A QUASI-WALRASIAN MODEL OF RATIONING AND LABOR SUPPLY: THEORY AND ESTIMATION

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ABSTRACT

This paper explores the microeconomic foundations of two essential features of the disequilibrium model: the assumption of wage rigidity and the assumption that the effective demand and supply expressed in a market are invariant with respect to the presence of disequilibrium in that market. A model similar in some respects to the Baily (1974) and Azariades (1975) implicit contract models is shown to imply wage and price rigidity. Unlike the Baily-Azariades models, our model implies that firms will hire, when possible, along their *ex post* marginal product of labor curves as is assumed by the disequilibrium specification. However, the presence of transactions costs may cause workers to modify their labor supply behavior when disequilibrium is possible. The econometric implications of this modification for the disequilibrium econometric specification are described. The econometric model is then applied to time series data on the U.S. labor market. The presence of disequilibrium is found to have a small but significant effect on the U.S. labor supply in the period 1930-1973.

1. Introduction

A number of authors, most prominently Clower (1965) and Leijonhofvud (1968), have interpreted Keynes' theory of unemployment as a disequilibrium model. Nominal price and wage rigidity prevent the labor market from clearing. The actual level of employment is given by the minimum of the effective demand for labor and the effective supply. In this paper we consider two essential features of the disequilibrium approach. One is the assumption of real wage rigidity; changes in exogenous variables shift supply and demand schedules without affecting the real wage. The other is that the effective demand and supply for labor may be defined independently of the existence of disequilibrium in the labor market itself; that is, the existence of disequilibrium in the labor market does not cause firms to modify their demand for labor nor workers their supply.

A number of theoretical and empirical studies have incorporated these two features as assumptions. "General disequilibrium" models of income and employment determination have been developed by Barro and Grossman (1976) and Malinvaud (1977). The model has been extended to the open economy by Dixit (1978) and to a two-period, rational expectations context by Neary and Stiglitz (1979). The econometric implications of the disequilibrium approach have been developed by Goldfeld and Quandt (1972), Maddala and Nelson (1974) and Rosen and Quandt (1978), among others. In summary, the disequilibrium approach has provided a fruitful context for analyzing macroeconomic behavior.

Disequilibrium models are open to the criticism that they assume real wage rigidity and labor demand and supply schedules invariant with respect to the degree of labor market disequilibrium rather than derive these features from optimizing behavior. Even if these assumptions seem empirically valid, without a theory of labor market behavior which derives these features from fundamentals, policy and forecasting implications of these models are subject to the Lucas (1976) critique: contemplated changes in policy may change the price determination process itself. Furthemore, effective labor demand and supply functions may change if there is a large change in the degree of disequilibrium.

Recently, a class of models has been developed in which problems of imperfect information and transactions costs create situations in which prices cannot continuously clear markets. Calvo (1979) surveys this literature, referring to these models as "quasi-Walrasian" theories of unemployment. Included in this set of models are the implicit contract theories of Baily (1974) and Azariades (1975), the models of turnover costs of Stiglitz (1974) and Salop (1979) and the models of supervision costs of Calvo (1980). Unlike the disequilibrium approach these models derive real wage rigidities explicitly from optimizing behavior and technology. However, the implications of this body of theory for macroeconomic modelling and econometric specification have not been fully worked out. The extent to which these models imply behavior which is consistent with the disequilibrium approach is, to our knowledge, not well known.

This paper develops a model of labor market behavior similar in spirit to the implicit contract models of Baily (1974) and Azariades (1975). Unlike their models, however, ours implies behavior which is consistent with a modified version of the disequilibrium model. A modification is necessary because the presence of transactions costs implies that potential workers may not enter the labor market because they perceive the probability of unemployment as high.

In a competitive Walrasian system the notional supply of labor is defined as the amount of labor a worker wishes to provide when he can buy and sell as much as he would like in all markets, subject only to his budget constraint and given prices and wages. The notional supply of labor, then, is given by ℓ^* where $[\ell^*,c^*]$ achieves the indirect utility function

$$g(c_0,w,p) = \sup_{\ell,c}\{[u(\ell,c)],c_0+w\ell\geq pc\} \tag{1-1}$$

where u is a utility function, ℓ the labor supply, w the wage, p a commodity price vector, c_0 nonwage income, and c a vector of commodities consumed.

A common definition of the effective labor supply (see, for example, Clower, 1965) is that amount of labor a worker would like to provide at the given wage and at given commodity prices, subject to any quantity constraints in all but the labor market, i.e. the effective labor supply $[\ell^e,c^e]$ attains the indirect utility function

$$h(c_0,p,w,q') = \sup_{\ell,c}\{[u(\ell,c)],c_0+w\ell\geq pc,c'\leq q'\} \tag{1-2}$$

where c' denotes the components of c subject to quantity constraints and q' the maximum amounts available to the consumer. In the absence of transaction costs and given a non-manipulative rationing scheme[1] one would observe a typical worker providing ℓ^e to the market even if fewer than ℓ^e jobs were available in the market.

In the presence of transactions costs, however, the amount a worker will provide in the market may not be given by ℓ^e if the possibility of rationing in the labor market itself exists. If a worker bears a transactions cost in placing his labor on the market, he may be dissuaded from doing so by the possibility that he cannot sell as much as he would like at the prevailing wage. This seems to state the essence of the discouraged worker phenomenon, that in times of excess supply of labor some workers will not enter the labor force because they perceive the probability of obtaining a job as low. Nevertheless, these workers fit the definition of unemployed workers in that they would accept a job at the prevailing wage.

This phenomenon presents a problem in estimating the aggregate labor supply. Some workers are not observed in the labor market because the probability of employment is low. Something less than ℓ^e defined by (1-2) is observed. The purpose of this paper is to provide a model of the labor market when transactions costs are present and to estimate that model with aggregate U.S. time series data.

Section 2 derives the expected utility maximizing response of workers to a demand for labor which is stochastic at a given real wage. Section 3 demonstrates that the labor demand behavior assumed in this section can be consistent with

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[1]Benassy (1977) defines a non-manipulative rationing scheme as one in which the amount a worker attempts to sell in the market will not affect the demand for his labor. If the rationing scheme is non-manipulative then the worker has no incentive to dissemble in choosing ℓ^e.

profit maximizing behavior of firms. Together sections 3 and 4 constitute a model of the labor market based on optimizing behavior in which the real wage fails to adjust to equate demand and supply *ex post*. As in existing implicit contracts models, this result arises because workers must allocate themselves among firms before all randomness in the production process is resolved. Thus wage-employment packages used to attract workers cannot reflect all information about the ultimate marginal product of labor. In contrast with existing models, however, our model implies that in situations of unemployment the level of employment will be that level which equates the marginal physical product of labor to the real wage. In the Baily-Azariades models firms hire more than this amount when unemployment occurs. Hence their models, unlike ours, do not imply a disequilibrium specification.

Section 4 describes the procedure used to estimate labor demand and supply in a model when the probability of excess supply affects the level of supply itself. Results based on a time-series analysis of the U.S. labor market using this procedure appear in Section 5. The supply of labor is found to respond significantly to the probability of excess supply.

Since it has received most attention in the literature, we address our theoretical model and the econometric specification it implies to the market for labor. We feel, however, that both could be applied equally well to a number of markets where prices do not adjust to equate demand and supply in the shortest run and excess demand and supply are observed. The markets for most retail items and restaurants come to mind. Indeed, only where a number of buyers and sellers are physically together so that transactions costs involved in finding a new seller or buyer are negligible, do prices appear to be set in a way which approximates a truly Walrasian process to reflect all information.

2. Transactions Costs and Discouraged Workers: The Supply of Labor

We consider a model of considerable simplicity. Workers consume a composite commodity and leisure. An attempt to find a job is assumed costly in utility terms. Let $y = 0$ if the consumer makes no effort to find a job and $y = 1$ otherwise. We assume that a typical worker's utility function is

$$u(c,\ell,y) = v(c,\ell) - \gamma y; \; v_1 > 0, \; v_{11} \leqq 0, \; v_2 < 0, \; c \geqq 0, \; 1 \geq \ell \geq 0 \qquad (2\text{-}1)$$

where c denotes consumption of commodities and ℓ hours worked. We also assume that the function v and the value of γ is identical for all members of the labor force. The wage is given at w and consumers have an initial endowment of commodities of c_0 and time of 1. Consumption of commodities by the typical consumer is therefore $c_0 + w\ell$. We define the worker's Walrasian or unconstrained supply of hours of labor as ℓ^* where ℓ^* maximizes $v(c_0+w\ell,\ell)$ subject to $\ell \in [0,1]$. We characterize the labor force as a continuum of individuals distributed uniformly on the interval [0,1].

In the event that the number of hours of labor supplied exceeds the number demanded we assume that workers are hired according to an exogenously determined index λ which is uniformly distributed across the labor force on [0,1]. Workers with low values of λ receive jobs first. The index λ may correspond to seniority or distance from the firm or it may simply reflect the worker's physical ability to obtain a place in a queue.

A worker who is hired may work as much as he desires. For an individual worker there are three possible outcomes: he may seek a job and find one; he may seek one and fail; or he may not attempt to find one. The indirect utilities in each case are

$$v^1 \equiv v(c_0,w|\text{successful entry into labor force}) = v(c_0+w\ell^*,\ell^*) - \gamma$$
$$v^2 \equiv v(c_0|\text{unsuccessful entry into labor force}) = v(c_0,0) - \gamma$$
$$v^3 \equiv v(c_0|\text{no entry into labor force}) \equiv v(c_0,0) \tag{2-2}$$

We assume that w and v are such that $v^1 \geqq v^3$. It follows that $v^1 - v^2 \geqq \gamma$.

Now assume that $\bar{\ell}$, the total number of hours of work demanded, is a random variable with pdf $f(\bar{\ell})$ and distribution $F(\bar{\ell})$.

Consider the labor supply decision of a worker i for whom $\lambda = \lambda^i$. If all workers for whom $\lambda < \lambda^i$ have entered the labor force he will receive a job if $\bar{\ell} > \lambda^i\ell^*$ and not otherwise. We denote the probability of worker i not receiving a job upon entering the labor force conditional on all senior workers entering the labor force as P^i where

$$P^i = F(\lambda^i\ell^*) \tag{2-3}$$

Knowing that all senior workers are seeking jobs, worker i will seek a job if and only if

$$[1-F(\lambda^i\ell^*)]v^1 + F(\lambda^i\ell^*)v^2 \geqq v^3 \tag{2-4}$$

and not otherwise. Clearly, since the left hand side is decreasing in λ^i while the right hand side is independent of λ^i, if (2-4) obtains for worker i it obtains for any senior worker.

Denote by λ^* the value of λ at which the left and right hand sides of (2-4) are equal. Thus the probability that $\lambda^*\ell^* > \bar{\ell}$ is given by

$$F(\lambda^*\ell^*) = 1 - \frac{\gamma}{v^1 - v^2} \qquad (2\text{-}5)$$

which, since $v^1 - v^2 \geq \gamma$, defines a value of λ^* greater than or equal to 0. If λ^* exceeds 1, all workers will seek employment. The aggregate effective labor supply will be ℓ^* and the probability that labor supply exceeds labor demand is

$$P^* = F(\ell^*) \qquad (2\text{-}6)$$

Otherwise the effective labor supply will be $\lambda^*\ell^*$ and the probability that there will be an excess supply of labor is

$$P^* = F(\lambda^*\ell^*) \qquad (2\text{-}7)$$

According to this model the composition of the labor force may be described by the following characteristics.

C1. The actual number of hours worked is given by

$$\min[\lambda^*\ell^*, \ell^*, \bar{\ell}] \qquad (2\text{-}8)$$

C2. The proportion of unemployed workers in the labor force is given by

$$\max\{\min[\lambda^*-\bar{\ell}/\ell^*,1-\bar{\ell}/\ell^*],0\} \qquad (2\text{-}9)$$

C3. The number of discouraged workers outside the labor force is

$$\max[(1-\lambda^*),0] \qquad (2\text{-}10)$$

If $\lambda^* \geq 1$, then the labor supply is ℓ^* which depends simply on the utility function v, the level of nonwage income c_0, and the wage w. If $\lambda^* < 1$, however, the effective labor supply, $\lambda^*\ell^*$, depends also on the probability of rationing, P^*, and the distribution of $\bar{\ell}$. The following propositions characterize this relationship:

Proposition 2.1: If $\lambda^* < 1$, a rightward shift in $F(\bar{\ell})$, which for any given $\bar{\bar{\ell}}$ increases the probability that $\bar{\ell} > \bar{\bar{\ell}}$, will increase λ^*.

Proof: If F shifts right, the left hand side of (2-5) will become less than the right hand side at the initial value. Since F is increasing in λ, equality is restored at a higher level of λ. Q.E.D.

This proposition establishes the possibility of a negative relationship between the probability of rationing P^* and the effective labor supply, $\lambda^*\ell^*$. It follows from (2-5) that in the range $\lambda^* < 1$, the labor supply is perfectly elastic with respect to P^*. When $\lambda^* \geq 1$ the labor supply is perfectly inelastic.[2]

3. Implicit Contracts and Discouraged Workers: The Demand for Labor

The model in Section 2 derived the optimal response of workers to a regime in which (1) they face a certain wage, (2) they face an uncertain demand for their labor at that wage, and (3) jobs are assigned on the basis of an exogenously determined order. The assumption of a real fixed wage was imposed and no analysis was made of the determinants of the distribution of the demand for

[2] Denote the distribution of $\bar{\ell}$ as $F(\bar{\ell},\alpha)$, where α is a parameter of F, and define $\ell^S \equiv \min(\lambda^*\ell^*,\ell^*)$. Let ε be the elasticity of labor supply with respect to P^*, $[d\ell^S/d\alpha)P^*]/[(dP^*/d\alpha)\ell^S]$. From (2-5), $dP^*/d\alpha = 0$ if $\lambda^* \leq 1$ while $d\ell^S/d\alpha = 0$ if $\lambda^* > 1$. Thus for $\lambda^* \leq 1$, $\varepsilon = \infty$ while $\varepsilon = 0$ for $\lambda^* > 1$.

hours worked or the real wage itself. This section derives a model of expected profit maximizing behavior on the part of firms which justifies these assumptions.

The implicit contracting models of Baily (1974) and Azariades (1975) do provide one framework in which firms may set a real wage and vary the number of active workers. This outcome can occur when technology is random and workers cannot change jobs after the randomness is resolved. Firms must offer workers a wage and probability of employment which provides a level of expected utility equivalent to that available elsewhere in the economy. Risk aversion on the part of workers, but not of firms, will lead to an equilibrium in which the same real wage is paid in any outcome.[3]

A crucial assumption in these models is that workers and firms have equal access to information about the state of nature which has materialized. Alternatively, one might hypothesize that information about the state of nature is revealed only to firms. If this is so a potential problem of moral hazard arises and contracts of the Baily-Azariades types may be unenforceable. If, for instance, the firm has contracted to provide the same compensation in any state of nature but has provided leave in unfavorable states, then it will always have an incentive to report a favorable state of nature. If, on the other hand, the firm has agreed only to provide compensation if the worker is working, it will have an incentive to employ workers only up to the point at which the marginal product of labor equals the real wage, not above that point as in the Baily-Azariades model.

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[3] Feldstein (1976) has pointed out that in the context of these models and in the absence of unemployment insurance with imperfect experience rating, the firm will find it optimal to pay the worker even if he is not actively working. If the utility function is separable in income and leisure the firm will provide the same level of income in any state of nature regardless of whether or not the worker is working.

We assume, instead, that workers do not observe the true marginal product of labor. Enforceable contracts cannot require state-dependent actions, then, unless the actions agreed upon for the firm in any state of nature are optimal for the firm *ex post*. We also assume that probabilistic contracts are not enforceable, i.e., that a firm cannot be held to a promise to provide a wage or job with some probability less than one. Implicitly we ignore the possibility that a firm will try to build a reputation as a good employer by taking actions that are *ex post* not in its short-run best interests.[4] There must be, however, non-probabilistic elements of a contract which are enforceable. Otherwise, workers would expect the firm to exploit fully its *ex post* monopsony position and would not join its labor pool. The firm may, for instance, make a prior commitment to employ a worker at a predetermined wage in any state of nature. Alternatively, it may simply promise a wage but not guarantee employment. In this case the firm will simply hire *ex post* the number of workers which equates the marginal product of labor to the predetermined wage. In this section we show that the optimal policy of the firm may involve guaranteeing workers both a wage and a job or simply guaranteeing (a necessarily higher) wage but varying employment to maximize profits *ex post*.

The crucial assumptions of the analysis that follow are that (1) a set of identical, competitive firms maximize expected after-tax profits, (2) firms face a stochastic production function which generates a stochastic marginal product of labor, (3) workers cannot change jobs once randomness in production is resolved, (4) firms can change the level of employment once randomness is resolved, (5) workers cannot observe the outcome of the stochastic process generating the randomness in production but know its distribution and the production function, (6) workers cannot observe the level of employment in the firm, (7) firms face a recruitment cost that is increasing in λ , (8) firms

[4]Such an assumption is more appropriate, of course, to employers who do not expect to hire frequently or when the market is too large for employers to develope individual reputations. (i.e., the reputation may apply to the industry rather than to an individual firm so that maintaining a reputation becomes a public good.) See Dybvig and Spatt (1980) for an analysis of firm incentives to maintain a reputation for product quality in a dynamic context.

must pay all workers the same wage in any particular state of nature and cannot charge an application fee, and (9) the index λ is firm-specific. A worker who enters the labor force of another firm has $\lambda = 1$ at his new firm.

Assumptions (1) through (4) are standard assumptions in the theory of implicit contracts developed by Baily (1974) and Azariades (1975). Assumptions (5) and (6) introduce a problem of moral hazard not present in these models.[5] Assumptions (7) and (8) resemble those in the models of Stiglitz (1974) and Salop (1979). Firms must pay a differential cost to bring different workers to the firm, but once a worker is at the firm, he is indistinguishable from all other workers.

The value of output of a representative firm is given by the twice differentiable function $g(\ell,\theta)$ where θ represents a random variable uniformly distributed on $[0,1]$ and ℓ is the employment level. We assume that $g_1 > 0$, $g_{11} < 0$, $g_2 > 0$, $g_{12} > 0$. The recruitment cost is given by $\alpha[\max(\lambda^i)]$; i.e., the cost is proportional to the index of the

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[5] Our purpose is making these two assumptions is to remove the possibility of state and employment-level contingent contracts. The Azariades-Baily models incorporate state-contingent contracts. Hall and Lilien (1979) present a model in which contracts are not contingent on states of nature directly, (for the same reason they are not in ours--they are unobserved) but which does allow employment level contingent contracts. The results we arrive at in our model would follow if we replaced our assumptions (5) and (6) with the assumption that monitoring costs and enforcement costs are sufficiently high to render contracts contingent on these outcomes infeasible. A worker and a court can easily observe if the worker is not paid a promised wage or that he has been fired. Determining the aggregate level of employment in a labor pool might be more difficult. Furthermore, the design of total-employment contingent contracts represents an intricate problem (see Hall and Lilien) which an individual worker might find too costly to negotiate. Indeed, Hall and Lilien apply their model to contracts arrived at through collective rather than individual bargaining.

least senior workers.[6]

The firm's problem, then, is to set an employment level $\ell(\theta)$ and a wage $w(\theta)$ to maximize.

$$\pi = \int_0^1 [g(\ell(\theta),\theta)-w(\theta)\ell(\theta)-\alpha\max(\lambda^i)d\theta \qquad (3\text{-}1)$$

where π denotes expected profit.

Clearly the firm will hire workers with low values of λ first so that if $\ell(\theta)$ workers are hired, the recruitment cost is $\bar{\alpha}\ell(\theta)$. Thus, (3-1) may be written

$$\pi = \int_0^1 [g(\ell(\theta),\theta)-w(\theta)\ell(\theta)-\alpha\ell(\theta)/\ell^*]d\theta \qquad (3\text{-}1')$$

The following two propostions follow immediately from our assumptions.

<u>Proposition 3.1</u>: The firm must pay the same wage in all states of nature.

<u>Proof</u>: Irrespective of the existence of wage/employment contracts offering different wages, by assumption (5) the firm could always claim that the state of nature had materialized for which the lowest wage had been contracted.

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[6] This assumption about recruitment cost leads to the simplest specification in which the firm hires according to seniority. However, qualitatively similar results would arise under any specification of recruitment cost that is monotonically increasing in λ.

Nor can workers infer the state from the level of employment which they cannot observe by assumption (6). Q.E.D.

We denote the wage the firm offers as w.

Proposition 3.2: The firm must either guarantee employment to all workers in all states of nature or else the level of employment must not exceed that given by $\hat{\ell}(\theta)$ where

$$g_1(\hat{\ell}(\theta), \theta) = w + \alpha/\ell^* \qquad (3\text{-}2)$$

Proof: If the firm does not guarantee employment in every state of nature then the firm will never hire employees beyond the point at which (3-2) obtains and it will inform unemployed workers that a state of nature for which less than full employment had been bargained for had occurred. Note that since $g_{11} < 0$ and $g_{12} > 0$, (3-2) defines a unique $\hat{\ell}(\theta)$ which is an increasing function of θ. Q.E.D.

In the remainder of the analysis much simplicity obtains by assuming that ℓ^*, the notional labor supply, is insensitive to the wage w. We thus set $\ell^* = 1$. Relaxing this assumption complicates the analysis considerably while adding little to it.

If the firm follows a policy of hiring all workers in its labor pool in all states of nature, the expected utility of any worker is, setting $\ell^* = 1$, $v(c_0+w,1)$. If there is a positive number of discouraged workers, as we will assume throughout the remainder of this section, the wage under this regime will be given by w^{FE} where[7,8]

$$v(c_0+w^{FE},1)-\gamma = v(c_0,0) \tag{3-3}$$

If instead, the firm follows a policy of hiring different numbers of workers in different states of nature, the number of workers the firm would want to hire is given by $\hat{\ell}(\theta)$ where, from (3-2) and given w, $\hat{\ell}(\theta)$ is an increasing function of θ. Define θ^i, corresponding to the ith worker with seniority index λ^i,

[7] If, instead there were no discouraged workers, the firm would have to provide a wage w^{FE} such that $v(c_p+w^{FE},1) - \gamma = \bar{u}^1$, where $\bar{u}^1$ represents the level of utility a workers with $\gamma = 1$ would obtain elsewhere in the economy. The level of $\bar{u}^1$ would be determined by equating the aggregate demand and supply of labor as in the Azariades (1975) model

[8] The firm will guarantee jobs to only a finite number of workers and discouraged workers may remain. The number of workers the firm will hire at a wage w^{FE} is given by $\tilde{\ell}$ attaining

$$\max_{\ell} \int_0^1 [g(\ell,\theta) - (w^{FE}+\alpha)\ell]d\theta$$

If $\tilde{\ell} < 1$ then $1 - \tilde{\ell}$ discouraged workers will remain. Thus, even if a firm maintains a policy of hiring the same number of workers in any state of nature discouraged workers may exist. Any worker for whom $\lambda^i > \tilde{\ell}$ knows that he will receive a job with probability zero.

as that value of θ for which $\lambda^i \ell^* = \hat{\ell}(\theta^i)$.[9] For worker i, expected utility from entering the labor force is, given w and $\ell^* = 1$,

$$E[u^i(w)] = \theta^i v(c_0,0) + (1-\theta^i)v(c_0+w,1) - \gamma \tag{3-4}$$

which, since $\hat{\ell}(\theta^i)$ is increasing in θ^i, falls as λ^i rises. Define θ^* as the value of θ for which

$$\theta^* v(c_0,0) + (1-\theta^*)v(c_0+w,1) - \gamma = v(c_0,0) \tag{3-5}$$

and λ^* as the value of λ for which

$$\lambda^* \ell^* = \hat{\ell}(\theta^*) \tag{3-6}$$

i.e., λ^* is the index of the worker who is just indifferent between seeking and not seeking a job.[10]

By (3-4) the expected utility of any worker for whom $\lambda < \lambda^*$ exceeds $v(c_0,0)$. By offering a wage w, then, the firm can expect to attract $\lambda^* \ell^*$ hours of work since workers for whom $\lambda > \lambda^*$ will not join the labor force.

When $\ell^* = 1$, the firm's problem is to choose w to maximize

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[9] The total supply of workers with index λ^i or less is $\int_0^{\lambda^i} \ell^* d\lambda = \lambda^i \ell^*$.

[10] Again, if there were no discouraged workers, $E[u^i(w)] = \bar{u}^i$ would replace (3-5).

$$\pi = \int_0^{\theta^*} [g(\hat{\ell}(\theta),\theta)-(w+\alpha)\hat{\ell}(\theta)]d\theta + \int_{\theta^*}^1 [g(\lambda^*,\theta)-(w+\alpha)\lambda^*]d\theta =$$

$$= \int_0^{\theta^*} [g(\hat{\ell}(\theta),\theta)-(w+\alpha)\hat{\ell}(\theta)]d\theta + \int_{\theta^*}^1 g(\lambda^*,\theta)d\theta-(1-\theta^*)(w+\alpha)\lambda^* \qquad (3\text{-}1'')$$

subject to

$$v(c_0,0) \leq \theta^* v(c_0,0) + (1-\theta^*)v(c_0+w,1) - \gamma \qquad (3\text{-}7)^{11}$$

Expected profit is thus maximized with respect to the wage and a critical value of the variable that indexes states of nature. If (3-7) held as a strict inequality, the firm would reduce w as long as the number of discouraged workers is positive. Hence we assume that (3-7) is satisfied as an equality.[12] Since as an equality (3-7) defines a monotonic relationship between w and θ^*, choosing a value of w is equivalent to choosing a value of θ^*. We find it more convenient to consider the firm's problem as one of choosing θ^* optimally even though w represents the direct control variable.
The first-order condition for a maximum then is

$$\frac{d\pi}{d\theta^*} = [-\int_0^{\theta^*} \hat{\ell}(\theta)d\theta-(1-\theta^*)\lambda^*](\frac{dw}{d\theta^*})$$

$$+ [\int_{\theta^*}^1 g_1(\lambda^*,\theta)d\theta - (1-\theta^*)(w+\alpha)] \frac{d\lambda^*}{d\theta^*} \qquad (3\text{-}8)$$

where, from (3-7),

$$\frac{dw}{d\theta^*} = \frac{v(c_0+w,1) - v(c_0,0)}{(1-\theta^*)v_1(c_0+w,1)} > 0 \qquad (3\text{-}9)$$

and, from (3-2) and (3-6),

[11] If there were no discouraged workers $\bar{u}^{-1}$ would replace $v(c_0,0)$ on the left-hand side of (3-7).

[12] Again, if there were no discouraged workers, the firm would reduce w until the right-hand side of (3-7) equalled $\bar{u}^1$.

$$\frac{d\lambda^*}{d\theta^*} = \frac{dw/d\theta^* - g_{12}}{g_{11}} \tag{3-10}$$

which is ambiguous in sign. Evaluating (3-8) at $\theta^* = 0$ yields

$$\frac{d\pi}{d\theta^*} = \frac{1}{v_1(c_0+w^{FE},1)}\left\{\lambda^*\gamma+\left[\int_0^1 g_1(\lambda^*,\theta)d\theta-(w+\alpha)\right](\gamma-v_1(c_0+w^{FE},1)g_{12})/g_{11}\right\} \tag{3-11}$$

where γ is the market transactions cost and where we employed (3-3). Assume as an example that

$$g(\ell,\theta) = \theta ln\ell \tag{3-12}$$

in which case

$$\hat{\ell}(\theta) = \frac{\theta}{w+\alpha} \tag{3-13}$$

The limit of (3-11) as $\theta^* \to 0$ can be shown to be

$$[\int_0^1 g_1(0,\theta)d\theta-(w+\alpha)]/(w+\alpha) > 0 \tag{3-14}$$

since $\hat{\ell}(0) = 0$. Thus the firm will set $\theta^* > 0$. Similarly, at $\theta^* = 1$, $\frac{dw}{d\theta^*}$ becomes infinite by (3-9) and (3-8) is negative. Under a plausible assumption about production, an interior solution for θ^* can occur.

It remains to show that a strategy of varying employment at a relatively high wage can dominate one of holding employment constant at a low wage. We consider the production function given by (3-12).

Expected profit given a decision to hold employment constant is given by

$$\max_{\ell}[\frac{1}{2}ln\ell-(w^{FE}+\alpha)\ell] \tag{3-15}$$

or

$$\pi^{FE} = -\frac{1}{2}[ln2+ln(w^{FE}+\alpha)+1] \tag{3-15'}$$

Expected profit given a decision to vary employment is given by

$$\pi^{*VE} = \max_{\theta^*\varepsilon(0,1)} \pi^{VE}(\theta^*) = \max_{\theta^*\varepsilon(0,1)} \frac{1}{2}[ln\theta^*-ln(w+\alpha)+(\theta^*)^2/2] - \theta^* \tag{3-16}$$

subject to (3-7).

To show that $\pi^{*VE} > \pi^{FE}$ can occur we evaluate the maximand of (3-16) at $\theta^* = .5$ yielding

$$\pi^{VE}(.5) = -\frac{1}{2}[ln2+ln(w+\alpha)] - \frac{7}{16} \tag{3-17}$$

Thus,

$$\pi^{VE}(.5) - \pi^{FE} = \frac{1}{2}[ln(w^{FE}+\alpha)-ln(w+\alpha)] + \frac{1}{16} \tag{3-18}$$

which is positive if

$$ln(w+\alpha) - ln(w^{FE}+\alpha) < \frac{1}{8} \tag{3-19}$$

The wage levels w^{FE} and w satisfy

$$v(c_0+w^{FE},1) = v(c_0,0) + \gamma \tag{3-20}$$

and

$$\frac{1}{2}v(c_0+w,1) + \frac{1}{2}v(c_0,0) = v(c_0,0) + \gamma \qquad (3\text{-}20)$$

and at $\gamma = 0$ (3-19) and (3-20) are solved by $w = w^{FE}$. If v is continuous and one-to-one, then

$$\lim_{\gamma \to 0} w = w^{FE} \qquad (3\text{-}21)$$

By choosing γ near zero the left-hand side of (3-19) can be made arbitrarily small and (3-19) will obtain. A variable employment policy with $\theta^* = .5$ dominates a fixed employment policy for small values of γ.

To illustrate this result more explicitly consider a utility function of the form

$$v(c,\ell) = ln[c+\beta(1-\ell)] \qquad (3\text{-}22)$$

which yields $\ell^* = 1$ for $w > \beta$. Assume that, by coincidence, $c_0 = \alpha$. Under these assumptions (3-19) and (3-20) imply

$$ln(w^{FE}+\alpha) = ln(\alpha+\beta) + \gamma \qquad (3\text{-}23)$$

$$ln(w+\alpha) = ln(\alpha+\beta) + 2\gamma \qquad (3\text{-}24)$$

Expression (3-18) thus becomes

$$-\frac{\gamma}{2} + \frac{1}{16}$$

which is positive when $\gamma < \frac{1}{8}$.

Figure 1 illustrates the determination of the equilibrium values of θ^*, λ^* and w. The expected profit π of the representative firm is measured on the positive portion of the vertical axis and θ^* is measured on the positive portion of the horizontal axis. The curve π^{VE} in the northeast quadrant relates θ^* and expected profit incorporating (3-2) and (3-7) as an equality. As shown in the text, π^{VE} slopes down at $\theta^* = 1$ and if $g(\ell,\theta) = \theta \ln \ell$, for instance, up at $\theta^* = 0$. The negative portion of the vertical axis measures λ, the demand for labor. The curve $\hat{\ell}(\theta,w)$ in the southeast quadrant relates θ to λ via the marginal productivity relation (3-2). In general it slopes downward. When $g(\ell,\theta) = \theta \ln \ell$ it is a straight line with slope $-1/(w+\alpha)$. An increase in w will shift $\hat{\ell}(\theta,w)$ upward; fewer workers will be employed at each value of θ.

The negative portion of the horizontal axis measures expected worker utility. The curve $u(w)$ in the southwest quadrant relates expected utility of entrants to the labor force to seniority, measured along the negative vertical axis, given w and the distribution of $\bar{\ell}$. Since workers for whom λ is low are hired first, senior workers enjoy higher expected utility and the curve slopes downward. The line v^3, defined in (2-2), measures utility of non-entrants to the labor force. The size of the labor force λ^* is given by the vertical distance between the origin and intersection of $u(w)$ and v^3. Workers for whom $u(w) < v^3$ do not enter the labor force while those for whom $u(w) \geq v^3$ do.

An increase in w, given $F(\bar{\ell})$, shifts $u(w)$ leftward and increases number of workers in the labor force. Given w, a rightward shift of $F(\bar{\ell})$, increasing the probability of a high demand for labor, also shifts the $u(w)$ curve leftward, increasing the labor supply.

Given θ^*, the equilibrium wage equates the demand for labor at θ^*, $\hat{\ell}(\theta^*,w)$,

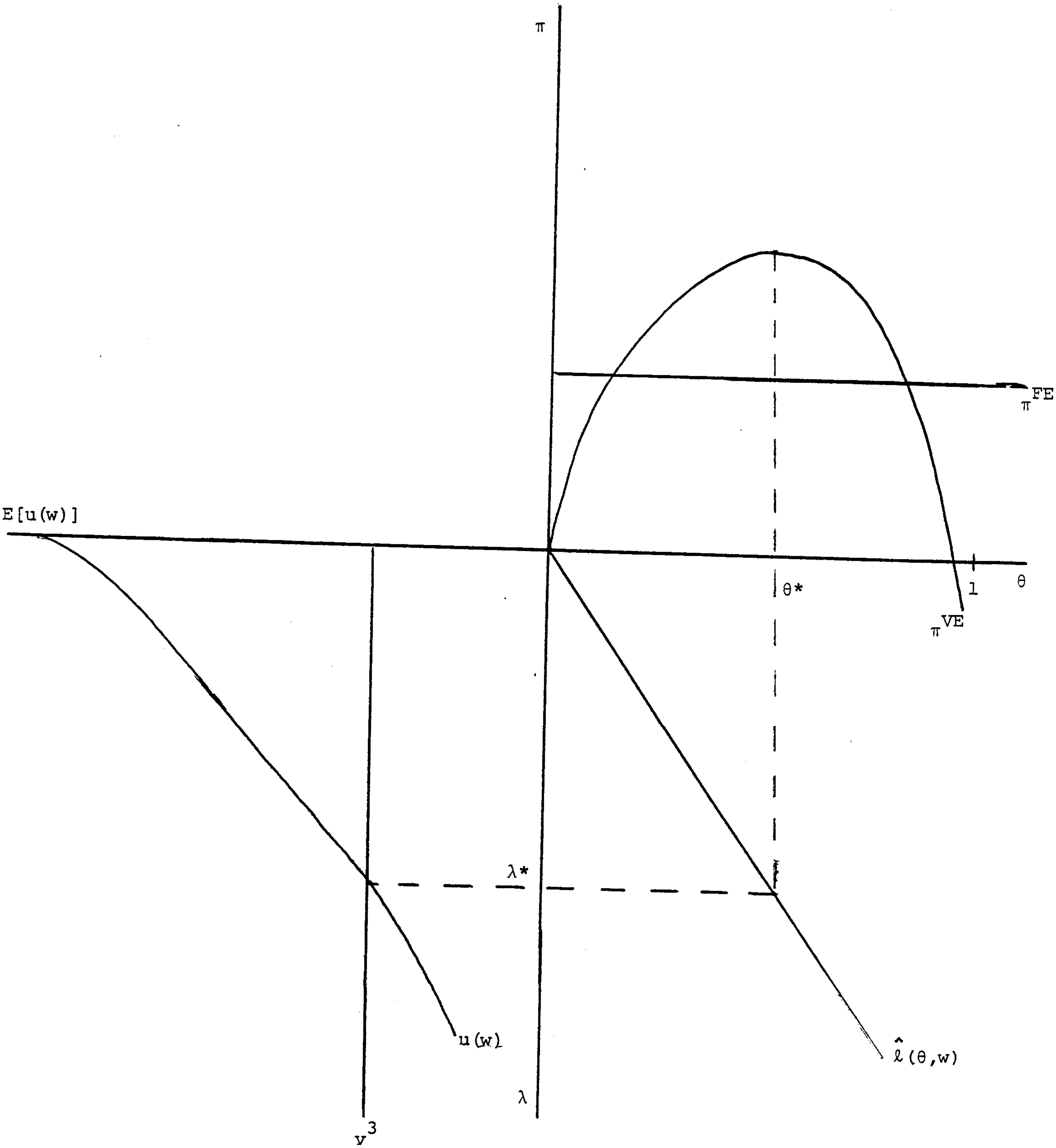

Figure 1

to the number of workers who enter the labor force. A reduction in w from equilibrium will shift the $\hat{\ell}(\theta,w)$ curve downward and increase the number of workers demanded at θ^*. The reduction in w will have the direct effect of shifting the $u(w)$ curve rightward but at the lower wage the demand for labor will be higher at any value of θ, shifting the $u(w)$ curve leftward. If θ^* is an optimum, however, then the net effect must be to shift $u(w)$ rightward. A reduction in w must reduce the labor supply. Otherwise, the firm could reduce w and increase λ^*, thereby raising expected profits.[13] The fall in w thus creates an excess demand for labor. The firm cannot hire the number of workers it desires at θ^* so that θ^* no longer can represent the critical value of θ. Given θ^*, then, there is a unique w which clears the labor market.

The curve π^{FE} represents the expected profit level of a firm which guarantees workers jobs in any state of nature. As we have shown above, it may lie above or below the point at which π^{VE} achieves a maximum.

We have shown, by way of an example, the possibility that the optimal hiring policy of a firm may indeed involve announcing a fixed real wage and varying employment in different states of nature. While a similar result arises in the implicit contract literature as represented by Baily (1974) and Azariades (1975), there are some significant differences. First, the justification for the fixed real wage arises not from workers' risk aversion but from their inability to observe the outcome of the random process generating movements in the marginal product of labor. Secondly, in the Baily-Azariades models the level of employment in unfavorable states of nature exceeds the level at which the marginal product of labor equals the real wage. The problem of moral hazard and the

[13]Mathematically, this is shown by the first-order condition (3-8). The first term is necessarily negative. The second terms can be positive only if $d\lambda^*/d\theta^* > 0$. Since $dw^*/d\theta^* > 0$ an interior maximum can occur only when $d\lambda^*/dw^* > 0$.

inability of individual workers to observe the total employment level and the marginal product of labor precludes such a possibility in the present model. If the firm varies employment at all, the level of employment is given by the minimum of the supply and demand for labor where the demand for labor is that level which equates the marginal product and the wage. Thirdly, because of moral hazard the present model does not admit the possibility of firms providing paid leave as a form of compensation. In the absence of a welfare or unemployment compensation scheme which made unemployment advantageous, this possibility would preclude equilibria in which the level of employment varied. Fourthly, in the current model differential recruitment costs lead to hiring by seniority rather than randomly.

4. Econometric Implementation

The foregoing sections have provided a theoretical justification for the claims that (1) firms may find it profitable to fix the wage and allow employment to vary in response to stochastic variations in marginal product curves; (2) in response to random variations in employment possibilities, workers' effective demand responds to the probability of rationing. The present section discusses how such a model may be implemented econometrically.

The Likelihood Function. The standard disequilibrium model for a single market is given by

$$D_t = \beta_1' x_{1t} + u_{1t} \qquad (4\text{-}1)$$

$$S_t = \beta_2' x_{2t} + u_{2t} \qquad (4\text{-}2)$$

$$Q_t = \min(D_t, S_t) \qquad (4\text{-}3)$$

where x_{1t}, x_{2t} are vectors of exogenous variables, β_1, β_2 vectors of parameters and where D_t, S_t are not observed but Q_t is observed.[14] It is usually also assumed that $(u_{1t},u_{2t}) \sim N(0,\Sigma)$ independently of t; according to the most frequent usage Σ is assumed to be diagonal.[15] In more complicated versions of the disequilibrium model a price-adjustment equation may be appended to (3-1) through (3-3); treatment of this generalization is deferred to Section 5.

In the present section we assume without loss of generality that the buyer is rationed. Denote by P the probability that the buyer will be rationed. The simplest form in which the presence of P in the effective demand function can be implemented is to introduce it linearly into (4-1), replacing that equation by

$$D_t = \beta_1' x_{1t} + \rho P_t + u_{1t} \tag{4-4}$$

If the probability of being rationed has a dampening effect on the quantity demanded, as is suggested by the model of Section 2, ρ will be negative.

An important question is how, precisely, P_t is to be interpreted. In the disequilibrium model there are two possible interpretations. In order to indicate these, define the event $D_t \geq S_t$ by R_t; the conditional density of Q_t, conditional on the event R_t, by $f(Q_t | D_t \geq S_t)$; the probability density functions of D_t and S_t from (3-1) and (3-2) by $f_1(D_t)$, $f_2(S_t)$; and the corresponding cumulative distribution functions by $F_1(D_t)$, $F_2(S_t)$ respectively. Then we can write

[14] Theoretical and computational aspects of this model are discussed in Maddala and Nelson (1974), Goldfeld and Quandt (1979), Quandt (1978), Laffont and Monfort (1976) and others.

[15] For some computational difficulties with nondiagonal Σ see Goldfeld and Quandt (1979).

$$h(Q_t) = f(Q_t|D_t<S_t)\Pr\{D_t<S_t\} + f(Q_t|D_t\geq S_t)\Pr\{D_t\geq S_t\}$$
$$= f_1(Q_t)(1-F_2(Q_t)) + f_2(Q_t)(1-F_1(Q_t)) \tag{4-5}$$

The probability of the event R_t can then be interpreted either unconditionally as

$$\Pr\{R_t\} = \Pr\{D_t\geq S_t\} \tag{4-6}$$

or conditionally on Q_t as

$$\Pr\{R_t|Q_t\} = \frac{\Pr\{Q_t|R_t\}\Pr\{R_t\}}{\Pr\{Q_t\}} = \frac{f_2(Q_t)(1-F_1(Q_t))}{f_1(Q_t)(1-F_2(Q_t)) + f_2(Q_t)(1-F_1(Q_t))} \tag{4-7}$$ [16]

The spirit of the model in Section 2 suggests that the unconditional (i.e., prior) probability in (4-6) is the appropriate one. It is also clear that using (4-7) in (4-4) would tend to make the problem intractable since the pdf of Q_t is obtained by integrating the joint pdf of D_t and S_t with respect to D_t and S_t over part of the range; but D_t and S_t, in turn, have a pdf which depends on Q_t through P_t! We shall thus define

$$P_t = \Pr\{D_t\geq S_t\} = \Pr\{\beta_1'x_{1t}+\rho P_t+u_{1t}\geq\beta_2'x_{2t}+u_{2t}\} = \Pr\{\beta_1'x_{1t}-\beta_2'x_{2t}+\rho P_t\geq u_{2t}-u_{1t}\} =$$

$$\int_{-\infty}^{\beta_1'x_{1t}-\beta_2'x_{2t}+\rho P_t} \frac{1}{\sqrt{2\pi}\sigma} e^{-v^2/2\sigma^2} dv \tag{4-8}$$

where $\sigma^2 = \sigma_1^2 + \sigma_2^2$ for independent u_{1t}, u_{2t}.

We have the following

Proposition 4-1: For any β_1, β_2, σ, x_{1t}, x_{2t} Equ. (4-8) determines a unique P_t, $0 < P_t < 1$.

Proof: For $P_t = 0$ the right hand side of (4-8) is positive and less than

[16] See Gersovitz (1978). The same distinction can be made in related models such as switching regression models in which nature selects between two regimes with constant probabilities λ and $1-\lambda$.

unity. The right hand side is a monotone decreasing function of P_t since $\rho < 0$. Since the left hand side is zero at the origin and has constant slope = 1, the mapping from the right hand side to the left possesses a fixed point in the open interval (0,1).

Corollary: A unique solution exists if $\rho > 0$. In this case the right hand side is an increasing concave function greater than zero at the origin and less than unity at $P_t = 1$; hence a unique fixed point exists.

The probability P_t depends only on parameters and on the exogenous variables x_{1t}, x_{2t}. The pdf of Q_t can then be derived in exactly the same fashion as in the standard case by substituting in (4-5) the appropriate normal densities and distribution functions. We then have

$$h(Q_t) = \frac{1}{\sqrt{2\pi}\sigma_1}\exp\{-\frac{(Q_t-\beta_1'x_{1t}-\rho P_t)^2}{2\sigma_1^2}\}(1-\Phi(\frac{Q_t-\beta_2'x_{2t}}{\sigma_2})) + \frac{1}{\sqrt{2\pi}\sigma_2}\exp\{-\frac{(Q_t-\beta_2'x_{2t})^2}{2\sigma_2^2}\}(1-\Phi(\frac{Q_t-\beta_1'x_{1t}-\rho P_t}{\sigma_1})) \tag{4-9}$$

where $\Phi(\ )$ is the standard normal cumulative distribution and where P_t is determined from (4-8). The likelihood function is

$$L = \prod_t h(Q_t) \tag{4-10}$$

Sampling Experiments. The purpose of the experiments described below is primarily to verify that maximum likelihood estimates can be derived for the model expressed by (4-2) to (4-4). They are meant to be merely illustrative: because of the high cost of the computations the number of replications was kept very small. Numerical optimization involves the repeated evaluation of the logarithm of the function (4-10); an unusual feature of the present problem

Table 1. Features of Experiments

Variables or Coefficients	Case 1 N=60	Case 2 N=60	Case 3 N=30
ρ	-30	-50	-30
x_{1t}	U(4,9)	U(4,9)	U(4,9)
x_{2t}	U(35,55)	U(30.70)	U(35,55)
x_{3t}	U(7.5,12.5)	U(2.5,17.5)	U(7.5,12.5)

is that every function evaluation requires the solution of n transcendental equations of the form (4-8) where n is the sample size.

Data were generated from the model

$$D_t = \beta_0 + \beta_1 x_{1t} + \beta_2 x_{2t} + \rho P_t + u_{1t} \tag{4-11}$$

$$S_t = \beta_4 + \beta_5 x_{1t} + \beta_6 x_{3t} + u_{2t} \tag{4-12}$$

$$Q_t = \min(D_t, S_t) \tag{4-13}$$

The error terms u_{1t}, u_{2t} were normally distributed, independently of one another and of t, with mean zero and variances σ_1^2, σ_2^2.[17] The parameter values which were the same in all experiments were $\beta_0 = 110$, $\beta_1 = -8$, $\beta_2 = 1$, $\beta_4 = 10$, $\beta_5 = 10$, $\beta_6 = 2$, $\sigma_1^2 = 10$, $\sigma_2^2 = 10$. The remaining features of the experiments are described in Table 1 where U(a,b) indicates the uniform distribution between a and b. The samples of x's were chosen once-and-for-all in each experiment and were kept identical under the replications.

An important feature of the present model is that a relatively small fraction of the observations is noticeably different from those that would have been obtained if ρ had been identically zero. The reason for this is as follows. First,

[17]The normal deviates were obtained by applying the Box-Muller transformation to pseudo-random uniformly distributed numbers on (0,1) generated by a congruential method.

if the realizations of error terms are such that $\beta_1 x_{1t} + u_{1t} > \beta_2 x_{2t} + u_{2t}$, then if ρ were zero in reality, S_t would be selected as Q_t. If one were to introduce a negative ρ, there will be many instances in which $\beta_1 x_{1t} + \rho P_t + u_{1t}$ is still greater than $\beta_2 x_{2t} + u_{2t}$ and hence for these observations exactly the same Q_t is observed. In the reverse case D_t is selected as Q_t so that if one now were to allow for term ρP_t with a negative ρ, it would appear that D_t would have to diminish and thus lead to a different (smaller) Q_t. This is, however, frequently not the case, since if $\beta_2 x_{2t} + u_{2t}$ exceeds $\beta_1 x_{1t} + u_{1t}$ sufficiently, the solution to (4-8) will be $P_t \approx 0$ and hence there again is no effect on Q_t. On the average only about one fifth of the observations showed a difference from what they would have been if ρ had been zero. This rests the ability of the model to provide estimates of ρ on a small number of observations and suggests that the performance of the model in small samples may not be very good.[18]

For each replication of each experiment the likelihood function (4-10) was maximized. In addition we also maximized the corresponding (simple disequilibrium) likelihood obtained by setting ρ identically to zero.[19] Computations were extremely expensive and only few replications were obtained in each case. Salient results are described in Table 2. Approximately half of all the observations came from the demand side and half from the supply side. The number of computational failures was large; clearly this number would have been reduced if further analysis had been undertaken of each failed replication.

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[18] A second difficulty is that $P(\rho)$, the probability written as a function of ρ, has a numerically very small derivative with respect to ρ except in the neighborhood of zero. As a result the likelihood function tends to be somewhat flat with respect to ρ.

[19] Optimization in each case employed first the DFP algorithm and was followed by reoptimization using the Goldfeld-Quandt-Trotter GRADX algorithm (see Goldfeld and Quandt (1972)). Solution of the transcendental equation (3-8) was by the method of false position (regula falsi). No further analysis of a replication was undertaken if computational failure occurred. Derivatives required during optimization were obtained by numerical approximation.

Given the small number of replications, meaningful comparisons of the more general method with the likelihood function (4-10) with the simple disequilibrium model are not possible. Of the 24 possible comparisons of mean square errors, the more general method has smaller MSE in 13 cases. Several outliers produced by the maximization of (4-10) are in part responsible for this result. Finally, one can also examine the fraction of replications in which the standard likelihood ratio test rejects the null hypothesis $H_o: \rho = 0$. Testing $-2\log\lambda$ against the critical values of $\chi^2(1)$, at the .10 level, H_o is correctly rejected in 87, 100 and 36 percent of the cases. In view of the difficulties of estimating the model, and the relatively small sample size for all cases, but particularly Case 3, this appears to be satisfactory power.

5. An Economic Example

Recently Rosen and Quandt (1978) estimated demand and supply functions for an aggregate labor market employing the following disequilibrium specification:

$$lnD_t = \alpha_0 + \alpha_1 lnw_t + \alpha_2 lnQ_t + \alpha_3 t + u_{1t} \tag{5-1}$$

$$lnS_t = \beta_0 + \beta_1 lnw_{nt} + \beta_2 lnA_{nt} + \beta_3 lnH_t + u_{2t} \tag{5-2}$$

$$lnL_t = \min(lnD_t, lnS_t) \tag{5-3}$$

$$lnw_t - lnw_{t-1} = \gamma_1(lnD_t - lnS_t) + \gamma_2 V_t + u_{3t} \tag{5-4}$$

where L_t is the actual amount of labor, D_t and S_t are labor demand and supply, w_t is the wage, w_{nt} is the net wage, Q_t is GNP, H_t is potential man-hours, A_{nt} is net unearned income, t is a time trend, and V_t is alternately chosen to be the fraction of the labor force that is unionized (U_t), or the change in the quantity $(U_t - U_{t-1})$, or simply a constant (1.0). The error terms were assumed to be normally and independently distributed with mean zero and variances σ_1^2, σ_2^2, σ_3^2. Estimates for the model were obtained using U.S. data for the years 1930-73.[20] Reasonable coefficients

[20] See Rosen and Quandt (1978) for the sources of data.

Table 2. Results of Experiments

	Case 1		Case 2		Case 3	
	Mean Biases					
	$\beta_3 \neq 0$	$\beta_3 \equiv 0$	$\beta_3 \neq 0$	$\beta_3 \equiv 0$	$\beta_3 \neq 0$	$\beta_3 \equiv 0$
β_0	9.16	3.24	1.48	-13.73	12.04	12.02
β_1	-.04	.95	-.45	.40	.05	.08
β_2	.04	-.02	.05	.01	.04	.05
β_3	-18.42	-	-133.56	-	-66.63	-
β_4	3.07	6.32	4.08	6.65	.58	7.13
β_5	-.45	-1.32	-.02	-1.20	.18	1.31
β_6	-.04	-.01	-.08	-.16	.12	.10
σ_1^2	-1.01	.00	-1.38	.89	3.10	2.89
σ_2^2	-1.99	-2.50	21.93	-1.92	14.15	1.94
	Mean Square Errors					
β_0	98.77	91.26	6.19	38.10	610.86	449.23
β_1	1.16	1.84	1.13	1.32	9.84	9.80
β_2	.50	.50	.02	.02	.08	.10
β_3	3.89×10^3	-	1.11×10^5	-	41.27×10^5	-
β_4	37.29	63.03	45.42	53.92	112.34	88.31
β_5	.55	2.24	.40	1.69	2.10	2.27
β_6	.14	.11	.02	.03	.30	.16
σ_1^2	8.62	7.66	6.19	12.56	21.60	24.28
σ_2^2	17.20	9.41	331.27	6.79	835.15	9.31

	Case 1	Case 2	Case 3
Successful Replications	8	8	19
Number of Failures	5	2	10
Number of Observations with $D \geq S$	39.63	25.13	22.16

emerged from the estimations and the hypothesis of equilibrium was strongly rejected in favor of the disequilibrium interpretation. The only unsatisfactory nature of the results was that the unemployment of the 1930's was not predicted when the excess demand function was calculated by substituting the estimated coefficients in the structural equations.

It appears reasonable to reformulate the present model to account for the possibility that the probability that labor supply will be rationed affects the supply of labor itself. Accordingly (5-2) was reformulated as

$$\ln S_t = \beta_0 + \beta_1 \ln w_{nt} + \beta_2 \ln A_{nt} + \beta_3 \ln H_t + \beta_4 P_t + u_{2t} \qquad (5\text{-}2')$$

where $P_t = \text{Prob}\{S_t > D_t\}$.[20] The maximum likelihood estimates from the procedure discussed in Section 3 are displayed in Table 3 in the columns labelled EQ. The original estimates due to Rosen and Quandt are shown in the columns labelled RQ.

Cases 1 and 3 show practically no change in either the coefficients or in the value of the likelihood function and the estimated values of β_4 are not significant. Case 2, which was economically the most plausible because of its use of $V_t = U_t - U_{t-1}$, had done worst originally in terms of the value of the likelihood function (although a straightforward comparison of likelihood function values across cases is not strictly valid for these represent nonnested hypotheses). Introducing $\beta_4 P_t$ into the supply function causes a statistically significant improvement in the likelihood function value and the resulting estimate of β_4 has the a priori expected sign and is statistically significant. The numerical magnitude of the coefficient and its standard error are themselves

[20] Note that in the present case the computation of this probability requires that one first obtain the reduced form from (5-1), (5-2) and (5-4).

Table 3. Estimates of the Aggregate Labor Market Model*

Coefficient	Case 1 $V_t=U_t$ RQ	 EQ	Case 2 $V_t=U_t-U_{t-1}$ RQ	 EQ	Case 3 $V_t=1.0$ RQ	 EQ
α_0	-1.330 (.199)	-1.313 (0.47)	-1.641 (.404)	-2.350 (0.64)	-1.460 (.261)	-1.464 (.047)
α_1	-.984 (.105)	-.983 (.094)	-1.085 (.120)	-1.246 (.139)	-.974 (.094)	-.975 (.096)
α_2	1.095 (.038)	1.092 (.007)	1.153 (.074)	1.297 (.012)	1.122 (.051)	1.122 .00
α_3	-.003 (.003)	-.003 (.003)	-.003 (.002)	-.005 (.004)	-.004 (.003)	-.004 (.003)
β_0	.209 (.496)	.258 (.079)	1.132 (1.199)	-.056 (.119)	.059 (1.196)	.047 (.079)
β_1	.008 (.040)	.008 (.043)	.031 (.051)	.051 (.041)	-.0001 (.051)	-.0002 (.039)
β_2	.490 (.046)	.491 (.050)	.482 (.049)	.428 (.049)	.492 (.048)	.492 (.047)
β_3	.871 (.091)	.862 (.014)	.700 (.219)	.914 (.021)	.899 (.219)	.901 (.014)
β_4	- -	.003 (.022)	- -	-.224 (.094)	- -	-.0007 (.021)
γ_1	.182 (.058)	.181 (.058)	.207 (.103)	.249 (.075)	.125 (.056)	.135 (.057)
γ_2	.002 (.0003)	.002 (.0003)	.006 (.007)	.001 (.008)	.030 (.006)	.030 (.006)
logL	202.64	202.65	187.86	193.15	199.91	199.91

*asymptotic standard errors in parentheses interesting for we can examine the impact upon labor supply of various hypothetical probabilities of being rationed. Some of these results in Table 4. Thus, for example, if the probability of not finding a job is .1, the point estimate for β_4 implies a 2 percent reduction in the supply of labor.

Table 4. Percentage Reduction in Labor Supply Due to Probability of Being Rationed

	$P_t=1$	$P_t=.5$	$P_t=.1$
At point estimate for coefficient minus two standard deviations	34%	19%	4%
At point estimate for coefficient	20%	11%	2%
At point estimate for coefficient plus two standard deviations	4%	2%	.4%

The original versions of the model produced poor predictions of the excess supply of labor and of the implied unemployment rate. A significant improvement in this respect was effected by Romer (forthcoming) who reestimated Case 3 while omitting the nonlabor income variable from the labor supply equation. The resulting implied unemployment rates track the official unemployment figures closely. Introducing the term $\beta_4 P_t$ into the Romer version yields a significant improvement in the likelihood and a significant value for β_4 (-.514) and substantially reproduces the unemployment predictions of the Romer version. Thus, on the whole, the present technique appears useful in a concrete case and leads to reasonable interpretations.

6. Conclusion

We have shown that disequilibrium in a market need not rest on an ad hoc assertion of price or wage rigidity but may emerge as a result of optimizing behavior by firms subject to stochastic variation of their production functions. Under these circumstances, plausible assumptions about workers' utility functions lead the effective supply of labor depending on the probability of rationing. The paper uses these theoretical underpinnings to formulate econometric models and applies maximum likelihood estimation to an aggregate labor market in which labor supply is a function of the probability of unemployment. The numerical estimates support the hypothesis that a high likelihood of unemployment reduces the effective labor supply.

The theoretical model and estimation techniques described in this paper may be relevant to a number of diverse problems. For instance, a very similar framework might be used to examine the demand for transportation services when congestion costs are significant. For another example, the claim has been made that women and minorities have been discouraged from seeking jobs in some sectors by the perceived low probability of hiring based on previous employment practices. Our model and technique suggest a way of detecting the extent to which this is true.

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