)$ to account for seasonality. This implies that on a steady state growth path, the average propensity to consume will not only be a declining function of g , but also of the inflation rate $\Delta_4 \ln P_t$. The impact elasticity is .26 only. The total effect rises to .47 after one quarter but the remaining .53 will need much time to occur. It should be noted that the inflation effect could be derived directly by expressing the dynamic function in nominal terms:

$$\Lambda_1(L) \ln CU_t = \Lambda_2(L) \ln YDU_t$$

which means that the consumer reacts slowly to changes in nominal income. While in the first formulation one assumes the consumer thinks in real terms, one now postulates he is aware of nominal values only, which is not unrealistic. In that case, however, the coefficient of inflation should have the same absolute

value than the coefficient of real income growth .47, instead of .13.

The results summarized thus far do not show any clear role for unemployment. Maybe this is not surprising as unemployment has always been introduced in an informal way. In the following, I shall try to specify the model more accurately by using a particular utility function and introducing the unemployment constraint from the start. I need to introduce for that purpose the theoretical results of LLUCH (1973). This is done in the next section.

III. THE EXTENDED LINEAR EXPENDITURE SYSTEM

Let us have a closer look at the intertemporal optimization problem of the consumer, starting with Lluch's case of an exogenous labor income. A classical formalization of the problem is: Choose X_T such that

$$\int_{T=0}^n e^{-\rho T} u(X_T) dT + B(A_n)$$

is maximized subject to

$$\begin{aligned} \dot{A}_T &= R \cdot A_T + W \cdot \overline{HW}_T - P' \cdot X_T \\ X_T &\geq 0 \end{aligned}$$

where n is the length of life

X is a vector of consumption goods

P is a vector of prices

$B(\cdot)$ accounts for the bequest motive .

Other notation follows Section II⁶. The statement of the problem entails the following postulates:

- A4 - Intertemporal additivity of the utility functional
- A5 - The instantaneous utility function is stationary

- A6 - The time preference discount rate ρ is constant
- A7 - Expectations on R , W , P and \overline{HW} are held with certainty
- A8 - R and P are expected to remain constant

The inequality constraint on X_T is usually avoided by:

- A9 - The marginal utility of any commodity is tending towards infinity when its consumption is declining towards zero.

LLUCH (1973) specializes the framework even more by assuming:

- A10 - There is no bequest motive: $B(A_n) = 0$
- A11 - The planning horizon is infinite: $n = \infty$

The Hamiltonian has the form:

$$H(A, X, \eta, \tau) = e^{-\rho\tau} u(X_T) + \eta_T (R \cdot A_T + \overline{HWU}_T - P' \cdot X_T)$$

From the Maximum Principle, we obtain, in addition to the budget constraint, the following necessary conditions for an optimal plan:

$$\frac{\partial H}{\partial X_T} = e^{-\rho\tau} u'(X_T) - \eta_T P = 0 \tag{3.1}$$

$$\frac{\partial H}{\partial A_T} = R\eta_T = -\dot{\eta}_T \tag{3.2}$$

Equation (3.2) implies

$$\eta_T = \eta_0 e^{-R\tau}$$

Substitution for this expression into (3.1) and recasting the budget constraint in terms of flows allows us to rewrite the set of necessary conditions as:

$$u'(X_T) = \eta_0 e^{(\rho-R)\tau} P \tag{3.3}$$

$$A_0 + \mathcal{L}(\overline{HWU}) = P' \mathcal{L}(X) \quad (3.4)$$

where $\mathcal{L}(Z) = \int_{\tau=0}^{\infty} e^{-R\tau} Z_{\tau} d\tau$ is the Laplace transform operator. At this stage, Lluch introduces one more assumption:

A12 - The consumer replans continuously,

which means that "only at time $\tau = 0$ do planning and historical time coincide: at $\tau = 0$, the plan is actually implemented." It has to satisfy

(3.3) - (3.4) rearranged as:

$$u'(X) = \eta P \quad (3.5)$$

$$P' X = (R \cdot A + \overline{HWU} + \mathcal{L}(\overline{\dot{HWU}})) - P' \mathcal{L}(\dot{X}) \quad (3.6)$$

The subscript "0" has been dropped and use has been made of the identity $\mathcal{L}(\dot{Z}) = R \cdot \mathcal{L}(Z) - Z$. Assumption A12 allows us to solve (3.5) - (3.6) and obtain behavioral relationships, provided $\mathcal{L}(\overline{\dot{HWU}})$ and $\mathcal{L}(\dot{X})$ are specified. For $\mathcal{L}(\overline{\dot{HWU}}) = P' \mathcal{L}(\dot{X})$, the solution corresponds to the traditional static model; there are no savings. More generally, the specification $\mathcal{L}(\overline{\dot{HWU}})$ will follow from exogenous information (remember leisure was not introduced in the utility function), while the anticipated optimal rate of change of the consumption plan, $\mathcal{L}(\dot{X})$, is determined within the model. From (3.3), Lluch derives

$$\dot{X}_{\tau} = \eta_0 (\rho - R) e^{(\rho-R)\tau} U_{\tau}^{-1} P$$

where U_{τ} is the Hessian of the utility function at time τ , as seen at $\tau = 0$.

Lluch applies those results to the Stone-Geary utility function:

$$u(X_{\tau}) = \beta_1' \ln(X_{\tau} - \gamma_1)$$

defined for $\gamma_1 \ll X_T$. A classical interpretation is that γ_1 represents the minimum expenditures needed to stay alive. Application of the preceding results to this particular utility function gives the following system of demand equations:

$$\hat{P} X = \hat{P} \gamma_1 + (\mu/i' \beta_1) \beta_1 (R \cdot A + \overline{HWU} + \mathcal{L}(\overline{HWU}) - P' \gamma_1) \quad (3.7)$$

where $\mu = \rho/R$

i is a vector of unit elements

\hat{P} is a diagonal matrix formed with the elements of P .

Generalization to the case of endogenous working time is straightforward.

Leisure is introduced into the utility function:

$$u(X_T, HL_T) = \beta_1' \ln(X_T - \gamma_1) + \beta_2 \ln(HL_T - \gamma_2^*)$$

while the budget constraint now takes the form:

$$\dot{A}_T = R \cdot A_T + W (HT - HL_T) - P' X_T$$

where W is expected to remain constant. An additional constraint is added:

$HL_T \leq HT$. Corresponding to (3.7), we obtain the following demand equations:

$$\hat{P} X = \hat{P} \gamma_1 + (\mu/\beta_2 + i' \beta_1) \beta_1 (R \cdot A - P' \gamma_1 + W(HT - \gamma_2^*))$$

$$W \cdot HL = W \gamma_2^* + (\mu/\beta_2 + i' \beta_1) \beta_2 (R \cdot A - P' \gamma_1 + W(HT - \gamma_2^*))$$

The system can be rewritten in terms of working hours, using $HW = HT - HL$

and $\gamma_2 = HT - \gamma_2^*$:

$$\hat{P} X = \hat{P} \gamma_1 + (\mu/\beta_2 + i' \beta_1) \beta_1 (R \cdot A - P' \gamma_1 + W \gamma_2) \quad (3.8)$$

$$W \cdot HW = W \gamma_2 - (\mu/\beta_2 + i' \beta_1) \beta_2 (R \cdot A - P' \gamma_1 + W \gamma_2) \quad (3.9)$$

γ_2 represents the maximum feasible number of working hours; the equations are valid for $0 \leq HW < \gamma_2$ only.

IV. THE LONG-RUN AGGREGATE CONSUMPTION FUNCTION

Two types of aggregation are involved in deriving a macroeconomic consumption function. Aggregation over goods will be treated in the same simple way as in Section II: I shall assume from the start there is only one consumption commodity, CO , the price of which is the retail price index P . As I want to derive precisely the effect of unemployment on consumption, leisure will be added as a second argument in the individual utility function. I take this function to be of the Stone-Geary type:

$$A13 - u_i = \beta \ln (CO_i - \gamma_1) + (1 - \beta) \ln (HL_i - \gamma_2^*)$$

defined for $CO_i > \gamma_1$ and $HL_i > \gamma_2^*$. Representing the behavior of the individual consumer by the intertemporal optimization of this function means that the consumer first determines both the amount of work he is willing to perform and the amount of money he is willing to spend. Allocation of this money between different consumption goods is only determined in a second step and will not be our concern here.

Aggregation over individuals will prove more difficult, as a result of the distinction I want to make between employed and unemployed persons. First, the notion of "individual consumer" is not straightforward at all. It actually represents an "average" household, one member of which is a potential worker (either employed or not) while others are not (children and retired persons, i.a.). This implies that all changes in available labor force NA_t are considered exogenous, even if they result from retirement decisions or increased female participation. Second, the fact that each household's situation may

switch at any time from unemployment to employment or vice versa makes aggregation impossible without the following strong assumption:

A14 - Each household has the same preferences.

Bearing those simplifications in mind, I now turn to the derivation of the consumption function and its associated labor supply function. As dynamic features will be introduced in the next section only, I shall delete any time subscript from this section.

IV.1. Individual Functions

I first consider the case of an unconstrained household. Effective and notional offers coincide and are obtained from (3.8) - (3.9) rewritten as:

$$CU_i = CU_i^W = \gamma_1 P + \mu\beta(R \cdot A_i - \gamma_1 P + \gamma_2 W) \quad (4.1)$$

$$HWU_i = HWU_i^W = \gamma_2 W - \mu(1 - \beta)(R \cdot A_i - \gamma_1 P + \gamma_2 W) \quad (4.2)$$

Equation (4.2) gives two alternative expressions for $\gamma_2 W$:

$$\gamma_2 W = (1 - \mu(1 - \beta))^{-1} (HWU_i^W + \mu(1 - \beta)(R \cdot A_i - \gamma_1 P))$$

$$\text{or } \gamma_2 W = (\mu\beta)^{-1} (HWU_i^W + \mu(1 - \beta)(R \cdot A_i - \gamma_1 P) - (1 - \mu)\gamma_2 W)$$

Substituting the first one into (4.1) gives the Walrasian consumption level as a linear function of "Walrasian" disposable income:

$$CU_i^W = m_0^* P + m_1^* YDU_i^W \quad (4.3)$$

$$\text{where } m_0^* = \gamma_1 (1 - \mu) / 1 - \mu(1 - \beta)$$

$$m_1^* = \mu\beta / 1 - \mu(1 - \beta)$$

The substitution of the second expression yields:

$$CU_i^W = (1 - \mu) (\gamma_1 P - \gamma_2 W) + \mu R \cdot A_i + HWU_i^W \quad (4.4)$$

which will prove useful later on.

If the household is constrained on the labor market and expects to remain constrained forever, its demand for consumption goods will be represented by the equivalent of (3.7):

$$CU_i = m_0 P + m_1 \overline{YDU}_i^P \quad (4.5)$$

where $m_0 = (1 - \mu) \gamma_1$

$$m_1 = \mu$$

$$\begin{aligned} \overline{YDU}_i^P &= R \cdot A_i + R \cdot \mathcal{L}(\overline{HWU}_i) + r(HWU_i^W - R \cdot \mathcal{L}(\overline{HWU}_i)) \\ &= R \cdot A_i + \overline{HWU}_i^P + r(HWU_i^W - \overline{HWU}_i^P) \end{aligned}$$

In this reformulation, account has been taken of unemployment benefits. It follows from (4.3) and (4.5) that

$$\begin{aligned} m_0 &< m_0^* \\ m_1 &> m_1^* \end{aligned}$$

for reasonable values of μ and β (.8, e.g.). The Keynesian individual consumption function (4.5) can be recast in terms of the Walrasian one, according to

$$\begin{aligned} CU_i &= CU_i^W + (CU_i - CU_i^W) \\ &= CU_i^W + f(\overline{HWU}_i^P, HWU_i^W) \end{aligned}$$

Using alternatively (4.1), (4.3) and (4.4) in combination with (4.5), we derive the following three equivalent expressions for $f(\cdot)$:

$$f(\cdot) = - (1 - \beta) \mu \gamma_1 P - \beta \mu \gamma_2 W + (1 - \beta) \mu R \cdot A_i + \mu (\overline{HWU}_i^P + r(HWU_i^W - \overline{HWU}_i^P)) \quad (4.6)$$

$$f(\cdot) = \frac{\mu(1-\mu)(1-\beta)}{1-\mu(1-\beta)} (-\gamma_1 P + R \cdot A_i) + \mu(\overline{HWU}_i^P + r(HWU_i^W - \overline{HWU}_i^P)) - \frac{\beta}{1-\mu(1-\beta)} HWU_i^W \quad (4.7)$$

$$f(\cdot) = (1-\mu) \gamma_2 W + \mu(\overline{HWU}_i^P + r(HWU_i^W - \overline{HWU}_i^P)) - HWU_i^W \quad (4.8)$$

With respect to the labor supply, no difficulty arises, as our initial assumptions allow us to express it again as (4.2).

The complete system of behavioral relationships for household i can now be put more compactly as:

$$CU_i = CU_i^W + \delta_i f(\overline{HWU}_i^P, HWU_i^W) \quad (4.9)$$

$$HWU_i = HWU_i^W \quad (4.10)$$

where $\delta_i = 0$ for i unconstrained on the labor market (denoted $i \in \mathcal{E}$)
 $= 1$ for i unemployed (denoted $i \in \mathcal{U}$)

The forms of CU_i^W , HWU_i^W , \overline{HWU}_i^P and $f(\cdot)$ are as specified above.

IV.2. Aggregation Over Individual Households

The difficulties encountered when aggregating over individuals may differ widely according to the type of rationing scheme that prevails on the labor market. When the rationing scheme is such that every worker will be constrained as soon as there is an excess aggregate supply of labor, the model takes the form of the switching regression model. Aggregation is not a problem at all as all the households have the same behavioral pattern at the same time. Moreover if one assumes there is always some frictional unemployment, then all workers are always constrained and we come down to the aggregate Keynesian consumption model:

$$CO = m_0 NA + m_1 \overline{YDO}^P$$

obtained by summing (4.5) over i . It seems more realistic however to assume that the burdens of the labor constraint are shared by a few people only who will experience a period of full unemployment. This amounts to distinguish two classes of households, \mathcal{E} and \mathcal{U} , respectively the set of fully employed and fully unemployed households. In this context, the aggregate Keynesian function can only be an approximation of the type described in Section II, as it will soon become clear.

As the purpose of this section is the derivation of the long-run consumption function, I may consider the simple case where $\overline{HWU}_i^P = 0$ for $i \in \mathcal{U}$. Summing (4.9) over i results in the following three equivalent expressions:

$$CU = (1 - \mu\beta)\gamma_1 P \cdot NA + \mu\beta\gamma_2 W \cdot NA + \mu\beta R \cdot A - (1 - \beta)\mu\gamma_1 P \cdot NA \cdot UR - \beta\mu\gamma_2 W \cdot NA \cdot UR + (1 - \beta)\mu R \cdot A_{\mathcal{U}} + \mu \cdot r \cdot HWU_{\mathcal{U}}^W \quad (4.11)$$

$$CU = (1 - \mu(1 - \beta))^{-1} (\gamma_1 (1 - \mu) P \cdot NA + \mu\beta(R \cdot A + HWU_{\mathcal{E}}^W) + \mu(1 - \mu) (1 - \beta) (-\gamma_1 P \cdot NA \cdot UR + R \cdot A_{\mathcal{U}}) + \mu(1 - \mu(1 - \beta))r \cdot HWU_{\mathcal{U}}^W) \quad (4.12)$$

$$CU = (1 - \mu) (\gamma_1 P - \gamma_2 W) NA + \mu R \cdot A + HWU_{\mathcal{E}}^W + (1 - \mu)\gamma_2 W \cdot NA \cdot UR + \mu r \cdot HWU_{\mathcal{U}}^W \quad (4.13)$$

Equations (4.11), (4.12) and (4.13) correspond to (4.6) - (4.7) - (4.8) respectively. Subscripts \mathcal{E} and \mathcal{U} mean respectively aggregation over unconstrained and constrained households such that $A = A_{\mathcal{U}} + A_{\mathcal{E}}$ and $HWU^W = HWU_{\mathcal{E}}^W + HWU_{\mathcal{U}}^W$. These equations can be used for empirical purposes, provided one is ready to accept the approximation of permanent values by current ones. In that case \mathcal{U} is the set of currently fully unemployed workers and

$$HWU_{\mathcal{E}}^W = \overline{HWU}, \quad r \cdot HWU_{\mathcal{U}}^W = UBU, \quad R \cdot A = RAU$$

Notice that (4.11) - (4.12) have the unobserved A_{μ} as regressor and will not be useful until we are ready to make a strong assumption on income distribution. This is not true for (4.13) however and some interesting results can be derived from it. Let us approximate:

$$\overline{HWU} + \mu (RAU + UBU) \approx \overline{HWU} + RAU + UBU = YDU$$

and

$$\frac{W}{P} \approx k \frac{YDO}{NA},$$

such that (4.13) can be rewritten (in real terms):

$$CO \approx (1 - \mu) \gamma_1 NA + (1 - \mu) \gamma_2 k YDO \cdot UR + (1 - (1 - \mu) \gamma_2 k) YDO$$

Neglecting the effect of unemployment by incorporating its average impact into the constant term results in the traditional consumption function linear in disposable income. Yet the coefficient of income cannot be interpreted as a marginal propensity to consume. This gives a new view to equation (2.4) and highlights the consequences of the underlying simplifications. The strange results obtained from equation (2.5) can be explained in a similar way. From (4.13) it follows

$$CO = (1 - \mu) \gamma_1 NA + (-(1 - \mu) \gamma_2 \frac{W}{P} NA + \mu RAO + \overline{HWO}) \\ + ((1 - \mu) \gamma_2 \frac{W}{P} NA \cdot UR + \mu UBO)$$

Using $(\mu RAO + \overline{HWO}) \approx (RAO + \overline{HWO})$ and $\frac{W}{P} \approx k_0 (RAO + \overline{HWO})/NA$ in one case and $\frac{W}{P} \approx k_1 + k_2 \frac{UBO}{NA \cdot UR}$ in the other, we get:

$$CO \approx (1 - \mu) \gamma_1 NA + (-(1 - \mu) \gamma_2 k_0 (RAO + \overline{HWO}) + (RAO + \overline{HWO})) \\ + ((1 - \mu) \gamma_2 (k_1 NA \cdot UR + k_2 UBO) + \mu UBO) \\ \approx (1 - \mu) \gamma_1 NA + (1 - \mu) \gamma_2 k_1 NA \cdot UR + (1 - (1 - \mu) \gamma_2 k_0) (RAO + \overline{HWO}) \\ + (\mu + (1 - \mu) \gamma_2 k_2) UBO.$$

Once more the coefficient of unemployment benefits is not a marginal propensity to consume and may be much higher, depending on the value of $(1 - \mu) \gamma_2 k_2$.

V DYNAMICS

There are at least two reasons to introduce a dynamic structure into the model developed in Section IV. First because the assumption of a stationary utility function may be false. What people consider to be a minimum consumption level may vary through time, as well as the maximum amount of work they are willing to perform. It is realistic to believe these amounts are greatly affected by the past history of the households and are strongly characterized by habit formation processes. This argument favors a dynamic specification of parameters γ_1 and γ_2 . A simple one is:

$$\gamma_{1t} = c_{10} + c_{11} CO_{t-1}/NA_{t-1}$$

$$\gamma_{2t} = c_{20} - c_{21} HW_{t-1}/NA_{t-1}$$

as suggested by POLLAK-WALES (1969). Attempts with this specification failed to give satisfactory results. As the more general dynamic specification introduced by PHILIPS (1972) in an atemporal context is too complicated for an intertemporal model, I shall keep the stationarity assumption. As the estimation period is relatively short (1954 - 1976), this should not distort too much the empirical findings.

A second and stronger motive calling for a dynamic structure arises from the intertemporal nature of the model. Nothing has been said yet on the values of R , W , P , the expectations of the household on the interest rate, the nominal wage rate, the price of the consumption commodity respectively and on \overline{HWU}_i^p , the permanent labor constraint. In deriving the long-run model, I simply took them equivalent to current values. I shall now try to give them a

dynamic specification by using the approach of DAVIDSON et.al., (1978) presented in Section II and applying it to the behavioral model (4.9) - (4.10).

This leads to:

$$CU_{it} = \Lambda(L) CU_{it}^W + \Lambda(L) (\delta_{it} f(\overline{HWU}_{it}, HWU_{it}^W)) \quad (5.1)$$

$$HWU_{it} = \Lambda(L) HWU_{it}^W \quad (5.2)$$

where CU_{it}^W , HWU_{it}^W and $f(\cdot)$ are as defined in the preceding section, except that expectations and permanent values are replaced by current ones.

Expanding (5.1) - (5.2) reveals their underlying meaning. First expand CU_{it}^W as in (4.1):

$$\Lambda(L) CU_{it}^W = \gamma_1 \Lambda(L) P_t + \mu \beta (\Lambda(L) RAU_{it} - \gamma_1 \Lambda(L) P_t + \gamma_2 \Lambda(L) W_t)$$

which shows that expectations on the price and wage levels are represented by a weighted sum of past levels, while permanent property income is approximated by an average of past realized returns. The same interpretation arises from the development of $\Lambda(L) HWU_{it}^W$ as in (4.2). The meaning of the second term in (5.1) will show through by the use of (4.8) and (5.2):

$$\begin{aligned} F_t(\overline{HWU}_{it}, HWU_{it}^W) &= \Lambda(L) (\delta_{it} f(\overline{HWU}_{it}, HWU_{it}^W)) \\ &= \Lambda(L) (\delta_{it} ((1 - \mu) \gamma_2 W_t + \mu (\overline{HWU}_{it} + UBU_{it}))) \\ &\quad + (1 - \delta_{it}) HWU_{it}^W - \Lambda(L) HWU_{it}^W \\ &= (1 - \mu) \Lambda(L) (\delta_{it} \gamma_2 W_t + (1 - \delta_{it}) HWU_{it}^W) \\ &\quad + \mu \Lambda(L) (\delta_{it} (\overline{HWU}_{it} + UBU_{it}) + (1 - \delta_{it}) HWU_{it}^W) - HWU_{it} \\ &= (1 - \mu) \Lambda(L) ((\delta_{it} \gamma_2 + (1 - \delta_{it}) HWU_{it}^W) \cdot W_t) \\ &\quad + \mu (\Lambda(L) (\delta_{it} \overline{HWU}_{it} + (1 - \delta_{it}) HWU_{it}^W) + \Lambda(L) \delta_{it} UBU_{it}) - HWU_{it} \end{aligned}$$

From the comparison with (4.8) it follows that we estimate the permanent labor constraint by

$$\Lambda(L) (\delta_{it} \overline{HWU}_{it} + (1 - \delta_{it}) HWU_{it}^w)$$

If the household has always been constrained on the labor market ($\delta_{it} = 1, \forall t$), the permanent labor constraint is equal to a weighted average of past constraints. Perceived constraint and desired employment become equal provided the household has never been rationed ($\delta_{it} = 0, \forall t$). In that case also $F_t(\cdot) = 0$. This specification allows us to differentiate households having experienced in the past different degrees of labor rationing.

Aggregate dynamic functions are obtained by summing over i as in Section IV⁷, but with the simplification $\overline{HWO}_i^p = 0$ for $i \in \mathcal{U}$ replaced by $\overline{HWO}_{it} = 0$ for $i \in \mathcal{U}_t$. Using the definition $\Lambda(L) = \Lambda_2(L)/\Lambda_1(L)$, we derive the three alternative forms of the consumption function:

$$\begin{aligned} \Lambda_1(L) \frac{CU_t}{NA_t} &= \Lambda_2(L) (\gamma_1 P_t + \mu\beta \left(\frac{RAU_t}{NA_t} - \gamma_1 P_t + \gamma_2 W_t \right) \\ &\quad - (1 - \beta)\mu \gamma_1 P_t \cdot UR_t - \beta\mu \gamma_2 W_t \cdot UR_t \\ &\quad + (1 - \beta)\mu \frac{RAU_{\mathcal{U}_t}}{NA_t} + \mu \frac{UBU_t}{NA_t} \end{aligned} \quad (5.3)$$

$$\begin{aligned} \Lambda_1(L) \frac{CU_t}{NA_t} &= (1 - \mu(1 - \beta))^{-1} (\Lambda_2(L) (\gamma_1 (1 - \mu) P_t + \mu\beta \frac{RAU_t}{NA_t}) \\ &\quad + \mu\beta\Lambda_1(L) \frac{HWU_t}{NA_t} + \Lambda_2(L) (\mu(1 - \mu)(1 - \beta) (-\gamma_1 (P_t \cdot UR_t) \\ &\quad + \frac{RAU_{\mathcal{U}_t}}{NA_t}) + \mu(1 - \mu(1 - \beta)) \frac{UBU_t}{NA_t} - \mu\beta \frac{HWU_{\mathcal{U}_t}^w}{NA_t}) \end{aligned} \quad (5.4)$$

$$\begin{aligned} \Lambda_1(L) \frac{CU_t}{NA_t} &= \Lambda_2(L) ((1 - \mu) \gamma_1 P_t - (1 - \mu) \gamma_2 W_t + \mu \frac{RAU_t}{NA_t}) \\ &\quad + \Lambda_1(L) \frac{HWU_t}{NA_t} + \Lambda_2(L) ((1 - \mu) \gamma_2 (W_t \cdot UR_t) \\ &\quad + \mu \frac{UBU_t}{NA_t} - \frac{HWU_{\mathcal{U}_t}^w}{NA_t}) \end{aligned} \quad (5.5)$$

In (5.4) - (5.5), the identity $\Lambda_1(L) \frac{HWU_t}{NA_t} = \Lambda_2(L) \frac{HWU_t^W}{NA_t}$ has been used. Notice that in these two equations $\Lambda_2(L) HWU_t^W$ does not reduce to $\Lambda_1(L) HWU_t$, as a consequence of

$$\Lambda_2(L) \left(\sum_{i \in \mathcal{U}_t} HWU_i^W \right) \neq \sum_{i \in \mathcal{U}_t} \Lambda_2(L) HWU_i^W$$

HWU_t^W being unknown, it means that (5.4) - (5.5) cannot be used empirically. Equation (5.3) will not be more useful, unless we assume:

A15. - non-labor income (which excludes unemployment benefits)
is evenly distributed among households.

It follows that the unobserved RAU_t can be replaced by $RAU_t \cdot UR_t$. The aggregate supply of labor is given by:

$$\Lambda_1(L) \frac{HWU_t}{NA_t} = \Lambda_2(L) \left((1 - \mu(1 - \beta)) \gamma_2 W_t - \mu(1 - \beta) \left(\frac{RAU_t}{NA_t} - \gamma_1 P_t \right) \right) \quad (5.6)$$

From (5.3) - (5.6), elasticity formulas can be obtained. Defining $\Lambda_1(L) = 1 - \lambda_1 L$ and $\Lambda_2(L) = \lambda_2 + \lambda_3 L$ and neglecting unemployment, we get with respect to the consumption function:

$$\epsilon_{CP} = \frac{\partial CO}{\partial P} \frac{P}{CO} = \lambda_2 (1 - \mu\beta) \gamma_1 \frac{NA}{CO} - 1$$

$$\epsilon_{CW} = \frac{\partial CO}{\partial W} \frac{W}{CO} = \lambda_2 \mu\beta\gamma_2 \frac{W}{P} \frac{NA}{CO}$$

$$\epsilon_{CA} = \frac{\partial CO}{\partial RAO} \frac{RAO}{CO} = \lambda_2 \mu\beta \frac{RAO}{CO}$$

and similarly with respect to the labor supply:

$$\epsilon_{HP} = \frac{\partial HW}{\partial P} \frac{P}{HW} = \lambda_2 \mu (1 - \beta) \gamma_1 \frac{NA}{HWO}$$

$$\epsilon_{HW} = \frac{\partial HW}{\partial W} \frac{W}{HW} = \lambda_2 (1 - \mu(1 - \beta)) \gamma_2 \frac{NA}{HW} - 1$$

$$\epsilon_{HA} = \frac{\partial HW}{\partial RAO} \frac{RAO}{HW} = \lambda_2 \mu (1 - \beta) \frac{RAO}{HWO}$$

Long-run elasticities are obtained for $\lambda_2 = 1$. Marginal propensities to consume will be derived from equation (5.4). The marginal propensity to consume out of labor income (for employed workers) is the coefficient of HWU_t/NA_t :

$$m_{Hi} = \mu\beta/1 - \mu(1 - \beta) \quad i \in \mathcal{E}_t$$

Long-run and short-run values coincide. The marginal propensity to consume out of unearned income differ for employed and unemployed persons:

$$m_{Ai} = \lambda_2 \mu\beta/1 - \mu(1 - \beta) \quad i \in \mathcal{E}_t$$

$$m_{Ai} = \lambda_2 \mu \quad i \in \mathcal{U}_t$$

As suggested in Section II, the dynamic specification adopted in (5.1) - (5.2) postulates implicitly that households are mostly aware of nominal quantities and not of their real counterparts. The implications of this formulation will appear clearly if we consider the case of an economy characterized by pure inflation and zero growth. Then (5.4) can be rewritten (neglecting unemployment)

$$\begin{aligned} \frac{CO}{NA} \Lambda_1(L) P_t &= (1 - \mu(1 - \beta))^{-1} ((\gamma_1(1 - \mu) + \mu\beta \frac{RAO}{NA}) \Lambda_2(L) P_t \\ &\quad + \mu\beta \frac{HWO}{NA} \Lambda_1(L) P_t) \end{aligned}$$

$$\text{or: } \frac{CO}{NA} = (1 - \mu(1 - \beta))^{-1} \left(\frac{\Lambda_2(L) P_t}{\Lambda_1(L) P_t} (\gamma_1(1 - \mu) + \mu\beta \frac{RAO}{NA}) + \mu\beta \frac{HWO}{NA} \right)$$

while (5.6) becomes:

$$\frac{HWO}{NA} = \frac{\Lambda_2(L) P_t}{\Lambda_1(L) P_t} (\mu(1 - \beta) \gamma_1 + (1 - \mu(1 - \beta)) \gamma_2 \frac{W}{P} - \mu(1 - \beta) \frac{RAO}{NA})$$

The multiplier $\frac{\Lambda_2(L) P_t}{\Lambda_1(L) P_t}$ appearing in both expressions can be developed as

$$\frac{\Lambda_2(L) P_t}{\Lambda_1(L) P_t} = \frac{\lambda_2 P_t + \lambda_3 P_{t-1}}{P_t - \lambda_1 P_{t-1}} = \frac{\lambda_2(1 + \dot{P}) + \lambda_3}{1 + \dot{P} - \lambda_1}$$

where I used $\dot{P} = (P_t/P_{t-1}) - 1$. Introduction of the constraint

$\lambda_1 + \lambda_2 + \lambda_3 = 1$ allows us to rewrite the multiplier as:

$$\frac{\Lambda_2(L) P_t}{\Lambda_1(L) P_t} = 1 - \frac{(1 - \lambda_2) \dot{P}}{\dot{P} + 1 - \lambda_1} \quad (5.7)$$

The assumption that households are not aware of real quantities entails that continuous inflation will decrease both the consumption out of unearned income and the supply of labor by a factor $(1 - \lambda_2) \dot{P}/(\dot{P} + 1 - \lambda_1)$. The fact that the proportion of labor income devoted to consumption remains unaffected may explain why the coefficients of the inflation rate in (2.6) were lower than expected. The effect of inflation can be avoided by assuming households know what real quantities are and respecifying (5.1) - (5.2) as

$$CO_{it} = \Lambda(L) CO_{it}^W + \Lambda(L) (\delta_{it} f(\overline{HWO}_{it}, HWO_{it}^W))$$

$$HWO_{it} = \Lambda(L) HWO_{it}^W$$

Most results carry over provided real values are substituted for nominal ones. The only changes concern the price elasticities of consumption demand and labor supply, which are now equal to:

$$\epsilon_{CP} = -\lambda_2 \mu \beta \left(\frac{RAO}{NA} + \gamma_2 \frac{W}{P} \right) \frac{NA}{CO}$$

$$\epsilon_{HP} = 1 - \lambda_2 \left((1 - \mu(1 - \beta)) \gamma_2 \frac{W}{P} - \mu(1 - \beta) \frac{RAO}{NA} \right) \frac{NA}{HWO}$$

VI. EMPIRICAL RESULTS

Equations (5.3) and (5.6) and their constant price equivalent have been estimated on Belgian 1954-1976 annual data. Data on total expenditures, including durables, have been used for CU and the retail price index was scaled to one in 1970. Other data are described in the Appendix. The following specifications of $\Lambda_1(L)$, $\Lambda_2(L)$ have been adopted:

$$\Lambda_1(L) = 1 - \lambda_1 L ; \Lambda_2(L) = \lambda_2 + \lambda_3 L .$$

The weights λ_i are restricted to sum to unity. Rearranging the terms of (5.3), (5.6), we get:

$$\begin{aligned} \Delta_1 \frac{CU_t}{NA_t} = & \lambda_2 ((1 - \mu\beta)\gamma_1 \Delta_1 P_t + \mu\beta \Delta_1 \frac{RAU_t \cdot (1 - UR_t)}{NA_t} + \mu\beta \gamma_2 \Delta_1 W_t (1 - UR_t) \\ & - \mu(1 - \beta) \gamma_1 \Delta_1 (P_t \cdot UR_t) + \mu \Delta_1 \frac{UBU_t + RAU_t \cdot UR_t}{NA_t}) \\ & + (1 - \lambda_1) ((1 - \mu\beta) \gamma_1 P_{t-1} + \mu\beta \frac{RAU_{t-1} (1 - UR_{t-1})}{NA_{t-1}} \\ & + \mu\beta \gamma_2 W_{t-1} (1 - UR_{t-1}) - \mu(1 - \beta) \gamma_1 P_{t-1} \cdot UR_{t-1} \\ & + \frac{UBU_{t-1} + RAU_{t-1} \cdot UR_{t-1}}{NA_{t-1}} - \frac{CU_{t-1}}{NA_{t-1}}) \end{aligned} \quad (6.1)$$

$$\begin{aligned} \Delta_2 \frac{HWU_t}{NA_t} = & \lambda_2 (\mu(1 - \beta)\gamma_1 \Delta_1 P_t + (1 - \mu(1 - \beta))\gamma_2 \Delta_1 W_t \\ & - \mu(1 - \beta) \Delta_1 \frac{RAU_t}{NA_t}) + (1 - \lambda_1) (\mu(1 - \beta)\gamma_1 P_{t-1} \\ & + (1 - \mu(1 - \beta))\gamma_2 W_{t-1} - \mu(1 - \beta) \frac{RAU_{t-1}}{NA_{t-1}} - \frac{HWU_{t-1}}{NA_{t-1}}) \end{aligned} \quad (6.2)$$

These two equations (and also their constant price equivalent) were estimated jointly using a weighted least squares criterion. The error terms were assumed to be non-autocorrelated; heteroskedasticity has been allowed for by dividing both sides of the consumption equation by nominal (real for the constant price version) disposable income and both sides of the labor equation by nominal (or real) wages.

Estimates of the structural coefficients are reported in Table 1; figures in parentheses are standard errors. The estimates of three out of the four behavioral parameters seem rather independent of the dynamic formulation; they are: γ_1 , the minimum expenditure level ($\gamma_1 \approx 110,000$ B. F₁₉₇₀ or $\gamma_1 \approx 3,667$ \$₁₉₇₀) ; γ_2 , the maximum number of working hours

($\gamma_2 \approx 2,600$ hours a year); μ , the ratio of the subjective discount rate and the rate of interest on non-human wealth ($\mu \approx .83$). Coefficient β however takes very different values. While in the first case, its value remains close to traditional estimates (ABBOT-ASHENFELTER (1976) report a value of .879 for the United States, PIERAERTS-PHILIPS (1978) .805 for Belgium, both values computed within an atemporal model), it becomes much lower in the second case. This corresponds to a much higher preference for leisure and a greater reaction of labor supply to wage or property income changes. A fast adjustment to new equilibrium states is the second characteristic of the constant price model: 83% of the adjustment is realized within the first period, compared with only 45% in the first model. This last figure is much closer to the findings of Davidson et.al. However, according to (5.7), it implies that a permanent 5% inflation rate will decrease the labor supply by as much as 15%. While in the constant price model, the decline in working time is mainly attributed to a strong preference for leisure, the phenomenon is here explained by continuous inflation which makes it easier for the household to obtain its nominal labor income target. These differences translate in the estimates of the elasticity coefficients⁸ and of the marginal propensities to consume. In the current price model, long-run elasticities of the supply of labor are always very low in absolute value, while long-run propensities to consume are rather high. This result fits the discovery made by KUZNETS (1942) of a stable and a high long-run average propensity to consume. Notice also that marginal propensities of employed and unemployed workers are very similar in the first model, but quite different in the second. These rather mixed results make it difficult to discriminate between the two dynamic specifications. The traditional statistical criteria reported at the bottom of Table 1 are not much more helpful. The Durbin-Watson and the R^2 statistics, computed separately for the consumption

	Equations in	
	current prices	constant prices
γ_1	106.4 (44.99)	114.0 (14.61)
γ_2	2.668 (.3168)	2.625 (.2220)
β	.8248 (.1451)	.5828 (.0673)
μ	.8765 (.0564)	.8105 (.0296)
λ_1	.8602 (.0841)	.7567 (.1209)
λ_2	.4463 (.0992)	.8293 (.1845)
λ_3	-.3065 (.0995)	-.5860 (.2047)
D.W.: (6.1)	2.758	2.129
(6.2)	1.537	1.591
R^2 : (6.1)	.9704	.8005
(6.2)	.9630	.7197
SEE: CO	.0129	.0155
HW	.0113	.0168

Table 1

	Equations in	
	current prices	constant prices
ϵ_{CP} short	-.9411	-.5602
long	-.8681	-.6755
ϵ_{CW} short	.2970	.4278
long	.6656	.5159
ϵ_{CA} short	.0904	.1324
long	.2025	.1596
ϵ_{HP} short	.0403	.3820
long	.0935	.2548
ϵ_{HW} short	-.5703	-.2659
long	.0372	.1146
ϵ_{HA} short	-.0237	-.1160
long	-.0532	-.1399
$m_{Hi\epsilon}$ short	.8541	.7137
long	.8541	.7137
$m_{Ai\epsilon}$ short	.3812	.5919
long	.8541	.7137
$m_{Ai\epsilon}^u$ short	.3912	.6721
long	.8765	.8105
s short	.3432	.4212
long	.7689	.5079

Table 2

demand and labor supply functions were calculated from the equations in their estimated form ((6.1) and (6.2) divided by disposable income and the wage rate respectively). The last statistic is the standard error of the estimate (SEE) computed in both models with respect to real consumption and working hours. None of these results is clear-cut enough to allow the rejection of one of the models.

The analysis of the effect of involuntary unemployment on consumption will be carried with the help of equations (4.3) and (4.5). In real terms, we get

$$CO_i = 15.524 + .8541 YDO_i^W = CO_e \quad i \in \mathcal{E}$$

$$CO_i = 13.140 + .8765 \overline{YDO}_i = CO_u \quad i \in \mathcal{U}$$

for the current price model and

$$CO_i = 32.640 + .7137 YDO_i^W = CO_e \quad i \in \mathcal{E}$$

$$CO_i = 21.603 + .8105 \overline{YDO}_i = CO_u \quad i \in \mathcal{U}$$

for the constant price model. The intercepts are expressed in thousands of 1970 Belgian francs. In both models, the first equation represents the behavior of a household which has never been rationed in its labor supply, while the second represents the behavior of a household which has always been fully unemployed. Moreover, it is implicitly assumed there is no inflation and that disposable income of employed and unemployed workers has always remained at the same level. I now concentrate on the income and consumption patterns of a fictitious household i which never experienced unemployment in the past and suddenly became fully unemployed for two years (labeled year 2 and 3). Its demand for consumption goods is given by:

$$\begin{aligned}
 CO_{it} &= CO_e && \text{for } t < 2 \\
 CO_{it} &= \lambda_2 CO_u + (1 - \lambda_2) CO_e && \text{for } t = 2 \\
 CO_{it} &= (\lambda_2 + (\lambda_1 \lambda_2 + \lambda_3)) CO_u + (1 - \lambda_2 - (\lambda_1 \lambda_2 + \lambda_3)) CO_e && \text{for } t = 3 \\
 CO_{it} &= (\lambda_1 \lambda_2 + \lambda_3) (1 + \lambda_1) CO_u + (1 - (\lambda_1 \lambda_2 + \lambda_3) (1 + \lambda_1)) CO_e && \text{for } t = 4 \\
 &\vdots && \\
 CO_{it} &= CO_e && \text{for } t \rightarrow \infty
 \end{aligned}$$

where the properties $\Lambda_2(L)/\Lambda_1(L) = (\lambda_2 + \lambda_3 L) (1 + \lambda_1 L + \lambda_1^2 L^2 + \dots)$

$$= \sum_{\ell} \pi_{\ell} L^{\ell}$$

and $\sum_{\ell} \pi_{\ell} = 1$ have been used. Figures 1 and 2 reproduce the evolution of consumption in both models using the real income data observed in 1975, calculated as:

$$\begin{aligned}
 YDO_i^w &= \frac{RAO}{NA} + \frac{\overline{HWO}}{NA(1 - UR)} = 304.454 && \text{for } t \neq 2,3 \\
 \overline{YDO}_i &= \frac{RAO}{NA} + \frac{UBO}{NA \cdot UR} = 182.274 && \text{for } t = 2,3
 \end{aligned}$$

Accordingly, the equilibrium consumption level of an employed worker is much higher in the first model than in the second (275.558 against 249.929) while the equilibrium consumption level of an unemployed worker is almost the same in both models (172.903 compared with 169.336). This means that the total effect of unemployment on consumption is higher in the first, current model. From the figures however we can see that the impact effect is much larger in the second one. A priori, the pattern appearing in Figure 1 seems the most satisfactory; it is intuitively appealing that the effect of an unemployment spell spreads over many years through temporary dissavings. Those differences between the current and constant price models are quantified in the short-run and long-run spill-over coefficients shown at the bottom of Table 2. For the

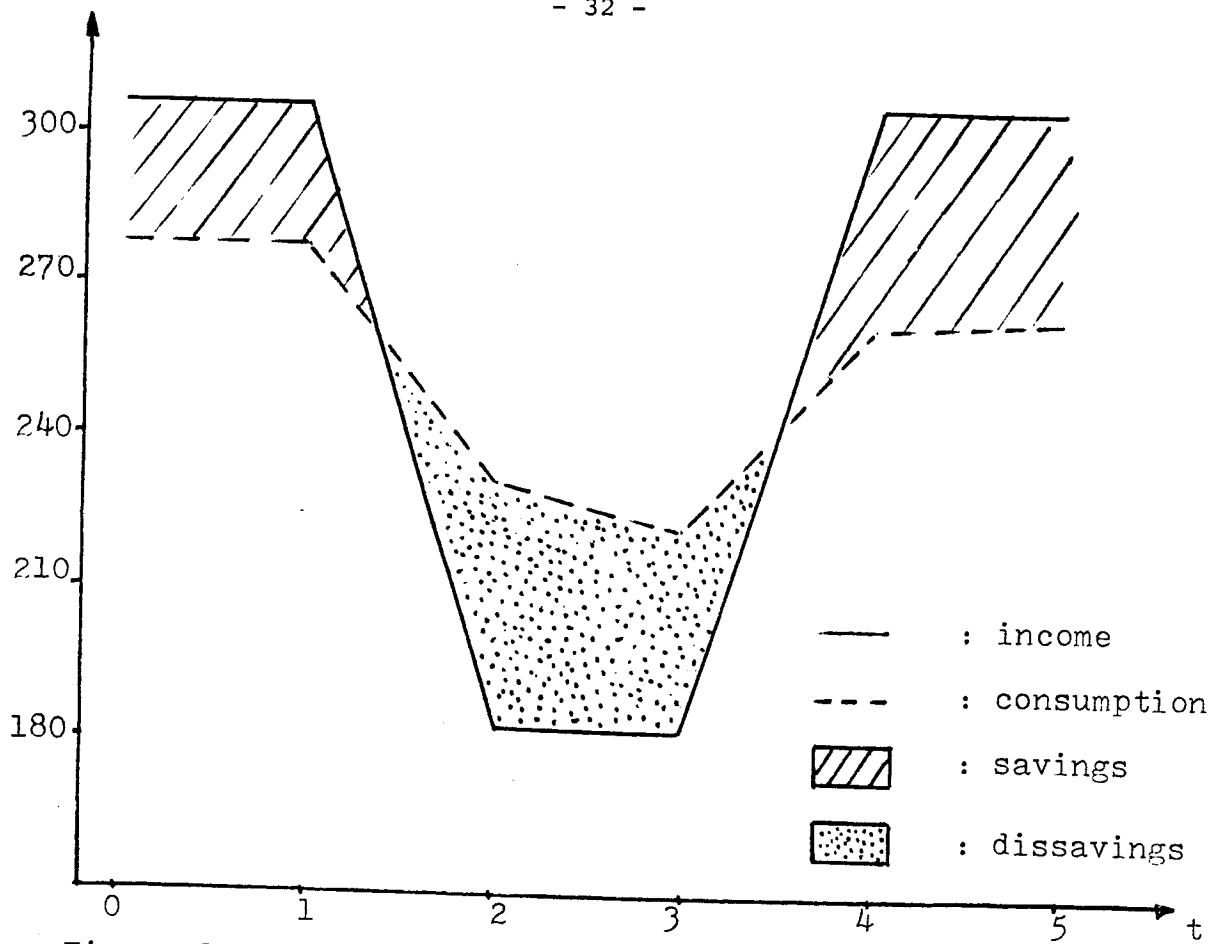


Figure 1 : Effect of unemployment on consumption in the current price model.

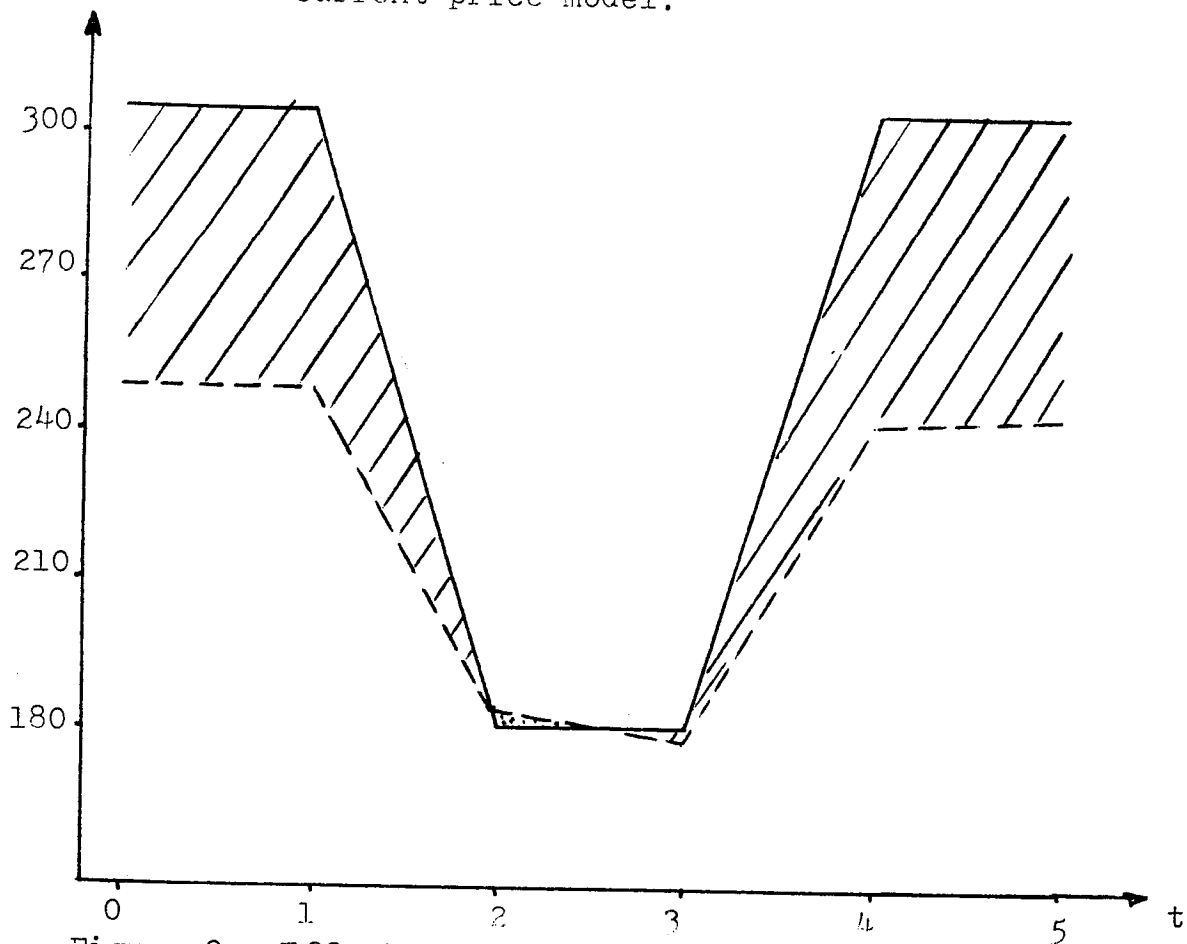


Figure 2 : Effect of unemployment on consumption in the constant price model.

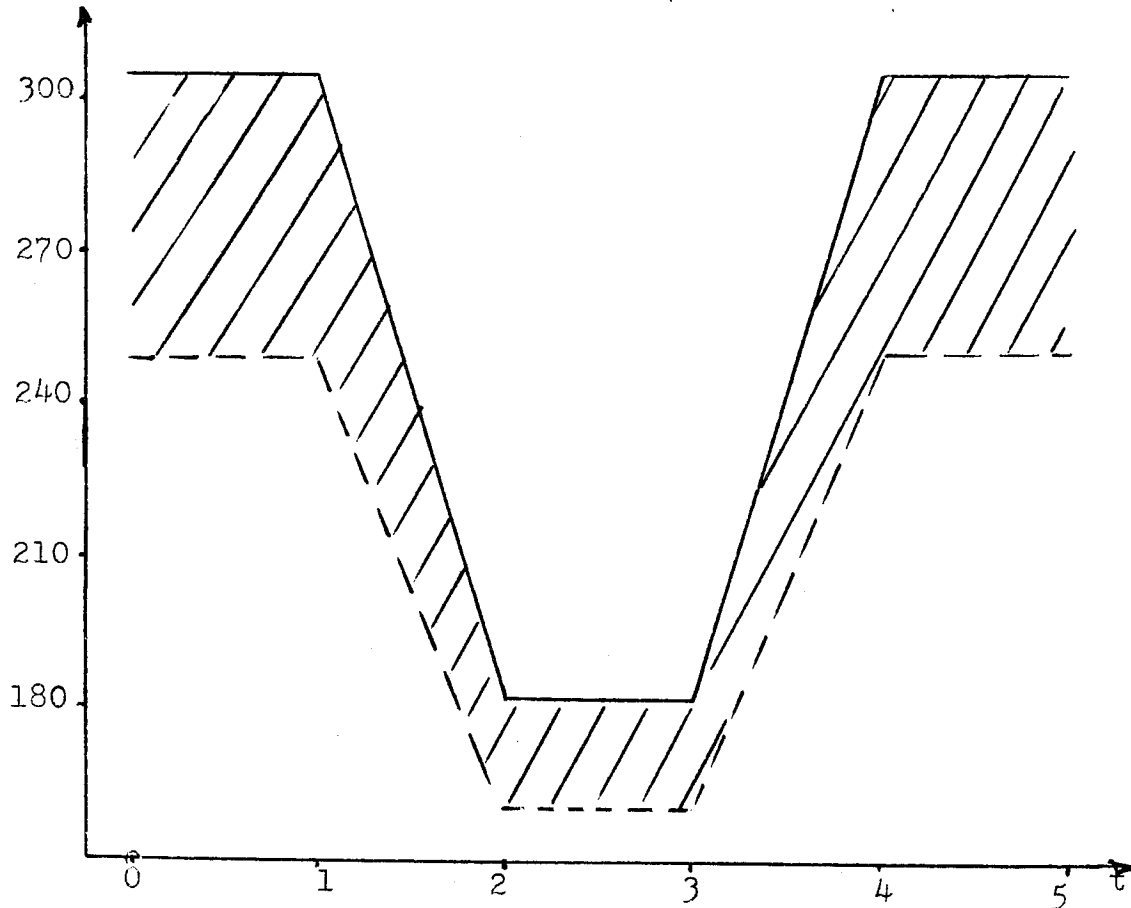


Figure 3 : Effect of unemployment on consumption in the static linear function (2.4).

sake of comparison, Figure 3 shows the evolution of consumption according to the traditional linear consumption function (2.4). It seems even less satisfying than Figure 2. The use of income data from years before 1975 would reveal the same discrepancies between the alternative models. However in the constant price model, the estimated consumption of a household experiencing its first year of unemployment always remain below the minimum expenditure level γ_1 during approximately the first ten years of the sample period. This also seems to favor the current price formulation.

VII. CONCLUSION

Deriving the effect of involuntary unemployment on the consumer's behavior from aggregate time-series data is a perilous exercise. It is much like trying to solve a system containing more variables than equations: it cannot be done without the use of extraneous information. In this paper, I tried to use as much as possible the information provided by economic theory. This leads of course to strong unrealistic assumptions on individual preferences and income distribution, but it also turned out that simply avoiding those difficulties might result in even more stringent implicit assumptions and in uninterpretable relationships. On the contrary, introducing them explicitly allowed us to obtain a sensible representation of the unemployment effect and also to explain some past failures. Confirmation from time-series of other countries or from cross-section data should now be desirable.

APPENDIX and DATA SOURCES

1. Basic Data

The following data were taken from Belgian National Accounts:

- CO - private consumption (including durables) at constant prices
(in thousands of 1970 B.F.)
- CSU - contributions to Social Security (in thousands of B.F.)
- CU - private consumption (including durables) at current prices
(in thousands of B.F.)
- DTU - direct taxes (in thousands of B.F.)
- EIU - private entrepreneurial income (in thousands of B.F.)
- PIU - private property income (in thousands of B.F.)

TU - social security and government transfer to the private sector
(in thousands of B.F.)

WBU - wage bill (in thousands of B.F.)

Labor statistics were kindly provided by the Belgian "Bureau du Plan."

AHW - average effective working time in quarries and manufacturing
industries (in thousands of hours per year)

E - number of persons engaged in production, including partially
unemployed persons (in millions)

UC - number of fully unemployed persons (in millions)
These data have been taken from SCHUTTRINGER-TOLLET (1978) who
corrected the original data for changes in the counting procedure

UP - number of partially unemployed persons, in terms of full unemploy-
ment equivalent (in millions)

Data on unemployment benefits were taken from the Belgian "Annuaire
Statistique" and the 1976 "Rapport annuel de l'ONEM."

2. Derived data

DTR - direct taxation rate; $DTR = DTU / (PIU + WBU + EIU - CSU)$

HW - labor supply (in thousands of hours per year): $HW = AHW \cdot E \cdot NA / (E - UP)$

NA - available labor force (in millions); $NA = E + UC$

P - private consumption price index; $P = CU / CO$

RAU - net unearned income (in thousands of B.F.): $RAU = (1 - DTR)PIU + TU$

UR - unemployment rate: $UR = (UC + UP) / NA$

W - net hourly wage rate: $W = (1 - DTR)(WBU + EIU - CSU) / (AHW \cdot E)$

FOOTNOTES

- 1 Throughout the paper, postfix "0" or "U" will refer to quantities at constant or current prices respectively. Accordingly, $CU = CO \cdot P$ and $HWU = HWO \cdot P = HW \cdot \frac{W}{P} \cdot P$, where P and W refer to consumption prices and nominal wages respectively.
- 2 The effective demand for commodity j is said to be of the Clower type if it is a function of the quantity constraints prevailing on all the other markets except the one prevailing on market j itself.
- 3 This may seem an unusual way to introduce unemployment benefits as actual compensations are calculated by reference to the number of hours worked by employed workers and not by reference to the number desired by unemployed people themselves. However, we shall later on be forced to assume that all workers have identical preferences and non-labor income (assumptions A14 - A15) and this will eliminate any discrepancy between both measures.
4. Aggregate values are represented by simply deleting subscript i .
5. For the sake of simplicity, Vankeerberghen used data on aggregate expenditures instead of aggregate consumption (which excludes purchases of durables). Moreover she performed the estimation on data divided by total population N_t . Results are expressed in terms of available labor force by multiplying the first coefficient by the average value of the ratio N_t/NA_t . Standard errors are given between parenthesis for the unchanged coefficients only.
6. As all this section is concerned with the behavior of a single household, the subscript i has been dropped. This should not be interpreted as a use of aggregate values. I use the time indice τ instead of t to emphasize Lluch's distinction between planning and historical time.
- 7 It is to be noted that in this rationing context, aggregating individual dynamic functions is not equivalent to dynamizing an already aggregated function.
- 8 Elasticities have been computed using the formulas derived in Section V. In each of them, the average observed values of CO and HW (660.4 and 7.332 respectively) have been replaced by

$$CO = (1 - \mu\beta)\gamma_1 NA + \mu\beta(RAO + \gamma_2 NA \cdot \frac{W}{P})$$

$$HW = (1 - \mu(1 - \beta))\gamma_2 NA - \mu(1 - \beta) \left(\frac{RAU}{W} - \gamma_1 \frac{P}{W} NA \right)$$

where the regressors are given their average observed values. This allows us to avoid the effect of permanent real growth or inflation on CO and HW . Values obtained are

$$CO = 834.39 \quad HW = 8.758$$

$$\text{and } CO = 692.015 \quad HW = 7.329$$

with the first and second model respectively.

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