ESTIMATION OF RATIONAL EXPECTATIONS MODELS

Gregory C. Chow*

Abstract: This paper considers the estimation of linear rational expectations models when the objective function of the decision maker is quadratic. It presents methods for maximum likelihood estimation in the general case and in a special case when the decision maker's action is assumed to have no effect on the environment (as under perfect competition). It proposes a family of consistent estimators for the general case. It also comments on the assumptions of rational expectations models, and extends the above methods to estimating nonlinear models.

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1. INTRODUCTION

In an optimal control problem where the model is linear

$$y_t = Ay_{t-1} + Cx_t + b + u_t \tag{1}$$

and the objective function to be maximized is quadratic

$$-E_o \sum_{t=1}^{T} (y_t - a_t)' K_t (y_t - a_t) , \tag{2}$$

the optimal feedback rule for the vector x_t of the control variables is linear in the state variables y_{t-1} [cf. Chow (1975)],

$$x_t = G_t y_{t-1} + g_t . \tag{3}$$

In this paper, we assume $K_t = \beta^t K$ and $a_t = \phi^t a$, β being a discount factor and ϕ being a diagonal matrix, with some diagonal elements known to be unity if the targets in a_t are time-invariant. We will be concerned with the estimation of the parameters β, K, ϕ, and a in the objective function and the parameters A, C, and b of the model, using data on (y_t, x_t).

In the literature of macroeconomic policy analysis following the tradition of Theil (1958) and Friedlander (1973), this note would be entitled the estimation of government preference functions in policy optimization problems. Its present title is motivated by the more recent literature on macroeconomic modelling and analysis which has been stimulated by the works of Muth (1961)

and Lucas (1976), and further extended by Sargent (1978, 1979), Hansen and Sargent (1980), and Taylor (1979), among others. Consider economic agents (firms, households) facing a stochastic environment described by (1) and having an objective function (2). They are assumed to derive their behavioral equations (the demand equations for inputs, the consumption functions, etc.) given by (3) through the maximization of (2) subject to the constraint (1). Under the assumption of rational expectations, the econometrician shares the same functions (1) and (2) with the economic agents. The econometrician's problem is to estimate (1) and (2) by observing the data on x_t and y_t. This is really a classic problem in econometric modelling, except that the economic agents are assumed to maximize a multiperiod objective function under a stochastic environment (with uncertainty). More on this point in Section 5.

This paper presents methods for the maximum likelihood estimation of linear rational expectations models just described, covering the general case and the special case when the agent's action x_t does not affect the economic environment as in the model of perfect competition. The special case is exemplified by the models used by Sargent (1978,79) and by Hansen and Sargent (1980). We obtain explicit expressions for the coefficients in the agent's behavioral equation (3) in terms of the parameters of (1) and (2) using the known results on stochastic control theory in Chow (1975). To ease the computations in the general case, we propose a family of consistent estimators which are analogous to the methods of limited-information maximum likelihood and two-stage least squares for the estimation of linear simultaneous equations. In this paper, we will frequently be interested in estimating the parameters when the coefficient matrix G_t in (3) reaches a steady state G (the rational expectations equilibrium). The results will be extended to estimating nonlinear models.

2. MAXIMUM LIKELIHOOD ESTIMATION IN THE GENERAL CASE

Our problem is to estimate the parameters of (1) and (2) using observations on y_t and x_t. It is understood that a system involving high-order autoregressive and moving average processes can be written in the form (1) where u_t are serially uncorrelated and identically distributed, as is done in Chow (1975). If one is willing to add a random residual to (3) and assume a multivariate normal distribution for this residual and u_t, the likelihood function based on (1) and (3) is well-known. It has A, C, b, G_t, g_t and the covariance matrix of the residuals as arguments. If (1) is a set of reduced-form equations derived from a system of linear simultaneous structural equations, the parameters A, C, b and the covariance matrix of u_t will be replaced by the corresponding structural parameters as arguments in the likelihood function.

What makes our problem different from the standard problem of estimating the parameters of a system of linear structural equations is that we need to maximize the likelihood function with respect to the parameters β, K, ϕ, and a of the objective function (with $K_t = \beta^t K$ and $a_t = \phi^t a$) instead of the coefficients G_t and g_t in the behavioral equation (3). To apply any gradient or conjugate gradient method for maximization [cf. Goldfeld and Quandt (1972)], it is first required to evaluate the likelihood function in terms of the parameters A, C, b, β, K, ϕ, and a (after the covariance matrix of the residuals has been concentrated out), where A, C, and b will further be written as functions of the coefficients of the structural equations if necessary. The problem then boils down to the convenient expression of G_t and g_t as functions of A, C, b, β, K, ϕ and a.

The coefficients of (3) as solution to the optimal control problem (1)-(2) are given in Chow (1975, pp. 178-179):

$$G_t = -(C'H_tC)^{-1}C'H_tA \qquad (4)$$

$$H_t = K_t + (A + CG_{t+1})'H_{t+1}(A + CG_{t+1}) \qquad (5)$$

$$g_t = -(C'H_tC)^{-1}C'(H_tb_t - h_t) \qquad (6)$$

$$h_t = K_ta_t + (A + CG_{t+1})'(h_{t+1} - H_{t+1}b_{t+1}) \qquad (7)$$

with conditions $H_{t+N} = K_{t+N} = \beta^N K_t$ for (5) and $h_{t+N} = K_{t+N}a_{t+N} = K_{t+N}\phi^N a_t$ for (7) if the planning horizon is N. To compute G_t, we evaluate the right-hand sides of (4) and (5) backward in time starting from t+N, using the initial condition $H_{t+N} = \beta^N K_t$. Having completed these calculations, we compute g_t by evaluating the right-hand sides of (6) and (7) backward in time starting from t+N, using the initial condition $h_{t+N} = K_{t+N}\phi^N a_t$.

Even for fairly large N, these computations are inexpensive provided that the (symmetric) matrix H_t is not too large, say with order less than one hundred. Some computational experience is recorded in Chow and Megdal (1978). The computations consist mainly of matrix multiplications. The matrix $C'H_tC$ to be inverted is of the same order as the number of control variables, which is very small as judged by the cost of matrix inversion using a modern computer. Furthermore, even if N is very large, experience shows that a steady-state solution for G_t and H_t from (4) and (5) is often reached after 4 or 5 time periods backward from t+N, as illustrated in Chow (1975, pp. 208, 270). Thus only several evaluations of (4) and (5) are required. If (4) and (5) do converge slowly, the model of rational expectations adopted to derive a steady-state G in equation (3) should itself be questioned. The failure for (4) and (5) to converge would mean that a rational expectations equilibrium does not exist for the behavior of the economic agent. A slow convergence means that the economic agent needs to plan many periods ahead under the

questionable assumption of a constant economic structure for all future periods (the same matrices A and C being used in the calculations of (4) and (5) for all future periods). We thus argue that in practice the coefficients G and g_t in (3) can frequently be computed inexpensively from the parameters A, C, b, β, K, φ and a.

Since the computation of G and g_t is only a first step (the step of evaluating the likelihood function) in the method of maximum likelihood, the second step being to maximize numerically, it would be very desirable if G could be expressed explicitly as a function of the parameters without resorting to repeated calculations of (4) and (5). In the next section, we treat a special case where this can be done.

3. ESTIMATION WHEN ENVIRONMENT IS UNAFFECTED BY AGENT'S ACTION

Let the environment be described by

$$\tilde{y}_t = A_1 \tilde{y}_{t-1} + \tilde{u}_t \tag{8}$$

which is not affected by the agent's action x_t. This special case includes the examples given by Sargent (1978, 1979) and Hansen and Sargent (1980). These references use an example of a firm trying to determine its optimal employment of an input while facing a set of stochastic difference equations (8) which explain the price of the input and a technological coefficient. To allow for the costs of the control variables and their changes, we introduce x_t and Δx_t as state variables in the objective function and write the model as

$$\begin{bmatrix} \tilde{y}_t \\ x_t \\ \Delta x_t \end{bmatrix} = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -I & 0 \end{bmatrix} \begin{bmatrix} \tilde{y}_{t-1} \\ x_{t-1} \\ \Delta x_{t-1} \end{bmatrix} + \begin{bmatrix} 0 \\ I \\ I \end{bmatrix} x_t + \begin{bmatrix} \tilde{u}_t \\ 0 \\ 0 \end{bmatrix} \tag{9}$$

which is a special case of (1) with

$$A = \begin{bmatrix} A_1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & -I & 0 \end{bmatrix} ; \qquad C = \begin{bmatrix} 0 \\ I \\ I \end{bmatrix} .$$

Note the special feature of the maxrix C allowing for no effect of x_t on $\tilde{y}_t$.

The objective function is given by (2) with

$$K_t = \beta^t K = \beta^t \begin{bmatrix} K_{11} & K_{12} & 0 \\ K'_{12} & K_{22} & 0 \\ 0 & 0 & K_{33} \end{bmatrix}$$

where K_{22} and K_{33} are assumed to be diagonal, the former capturing increasing marginal costs of using the inputs x_t in the example on the demand for inputs, and the latter measuring the adjustment costs of changes in the inputs. We are concerned with the steady-state solution of (4) and (5), namely

$$G = -(C'HC)^{-1}C'HA \tag{10}$$

$$H = K + \beta(A+CG)'H(A+CG)$$

$$= K + \beta A'H(A+CG) \tag{11}$$

where the second equality sign of (11) is due to (10).

Using equation (10) and the definitions for A and C, with (symmetric) H partitioned into 3 by 3 blocks corresponding to K, we have

$$G = - [H_{22}+H_{23}+H'_{23}+H_{33}]^{-1}[(H'_{12}+H'_{13})A_1 \qquad -(H_{23}+H_{33}) \qquad 0] \qquad (12)$$

Since A' has all zeros in its last row, so does $\beta A'H(A+CG)$. By equation (11) the last row of H equals the last row of K, i.e.,

$$H'_{13} = K'_{13} = 0 \; ; \quad H'_{23} = K'_{23} = 0 \; ; \quad H_{33} = K_{33} \; . \qquad (13)$$

Using (13), we write (12) as

$$G = - [H_{22}+K_{33}]^{-1}[H'_{12}A_1 \qquad -K_{33} \qquad 0] \qquad (14)$$

We need to find only H_{22} and H_{12} to evaluate G. Using (14) and letting $\theta = [H_{22}+K_{33}]^{-1}$, we have

$$A'H(A+CG) = \begin{bmatrix} A'_1(H_{11}-H_{12}\theta H'_{12})A_1 & A'_1 H_{12}\theta K_{33} & 0 \\ K_{33}\theta H'_{12}A_1 & K_{33}-K_{33}\theta K_{33} & 0 \\ 0 & 0 & 0 \end{bmatrix} \qquad (15)$$

Equations (15) and (11) imply

$$H_{22} = K_{22} + \beta K_{33} - \beta K_{33}[H_{22}+K_{33}]^{-1}K_{33} \qquad (16)$$

$$H_{12} = K_{12} + \beta A'_1 H_{12}[H_{22}+K_{33}]^{-1}K_{33} \qquad (17)$$

Since K_{22} and K_{33} are diagonal by assumption, a diagonal H_{22} is a solution of (16), with its i-th diagonal element satisfying

$$h_{22,i} = k_{22,i} + \beta k_{33,i} - \beta \frac{k_{33,i}^2}{h_{22,i}+k_{33,i}} \qquad \text{or}$$

$$h_{22,i}^2 - (k_{22,i}+k_{33,i}\beta-k_{33,i})h_{22,i} - k_{22,i}k_{33,i} = 0 \qquad (18)$$

which can be solved for $h_{22,i}$. We take the smaller root of the quadratic

equation (18) as we wish to make $h_{22,i}$ as small as possible. The dynamic programming solution to the linear-quadratic control problem [cf. (Chow, 1975)] transforms a multiperiod maximization problem into many one-period problems. For each period t , one minimizes the expectation of a quadratic function in y_t involving $y_t'H_ty_t$, $H_t \geq 0$. Hence H_{22} should be diagonal with small elements. Having obtained H_{22} , we use (17) to compute $H_{12} = (h_{12,ij})$. Denoting the diagonal matrix $[H_{22}+K_{33}]^{-1}K_{33}$ by $D = \text{Diag}\{d_i\}$, and the elements of A_1 by a_{ij} , we have

$$h_{12,ij} = k_{12,ij} + \beta \sum_{\ell}a_{\ell i}d_{\ell}h_{12,\ell j} \quad . \tag{19}$$

The elements $h_{12,ij}$ in the j-th column of H_{12} satisfy a set of linear equations (19). We have thus provided an explicit expression for G as a function of A_1 , β and K by using formulas (14), (18) and (19).

As an illustration for a scalar x_t consider the example of Sargent (1979, p. 335) and Hansen and Sargent (1980) where x_t (our notation) denotes the demand for an input labor; y_{1t} is technology which satisfies a q-th order univariate autoregression

$$y_{1t} = a_{11}y_{1,t-1} + \cdots + a_{1q}y_{1,t-q} + u_{1t} \quad ;$$

$y_{2t} = y_{1,t-1},\cdots,y_{q+1,t} = y_{1,t-q}$ are introduced to make the model first-order; and $y_{q+2,t}$ is the wage rate which satisfies an r-th order multivariate autoregression. This model can certainly be written as our equation (1).

The objective function is, for the current period 0 ,

$$E_o \sum_{t=1}^{T} \beta^t [(\gamma_o+y_{1,t}-y_{q+2,t})x_t - \frac{\gamma_1}{2}x_t^2 - \frac{\delta}{2}(\Delta x_t)^2]$$

where $\gamma_1 = K_{22}$ and $\delta = K_{33}$ in our notation, both being scalars. Equations (16) and (18) are identical for a scalar x_t . They become

$$h_{22}^2 - (\gamma_1 + \delta\beta - \delta)h_{22} - \gamma_1\delta = 0 \ , \ \text{implying}$$

$$h_{22} = \frac{1}{2}[(\gamma_1 + \delta\beta - \delta) - \sqrt{(\gamma_1 + \delta\beta - \delta)^2 + 4\gamma_1\delta}]$$

The matrix H_{12} becomes a column vector consisting of the coefficients of the products of x_t and y_{it} in the objective function. Since $H_{22} + K_{33}$ in (17) is the scalar $h_{22} + \delta$, we can write the solution of (17) as

$$H_{12} = [I - \beta\delta(h_{22} + \delta)^{-1}A_1']^{-1}k_{12} \ .$$

The coefficient of x_{t-1} in the optimal feedback control equation (or a demand for labor equation) is $[H_{22} + K_{33}]^{-1}K_{33}$ according to equation (14), or $\delta/(h_{22} + \delta)$. This result agrees with the coefficient obtained by Sargent (1979, p. 336) and Hansen and Sargent (1980) using classic (pre-1970) control techniques. Their coefficient ρ_1 is the inverse of the (smaller) root of the quadratic equation

$$\delta\beta - (\gamma_1 + \delta + \delta\beta)z + \delta z^2 = 0 \ .$$

The explicit solution of this section breaks down when the matrix C does not have a submatrix of zeros, for then $(C'HC)^{-1}$ can no longer be written as $[H_{22} + K_{33}]^{-1}$ as in (14) and one cannot solve an equation corresponding to (16) explicitly for the elements of H_{22} even if K_{22} is diagonal.

4. A FAMILY OF CONSISTENT ESTIMATORS FOR THE GENERAL CASE

A family of consistent estimators is proposed for the general case. It is based on the observations that the least-squares estimator $\hat{G}$ of the coefficients G in the regression of x_t on y_{t-1} (which includes x_{t-1} as a subvector) is consistent, and that, if the rational expectations model is correct, G should satisfy equations (10) and (11). The situation is analogous to the estimation of structural parameters $(B \ \Gamma)$ in linear simultaneous equations by

the use of the least-squares estimates $\hat{\Pi}$ of the reduced-form coefficients Π . The latter are consistent, and, if the model is correct, Π satisfies $B\Pi=\Gamma$ which corresponds to (10) and (11) in the present problem. Therefore, if we solve (10) and (11) for H , K and β (the structural parameters) using the least-squares estimate $\hat{G}$ for G and consistent estimates $\hat{A}$ and $\hat{C}$ for A and C , we will obtain consistent estimates of the former, as we will obtain consistent estimates of B and Γ by solving $B\hat{\Pi}=\Gamma$.

As the first step of this method, we obtain least-squares estimates $\hat{G}$ of the coefficients in the multivariate regression of x_t on y_{t-1} . If the target vector a_t and the intercept b_t in the model are constant through time, h_t is also a constant satisfying equation (7) with the subscript $t+1$ replaced by t . We have $g_t=g$. Otherwise, the coefficients $\hat{G}$ will be estimated by adding some smooth trends in the regression equations.

Having obtained $\hat{G}$, we will find H , K and β to satisfy equations (10) and (11), but as in the case of overidentified structural equations, there may be more equations than unknowns. Defining $R = (r_{ij}) = \hat{A} + \hat{C}\hat{G}$, we write these equations as

$$C'HR = 0 \qquad\qquad\qquad (20)$$

$$K = H-\beta R'HR \qquad\qquad\qquad (21)$$

Let H be a symmetric $p \times p$ matrix with elements h_{ij} , and let C be a $p \times q$ matrix with elements c_{ij} . These two equations imply respectively

$$\sum_{i,j}^{p} c_{im}r_{j\ell}h_{ij} = 0 \qquad (m=1,\ldots,q; \quad \ell=1,\ldots,p) \qquad (22)$$

$$h_{m\ell} - \beta \sum_{i,j}^{p} r_{mi}r_{j\ell}h_{ij} = 0 \qquad \text{if } k_{m\ell} = 0 \qquad (23)$$

(22) and (23) are linear equations in $h_{ij} = h_{ji}$. Let h be the column vector consisting of the $p(p+1)/2$ elements $h_{ij}(i=1,\ldots,p; j \geq i)$. Write (22) and

(23) as

$$Qh = 0 \qquad\qquad (24)$$

Exact, over or under identification occurs according as the rank of Q is equal to, larger or smaller than $p(p+1)/2$ minus one. In the overidentified case, there will be more equations than unknowns in (24); the elements on its right-hand side cannot all vanish. Corresponding to the method of indirect least squares, one can suggest discarding extra equations in (24) and solving the remaining $p(p+1)/2$ homogeneous linear equations which are made nonhomogeneous by a normalization $h_{pp}=1$. This method is still consistent but it discards useful information. Corresponding to the method of two-stage least squares, according to the interpretation of Chow (1964), we normalize by setting $h_{pp}=1$ (or any $h_{ii}=1$), partition Q and h' respectively as $(Q_1 \; q_2)$ and $(h_1' \; 1)$ to write (24) as

$$Q_1 h_1 + q_2 = 0 \qquad\qquad (25)$$

and estimate h_1 by $\hat{h}_1 = - (Q_1'Q_1)^{-1}Q_1'q_2$ using the method of least squares.

Corresponding to the method of limited-information maximum likelihood, according to the interpretation of Chow (1964), we normalize symmetrically by setting $h'h = $ constant and find h to minimize $h'Q'Qh$ subject to this normalization constraint. The minimizing h is the characteristic vector associated with the smallest characteristic root of $Q'Q$. Unlike the method of two-stage least squares, this method yields a vector estimate of h which is invariant with respect to the choice of the variable for normalization. However, if the order of Q is very large, the symmetric normalization is not recommended as it is computationally expensive. If β is unknown, one has to find a scalar to minimize the appropriate sum of squares, be it $h_1'Q_1'Q_1h_1$ or $h'Q'Qh$, but this is an easy problem. Having obtained h and β, we use the remaining equations of (21), other than (23), to compute the nonzero elements

of K . Given H , $\hat{A}$ and $\hat{C}$, we can obtain a new estimate $\hat{G}_{(2)}$ from (10).

If the estimates of H , β and K by the method of this section are not accepted as final, they can serve as initial estimates to be used in the (more expensive) maximization of the likelihood function by the method of section 2. The consistent estimates of this section can be recommended if the numerical maximization of the likelihood function is too expensive.

5. THE ASSUMPTIONS OF RATIONAL EXPECTATIONS MODELS

Besides providing practical methods, the above discussion has pin-pointed the problems involved in the estimation of linear rational expectations models. It should be pointed out that even when the problems are overcome, the estimates by the method of section 2 will still not satisfy the assumptions of rational expectations.

If the economic agents and the econometrician share the same model (1) and (1) indeed is the true model of the economic environment (two strong assumptions), the optimal policy for maximizing the expectation of the objective function (2), correctly specified by the econometrician (another assumption), is not equation (3) with coefficients given by (4)-(7) because the economic agents do not know (and are not assumed to know) the numerical values of the parameters A, C and b exactly. Given uncertainty concerning A, C and b , equations (4)-(7) no longer specify the parameters of the optimal behavioral equation for the agents to maximize the expectation of (2). In fact, no one knows how to compute the truly optimal behavioral equation. Some perhaps nearly optimal solutions are given in chapters 10 and 11 of Chow (1975), for example. Equations (4)-(7) only specify the certainty-equivalent solution which is not optimal when A , C and b are uncertain. Strictly speaking, a true believer in rational expectations models should use the optimal behavioral equation which no one knows,

or at least the more complicated, but more nearly optimal behavioral equation as referenced above. Economists who build models other than rational expectations models have been criticized for their failure to take optimizing behavior into account. The question is how far one should push optimizing behavior in building economic models for multiperiod decision under uncertainty and where one should stop.

As it has been recognized, current practitioners of rational expectations models often ignore, or fail to model explicitly, the process of learning by the economic agents about the economic environment (1) and assume, as in the method of section 3, that a steady-state is always observed for the optimal behavioral equation (3). The modeling of learning will automatically be incorporated if one uses a behavioral equation which is more nearly optimal than the certainty-equivalent strategy by taking into account the uncertainty in the model parameters. Such behavioral equation incorporates the process of learning, is strictly speaking nonlinear in y_{t-1} and is time-dependent. The estimation of such models is much more difficult. Again, how far should one push the assumption of optimal behavior? How useful are the models based on approximate solutions (how approximate?) to optimal behavior as exemplified by the methods of this paper?

6. ESTIMATING NONLINEAR RATIONAL EXPECTATIONS MODELS

It is well recognized that the assumption of rational expectations makes the construction of nonlinear models difficult (because the expectation of a nonlinear function is not the nonlinear function of the expectation). Insofar as the world is nonlinear, it becomes an unattractive assumption to use. Since this assumption is not strictly followed by its practitioners even for linear models with uncertain coefficients, one may boldly apply the certainty-equivalent strategy to nonlinear stochastic models by first linearizing the models as suggested in Chow (1975, Chapter 12). The methods of this paper will then be applicable to

the estimation of nonlinear models by introducing the following modifications.

For the methods of sections 2 and 3:

(a) Starting with some estimates of the parameter vector θ of a nonlinear model (1) and the parameters β , K , ϕ and a of the objective function (2), linearize the model (1) to yield

$$Y_t = \hat{A}_t Y_{t-1} + \hat{C}_t x_t + \hat{b}_t + \hat{u}_t .$$

(b) Compute the coefficients G_t and g_t of the optimal linear feedback control equation (3) using the linear model and the parameters of (2). Note that equations (4) - (7) will have time subscripts for A and C .

(c) Evaluate the likelihood function for models (1) and (3).

(d) Take one step in a numerical maximization algorithm and return to (a).

For the method of section 4:

(a′) Using a consistent estimate $\hat{\theta}$ of the parameter vector of a nonlinear model (1), linearize the model as in (a) above.

(b′) Compute least-squares estimates $\hat{G}$ and $\hat{g}_t$ of the coefficients in a regression of x_t on Y_{t-1} and appropriate trends.

(c′) Define $R_t = (\hat{A}_t + \hat{C}_t \hat{G})$. For each t , follow the methods of section 4 to form $Q_t h_t = 0$, solve for h_t (not to be confused with the vector in equation (7)) to be used as elements of the matrix H_t , and proceed as before.

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