ON THE ECONOMETRICS OF QUANTITY RATIONING MODELS

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INTRODUCTION

Following the lead of Keynes (1936), Patinkin (1965, chap. 13), Clower (1965), mathematical economists have tried to formalize all the consequences of removing the assumption that trading never occurs out of a Walrasian equilibrium. This means that trading will take place even though the value of desired purchases is not equal to the value of desired sales and that realized transactions are determined by some allocation procedure or rationing scheme. Early works in that domain are Dreze (1975) and Benassy (1975) which analyze under which conditions an equilibrium with quantity rationing may exist. At the same time and though non-Walrasian economic theory was (and still is) far from satisfactory, econometricians tried to implement the new concepts in small econometric models, the so-called disequilibrium models. The word "disequilibrium" refers to the fact that demand and supply are not equal. Ironically enough, these models actually depict true equilibrium situations arising in the context of quantity rationing models. To avoid any confusion, the latter terminology is then to be preferred. One market partial equilibrium models were studied first and a maximum likelihood estimation procedure was proposed by Maddala-Nelson (1974). Later Ito (1977) and Gourieroux-Laffont-Monfort (1977) extended the model to the two market case and derived the corresponding likelihood function, taking account of the interactions between markets induced by the rationing (spill-over effects).

The purpose of this paper is two-fold. In the first place, it will seek to give an account of the theoretical foundations available to econometricians to build quantity rationing models (henceforth QRM). This is done in Section 1. Special emphasis is given to two recent works. The first one, Benassy (1977) develops the concept of perceived rationing scheme. The use of this concept enables us in Section 1.1 to formalize a general framework encompassing various existing theoretical models as special cases. The general model is characterized by three components, namely the effective demand, the rationing scheme and the perceived rationing scheme. The second paper, Svensson (1977) stresses the difficulties encountered in developing the concept of effective demand. It shows that the effective demand will not be uniquely defined unless some uncertainty and/or transaction costs are introduced (Section 1.2).

The second purpose of the paper is to exploit the theoretical concepts presented in Section 1 and to develop a framework for econometric two-market QRM. Two different strategies are possible. The first one is precisely the one that has been popularized by Gourieroux et al. (henceforth GLM) and Ito. The difficulty of specifying the three components mentioned above is escaped by the assumption that transactions never occur out of equilibrium and that the rationing scheme is efficient. In this way rationing scheme and perceived rationing scheme can be represented by simple identities and one is able to specify a QRM that remains rather general without becoming complicated. Unfortunately, this approach will be shown to entail unacceptable shortcomings (Section 2.1). The main defect, resulting from the equilibrium assumption and the subsequent loss of information, is that it will no longer be possible to derive a well-defined effective demand concept. Depending on the particular concept (arbitrarily) chosen by the econometrician, the empirical results may well be very different. Moreover it will not be easy in the GLM-Ito framework to account for a well-known phenomenon like frictional unemployment or to allow the possibility that, if traders have pessimistic expectations, the economy may get stuck at an inefficient equilibrium though a better one is compatible with prevailing prices. Last but not least, the estimation procedure associated with the GLM-Ito formulation is far from trivial and might well prevent the estimation of realistic two-market models.

The shortcomings of the traditional approach suggest an alternative strategy, presented in Section 2.2. The equilibrium assumption is removed and specifications are proposed for the rationing scheme and the perceived rationing scheme. This is done at some cost and does not allow us to stay at the level of generality enjoyed by GLM-Ito. However, by introducing uncertainty and transaction costs along the lines proposed by Svensson, one obtains a uniquely defined effective demand concept. The estimation method associated with this respecification is based upon Tishler-Zang (1978) and seems practically feasible. Other defects mentioned with the traditional approach can be avoided too. Section 2.3 concludes with a few remarks. It is shown that, despite its lack of generality, the new model seems to be an interesting starting point for future developments and that it is at least well-fitted for the estimation of an annual macroeconomic model.

1. THEORETICAL FOUNDATIONS

Assuming that an econcmy is permanently at "the" Walrasian equilibrium has very powerful consequences. For example, the behavior of any consumer given some prices can be inferred from knowledge of only his tastes and endowments. Otherwise stated, prices are all that a consumer needs to know to have a correct appraisal of his economic environment and make the best decisions. Decisions made by other consumers can influence neither his own choices, nor the particular way in which a market is organized. Moreover, as prices are defined as those equating supplies and demand, no explanation needs to be given as to how they are obtained. Any information about the institutional setting is superfluous provided we know that trading never occurs out of Walrasian equilibrium.

Unfortunately, reality is not that nice and actual prices will not generally be the Walrasian ones, i.e., the aggregate excess demands will generally not be zero and at least some markets will not clear. This will involve us in a much closer description of the working of the economy. The existence of non-Walrasian situations implies that not all trade offers can be satisfied, that the quantity transacted by a consumer may not coincide with his demand (or supply) and will remain unknown until we specify how the prevailing rationing scheme allocates available resources among agents. Additional information will also be required with respect to the consumer himself. We now have to know the way he will react to the existence of quantity rationing. A consumer who expects not to be able to buy the car he wants will decrease his purchase of gasoline but may also well replace the car by a substitute like a motorcycle if the waiting period appears too long. The mere knowledge of tastes and endowments is no longer sufficient to describe the behavior of the consumer. A correct representation will be impossible unless we specify how the agent reacts to the quantity signals he perceives and how these signals are generated. Finally, additional information is also needed about price formation. As prices can no longer be defined as those equating supply and demand, we must specify in another way how they become what they are. In some cases, for example, one could assume they are fixed by sellers while buyers act as price takers.

When mathematical economists started studying the properties of the Walrasian equilibrium, they did not claim that our economies are always at such an equilibrium. It was merely supposed that if a stationary state was to be reached, it should be a Walrasian equilibrium. The question whether a stationary state could be obtained or not was therefore identical to that of the existence or non-existence of a Walrasian equilibrium, while the question whether that stationary state was likely to be reached or not coincided with the question of the stability of the Walrasian equilibrium. However, once the economy is more carefully described and the above concepts (rationing scheme, quantity signals, price formation) introduced, it is not at all clear that the concept of Walrasian equilibrium remains the most natural one. Once we consider quantity rationing models, we may happen to observe a stationary state that is not the Walrasian one, even if prices are flexible. Developing a relevant equilibrium concept in this framework is not a straightforward matter. Much theoretical work is still to be done in that area.

In the sequel, I shall always restrict myself to pure quantity rationing models where price formation is left unexplained or, more simply, where prices are fixed. Even in this restricted domain there remain many unsolved problems and much disagreement in the way models are built. However, it appears that exploiting the concept of "perceived rationing scheme" developed by Benassy (1977) allows presentation of most existing models as special cases of a general one. This will be done in Section 1.1 and, it is hoped, will give a better understanding of what quantity rationing models really are. Section 1.2 is then devoted to the problem of getting well-defined effective demand (or supply) functions. While this is not necessarily a difficulty for equilibrium analysis, it may turn out to be a crucial one for the econometrician trying to implement a quantity rationing model.

1.1 The Structure of Quantity Rationing Models

a. Institutional setting: the rationing scheme

It will suffice for our purpose to restrict one's attention to the simpler case of an exchange economy. Introduction of a production sector would unnecessarily complicate the notation without adding any insight into the model. There are N non-monetary commodities, indexed $n = 1, 2, \ldots, N$.

Commodity 0 is money which is the sole medium of exchange: "money buys goods and goods buy money; but goods do not buy goods" (Clower (1967)). This allows the assimiliation of goods and markets: on market n, commodity n is exchanged against money. There are only N markets and no market for money which will never be rationed. Money is the sole store of value and acts as a buffer stock. It is always desired. If we normalize the price of money to one, the price vector can be represented by $(1,p)$ where $p \in R_+^N$ is the vector of prices of non-monetary commodities. There are I consumers indexed $i = 1, 2, \ldots I$. On market n, consumer i submits an offer z_{in} where

$z_{in} < 0$ means an offer to sell,

$z_{in} > 0$ means an offer to buy .

Trade offers of all the agents on market n are represented by the vector $z_n = (z_{1n}, \ldots, z_{in}, \ldots z_{In}) \in R^I$. Similarly the vector $z = (z_1, \ldots z_n, \ldots z_N) \in R^{IN}$ will be the vector of trade offers made by all the agents on all the N markets. The aggregate effective demand for commodity n, D_n, is obtained by

$$D_n = \sum_{i=1}^{I} \max (z_{in}, 0)$$

The corresponding aggregate supply is in absolute value:

$$S_n = \sum_{i=1}^{I} \max (-z_{in}, 0)$$

so that aggregate excess demand, defined by $EX_n = D_n - S_n$ will be positive if there is a supply shortage and negative if there is a demand shortage. Let us denote x_{in} the quantity actually transacted by agent i on market n, $x_n = (x_{1n}, \ldots, x_{In}) \in R^I$ the vector of all transactions realized on that market and finally $x = (x_1, \ldots x_N) \in R^{IN}$ the vector of all transactions on all markets.

The determination of x_{in} is not a problem in Walrasian models. By the definition of prices we know that the identities $EX_n = 0$ and $x_{in} = z_{in}$ will be satisfied for all i and n. In fix-price models, this will not generally be the case. If for example market n is characterized by a positive aggregate excess demand $(EX_n > 0)$, then obviously all the demands for product n cannot be satisfied. We then have to describe how the exchange process will take place. The allocation of the available quantity S_n among buyers can be done in numerous ways. The following example is taken from Benassy (1975). It considers that the distribution of available supply is organized as a priority system. There are J demanders, indexed $j = 1, 2, \dots J$. Assume the priority order coincides with the ranking $1, \dots, J$ (i.e., demander $(j-1)$ has priority over demander j). The quantity actually transacted by demander 1 is determined by:

$$x_{1n} = z_{1n} \qquad \text{if} \quad z_{1n} \leq S_n$$

$$= S_n \qquad \text{if} \quad z_{1n} > S_n$$

or more compactly

$$x_{1n} = \min(z_{1n}, S_n) .$$

when demander 2 comes to the market, the quantity still available for him is $(S_n - z_{1n})$ if $z_{1n} < S_n$, zero otherwise. His transaction is then determined by

$$x_{2n} = \min(z_{2n}, \max(0, Sn - z_{1n}))$$

In the same way, the transaction of demander j is

$$x_{jn} = \min(z_{jn}, \max(0, Sn - D_{jn})$$

where $D_{jn} = \sum_{j'<j} z_{j'n}$.

Obviously this allocation procedure is efficient, in the sense that all available quantities are sold. The quantity transacted by any supplier k is then simply $x_{kn} = z_{kn}$. If we now take the reverse assumption that market n is characterized by a lack of demand $(EX_n < 0)$, realized transactions will similarly be determined by

$$x_{jn} = z_{jn}$$

$$x_{kn} = - \min (-z_{kn}, \max (0, D_n - S_{kn})$$

where $S_{kn} = - \sum_{k'<k} z_{k'n}$. More compactly x_{jn} and x_{kn} can be written as the following functions:

$$x_{jn} = f_{jn} (z_{jn}, D_n, S_n, D_{jn})$$

$$x_{kn} = f_{kn} (z_{kn}, D_n, S_n, S_{kn})$$

This is one possible allocation procedure.Many others could be imagined. Without loss of generality (in our fix-price model), we may define:

D1.1. The rationing scheme operating on market n is a list of (possibly random) functions f_{in} whose arguments are the desired net trades z_{in} and which determine the quantity transacted by every agent i as

$$x_{in} = f_{in} (z_n)$$

such that

(i) $|f_{in}| \leq |z_{in}|$ and $f_{in} z_{in} \geq 0$ with probability one

(ii) $\sum_i f_{in} = 0$ with probability one.

The first restriction states that nobody can be forced to exchange more than he wishes, nor can be be forced to buy when he wants to sell or vice-versa (voluntary exchange restriction). The second one means that the rationing scheme succeeds in organizing the exchange process when supplies and demands are not equal (feasibility restriction). A rationing scheme will be called <u>deterministic</u> (as opposed to stochastic) when the functions F_{in} are non-random; it will be called <u>non-manipulable</u> when each agent i faces exogenous bounds on his trade. Given the upper- and lower-bounds $\bar{z}_{in} > 0$, $\underline{z}_{in} < 0$, a non-manipulable deterministic rationing scheme can be written as:

$$x_{in} = \min (\bar{z}_{in}, \max (z_{in}, \underline{z}_{in})) .$$

The priority system presented above is a member of this family which satisfies (i) and (ii). When the bounds $(\underline{z}_{in}, \bar{z}_{in})$ can be modified by the agent's trade offer z_{in} , the procedure is called manipulable. An example is the strictly proportional rationing scheme, which sets the quantity transacted by a buyer j equal to:

$$x_{jn} = z_{jn} \qquad \text{if } EX_n \leq 0$$

$$= z_{jn} \cdot \frac{S_n}{D_n} \qquad \text{if } EX_n > 0$$

and similarly for a seller k . As suggested by subscript n , the allocation procedure will in general vary from market to market. I shall denote $f_i(z)$ the vector of all rationing schemes relating to consumer i :

$$x_i = f_i(z) = (f_{i1}(z_1), \ldots, f_{in}(z_n), \ldots, f_{iN}(z_N))$$

The examples given above have the merit of illustrating what is meant by the function f_{in} . It should be remembered however they are a very poor illustration of the very complex and highly decentralized procedures that take place in actual economic systems. This point will be stressed again when discussing the efficiency and the econometric representation of actual rationing schemes.

b. Information structure of the consumer: the perceived rationing scheme

Let $m_i^0 > 0$ be the initial money holdings of consumer i and $W_i = (W_{i1}, \ldots W_{in}, \ldots W_{iN}) \in R_+^N$ the vector of his endowments in the commodities $1, \ldots n, \ldots N$ at the beginning of each period. We assume that the agent cannot store goods (which excludes durables from our analysis) and that his preferences can be represented by the strictly concave strictly increasing utility function

$$u_i \; (m_i, W_i + x_i)$$

where $x_i \in R_+^N$ is the vector of realized net trades. This single period utility function must be interpreted as being derived, through a backward dynamic programming technique, from the intertemporal optimization of a multi-period utility function in a world of uncertainty where money acts as a store of wealth. It is an indirect utility function in which money enters only as a result of this optimization over future periods. It is a short-run utility function in that it is conditional on expectations about future prices and constraints. In particular, if expectations about future constraints depends on current constraints, the form of u and the indirect utility of money will also be affected by those constraints. The correct formulation then becomes:

$$u_i \; (m_i, W_i + x_i \mid \sigma_i)$$

where σ_i are the quantity signals currently perceived by consumer i and will be defined below.

In a model without quantity rationing, this information would be sufficient to derive the set of optimal trade offers. In that case we know $x_i = z_i$ and the set of optimal z_i is simply obtained from

$$\max_{z_i} u_i\ (m_i, W_i + z_i)$$

subject to

$$m_i = m_i^0 - pz_i \geq 0$$

$$W_i + z_i \geq 0 .$$

The problem would not be more difficult in a quantity rationing model, provided we assume the agent has perfect knowledge of both the prevailing rationing scheme and the trade offers made by the other consumers. For then agent i knows that $x_i = f_i(z) = f_i(z_i, z_{\hat{i}})$ where $z_{\hat{i}} = (z_{\hat{i}1}, \ldots, z_{\hat{i}n}, \ldots, z_{\hat{i}N})$ and $z_{\hat{i}n} = (z_{1n}, \ldots, z_{i-1n}, z_{i+1n}, \ldots, z_{In})$. Assuming f_i is deterministic, his optimization problem, conditional on $z_{\hat{i}}$, is now:

$$\max_{z_i} u_i\ (m_i, W_i + f_i(z_i, z_{\hat{i}}))$$

subject to

$$m_i = m_i^0 - pf_i(z_i, z_{\hat{i}}) \geq 0$$

$$W_i + f_i(z_i, z_{\hat{i}}) \geq 0 .$$

This is of course a strong assumption. The agent may not know the true rationing schemes and/or the trade offers made by other consumers. A more realistic procedure is to introduce the possibility of imperfect knowledge of both f_i and $z_{\hat{i}}$. For this purpose, Benassy (1977) introduced a concept very similar to the perceived demand curve of monopoly theory, the concept of perceived rationing scheme.

D1.2 The perceived rationing scheme ϕ_{in} gives the transaction χ_{in} agent i expects to be able to realize on market n ; it is a (possibly random) function of the agent's trade offer z_{in} and of any relevant information σ_{in} (quantity signals) he has on the situation in that market:

$$\chi_{in} = \phi_{in}(z_{in}, \sigma_{in})$$

Function ϕ_{in} satisfies the voluntary exchange restriction:

(i') $|\phi_{in}| \leq |z_{in}|$ and $\phi_{in} z_{in} \geq 0$ with probability one.

In other words, ϕ_{in} is the subjective perception of f_{in} by agent i. Notice that the perceived rationing does not have to satisfy the feasibility restriction (ii). The perfect knowledge assumption corresponds to the special case

$$\phi_{in} = f_{in} , \qquad \sigma_{in} = z_{\hat{i}n}$$

Imperfect knowledge may originate in $\phi_{in} \neq f_{in}$ (for example, a manipulable rationing scheme may be perceived as non-manipulable) and/or in $\sigma_{in} \neq z_{\hat{i}n}$. The latter expression may be misleading however. For example, if the rationing scheme is deterministic and non-manipulable:

$$x_{in} = \min (\bar{z}_{in}, \max (z_{in}, \underline{z}_{in}))$$

and if it is perceived as such $(\phi_{in} = f_{in})$, the only additional information consumer i needs to have to enjoy perfect knowledge is $(\underline{z}_{in}, \bar{z}_{in})$. In this case, perfect knowledge will be written $\sigma_{in} = (\underline{z}_{in}, \bar{z}_{in})$; the consumer does not need to know the trade offer of each agent individually, the only relevant information to him is the set of fixed bounds he faces. Whatever the form of the rationing scheme, the expression $\sigma_{in} = z_{\hat{i}n}$ should then be interpreted as meaning merely that consumer i knows accurately the parameters of f_{in} or ϕ_{in} relevant to him. In most cases however, his information will be imperfect. The only information available to a trader will usually emerge out of his own past experience; this can be expressed by

$$\sigma_{in} = g_{in}\ (x_{in}\ (t-1))$$

or as

$$x_{in}\ (t-1) = f_{in}\ (z_n(t-1))\ :$$

$$\sigma_{in} = g^{*}_{in}\ (z_n\ (t-1))\ .$$

Obviously it is not necessarily implied that trader i knows all the elements of vector $z_n\ (t-1)$. Notice also that writing this expression defines the information structure of a consumer on a market by market basis. Nobody has enough information to know that changing his trade offer on some market may have an indirect effect (through the behavior of other agents) on his realized trades on other markets and result in a globally preferable situation. There is no exchange of information between traders and the information structure is then non-cooperative. No coalition will be formed that could prevent the existence of globally inefficient rationings. Finally, let us stress that both functions ϕ_{in} and g_{in} are subjective data which characterize individual i and will not be revised through time. It does not entail of course that the amount of information $\sigma_i = (\sigma_{i1} \ldots \sigma_{in} \ldots \sigma_{iN})$ itself is time-independent.

It is now straightforward to determine the optimal trade offers (or strategies) of a consumer. They result from the optimization program:

$$\underset{z_i}{\text{Max}}\ E[u_i(m_i,\ W_i + \phi_i(z_i,\ \sigma_i))]$$

subject to

$$m_i = m_i^0 - p\phi_i(z_i,\ \sigma_i) \geq 0 \quad \text{with probability one}$$

$$W_i + \phi_i(z_i,\ \sigma_i) \geq 0 \quad \text{with probability one}$$

As an illustration, we may consider the case of a particular consumer j who is a purchaser of two commodities 1 and 2 $(w_{j1} = w_{j2} = 0,\ m_j^0 > 0)$. The set of bundles he could actually afford is represented in Figure 1 by the triangle OBE where the line BE is defined by the set of (z_{j1}, z_{j2}) satisfying

$$m_j^0 = p_1\ z_{j1} + p_2\ z_{j2}$$

Assume also that the rationing schemes operating on both markets are of the priority type, as described in Section 1.1.a, and that our consumer is aware of it. Yet he does not have perfect knowledge of the upper-bounds facing him, but simply knows (or believes; he may be wrong) these upper-bounds are somewhere between A and C for commodity 1, G and F for commodity 2. This uncertainty can be represented by any probability distribution defined on those intervals. With these assumptions, the set of admissible trade offers K_j is shown in Figure 1 while the perceived rationing schemes perform the following mapping from the space positive trade offers z_j to the space of expected transactions X_j :

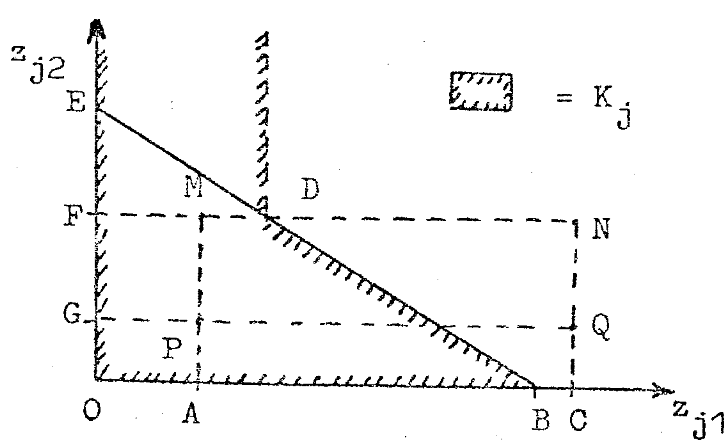

Figure 1

$$\begin{array}{lll}
\text{if } z_j \in OGPA, & \text{then} & \chi_j = z_j \\
\text{if } z_j \in GFMP, & \text{then} & \chi_{j1} = z_{j1}, \\
 & & G \leq \chi_{j2} \leq z_{j2} \\
\text{if } z_j \in APQC, & \text{then} & A \leq \chi_{j1} \leq z_{j1} \\
 & & \chi_{j2} = z_{j2} \\
\text{if } z_j \in PMNQ, & \text{then} & A \leq \chi_{j1} \leq z_{j1} \\
 & & G \leq \chi_{j2} \leq z_{j2} \\
\text{if } z_j \notin OFMNC, & \text{then} & A \leq \chi_{j1} \leq C \\
 & & G \leq \chi_{j2} \leq F .
\end{array}$$

When multiple outcomes are possible, the probability associated with each of them is determined by the joint probability distribution defined on the perceived upper-bounds. I now give the following general definition:

D1.3 Given some quantity signals σ_i, the set of optimal trade offers for consumer i is:

$$d_i(\sigma_i) = \{z_i \varepsilon K_i | E[u_i(m_i, W_i + \phi_i(z_i, \sigma_i))] \geq E[u_i(m_i', W_i + \phi_i(z_i', \sigma_i))] \ \forall \ z_i' \varepsilon K_i\}$$

As σ_i is a function g_i^* of $z(t-1)$, we can rewrite the effective demand correspondence as:

$$z_i \ \varepsilon \ d_i^*(z(t-1))$$

Any $z_i \ \varepsilon \ d_i(\sigma_i)$ is an optimal action which maximizes the expected utility of its consequences, the realized transactions. The definition of $d_i(\sigma_i)$ implies that an agent will never make a trade offer that embodies a positive subjective probability of bankruptcy. If ϕ_i, σ_i are such that any trade offer z_i has a positive chance to be satisfied, then the constraint

$$(m_i - m_i^0) + p_i \phi_i(z_i, \sigma_i) = 0 \qquad \text{with probability one}$$

implies

$$(m_i - m_i^0) + p\, z_i = 0$$

from which follows that the Walras law will be satisfied. This will not generally be the case, however.

c. Equilibrium analysis

In the preceding two paragraphs, we defined three functions f_i , g_i , d_i connecting three variables z_i , x_i , σ_i . These relations are illustrated in Figure 2. When a consumer has made a trade offer z_i , the rationing schemes operating on the N markets determine his actual transaction x_i . This result is used by the consumer as information about the situation prevailing on the markets and is translated into a set of quantity signals σ_i . Conditional on this new information, the consumer calculates a new trade offer maximizing his expected utility. This sequence is summarized in the simple expression $z_i \in d_i^*(z(t-1))$. The appropriate equilibrium concept is now straightforward:

D1.4. A list of trade offers $z^* = (z_1^*, \ldots, z_i^*, \ldots, z_I^*)$ is an equilibrium with quantity rationing if and only if $z_i^* \in d_i^*(z^*)$ for all i .

i.e., the equilibrium is defined as a stationary point in a dynamic process. At that point, no agent has an incentive to modify his trade offer; the actions of

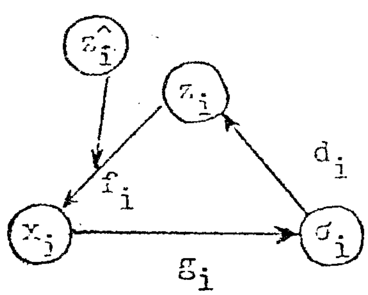

Figure 2: The structure of quantity rationing models.

the agents induce the same signals as before. This does not necessarily mean that at an equilibrium point agents enjoy perfect knowledge. As stressed by Hahn (1978), "the circumstance that the market signals that the agent has not made a mistake does not ensure that he is in fact not mistaken." Moreover, the precise equilibrium point that will be obtained depends upon the specification of ϕ_i and g_i as well as of u_i, w_i.

Definition D1.4. is rather general and existence theorems are not likely to be established for any specification of (f_i, ϕ_i, g_i). So far, only a few particular cases have been studied in detail. I shall briefly introduce three of them. In Bohm-Levine (1976), f_{in} may be any deterministic (manipulable or non-manipulable) rationing scheme, with the exception of strictly proportional schemes. The information structure of the agents is defined by:

$$\phi_{in} = f_{in} \qquad \forall \ i,n$$

and (at least at the equilibrium state):

$$\sigma_{in} = \hat{z}_{in} \qquad \forall \ i,n \ .$$

In other words, the functions g_{in} are such that a stationary state implies perfect knowledge of all the agents. Then a non-trivial $(z \neq 0)$ equilibrium will always exist provided f_{in} satisfies restrictions (i) and (ii) but also:

(iii) if $z_{in} EX_n < 0$, then $x_{in} = z_{in}$

This restriction states that an agent on the short side of the market is always able to realize his desired transaction. This is a kind of market by market efficiency assumption on the rationing schemes. Alternatively, it can be said that the markets are frictionless. Market by market, all possibilities of trade are exhausted so that buyers and sellers will never be rationed at the same time on the same market. This is not sufficient however to prevent the existence of globally inefficient equilibria. It allows one to classify the N markets according to the following characteristics:

- market n is a sellers' market if $EX_n > 0$
- market n is a buyers' market if $EX_n < 0$
- market n is a balanced market if $EX_n = 0$

Though efficient rationing schemes seem an acceptable characteristic of markets in equilibrium, it is questionable as a general requirement. A market should not be conceived as a central clearing-house with an auctioneer organizing the exchange process. On the contrary, trading is highly decentralized and the search of buyers by sellers or vice versa is a costly and time consuming activity so that (iii) need not be satisfied. Green (1978) is compatible with the latter point of view. Emphasizing the importance of aggregate market conditions in the determination of an equilibrium, Green considers a rationing procedure like:

$$x_{in} = f_{in}\ (z_{in}, D_n, S_n)$$

where f_{in} is stochastic and subject to (i), (ii) and

(iv) the distribution of f_{in} is the same for all i for each value of the arguments.

The latter requirement is one of anonymity. As in Bohm-Levine true and perceived rationing schemes coincide $(\phi_{in} = f_{in})$ and g_{in} is defined to ensure perfect knowledge at a stationary state. In this case, perfect knowledge simply means knowledge of aggregate demand and supply:

$$\sigma_{in} = (D_n, S_n) \qquad \forall\ i$$

This is one advantage of Green's definition of f_{in}. Green proceeds further showing his definition of f_{in} together with (i), (ii) and (iv) implies:

$$f_{in}(z_{in}, D_n, S_n) = z_{in} \cdot f^*_{in}(z_{in}, D_n, S_n)$$

where f^*_{in} depends on z_{in} only through its sign. In words, the rationing scheme must be of the manipulable type; the assertion that actual transactions are mainly determined by the value of aggregate demands and supplies is inconsistent with the assertion that the rationing schemes are non-manipulable. This suggests respecifying the model in a way similar to:

$$x_{in} = f_{in}(z_{in}, z_{\hat{i}n})$$

$$\chi_{in} = \phi_{in}(z_{in}, \sigma_{in}) \qquad \text{with} \quad \sigma_{in} = (D_n(t-1), S_n(t-1))$$

This specification retains the interesting feature of traders acting on the basis of their knowledge of aggregate conditions while it avoids the limitation to manipulable (true or perceived) rationing schemes. Because ϕ_{in} is not subject to restrictions (ii) and (iv), Green's result no longer applies. Of course, this reformulation will generally imply $\phi_{in} \neq f_{in}$ so that traders will never have perfect knowledge, even if, at a stationary state, they know the true values of aggregate demand and supply $(\sigma_{in} = (D_n, S_n))$. Benassy (1975) is, as far as I know, the only rationing model allowing imperfect information of traders at an equilibrium. The rationing scheme f_{in} may be any deterministic function satisfying (i), (ii) and (iii), but is always perceived (perhaps wrongly) as non-manipulable:

$$\Phi_{in}(z_{in}, \sigma_{in}) = \min(\bar{z}_{in}, \max(z_{in}, \underline{z}_{in}))$$

$$\sigma_{in} = (\underline{z}_{in}, \bar{z}_{in})$$

where $(\underline{z}_{in}, \bar{z}_{in})$ are the <u>subjective</u> lower- and upper-bounds a consumer perceives on his trade possibilities. These expectations are held with certainty, so that the g_{in} functions which generate them must be deterministic. Benassy assumes they satisfy the following restrictions:

(a) if $0 < x_{in}(t-1) < z_{in}(t-1)$, then $\bar{z}_{in} = x_{in}(t-1)$

if $z_{in}(t-1) < x_{in}(t-1) < 0$, then $\underline{z}_{in} = x_{in}(t-1)$

(b) if $z_{in}(t-1) = x_{in}(t-1)$, then $\underline{z}_{in} \leq x_{in}(t-1) \leq \bar{z}_{in}$

(c) if $z_{in}(t-1) \cdot EX_n(t-1) < 0$, then $\underline{z}_{in} < x_{in}(t-1) < \bar{z}_{in}$

Let us assume agent i is a net buyer on market $n (x_{in} > 0)$. Condition (a) means that if he happened to be rationed in period $(t-1)$, his perceived constraint in period t must be his last realized transaction. If he was not rationed, condition (b) states that his perceived upper constraint must be at least as large as his last purchase. Finally if he was on the short side of the market (which implies through (iii) that he was not rationed), then the upper-bound he perceives must be strictly larger than his last transaction. To establish the existence of at least one equilibrium in his framework, Benassy arbitrarily picked up one particular effective demand concept (namely the Clower concept, to be defined in section 1.2) among the whole set of possible values $d_{in}(\sigma_{in})$. This is a weak point; there is no objective reason to prefer one concept to the other. It raises the question, addressed in the next section, of how a model can be further specified as to make d_{in} single-valued.

1.2 Effective Demand Theory

According to the type of perceived rationing scheme used, the effective demand correspondence defined in D1.3 may be subject to two kinds of difficulties. The first difficulty is one of uniqueness and is associated with non-manipulable perceived rationing schemes. The second difficulty arises with manipulable schemes where the optimal trade offer may become unbounded.

The multivaluedness of d_{in} is not necessarily a problem in proving the existence of a rationing equilibrium. It is sufficient for that purpose to prove that the set of optimal trade offers contains at least one offer compatible with an equilibrium. The question whether the consumer would actually choose that particular value is not considered. However, it may turn to be a crucial one for future development of quantity rationing models. Price formation theory, for example, may need a reliable measure of aggregate excess demand. If we are to avoid the use of an arbitrary concept, we must consider the problem of how to specify ϕ_{in}, σ_{in} as to make $d_{in}(\sigma_{in})$ single-valued. No general theory is available so far. A very specific but illuminating case has been extensively studied by Svensson (1977). As his results constitute a useful introduction to problems encountered in section 2, I shall present them in detail. Svensson considers a three commodity (money plus two goods indexed 1, 2), two market economy and examines the behavior of a consumer whose characteristics (endowment, tastes and perceived rationing scheme) can be summarized in:

A1.1. (a) $m^0 > 0$, $w_1 = w_2 = 0$

(b) $u(m, x)$ is strictly concave, strictly increasing in each argument and twice differentiable: the Hessian of $u(m^0 - px, x)$ is negative definite.

(c) for $n = 1, 2$ $\chi_n = \min(z_n, \bar{z}_n)$ with probability λ_n

$= z_n$ with probability $(1-\lambda_n)$

For convenience, subscript i has been suppressed. Assumption (a) means that the consumer has no initial endowment but money. Accordingly, he will always be a net buyer of both commodities 1 and 2 $(z_n \geq 0, n = 1, 2)$. Assumption (b) describes the consumer's tastes by the usual indirect utility function. Assumption (c) refers to the perceived rationing schemes(1). According to it, the consumer believes the true rationing schemes are on both markets non-manipulable but stochastic. He considers that a demand smaller than the upper-bound $\bar{z}_n$ will certainly be satisfied while a demand superior to it will be satisfied with a probability $(1-\lambda_n)$. We can define $\sigma = (\bar{z}\ \lambda)$ where $\bar{z} = (\bar{z}_1\ \bar{z}_2)$, $\lambda = (\lambda_1\ \lambda_2)$. As from A1.1. (a) z_n is restricted to be positive, the lower-bound $\underline{z}_n$ does not come into play. In the limit case $\lambda = 0$, the consumer believes he will never be rationed; the optimal trade offer is then of course the Walrasian demand, denoted $z^W = (z_1^W\ z_2^W)$. For $\lambda = 1$, the perceived rationing schemes are deterministic. Svensson's results are derived conditional on the following assumption:

A1.2 The perceived constraints are binding $(\bar{z} << z^W)$ and

$$\text{satisfy} \quad \frac{du(m^0-px,\ x)}{d\ x_n}\bigg|_{x_n = \bar{z}_n} > 0 .$$

The case $\lambda = 1$ is analogous to the one found in Benassy (1975). As already mentioned, it does not allow a well-defined effective demand:

<u>Proposition 1</u>. If A1.1 and A1.2 hold together with $\lambda = 1$, then any trade offer z larger than or equal to $\bar{z}$ is an optimal action for the consumer:

$$d(\sigma) = \{z | z \geq \bar{z}\}$$

The so-called Clower and Dreze demand concepts are nothing but two special cases among infinitely many possibilities for formulating effective demand. The Clower demand on a market n denoted z_n^c, results from the maximization of the trader's preferences taking account of all quantity constraints except those prevailing on that market. In the context of Proposition 1, the Clower demand for commodity 1 is the first element of the vector $z = (z_1^c \bar{z}_2)$ obtained from:

$$\max_z \; u(m, z)$$

$$\text{subject to} \quad m = m^0 - p\, z \geq 0$$

$$z_2 \leq \bar{z}_2$$

which does not include any quantity constraint on commodity 1. z_2^c is computed similarly. Given our assumptions, the vector of Clower demands $z^c = (z_1^c\, z_1^c)$ satisfies $z^c \geq \bar{z}$ and is thus a member of $d(\sigma)$. The Dreze demands, denoted $z^d = (z_1^d\, z_2^d)$, are calculated simultaneously for commodities 1 and 2 by taking account of all constraints; they are the optimal solution to:

$$\max_z \; u(m, z)$$

$$\text{subject to} \quad m = m^0 - p \cdot z \geq 0$$

$$z \leq \bar{z}$$

which, given A1.1, is $\bar{z}$. It follows $z^d \varepsilon\, d(\sigma)$. The differences between the two concepts can be illustrated graphically. Figure 3 reproduces a hypothetical situation where the Walrasian demands are on the budget line so that $m^W = m^0 - p \cdot z^W = 0$. If the consumer faces the upper-bounds $\bar{z}$, his Clower demands z_1^c and z_2^c, computed separately as to satisfy the budget constraint, may well jointly violate it. This means the consumer could not even afford the quantities he demands. In our example, z^c is nevertheless

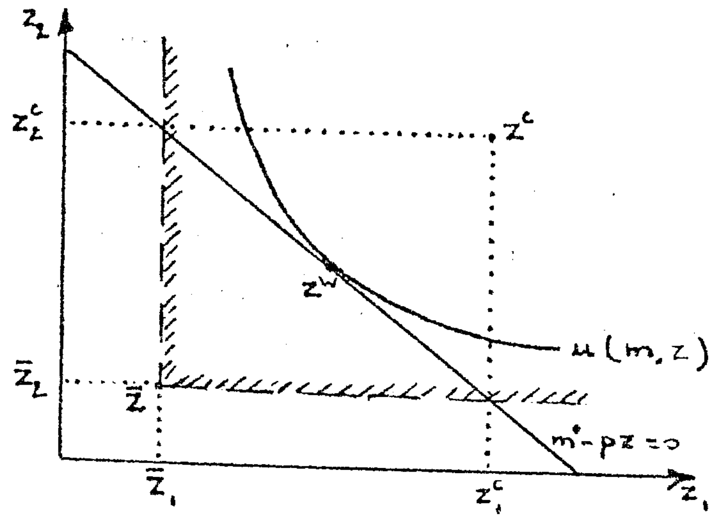

Figure 3: The set of optimal trade offers when $\lambda = 1$.

compatible with the definition of effective demand for the consumer is sure that, whatever his demands might be, he will never receive more than $\bar{z}$. If it was not the case $(\lambda < 1)$, a demand as large as z^c could well bring a positive probability of bankruptcy and therefore be unacceptable. The Dreze concept does not share these shortcomings but fails to give a measure of the discrepancy between demand and supply.

In some particular circumstances, however, the Clower and Dreze demands will make sense. Let us start with the following two results:

Proposition 2. If A1.1 and A1.2 hold with $\lambda_1 = 1$, $\lambda_2 < 1$, the set of optimal trade offers is then given by:

$$d(\sigma) = \{z | z_1 \geq \bar{z}_1, \ z_2 = z_2^c\}$$

Proposition 3. If A1.1 and A1.2 hold and if moreover the consumer is committed to pay prior to the occurrence of any trading a small non-refundable amount proportional to his effective demand, then the effective demand is uniquely defined and is a continuous function of λ satisfying

$$\lim_{\lambda_n \to 1} z_n = z_n^d \qquad n = 1, 2$$

This implies that, provided there are transaction costs as described in Proposition 3, the Dreze demand concept becomes the relevant one as soon as λ is close to unity. The way transaction costs are defined is crucial for the result. Neither a fixed transaction cost nor a down payment that is recovered in case of no rationing would modify the result of Proposition 1. The results of Propositions 2 and 3 can be combined in a special case that will be met in Section 2.2:

Corollary 4. Assume A1.1, A1.2 hold with $\lambda_1 = 1$ (or λ_1 sufficiently close to one), $\lambda_2 < 1$. Assume also market 1 is characterized by transaction costs of the type described in Proposition 3. Then the effective demand is uniquely defined and given by:

$$z = \{z_1^d, z_2^c\}$$

The proof is a straightforward extension of the one of Proposition 3 given in Svensson (1977). In the general case without transaction costs and with $\lambda << 1$, neither the Dreze concept, nor the Clower one are optimal strategies:

Proposition 5. Assume A1.1, A1.2 hold with $\lambda << 1$ (stochastic perceived rationing scheme). Then the effective demand is uniquely defined and is a continuous function of the probabilities λ_1 , λ_2 and of both rations $\bar{z}_1$, $\bar{z}_2$.

This is in contrast to the Clower concept where the demand for commodity 1 (alternatively 2) depends only on the ration on 2(1). Moreover it can be shown that if the Clower demands are not affordable (as in Figure 3), the optimal strategies defined in Proposition 5 are bounded away from and smaller than them. Provided there are no transaction costs, they are also bounded away from and larger than the Dreze demands.

So far we were concerned with non-manipulable perceived rationing schemes. It appeared that unless there is some uncertainty and/or transaction costs in the form of a proportional and non-refundable down payment, effective demands will not be uniquely defined. The problem has a different nature once we consider manipulable allocation procedures. Assume all traders believe the rationing is strictly proportional and deterministic. The allocation received by a buyer in a situation of excess demand becomes:

$$x_{in} = z_{in} \frac{S_n}{D_n} \qquad \text{for} \quad z_{in}, \quad EX_n > 0$$

Clearly the buyer has an advantage to exaggerate his demand in order to get the quantity he really wants. If all the buyers react in the same way, both z_{in} and D_n will soon become infinite. To avoid this situation and ensure boundedness, the model must be completed either by introducing some uncertainty, or by specifying that the market considers "credible" trade offers only.

A tentative general conclusion is that no specific concept of effective demand can be found that would be valid for all kinds of perceived rationing schemes. Inversely, not all perceived rationing schemes will give rise to well-defined effective demands. For each particular application, one will have to check that the effective demand formulation used is valid and uniquely defined.

2. ECONOMETRIC FORMULATION OF A TWO MARKET ECONOMY.

Once we come down to econometrics, it is by no means possible to remain at the level of generality we have enjoyed so far. We shall now have to limit our scope to a two market model and, perhaps more importantly, to ignore most distributional aspects. Still, it will be possible to save the typical three-component structure of quantity rationing models: effective demand, rationing scheme, perceived rationing scheme.

It has been emphasized in Section 1.2 that it will be almost impossible to find a formulation of a quantity rationing model that would be valid for every particular case. Accordingly, the discussion that will follow is strictly limited to production economies with an implicit reference to macroeconomic models. As before, commodity 0 is money which serves as means of exchange and has an indirect utility as a store of wealth. The output (say, aggregate consumption) is commodity 1, the production factor (say, labor) is commodity 2. I also introduce the assumption of specialized traders; traders can be divided into two groups, called producers (indexed k) and consumers (indexed j) respectively. Consumers are endowed with money and the production factor, but not with the consumption goods:

$$\overset{0}{m}_j,\ W_{2j} \geq 0 , \qquad W_{1j} = 0 \qquad \forall\ j$$

They have no production activity and are net buyers of the consumption goods and net sellers of the production factor:

$$z_{j1} \geq 0 , \qquad z_{j2} \leq 0 \qquad \forall\ j$$

Their goal is the maximization of their preferences by choosing the appropriate amount of present and future consumption of goods 1 and 2. As I do not want to introduce explicitly investment or inventories, the behavior of producers will be purely atemporal. Producers have no endowment of any kind. They buy commodity 2 from consumers and sell them their output of commodity 1 so as to maximize their profits.

To complete this rough picture of the economy, I need to describe briefly the true and perceived rationing schemes. I assume that both producers and consumers perceive the allocation procedures operating on both markets as non-manipulable. Because they are "specialized," they only need to have, in order to make the best decision, information about the upper-bound (alternatively lower-bound) they will face on the market on which they are buyers (sellers), i.e.:

$$\sigma_j = (\bar{z}_{j1}, \underline{z}_{j2}) \qquad \sigma_k = (\underline{z}_{k1}, \bar{z}_{k2})$$

Aggregate demands and supplies are then defined as:

$$D_1 = \Sigma_j\, d_{j1}(\sigma_j) \qquad D_2 = \Sigma_k\, d_{k2}(\sigma_k)$$

$$S_1 = -\Sigma_k\, d_{k1}(\sigma_k) \qquad S_2 = -\Sigma_j\, d_{j2}(\sigma_j)$$

which we assume equivalent to the following <u>aggregate effective demand</u> relations:

$$D_1 = d_1\ (\bar{D}_1, \bar{S}_2) \qquad D_2 = d_2(\bar{S}_1, \bar{D}_2)$$

$$S_1 = s_1\ (\bar{S}_1, \bar{D}_2) \qquad S_2 = s_2\ (\bar{D}_1, \bar{S}_2) \tag{2.1}$$

where $\bar{D}_n$, $\bar{S}_n$, $n = 1, 2$, are the absolute values of the aggregate perceived constraints on demand and supply respectively. These <u>aggregate perceived constraints</u> are defined by the aggregate equivalent of functions g_{in} of Section 1.1:

$$\bar{D}_1 = g_1\ (X_1(t-1)) \qquad \bar{D}_2 = g_2\ (X_2(t-1))$$

$$\bar{S}_1 = G_1\ (X_1(t-1)) \qquad \bar{S}_2 = G_2\ (X_2(t-1)) \tag{2.2}$$

where X_1, X_2 are the aggregate transactions resulting from the true allocation procedure. This result will in general depend on individual offers. It is assumed once more that the approximation in terms of aggregates is sufficiently accurate. Aggregate realized transactions are then defined as:

$$X_1 = f_1\ (D_1, S_1)$$
$$X_2 = f_2\ (D_2, S_2) \qquad (2.3)$$

A typical QRM will consist of the three groups of equations (2.1)-(2.2)-(2.3). This means a large number of equations to be specified and consequently significantly more specification problems than in classical models. In the traditional formulation of rationing models, this difficulty is bypassed by assuming that transactions never occur out of equilibrium. It will be shown however that this raises more problems than it solves. A (genuine) disequilibrium reformulation will be proposed which seems much more realistic and suitable for later extensions of the model. It will also highly simplify the estimation procedure.

2.1 The Traditional Formulation.

a. Specification

The traditional approach to QRM is based on the following postulates:

- The intertemporal behavior of consumers is represented by the indirect utility function:

$$U\ (M, X)$$

where M is the stock of money held by consumers. Implicitly in this formulation, the expectations about future constraints are fixed and independent of current constraints.

The behavior of producers is determined by the aggregate production function:

$$F\ (M',\ X_2)$$

where M' is the quantity of money used by producers in the production process. The introduction of money in the production function is meant to keep the model symmetric (by making the under-consumption equilibrium meaningful, as will be seen below) and to absorb the savings of consumers.

-The rationing schemes are defined as to fulfill restriction (i), (ii) (voluntary exchange feasibility) and also restriction (iii) (efficiency) so that their outcomes can simply be expressed as:

$$X_n = \min\ (D_n,\ S_n) \qquad n = 1,\ 2$$

-The expectations on actual constraints are held with certainty and are not invalidated by realized transactions (which does not imply they are correct).

This last statement entails that no agent has an incentive to change his trade offer, so that (conditional on the exogenous variables) the same trade offers and transactions will always be observed. This precisely defines a quantity rationing equilibrium. It is implicitly assumed there is some auctioneer who quotes the quantity constraints X_1 and X_2 (the tentative transactions), telling whether they apply to producers or to consumers. The auctioneer then registers the demand and supply for both goods. If they are compatible with the planned transactions and constrain the traders who were expected to be constrained, exchange may actually take place. Otherwise, the auctioneer quotes new constraints, asks for new trade offers and so on until an equilibrium is achieved. This highly centralized mechanism justifies the assumptions of almost perfect knowledge of the agents and of an efficient rationing scheme. Four types of equilibrium can be observed. As in Malinvaud (1977) and by reference to the macroeconomic example, the four equilibria will be called Keynesian unemployment, classical unemployment, repressed inflation and underconsumption equilibrium.

Given the four basic postulates, each regime can be characterized as follows:

- K-equilibrium	$X_1 = D_1$	$\bar{D}_1 \geq X_1$	$\bar{S}_1 = X_1$
	$X_2 = D_2$	$\bar{D}_2 \geq X_2$	$\bar{S}_2 = X_2$
- C-equilibrium	$X_1 = S_1$	$\bar{D}_1 = X_1$	$\bar{S}_1 \geq X_1$
	$X_2 = D_2$	$\bar{D}_2 \geq X_2$	$\bar{S}_2 = X_2$
- R-equilibrium	$X_1 = S_1$	$\bar{D}_1 = X_1$	$\bar{S}_1 \geq X_1$
	$X_2 = S_2$	$\bar{D}_2 = X_2$	$\bar{S}_2 \geq X_2$
- U-equilibrium	$X_1 = D_1$	$\bar{D}_1 \geq X_1$	$\bar{S}_1 = X_1$
	$X_2 = S_2$	$\bar{D}_2 = X_2$	$\bar{S}_2 \geq X_2$

The last equilibrium would be meaningless if the production function had been specified as a function of X_2 only. For in that case, the fourth regime would imply

$$X_1 < S_1 = F(X_2)$$
$$X_2 < D_2 = F^{-1}(X_1)$$

which are contradictory statements. It would also follow:

$$\frac{dD_2}{dX_1} \cdot \frac{dS_1}{dX_2} = \frac{d\,F^{-1}(X_1)}{dX_1} \frac{d\,F(X_2)}{dX_2} = 1$$

where dD_2/dX_1 , dS_1/dX_2 are the so-called spill-over coefficients.

It is to be noticed that not all perceived constraints and effective demands are defined. When, for instance, consumers are constrained on market 2 but not on market 1, then obviously their optimal demand for consumption goods is uniquely defined and is of the Clower-type. Assuming linearity, we write:

$$D_1 = D_1^W + \alpha_1(X_2 - S_2^W)$$

where the upper-script W denotes a Walrasian (or notional) trade offer and α_1 is the spill-over coefficient. Yet their optimal supply of commodity 2 is not defined. As consumers believe no trade offer could allow them to sell more than X_2, their supply may be any quantity larger than or equal to that amount. A similar reasoning applies to other cases so that the general form of QRM induced by the above postulates is:

<u>K-equilibrium</u>

$$D_1 = D_1^W + \alpha_1(X_2 - S_2^W)$$
$$S_1 \geq X_1$$
$$D_2 = D_2^W + \alpha_2(X_1 - S_1^W)$$
$$S_2 \geq X_2$$

<u>C-equilibrium</u>

$$D_1 \geq X_1$$
$$S_1 = S_1^W$$
$$D_2 = D_2^W$$
$$S_2 \geq X_2$$

<u>U-equilibrium</u>

$$D_1 = D_1^W$$
$$S_1 \geq X_1$$
$$D_2 \geq X_2$$
$$S_2 = S_2^W$$

<u>R-equilibrium</u>

$$D_1 \geq X_1$$
$$S_1 = S_1^W + \beta_1(X_2 - D_2^W)$$
$$D_2 \geq X_2$$
$$S_2 = S_2^W + \beta_2(X_1 - D_1^W)$$

where α_1, α_2, β_1, β_2 are the spill-over coefficients. Any specification adopted for the undefined perceived constraints and effective demands will be acceptable provided only it satisfies the required inequalities. Let us write

$$A^E\, Y = B^E\, Y^W$$

the structural form in a E-equilibrium of any such specification. The following notation has been used:

$$E \in \{K, C, R, U\}$$

$$Y = (D_1\ S_1\ D_2\ S_2)'$$

$$Y^W = (D_1^W\ S_1^W\ D_2^W\ S_2^W)'$$

A^E, B^E are the matrices of coefficients in a E-equilibrium. Then the reduced form will be:

$$Y = R^E\, Y^W \qquad\qquad E \in \{K, C, R, U\}$$

with $R^E = (A^E)^{-1}\, B^E$. For each equilibrium, we have:

$$R^K = (1-\alpha_1\alpha_2)^{-1} \begin{pmatrix} 1 & -\alpha_1\alpha_2 & \alpha_1 & -\alpha_1 \\ r_{21}^K & r_{22}^K & r_{23}^K & r_{24}^K \\ \alpha_2 & -\alpha_2 & 1 & -\alpha_1\alpha_2 \\ r_{41}^K & r_{42}^K & r_{43}^K & r_{44}^K \end{pmatrix}$$

$$R^C = \begin{pmatrix} r_{11}^C & r_{12}^C & r_{13}^C & r_{14}^C \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ r_{41}^C & r_{42}^C & r_{43}^C & r_{44}^C \end{pmatrix}$$

$$R^R = (1-\beta_1\beta_2)^{-1} \begin{bmatrix} r^R_{11} & r^R_{12} & r^R_{13} & r^R_{14} \\ -\beta_1\beta_2 & 1 & -\beta_2 & \beta_1 \\ r^R_{31} & r^R_{32} & r^R_{33} & r^R_{34} \\ -\beta_2 & \beta_2 & -\beta_1\beta_2 & 1 \end{bmatrix}$$

$$R^U = \begin{bmatrix} 1 & 0 & 0 & 0 \\ r^U_{21} & r^U_{22} & r^U_{23} & r^U_{24} \\ r^U_{31} & r^U_{32} & r^U_{33} & r^U_{34} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

In each case, the values of r^E_{ij} can be chosen arbitrarily. Both GLM and Ito have chosen to specify all the effective demands and supplies as Clower ones. However they use different definitions for the undefined perceived constraints. GLM (1977) consider that the (unbinding) constraint perceived on a market by an agent who is not currently rationed on that market is always larger than the Walrasian trade offer; for $n = 1, 2$, we have:

$$\bar{D}_n > D^W_n \qquad \text{if} \quad X_n = D_n$$
$$= X_n \qquad \text{otherwise}$$

$$\bar{S}_n > S^W_n \qquad \text{if} \quad X_n = S_n$$
$$= X_n \qquad \text{otherwise} .$$

This choice satisfies restrictions (a), (b), (c) of Benassy (1975). The model is now completely (though arbitrarily) defined and can be written as

K-equilibrium

$$D_1 = D_1^W + \alpha_1(X_2 - S_2^W)$$

$$S_1 = S_1^W$$

$$D_2 = D_2^W + \alpha_2(X_1 - S_1^W)$$

$$S_2 = S_2^W$$

C-equilibrium

$$D_1 = D_1^W + \alpha_1(X_2 - S_2^W)$$

$$S_1 = S_1^W$$

$$D_2 = D_2^W$$

$$S_2 = S_2^W + \beta_2(X_1 - D_1^W)$$

U-equilibrium

$$D_1 = D_1^W$$

$$S_1 = S_1^W + \beta_1(X_2 - D_2^W)$$

$$D_2 = D_2^W + \alpha_2(X_1 - S_1^W)$$

$$S_2 = S_2^W$$

R-equilibrium

$$D_1 = D_1^W$$

$$S_1 = S_1^W + \beta_1(X_2 - D_2^W)$$

$$D_2 = D_2^W$$

$$S_2 = S_2^W + \beta_2(X_1 - D_1^W)$$

Ito (1977), following Quandt (1978), assumes that perceived constraints are always equal to actual transactions:

$$\bar{D}_n = \bar{S}_n = X_n \qquad n = 1, 2$$

which is equivalent to deleting Benassy's restriction (c). In this way the QRM simplifies to:

$$D_1 = D_1^W + \alpha_1(X_2 - S_2^W)$$

$$S_1 = S_1^W + \beta_1(X_2 - D_2^W)$$

$$X_1 = \min(D_1, S_1)$$

$$D_2 = D_2^W + \alpha_2(X_1 - S_1^W)$$

$$S_2 = S_2^W + \beta_2(X_1 - D_1^W)$$

$$X_2 = \min\ (D_2, S_2)$$

Another admissible specification has been introduced by Portes (1977). Portes specifies the rationing model as[2]:

$$D_1 = D_1^W + \alpha_1(X_2 - S_2)$$

$$S_1 = S_1^W + \beta_1(X_2 - D_2)$$

$$X_1 = \min\ (D_1, S_1)$$

$$D_2 = D_2^W + \alpha_2(X_1 - S_1)$$

$$S_2 = S_2^W + \beta_2(X_1 - D_1)$$

$$X_2 = \min\ (D_2, S_2)$$

This specification relies upon the same definition of perceived constraints as Ito but not on the same concept of effective demands. It entails that the trade offer made on a given market is a function of the constraints perceived on both markets. This can be seen for example by substituting for S_2 in the demand function for commodity 1. The effective demand is now in the form:

$$D_1 = D_1^W + \alpha_{11}(X_1 - D_1^W) + \alpha_{12}(X_2 - S_2^W)$$

where α_{11}, α_{12} are restricted to

$$\alpha_{11} = -\ \alpha_1\beta_2(1 - \alpha_1\beta_2)^{-1} \qquad \alpha_{12} = \alpha_1(1 - \alpha_1\beta_2)^{-1}$$

and similarly for other cases. The restrictions on the coefficients make the reduced form compatible with the general model.

Undoubtedly, the postulates underlying the traditional approach to QRM are at best an approximation. Nobody will pretend they are perfect truth. Still they could be accepted as a fruitful simplification. Thanks to them, the specification of the rationing scheme and of the perceived rationing scheme were replaced by mere identities. The immediate cost of this simplification is that one is no longer able to derive reliable measures of demand pressure. Maybe this will not be crucial as long as we keep prices exogenous, but it may turn to have dangerous consequences in a later stage. The next section will highlight other, unacceptable, shortcomings.

b. <u>Coherency</u>

The theoretical framework underlying the traditional approach to QRM postulates that transactions always take place at an equilibrium. It tells nothing about the way this equilibrium was reached. The econometric model is a mere description of the equilibrium state. This is not peculiar to QRM . The same feature characterizes usual Walrasian econometric models; these models describe how equilibrium prices and quantities are interrelated, not how it was possible to reach the equilibrium. This is not a handicap for the econometrician provided the equilibrium is unique and can be retrieved from the knowledge of the exogenous variables. While this is usually the case in Walrasian models, it will not generally be so in QRM . Let us consider, for example, Ito's model and assume that the values of the exogenous variables are such that consumers' aggregate demand and supply functions may be represented as Figure 4. The full line represents the effective demand for the consumption commodity given a constraint on commodity 2, the dotted line represents the effective supply of commodity 2 given a constraint on commodity 1. These two lines intersect at a point $C = (D_1^W, S_2^W)$. What happens beyond this point is meaningless on the assumption that nobody can be forced to buy or sell more than he wishes. For $\alpha_1 \beta_2 < 1$, the dotted line will remain below the full line as long as $X < C$. Figure 5 brings together the preceding diagram and an equivalent one representing the behavior of producers. Accordingly $P = (S_1^W, D_2^W)$. It is easily verified that point E is a R-equilibrium and that no other equilibrium exist. However, by simply changing the values of coefficients α_1, β_2 in the consumers effective demand and supply, we may obtain a configuration like Figure 6. There are now two possible equilibria: a R-equilibrium at point E and a K-equilibrium at point E' .

GLM (1978) have derived necessary and sufficient conditions ensuring the existence and uniqueness of a quantity rationing equilibrium and the solvability of a QRM . Let us define the four open cones

$$C^K = (D_1 < S_1 , D_2 < S_2)$$

$$C^C = (D_1 > S_1 , D_2 < S_2)$$

$$C^R = (D_1 > S_1 , D_2 > S_2)$$

$$C^U = (D_1 < S_1 , D_2 > S_2)$$

defined in the vector space R^4 . The adherence of C^E is $\bar{C}^E$. Then GLM prove:

<u>Proposition 6</u>. If the two-market QRM

$$A^E Y = B^E Y^W \quad , \quad E \in \{K,C,R,U\}$$

satisfies

(i) A^E is invertible

(ii) $A^E Y$ is independent of E for $Y \in \overline{C}^K \cap \overline{C}^C \cap \overline{C}^R \cap \overline{C}^U$

(iii) $A^E(\overline{C}^E \cap \overline{C}^{E'}) = A^{E'}(\overline{C}^E \cap \overline{C}^{E'})$ for $E \neq E'$

(iv) $B^E = B$ is independent of E

Then the function $\sum_E A^E \mathbb{1}^{E(3)}$ is one-to-one from $\bigcup_E C^E$ to

$\bigcup_E A^E(C^E) = R^4 - A^K(\overline{C}^K \cap \overline{C}^C) \cup A^C(\overline{C}^C \cap \overline{C}^R) \cup A^R(\overline{C}^R \cap \overline{C}^U) \cup A^U(\overline{C}^U \cap \overline{C}^K)$

if and only if the determinants of the A^E matrices have the same sign.

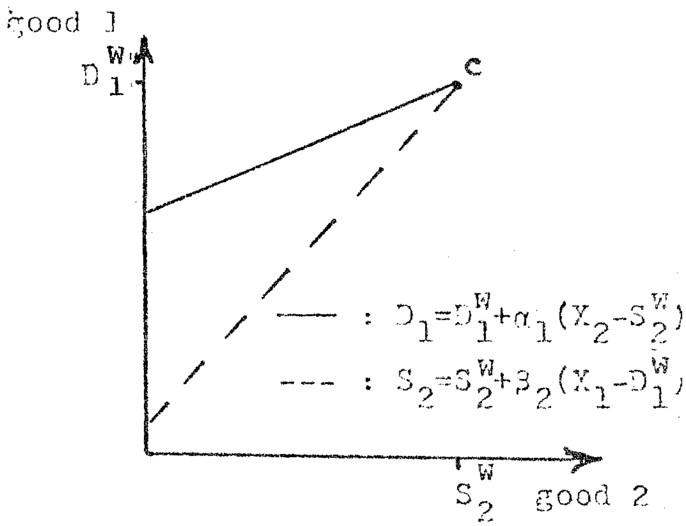

Figure 4

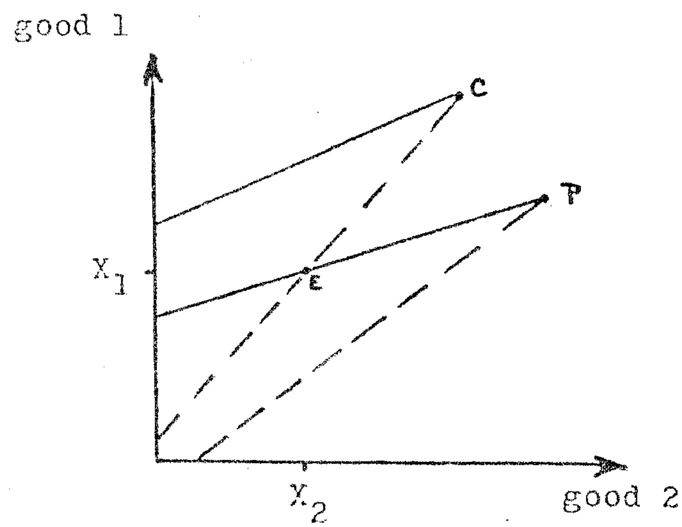

Figure 5

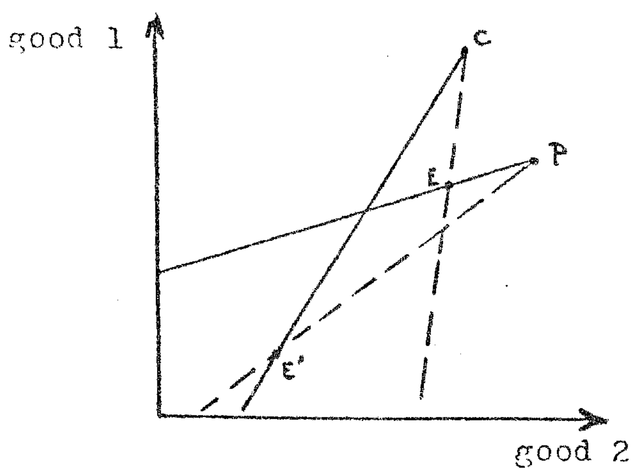

Figure 6

These conditions are fulfilled by the three particular models considered in Section 2.1.a provided their spill-over coefficients satisfy:

$$\alpha_1 \beta_2, \alpha_2 \beta_1, \alpha_1 \alpha_2, \beta_1 \beta_2 < 1$$

This result allows us to draw a diagram like the one in Figure 7. To each value of the exogenous variables (in this case the prices p_1 , p_2 for given values of the remaining exogenous variables) corresponds only to one type of equilibrium. The (p_1, p_2) plane can then be divided into four non-overlapping regions. For each value of (p_1, p_2) in the interior of these regions, the above rationing models have only one solution. Proposition 6 does not give any guarantee that the solution will remain unique for values of prices located on the boundaries. For example, assume the level of current prices ensures the matching of supply and demand on the consumption market but still creates an excess supply for the production factor (K-C boundary case). Then both submodels K and C of GLM's model can legitimately be used; still they will assign different values to the endogenous variables, even if they satisfy the restrictions of Proposition 6.

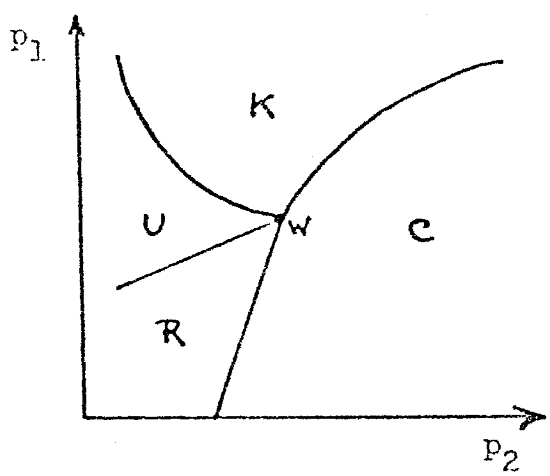

Figure 7

It must be emphasized that Proposition 6 leaves many problems unanswered:

1 - Not all (acceptable) effective demand concepts will allow us to derive a matrix B that is independent of E as to satisfy condition (iv). This will be the case for example if the undefined demands and supplies are specified to be of the Dreze type. Then knowledge of the exogenous variables gives no clue as to which equilibrium is prevailing.

2 - As noted by GLM (1978), two sets of effective demands and supplies will generate the same model if and only if

$$\min (D_n, S_n) = \min (D_n^*, S_n^*) \qquad n = 1, 2$$

and

$$D_n > S_n \iff D_n^* > S_n^* \qquad n = 1, 2$$

These conditions and the linearity assumption imply that within each regime, we must observe:

$$D_n^* - S_n^* = H_n^E (D_n - S_n) \qquad H_n^E > 0 , \qquad E \in \{K,C,U,R\}$$

This can be translated into restrictions on the reduced form. With respect to a K equilibrium, we obtain:

$$(R^*)^K = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 1-H_1^K & H_1^K & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1-H_2^K & H_2^K \end{pmatrix} R^K$$

Similar expressions hold for the other regimes. The three models defined by GLM , Ito, Portes satisfy this requirement and thus fall into the same equivalence class. Many other specifications however are acceptable which will not satisfy it and which will generate different observed transactions from the same exogenous variables. This means that (assuming the restrictions of Proposition 6 are met) for given values of the exogenous variables the regime that will be obtained depends on the particular model we have (arbitrarily) chosen.

3 - While it is legitimate to require that a QRM always has a solution it is less legitimate to impose this solution to be unique. In a macroeconomic context, it leaves no room for the Keynesian assertion that an economy may get stuck at a less than full employment equilibrium even though prices are not wrong prices, i.e., even though a better equilibrium is compatible with the same prices. This is due to the fact that the econometric models considered so far are strictly short-run models and do not introduce explicitly the effects of expectations about the future and of inter-period interactions. Only intra-period phenomena are accounted for.

c. Estimation

Provided the stochastic QRM is specified as[(4)]:

$$A^E Y = BY + W$$

where $W = (u_1\ v_1\ u_2\ v_2)'$ is distributed $N(0,\Omega)$, B and Ω are independent of E, and A^E satisfies the requirements of Proposition 6, the likelihood function can be written as

$$L(X_1, X_2) = \int_{S_1 > X_1} \int_{S_2 > X_2} h^K(X_1,S_1,X_2,S_2)dS_1dS_2$$
$$+ \int_{D_1 > X_1} \int_{S_2 > X_2} h^C(D_1,X_1,X_2,S_2)dD_1dS_2$$

$$+ \int_{D_1 > X_2} \int_{D_2 > X_2} h^R(D_1, X_1, D_2, X_2) dD_1 dD_2$$
$$+ \int_{S_1 > X_1} \int_{D_2 > X_2} h^U(X_1, S_1, D_2, X_2) dS_1 dD_2$$

where $h^E(\cdot)$ is the joint density of the endogenous variables in an E-equilibrium. Unlike in the one market case, it will not generally be possible to factor the densities as, e.g., $h^K(D_1, S_1, D_2, S_2) = h^K_{11}(D_1) \cdot h^K_{12}(S_1) \cdot h^K_{21}(D_2) \cdot h^K_{22}(S_2)$. The optimization problems appear to be formidable. GLM (1977) and Ito (1977) suggest completing the rationing model with deterministic price equations like

$$p_n(t+1) - p_n(t) = k_n(D_n(t) - S_n(t)) \qquad n = 1, 2$$

where $k_n > 0$. This would allow one to infer the type of equilibrium from the value of $(p_n(t+1) - p_n(t))$. These equations however have no theoretical foundations.

To summarize, the traditional approach to rationing econometrics can be criticized on two grounds. In the first place, it does not introduce any dynamics explicitly, i.e., it makes the assumption that trading never occurs before a short-run equilibrium has been reached. This has the undesirable consequence that it is not possible to give an appropriate definition of effective demands and supplies, and that requiring the fulfillment of the coherency conditions is equivalent to requiring that only one type of short-run equilibrium is compatible with the data. In the second place, the corresponding estimation procedure seems so complicated that it will prevent the building of richer models and may well force simplistic assumptions with respect to price formation. Nobody will claim that the perfect information assumption is a realistic one, but it could be defended as a handy approximation. However, when the "handy" approximation turns out to give problems both from a conceptual and an econometric point of view, its use becomes questionable. If more realistic features are not taken into account by the econometrician, the estimation of rationing models may well fail in most cases.

2.2 A disequilibrium reformulation

a. Specification.

This section is devoted to a respecification of rationing econometric models. The aim is to end up with a rationing model allowing genuine disequilibrium situations. This essentially means the removal of the perfect knowledge assumption for both traders and the fictitious auctioneer. The first assumption had one major advantage: thanks to it, the number of regimes in which an economy could find itself was limited to the 2^2 equilibrium states. When this simplification is abandoned and (possibly wrong) expectations are introduced the number of regimes may theoretically be as high as $(2^2)^3$. This follows from the fact that we now have three "min" conditions for each market, one for the rationing scheme and two for the traders expectations. Let us assume for example that the effective demand for the consumption commodity is written:

$$D_1 = D_1^W + \alpha_1 \ (\bar{S}_2 - S_2^W) \qquad \text{for} \quad \bar{S}_2 < S_2^W$$

$$D_1 = D_1^W \qquad \text{for} \quad \bar{S}_2 \geq S_2^W$$

This means that we have to compare $\bar{S}_2$ and S_2^W. If we define

$$\bar{\bar{S}}_2 = \min \ (\bar{S}_2, \ S_2^W)$$

we can rewrite the effective demand for good 1 more compactly as:

$$D_1 = D_1^W + \alpha_1 (\bar{\bar{S}}_2 - S_2^W)$$

The three "min" conditions on each market n are then:

$$\bar{\bar{D}}_n = \min(\bar{D}_n, D_n^W)$$

$$\bar{\bar{S}}_n = \min(\bar{S}_n, S_n^W)$$

$$X_n = \min(D_n, S_n)$$

At an equilibrium with perfect knowledge, the values of $\bar{\bar{D}}_n$, $\bar{\bar{S}}_n$ are straightforwardly deduced from the outcome of the rationing scheme so that one needs only one "min" condition per market. This is not true in the general disequilibrium case. For example, it is possible, when they have different and inaccurate information sources, that both consumers and producers believe they will be constrained on both markets and behave accordingly while the resulting transactions will actually constrain consumers on one market and producers on the other one. In this disequilibrium case both traders had wrong expectations. Other cases can be constructed in a similar way.

Fortunately it seems that in practice many disequilibria will generally be irrelevant and can reasonably be disregarded a priori. Which ones may actually be neglected will depend on the particular model considered. In a macroeconomic model of Western economies (and maybe in other cases too), the following assumptions seem realistic and will allow the reduction of the number of outcomes to be considered to four:

A2.1 Consumers always believe they will not be rationed on the consumption market.

A2.2 Trading occurs sequentially first on the production factor market, then on the consumption market.

A2.3 Producers and consumers perceive the allocation procedure on market 2 as non-manipulable and stochastic.

A2.4 Available data refer to exchanged quantities of commodity 2 and to produced quantities of commodity 1.

Assumption A2.1 states that rationing of the consumption goods has always been so rare and temporary that it is safe to assume it did not affect the supply of good 2. This can be written as:

$$\bar{\bar{D}}_1 = D_1^W$$

It partly follows from the existence of many substitutes to any single commodity. For instance, a consumer who is unable to buy his most preferred cigarettes will simply switch to another brand. This kind of rationing will not appear in aggregate data and will not affect the supply of the production factor. Moreover, in a small open economy, it can be considered that any domestic shortage is compensated by increased imports. A still more important argument favoring A2.1 is that the relevant constraint in the consumer's decision problem is the long-run, or "permanent" constraint and not the short-run one. This can best be seen by replacing the indirect utility formulation by the more explicit one:

$$\text{Max} \int_{\tau=t}^{\infty} e^{-\rho\tau} \, u(X_1(\tau) \, , \, X_2(\tau)) \, d\tau$$

$$\text{subject to } \; M(\tau) - M(\tau-1) = R \cdot M(\tau) + p_2 \, X_2(\tau) - p_1 \, X_1(\tau) \, .$$

where ρ is a subjective discount rate and R is the rate of interest.

In the absence of any (present and future) quantity rationing, this program defines the Walrasian demand and supply functions for commodities 1 and

2. If consumers are aware of the possible appearance of rationing, they will change their demand and supply, taking account of both present and future expected constraints. If they expect to be presently rationed in the consumption goods, their supply of good 2 will still remain almost unchanged provided they believe the rationing will not persist in the future. Accordingly, the relevant concepts for $\bar{\bar{D}}_1$, $\bar{\bar{S}}_2$ are not the short-run, present constraints, but the long-run or permanent expected ones(5). The main effect of assumption A2.2 is that when traders meet on the consumption market, they already have an accurate knowledge of at least the average constraint prevailing on market 2. It seems a natural assumption, implying that when producers go to the consumption market to sell their output, the production process is already taking place so that producers have a correct idea of what they can sell. Correspondingly, consumers already know what will be their average factor income. Assumption A2.3 will imply that the effective demand and supply of the production factor are of the Clower type and do not depend on $\bar{\bar{D}}_2$, $\bar{\bar{S}}_2$. These perceived constraints will only appear in the effective demand and supply functions for commodity 1. From A2.2, we are entitled to write them as:

$$E(\bar{\bar{S}}_2) = \Delta\ (L)\ E\ (X_2(t))$$

$$E(\bar{\bar{D}}_2) = E\ (X_2)$$

where E is the expected value operator. In defining $E(\bar{\bar{S}}_2)$, I have accounted for the intertemporal behavior of consumers by introducing the lag polynomial function

$$\Delta(L) = \delta_1 + \delta_2 L + \delta_3 L^2 + \ldots \text{ subject to } \Sigma\ \delta_i = 1$$

If consumers have never been constrained on market two, then $E(X_2(t)) = S_2^W$ for all t and $E(D_1)$ will be equal to D_1^W . The more they have been constrained in the past, the more their expected long-run constraint will diverge from S_2^W. In the sequel, I shall consider the simple case $\Delta(L) = \delta_1 + \delta_2 L$. Finally, assumption A2.4 again is verified in a macro-economic model where data on GNP are available while data on exchanged quantities could only be obtained by subtracting involuntary stock changes. The practical consequence of A2.4 is that the rationing scheme on market one can be ignored. Defining X_1^* as the quantity produced and using A2.4 we may write

$$X_1^* = F(X_2) + \text{error term.}$$

As it will be no longer useful, I have dropped the fiction of money as a factor of production. An autonomous sector must now be introduced which will absorb the savings of consumers. The autonomous sector has no endowment of any kind but has some fixed demand for the consumption commodity financed by the profits of the production sector and the savings of consumers. It has no production activity and will never be rationed on the consumption market. The global effect of A2.1 - A2.4 is the elimination of four out of the six "min" conditions (namely those for $\bar{\bar{D}}_1$, X_1, $\bar{\bar{D}}_2$, $\bar{\bar{S}}_2$), so that only 2^2 disequilibrium states (and the corresponding boundary cases) will be possible.

So far we have removed the hypothesis that traders know perfectly the upper-bounds they face. The next step is the deletion of the postulate of efficient and error free rationing schemes. Let us first go back to the models of Section 2.1 and discuss the implications of their specification of the rationing schemes:

$$X_n = \min (D_n , S_n)$$

To begin with, it should be realized that this entails that the auctioneer knows the value of disturbances which may not even exist. For instance, the knowledge of u_i (the error term associated with consumption demand) in an R-equilibrium means the knowledge of the exact amount of good 1 consumers would buy if they had not been constrained, while the knowledge of v_1 (the error term associated with consumption supply) in a K-equilibrium means that producers and the auctioneer know exactly what the production level would be if the constraint imposed by demand had been removed, i.e., they know the exact output of an imaginary production process. Alternatively, once it is known that the demand side is binding on market one, the production sector is able to produce exactly the amount demanded. These remarks suggest the use of the alternative formulation:

$$X_1 = \min\ (E(D_1)\ ,\ E(S_1)) + W_1$$

which postulates that the allocation procedure is first scheduled on the basis of the expected values of demand and supply. When the actual trading then takes place, some unexpected disturbances W_1 may be observed. This specification would not imply that production is strictly equal to X_1 ; if it is postulated that producers know $E(D_1)$, it would merely imply that production is equal to $\min\ (E(D_1)\ ,\ E(S_1)) + W_1^*$, where W_1^* is different from W_1 . This seems compatible with the important role inventories play in practice in absorbing the discrepancies between the distribution and production processes. The preceding criticisms may also be applied to the production factor market. Let us assume that commodity two is labor. Then the traditional specifications of the rationing scheme postulates, e.g., that in a K-equilibrium the number of hours an unemployed worker would have performed if he had not been constrained is known with accuracy. On the contrary, a specification like:

$$X_2 = \min\ (E(D_2)\ ,\ E(S_2)) + W_2$$

has the appealing interpretation that labor contracts are made on the basis of the number of working hours employers expect to need and employees expect to perform. The amount of work actually performed will then be the sum of two terms, the first one reflecting traders' expectations, the second one being an error term that will be known only after the production process has actually taken place.

The above respecifications of the allocation procedures are attractive because they state explicitly that nobody has enough information to make the rationing schemes perfectly efficient. They still embody the idea that the rationing schemes are efficient with respect to the expected values. This may not even be true. It would contradict, e.g., the existence of frictional unemployment on the labor market. In such circumstances, the following formulation seems preferable[(6)]:

$$X_n = \min\ (E(D_n)\ ,\ E(S_n)) - \frac{a_n}{1 + b_n EX_n^2} + W_n$$

where EX_n is redefined as $E(D_n) - E(S_n)$ and a_n , b_n are positive parameters characterizing the efficiency of the allocation procedure on market n . For $a_n = 0$, we come down to the preceding formulation of a mean-efficient allocation procedure. For $b_n = 0$, the efficiency of the allocation procedure does not depend on the gap between expected demand and supply. Notice that for $b_n \neq 0$, the rationing scheme is manipulable. These concepts are illustrated in Figure 8. If we interpret it as representing the labor market, then the quantities AB and AC are respectively the level of vacancies and of unemployment that will in the average prevail for $p_n = \overline{p}_n$. For convenience, I shall always assume in the following that $b_n\, EX_n^2 \simeq 0$ and that the constant term a_n is

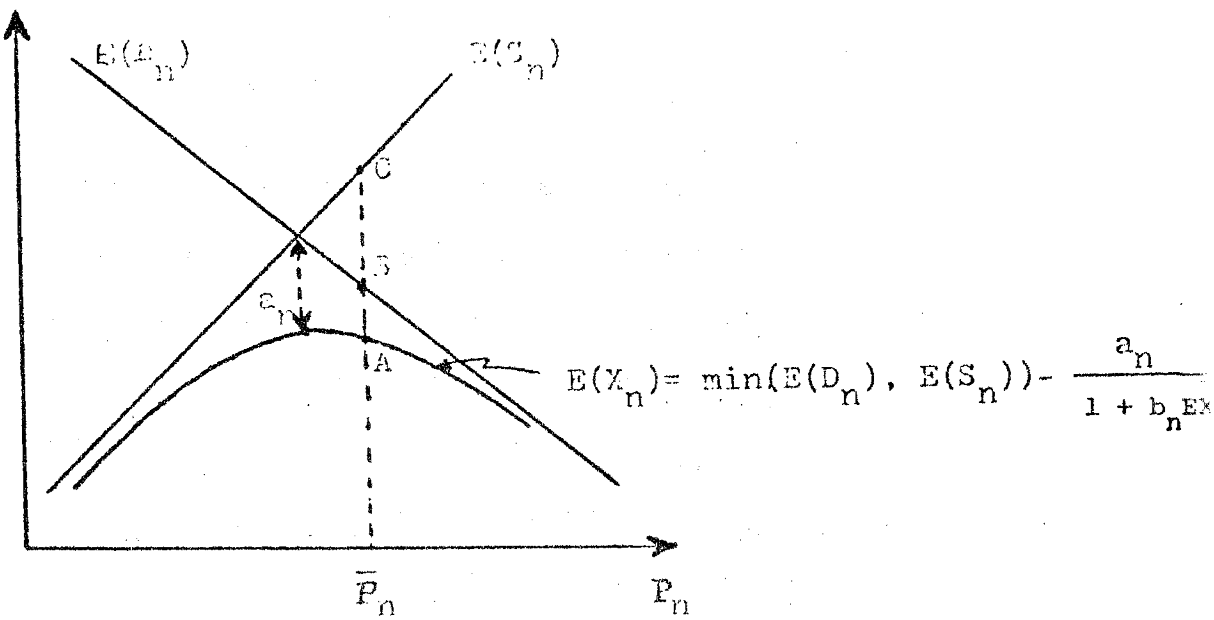

Figure 8: An inefficient rationing scheme.

incorporated in $E(D_n)$ and $E(S_n)$.

In a disequilibrium rationing model, "min" conditions will not only appear in the equations representing the allocation procedures but also in those defining expectations. I shall again specify them as, for example:

$$\bar{\bar{S}}_1 = \min(\bar{S}_1, S_1^W) + e_1$$

A first justification is that as $\bar{S}_1$ and S_1^W refer to the same agents there is no need to introduce two distinct error terms. Moreover it may be interesting to take account of divergent expectations among producers. In that case, we would have a phenomenon similar to the one described in Figure 8 and the comments made there apply directly. Finally, the existence of adjustment costs associated with changing the level of production $X_1^*(t-1)$ may well result in the binding constraint not just being the minimum of $\bar{S}_1$ and S_1^W but lying somewhere between that minimum and the preceding production level:

$$\bar{\bar{S}}_1 = u \min (\bar{S}_1 , S_1^W) + (1 - u) X_1^* (t - 1) + e_1$$

It will now be easy to complete the disequilibrium respecification of the quantity rationing model by deriving acceptable and uniquely defined effective demand and supply functions. It will be seen that the producers' behavior is most similar to the one of the consumer of Corollary 4. Let us interpret $E(\bar{\bar{S}}_1)$ as the minimum quantity producers believe they will certainly be able to sell on the consumption market $(\bar{z}_1$ in the notation of Section 1.2). Let us also assume that they consider there is always a small, but positive probability that a trade offer larger than $E(\bar{\bar{S}}_1)$ will be accepted by consumers $(\lambda_1$ close to unity in the terminology of Section 1.2). This implies they may not offer more than their expected output, otherwise they might well be unable to honor their trade offer. Simultaneously, it is not advantageous for them to produce more than $E(\bar{\bar{S}}_1)$ as the probability they will not be able to sell their output is high and as unsold production means decreasing profits. This means that, from the producers' point of view, market one is characterized by transaction costs of the kind described in Section 1.2. The optimal production plan for producers is then $E(\bar{\bar{S}}_1)$. Moreover, we know from assumption A2.3 that producers believe a given demand for the production factor always has a positive probability to be satisfied $(\lambda_2 < 1)$. When they go to market two, the producers' optimal demand for the production factor will be:

$$E(D_2) = D_2^W + \alpha_2 \; (E(\bar{\bar{S}}_1) - S_1^W) \; .$$

With respect to consumers, we know from A.2.1 they do not believe any rationing will occur on market one and from A2.3 that they consider their supply of the production factor always has a positive chance to be satisfied (compare with Proposition 2). Hence their optimal trade offer will be:

$$E(S_2) = S_2^W .$$

When thereafter producers and consumers meet on the consumption market, they both know their mean constraint on market two:

$$E(X_2) = \min (E(D_2) , E(S_2))$$

Both producers and consumers revise their plan accordingly. As consumers expect no rationing on the consumption goods, their optimal demand will be:

$$E(D_1) = D_1^W + \alpha_1 \ (\Delta(L) \ E(X_2(t)) - S_2^W)$$

where D_1^W is defined so as to incorporate the demand of the autonomous sector. Similarly the effective supply of producers is

$$E(S_1) = S_1^W + \beta_1 \ (E(X_2) - D_2^W)$$

where $\beta_1 = 1/\alpha_2$; this ensures that if producers are not constrained on the production factor market, their effective supply will coincide with the optimal production plan:

$$E(S_1) = S_1^W + \beta_1 \ [D_2^W + \alpha_2 \ (E(\bar{\bar{S}}_1) - S_1^W) - D_2^W]$$

$$= E(\bar{\bar{S}}_1) \qquad\qquad \text{iif} \quad \alpha_2 \ \beta_1 = 1$$

This shows that the effective supply of commodity 1 is of the Dreze type. From assumption A2.3 we may also write:

$$E(X_1^*) = E(S_1)$$

Like the equilibrium model of Section 2.1, this disequilibrium model admits four regimes. By analogy, I shall call them K- , C- , R- and U-disequilibria.

Which disequilibrium state will be obtained primarily depends on producers' expectations. For instance, the Keynesian disequilibrium is to be interpreted as meaning that producers expect to be constrained on the commodity market and that their corresponding demand for the production factor was not rationed. As it is a disequilibrium state, producers' expectations may turn out to be wrong so that a K-disequilibrium may eventually coincide with a positive excess demand for the consumption commodity. For each regime, the values of X_1^* and X_2 are given by:

K-diseq.: $\bar{S}_1 < S_1^W$; $E(D_2) < S_2^W$

$$X_1^* = \bar{S}_1 + W_1^*$$

$$X_2 = D_2^W + \alpha_2(\bar{S}_1 - S_1^W) + W_2$$

C-diseq.: $S_1^W < \bar{S}_1$; $E(D_2) < S_2^W$

$$X_1^* = S_1^W + W_1^*$$

$$X_2 = D_2^W + W_2$$

U-diseq.: $\bar{S}_1 < S_1^W$; $S_2^W < E(D_2)$

$$X_1^* = S_1^W + \beta_1(S_2^W - D_2^W) + W_1^*$$

$$X_2 = S_2^W + W_2$$

R-diseq.: $S_1^W < \bar{S}_1$; $S_2^W < E(D_2)$

$$X_1^* = S_1^W + \beta_1(S_2^W - D_2^W) + W_1^*$$

$$X_2 = S_2^W + W_2$$

The last two disequilibria result in the same observable variable, but correspond to different producers' expectations. The relevance of their distinction appears clearly once we consider the values of the excess demand functions in each regime. For that purpose, it will be convenient to postulate

$$\bar{S}_1 = \Gamma\ (L)\ E(D_1(t))$$

where $\Gamma\ (L)$ is a lag polynomial function:

$$\Gamma\ (L) = \gamma_1 + \gamma_2 L + \gamma_3 L^2 + \ldots \text{ subject to } \Sigma\gamma_i = 1 \ .$$

For $\gamma_1 = 0$, producers have no information on current demand except the indirect one from past effective demands. On the contrary, if $\gamma_1 = 1$, producers know accurately the average level of demand. For simplicity, I shall consider $\Gamma(L) = \gamma_1 + \gamma_2 L$; γ_2 may be interpreted as a measure of the potential inaccuracy of producers' expectations. For $\gamma_2 = 0$, expectations are correct. Provided we adopt the definitions $EX_n^W = D_n^W - S_n^W$ and $\dot{D}_1 = D_1(t) - D_1(t-1)$, the following relations are derived straightforwardly:

K-disequilibrium

$$EX_1 = \gamma_2 \, E(\dot{D}_1)$$
$$EX_2 = E(D_2) - S_2^W < 0$$

C-disequilibrium

$$EX_1 > \gamma_2 \, E(\dot{D}_1)$$
$$EX_2 = E(D_2) - S_2^W < 0$$

U-disequilibrium

$$EX_1 < \gamma_2 \, E(\dot{D}_1) + \alpha_2 \, EX_2^W$$
$$0 < EX_2 < EX_2^W$$

R-disequilibrium

$$EX_1 > \gamma_2 \, E(\dot{D}_1) + \alpha_2 \, EX_2^W$$
$$EX_2 = EX_2^W > 0$$

The R- and U-disequilibria have different excess demand functions. Notice also that the sign of EX_1 depends primarily on $\gamma_2 E(\dot{D}_1)$; for $\gamma_2 \neq 0$, EX_1 may differ widely from producers' expectations.

b. Coherency.

As for the equilibrium formulation presented in Section 2.1, it is important to know under which conditions the model will have a well-defined reduced form.

Proposition 7: The disequilibrium rationing model described in Section 2.2a and satisfying

$$\alpha_1 , \alpha_2 , \beta_1 > 0 \; ; \; \alpha_2\beta_1 = 1 \; ; \; 0 \leq \delta_1 , \gamma_1 \leq 1$$

will have a well-defined reduced form (including boundary cases) provided it also satisfies

$$\alpha_1 \alpha_2 \delta_1 \gamma_2 < 1$$

The proof is given in the Appendix. Notice that the mapping from the space of predetermined variables to the space of endogenous variables is not necessarily one-to-one and that the boundary case K-C-R-U does not necessarily coincide with the Walrasian equilbrium as it was the case in the equilibrium rationing model. A further and more important difference with the latter is that imposing coherency is no longer equivalent to imposing a unique equilibrium. A unique equilibrium point would be guaranteed only if $\alpha_1 \alpha_2 < 1$, which is not imposed by Proposition 7. This leaves a substantial role to expectations. It also raises the important problem of time aggregation. If the time interval between two observations is sufficiently small (what "sufficient" actually means may depend on the particular model considered), then δ_1 and γ_1 are likely to be small too so that the condition of Proposition 7 will be satisfied. If the time interval grows larger and larger, the values of δ_1, γ_1 will get closer and closer to unity and the condition $\alpha_1 \alpha_2 \delta_1 \gamma_1 < 1$ may well be violated. This stresses how much information can be lost by aggregating over time.

c. Estimation

Provided Proposition 7 is verified, the model can be rewritten more compactly as:

$$\begin{bmatrix} X_1^* \\ X_2 \end{bmatrix} = (1-r_1)(1-r_2)\begin{bmatrix} \bar{S}_1 \\ D_2^W+\alpha_2(\bar{S}_1-S_1^W) \end{bmatrix} + r_1(1-r_2)\begin{bmatrix} S_1^W \\ D_2^W \end{bmatrix} + r_2\begin{bmatrix} S_1^W+\beta_1(S_2^W-D_2^W) \\ S_2^W \end{bmatrix} + \begin{bmatrix} W_1^* \\ W_2 \end{bmatrix}$$

where $r_1 = 1$ if $\bar{S}_1 > S_1^W$, 0 otherwise

$r_2 = 1$ if $E(D_2) > S_2^W$, 0 otherwise

Assume(7): $W(t) = \begin{bmatrix} W_1^*(t) \\ W_2(t) \end{bmatrix} \sim IN\ (0,\Omega)$

The log-likelihood function is:

$$\mathcal{L} = -\frac{T}{2}\ln 2\pi - \frac{T}{2}\ln|\Omega| + \sum_{t=1}^{T}\ln|J(t)| - \frac{1}{2}\sum_{t=1}^{T} W'(t)\,\Omega^{-1}\,W(t)$$

where $w(t)$ is to be replaced by the expression in terms of observed variables given above and $|J(t)|$ is the Jacobian of that transformation. The inclusion of step functions in the likelihood makes it discontinuous. Tishler-Zang (1978) proposes then to replace r_n by the twice continuously differentiable approximation:

$$\tilde{r}_n = \begin{cases} 0 & \text{if } y \leq -\varepsilon \\ \frac{3}{16}\left(\frac{y}{\varepsilon}\right)^5 - \frac{5}{8}\left(\frac{y}{\varepsilon}\right)^3 + \frac{15}{16}\left(\frac{y}{\varepsilon}\right) + \frac{1}{2} & \text{if } -\varepsilon \leq y \leq \varepsilon \\ 1 & \text{if } \varepsilon \leq y \end{cases}$$

where $y = \bar{S}_1 - S_1^W$ if $n = 1$

$y = E(D_2) - S_2^W$ if $n = 2$

ε is a positive parameter that can be made arbitrarily small

The approximation is illustrated in Figure 9. By reducing ε, it is possible to achieve an arbitrarily close approximation to r_n. As $\tilde{r}_n$ is twice continuously differentiable second order maximization methods can be used.

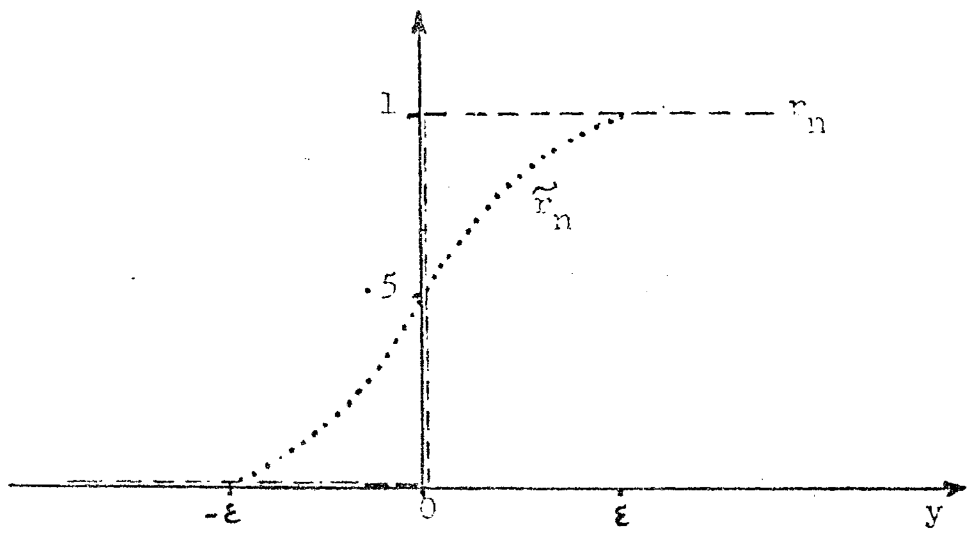

Figure 9.

2.3 CONCLUSIONS

The disequilibrium QRM that has been proposed does not intend to be a general formulation that would be valid for any application and would fit any kind of data. The type of approach however seems to have general validity. It stresses how important the specification problems are in implementing QRM. While in the equilibrium approach the emphasis was on estimation problems, it now lays on specification ones. Still the problems we encountered remained rather simple mainly because we ignored inventories and price changes. Our explicit assumptions were that producers are not able to influence demand by price changes (i.e., the rationing schemes are not price-manipulable) and that the production goods are perishable and cannot be stored. Maybe these shortcomings will not be too harmful in, e.g., an annual macroeconomic model. In that case, changes in the level of inventories give firms an accurate indication about the state of demand; on an annual basis, one may assume that the production level will not diverge by much from actual demand ($\gamma_2 \simeq 0$) so that involuntary stocking can safely be ignored (this would not necessarily apply to quarterly models) and prices can be seen as determined by longer run cost considerations. But this will not be the general case.

Because it explicitly allows the possibility of wrong expectations on behalf of producers, the disequilibrium reformulation seems well-fitted for future developments introducing price changes and inventories. Assume producers' expectations were too optimistic so that the level of production exceeds actual demand. The question arises then of knowing what will be the best reaction of producers, a question that could not even be raised in the equilibrium perfect knowledge framework and which seems to be the key to a better understanding of price and inventory policies. Two kinds of models can be distinguished. In the first type, the produced commodity is essentially perishable so that the first reaction of producers comes through prices (price-manipulable rationing schemes). This category includes most agricultural products. The Cobweb model is a limit case where prices instantaneously fluctuate as much as needed to erase surpluses. At some price level however producers may well find it more advantageous for them to destroy their output rather than accept lower prices. This would result in a model similar to the one built by Suits (1955) and examined in Goldfeld-Quandt (1975). The second type of model corresponds to non-perishable commodities. As in the macroeconomic example, prices are mainly determined by longer run considerations (which may explain downward price rigidity) while the level of undesired inventories becomes, together with expected demand, an important element in the determination of production targets.

These last considerations remain rather simplistic. Still, they provide support for the approach adopted in this paper and seem to present it as an interesting starting point for future research.

APPENDIX

<u>Proof of Proposition 7</u>: It must be proved that the two equations

$$\min(\overline{S}_1,\ S_1^W),\quad \min(E(D_2),\ S_2^W)$$

always generate one and only one regime. I first neglect the cases where $\overline{S}_1 = S_1^W$ and/or $E(D_2) = S_2^W$. The "min" conditions can then be rewritten as sign restrictions on functions of the predetermined variables. Let us define:

$$F_1 = EX_1^W + \alpha_1\gamma_1\delta_1 EX_2^W + \gamma_2[E(D_1(t-1))-D_1^W] + \alpha_1\gamma_1\delta_2[E(X_2(t-1))-S_2^W]$$

$$F_2 = \alpha_2 EX_1^W + EX_2^W + \alpha_2\gamma_2[E(D_1(t-1))-D_1^W] + \alpha_1\alpha_2\gamma_1\delta_2[E(X_2(t-1))-S_2^W]$$

$$F_3 = EX_2^W$$

$$F_4 = EX_1^W + \gamma_2[E(D_1(t-1))-D_1^W] + \alpha_1\gamma_1\delta_2[E(X_2(t-1))-S_2^W]$$

The following relations can now be verified:

<u>K-disequilibrium</u>

$$(\overline{S}_1 - S_1^W) = (1-\alpha_1\alpha_2\delta_1\gamma_1)^{-1}\ F_1 < 0$$

$$(E(D_2)-S_2^W) = (1-\alpha_1\alpha_2\delta_1\gamma_1)^{-1}\ F_2 < 0$$

<u>C-disequilibrium</u>

$$(\overline{S}_1 - S_1^W) = F_1 > 0$$

$$(E(D_2)-S_2^W) = F_3 < 0$$

<u>U-disequilibrium</u>

$$(\overline{S}_1 - S_1^W) = F_4 < 0$$

$$(E(D_2)-S_2^W) = F_2 > 0$$

<u>R-disequilibrium</u>

$$(\overline{S}_1 - S_1^W) = F_4 > 0$$

$$(E(D_2)-S_2^W) = F_3 > 0$$

Provided $\alpha_1\alpha_2\delta_1\gamma_1 < 1$, the signs taken by F_1, F_2, F_3, F_4 discriminate the alternative disequilibria. Notice that

$$F_4 = (F_4 - F_3)/\alpha_2 \tag{A.1}$$

so that in an R-disequilibrium $F_3, F_4 > 0$ implies $F_2 > 0$ and in a U-disequilibrium, $F_2 > 0$, $F_4 < 0$ implies $F_3 > 0$. Both regimes are thus characterized by $F_2, F_3 > 0$ and are distinguished by the sign of F_4 which plays no role with respect to the other regimes. Table 1 lists all the sign combinations that can be obtained from F_1, F_2, F_3 and gives the corresponding regimes. Only one regime is obtained at a time, but two sign combinations do not correspond to any disequilibrium. However as the F-functions are related by:

$$\alpha_2 F_1 - F_2 + (1-\alpha_1\alpha_2\gamma_1\delta_1) F_3 = 0 \tag{A.2}$$

and provided $\alpha_1\alpha_2\delta_1\gamma_1 < 1$, these two sequences will never be obtained. Consequently, for each value of the predetermined variables, one and only one regime will be obtained.

1	2	3	regime
+	+	+	R or U
+	+	−	C
+	−	+	———
+	−	−	C
−	+	+	R or U
−	+	−	———
−	−	+	K
−	−	−	K

Table 1

1	2	3	4	regime
0	−	−	+	K-C
0	+	+	∓	R or U
+	0	−	∓	C
−	0	+	−	U-K
+	+	0	+	C-R
−	−	0	−	K
+	+	+	0	R-U
−	−	−	0	K
0	+	+	0	R-U
0	−	−	0	K-C
0	0	0	0	K-C-R-U

Table 2

It is easy to establish in the same way that no indeterminancy arises when the F-functions may take zero values. All the possible additional combinations that may arise are listed in Table 2. Given (A.1), (A.2) the other cases are irrelevant. Among the new combinations we find the five boundary cases K-C , C-R , R-U , U-K , K-C-R-U . It is straightforward to check that when a boundary case is obtained, the two (or four) corresponding submodels generate the same expected values of the endogenous variables and are thus interchangeable, which means that the reduced form of the model is well-defined on boundaries too. Q.E.D.

FOOTNOTES

(1) Though Svensson (1977) does not explicitly use the concept of perceived rationing scheme, it is clear that his work must be interpreted in that framework. Accordingly, the critique made by Green stressing that Svensson's rationing scheme cannot satisfy the feasibility restriction (ii) does not destroy the argument.

(2) For this effective demand concept, Portes refers to Benassy (1975). His specification however seems to be a misinterpretation of an example given by Benassy (and reported in Section 1.1.a). Benassy (1975) only deals with Clower effective demands while the concept introduced by Portes is <u>not</u> a Clower demand.

(3) $\pi^E = 1$ if an E-equilibrium is actually observed

$\quad\;\; = 0$ otherwise

(4) Adding an error term

$$W \sim N(0,\Omega)$$

With Ω independent of E to a structural form of the type

$$A^E Y = B^E Y^W \qquad E \in \{K,C,R,U\}$$

or to the reduced form

$$Y = R^E Y^W \qquad E \in \{K,C,R,U\}$$

would result into a stochastic QRM that does not satisfy Proposition 6, even if the deterministic model does.

(5) In the traditional formulation, expectations about future constraints were assumed to be fixed and were implicitly incorporated in the Walrasian terms D_1^W, S_2^W.

(6) Muellbauer (1978) has proposed an alternative approach to inefficient rationing. He considers the aggregate market is made of several micro-markets on which efficient rationing schemes operate. The resulting aggregate relations are like those appearing in Figure 8. Our approach however seems to be both much easier and more natural.

(7) If the covariance matrix varies across regimes, Ω has simply to be redefined as:

$$\Omega = (1-r_1)^2(1-r_2)^2\Omega^K + r_1^2(1-r_2)^2\Omega^C + r_1^2 r_2^2\Omega^R + (1-r_1)^2 r_2^2\Omega^U$$

This will be the case if the true, regime independent, error term belongs to the reduced form while the likelihood function is expressed with respect to the structural form, or vice-versa.

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