

UNCERTAINTY, INDUSTRIAL STRUCTURE AND THE
SPEED OF R & D

by

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1. Introduction

In modern capitalist economies, competition assumes a variety of forms. Not only is there price competition -- the concern of much of traditional micro-economic theory -- but also product competition.^{1/} In this paper we shall be concerned with a third arena in which competition occurs -- in invention and innovation. While in the theory of product competition, the set of possible products is fixed and the technologies by which each is produced is given, the theory of invention and innovation focuses on the development of new products and new techniques of production. Theoretical explorations designed to analyze this form of competition have so far been very limited, despite the fact that an illuminating vision of such competition was described by Schumpeter some time ago (see, e.g. Schumpeter (1947).) Schumpeter perceived an environment in which industries experience a "perennial gale of creative destruction."

The traditional models of competitive equilibrium not only do not address themselves to this important form of competitive behavior and to the fundamental questions to which it gives rise, e.g. the relationship between market structure and the rate of technological progress, they employ assumptions (such as convexity of the technology) which will never be satisfied if there are R & D expenditures.^{2/}

Since the writings of Schumpeter, explorations in the relationship between the structure of an industry and R & D activity in the industry have, in large measure, been confined to the influence of the degree of concentration on R & D expenditure. The empirical literature, which has been surveyed admirably in Scherer (1970) and Kamien and Schwartz (1975), has, for the most part, explored a causal connection between the degree of concentration and R & D effort. For the short-run it is of course eminently sensible to treat the degree of concentration as a datum and therefore to seek a causal connection. But in the long run the structure of a given industry must itself be endogenous, and while one expects a relationship between R & D expenditure and industrial structure, presumably they depend on more basic ingredients, such as the technology of research, demand conditions, and the nature of the capital market.^{3/}

It is, of course, immediate that any attempt at relating industrial structure and R & D expenditure to such ingredients requires one to analyse not merely the form of competition in the product market, but also the form of competition in R & D. And this suggests that while competition in R & D will not be associated with perfect competition -- a point which was stressed most forcefully by Schumpeter -- it must be distinct as well from conventional monopolistic competition. In the theory of monopolistic competition the market power of a given firm is limited by the existence of close, but imperfect substitutes. By contrast, in the environment we are concerned with here, a firm producing a commodity may, for example, face no effective competition at a given moment of time. This may, for example, be because the firm enjoys a patent on the technology for producing the commodity for whose development it had incurred costs

in the past.^{4/} But other firms now compete to share the market in the future by undertaking R & D expenditure, while the original firm continues to engage in R & D activity in an attempt to maintain its monopoly position. For unless it competes through further R & D activity its monopoly position will be eroded. In such an environment the central form in which competition occurs is in R & D.^{5/}

Our concern in this and an earlier paper (Dasgupta and Stiglitz (1979)) has been to analyse the relationship between industrial structure and various characteristics of R & D activity. In Dasgupta and Stiglitz (1979) it was noted in the context of a model of process innovation that so long as industrial concentration is not too great in market equilibrium, there is a positive association between industrial R & D expenditure and the degree of concentration -- an association that has usually been given a causal interpretation in the empirical literature; (see Scherer (1970)). We noted as well that there is some presumption of excessive duplication of R & D activity in a market economy, in the sense that while each firm undertakes less than the socially optimal level of R & D activity, market equilibrium sustains too many firms, so that industry-wide R & D expenditure is excessive. In this paper we shall be concerned with two additional issues: the speed with which R & D occurs and the extent to which risk is undertaken in a market economy.

We conduct our analysis in the context of two polar cases. The first, (section 3), supposes that the uncertainties faced by different research units are perfectly correlated in a sense that will be made precise below. The second (sections 4 and 5) supposes that the uncertainties are completely independent of one another. These two hypotheses lead to totally different

consequences on the number of units engaged in R & D. The model to be analysed in section 4 is relatively simple. The stochastic process characterizing the R & D technology is assumed to be a Poisson one. This facilitates analysis greatly. But the limitations of the Poisson process for analysing the degree of risk-taking are obvious. Consequently in section 5 we postulate a general diffusion process and obtain some partial answers to the questions asked.

Our reason for concentrating our attention on these two polar cases is that they provide the simplest background for analysing the critical role that is played by the set of available research strategies on the nature of the market equilibrium which emerges. In a more general model, the degree of correlation between research strategies would itself be endogenous and one might then ask whether a market economy engages in sufficient diversification of research strategies or whether firms imitate one another unduly. It will be noted that there is a strong parallel between monopolistic competition in product space (i.e. product differentiation) and competition in the space of research strategies.

We are aware, of course, that the concept of pure process innovation on which we focus here is itself an idealized construct; that industrial innovations are a mixture of product and process innovations. What we have in mind are innovations that frequently occur in the manufacture of commercial aircrafts, on occasion in the field of electronics and in photographic equipments, where it may not be unreasonable to ignore the issue of product differentiation. There is no question, though, that a more complete theory must address itself to the mix of product and process innovation and its relationship to the structure of an industry.

2. A Preliminary Result

The analysis that follows is partial equilibrium in nature. We consider the market for a given commodity. If Q is the total output, $u(Q)$ is the social utility. We suppose $u'(Q) > 0$ and $u''(Q) < 0$. Consumer willingness to pay (equal to the market price) is by assumption

$$p(Q) = u'(Q). \quad (2.1)$$

If c is the unit cost of production associated with the current best-practice technique, then current net social surplus is

$$s(c) = \max_{\{Q\}} u(Q) - cQ.$$

We denote the value of Q which maximizes social welfare when costs are c as $Q_s(c)$. Suppose that an innovation occurs which reduces the unit cost of production from \underline{c} to c^* . If the market is socially managed then the gain in net social surplus per period is

$$\begin{aligned} \pi_s &= s(c^*) - s(\underline{c}) \\ &= \int_{\underline{c}}^{c^*} \frac{\partial s}{\partial c} dc = \int_{\underline{c}}^{c^*} Q_s(c) dc. \end{aligned} \quad (2.2)$$

π_s is the flow of "social profits" from the invention. We can define

$$V_s = \pi_s / r$$

as the present discounted value of social profits, where r is the (social) rate of discount. Note that implicitly in the analysis, we have assumed that there are no durable capital goods; introducing these would complicate the analysis but would not change any of the qualitative results.

We also assume that the present invention will not be replaced by a subsequent invention. This is obviously an unattractive assumption; again, it can be removed, but at the expense of some complication to the analysis. (See Gilbert and Stiglitz (1979), Dasgupta and Stiglitz (1977)).

Suppose next that the industry is controlled by a pure monopolist (i.e. there are barriers to entry). We assume that net marginal revenue is declining in output. When the cost of production is c , the monopolist's profit per period is

$$P(c) = \max_Q p(Q)Q - cQ$$

We denote the monopoly output when costs are c by $Q_m(c)$ and the gain in the monopolist's profit from lowering costs from \underline{c} to c^* is just

$$\begin{aligned} \pi_m &= P(c^*) - P(c) \equiv \int_{\underline{c}}^{c^*} \frac{\partial P}{\partial c} dc & (2.3) \\ &= \int_{c^*}^{\underline{c}} Q_m(c) dc \end{aligned}$$

Again, we can define the present discounted value of the increment to monopoly profits from the invention as

$$V_m = \pi_m / r$$

We assume a perfect capital market, with the rate of interest faced by firms equal to the social rate of discount.

Thirdly, we consider the case where the present technology is freely available, but the inventor of the new technology is awarded a patent on a technology for which the cost of production is c . Then, during the

length of the patent, the profit per period accruing to the patent holder is (letting subscript "e" denote the entering firm)

$$\pi_e(c) = \max_{\{Q\}} [p(Q)Q - cQ] \quad (2.4)$$

$$\text{s.t. } p(Q) \leq \underline{c}.$$

When the elasticity of demand for the product is less than unity, the patent owner always engages in limit pricing, i.e. $p = \underline{c}$. If the elasticity of demand for the product is greater than unity, the patent owner may, if \underline{c} is small enough, lower the price.

Clearly, $\pi_e(\underline{c}) = 0$, so

$$\pi_e(c) = \int_{\underline{c}}^{c^*} \frac{\partial \pi_e}{\partial c} dc = \int_{\underline{c}}^{c^*} Q_e(c) dc \quad (2.5)$$

The present discounted value of the invention depends on the length of life of the patent, T^* :

$$V_e = \frac{\pi_e}{r} (1 - e^{-rT^*}) \quad (2.6)$$

Still a fourth environment in which we shall be interested is that where the present technology (\underline{c}) is controlled by one monopolist, and the patent is won by some other firm. This is perhaps the market environment closest to that which Schumpeter had in mind, where one monopolist is succeeded -- temporarily -- by another.^{6/} The market after the invention is then characterized by duopoly, and the profits accruing to the winner of the patent will depend on the interactions between the two duopolists.

In our later discussion, we shall show that we can obtain certain general results which are independent of any particular assumptions concerning the nature of the interactions between the duopolists.

In the next section, we shall contrast the equilibrium in each of the three market situations (a single monopoly, competition in the initial technology, monopoly in the patent, monopoly in the initial technology, with a possibly different monopolist in the new technology) with the socially optimal pattern of R & D. We shall be concerned with the number of research units (firms) engaged in R & D activity and with the rate at which the innovation is expected to occur.

Since (gross) payoffs are different under different market arrangements, outcomes will be different. The following set of inequalities will prove useful in our subsequent analysis:

$$\pi_s > \pi_e > \pi_m \quad (2.7)$$

or, equivalently, with an infinite lived patent,

$$V_s > V_e > V_m \quad (2.7')$$

The relationship between these variables is depicted in figure 1. (We postpone until section 3 the presentation of the fundamental inequalities pertaining to the duopoly situation.) (2.7) follows immediately upon observing that for any c ,

$$Q_s(c) > Q_e(c) \geq Q_m(c)$$

and using (2.2), (2.3), and (2.5).

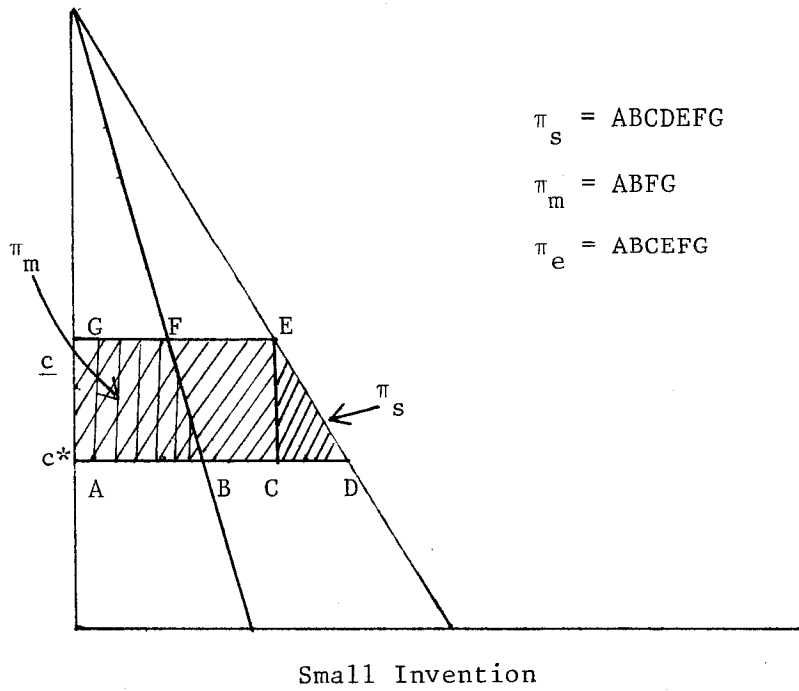
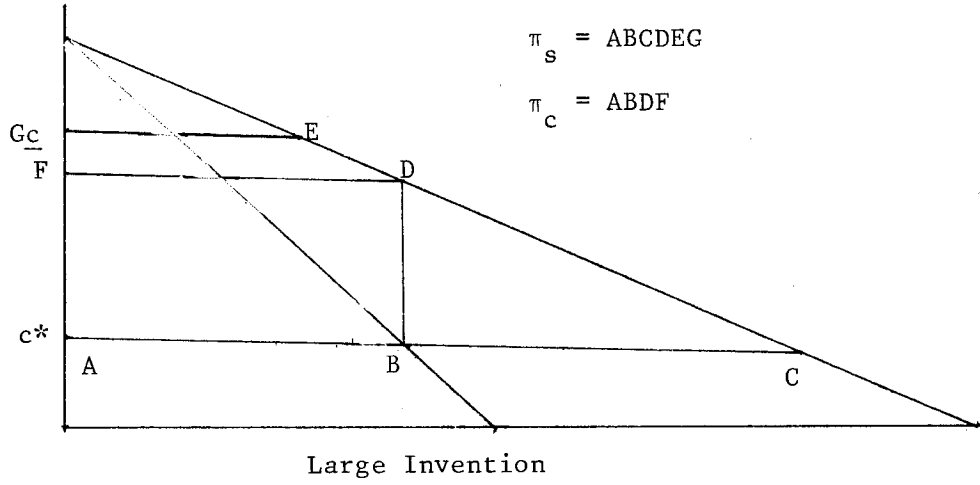


Figure 1

The fact that social payoffs (π_s) exceeded competitive payoffs (π_e) which in turn exceeded monopoly payoffs suggested that in competitive markets there will be too little research, while in monopoly markets there will be even less research. (The latter result seems, moreover, to contradict traditional Schumpeterian analysis which argued that at least some degree of monopoly had a positive effect on R & D.) Such conclusions, however, are at best suspect: to assess their validity we shall need to model formal competition in the R & D activity itself, and to distinguish between competition in the product market and competition in R & D. To this we now turn.

3. Incentives for Innovation under Certainty

3.1 The Socially Managed Market

If a research unit invests x at $t = 0$ it solves the entire set of research problems at $T(x)$,^{7,8/} i.e. it will "make" the invention at $T(x)$. We take it that $T(x) > 0$ for all $x \geq 0$ and that investing more brings forward the discovery date. In order to minimize difficulties with boundary solutions, we assume that $T(\infty) = 0$ and $T(0) = \infty$ but these assumptions may easily be dropped. We assume in addition diminishing returns in R & D technology; i.e. that $T''(x) > 0$ (See Figure 2).⁹

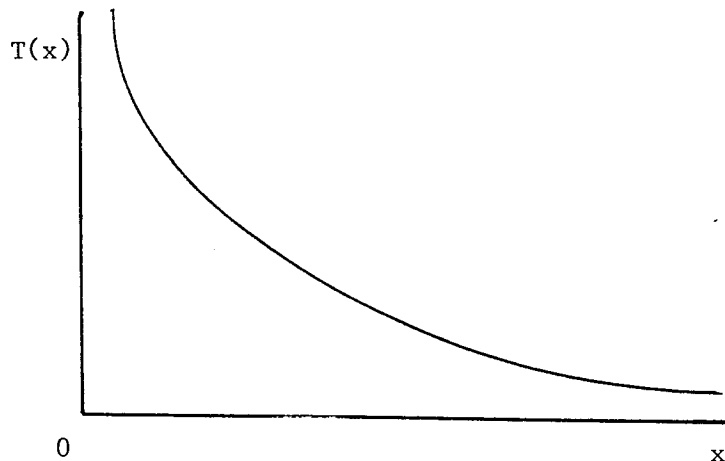


Figure 2

Consider first the socially managed market. It is clear that the social planner's problem is to:

$$\begin{aligned} &\text{maximize } (V_s e^{-rT(x)} - x). \\ &x \geq 0 \end{aligned} \tag{3.1}$$

A glance at (3.1) shows that the maximum is in general **not** a concave function, despite the assumption of diminishing returns in R & D technology. This is a general feature of process innovations. The social optimum is depicted in figure 3.

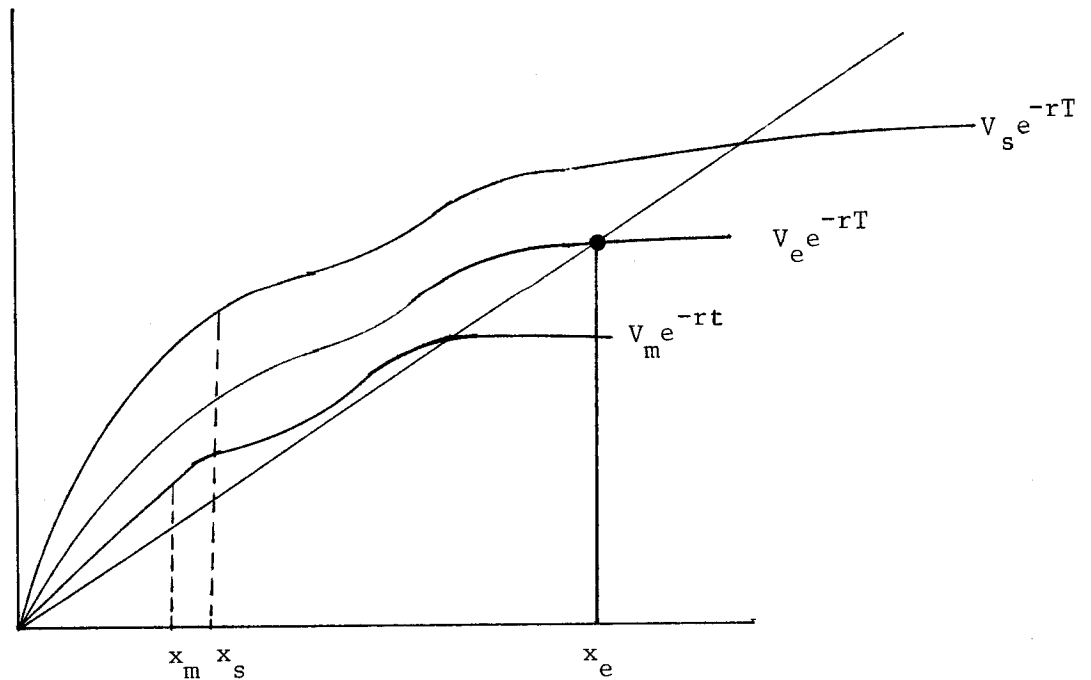


Figure 3

3.2 The Pure Monopolist

It is now easy to compare the social incentives for undertaking such an innovation with the incentives that a monopolist possesses within a regime where the monopolist is protected by barriers to entry in the field of R & D; (in section 3.4 we shall analyse the incentives that a monopolist has for undertaking R & D expenditure when there are no barriers to entry into R & D activity). The monopolist's problem is to:

$$\begin{array}{l} \text{maximize } (V_m e^{-rT(x)} - x). \\ x \geq 0 \end{array} \quad (3.2)$$

Let x_m be the solution of (3.2). Since $V_s > V_m$, we may now conclude from (3.1) and (3.2) that $x_s > x_m$.^{10/} Thus, the volume of R & D expenditure, and therefore the speed of research undertaken by a monopolist protected by entry barriers in the research sector is less than what is socially desirable.

3.3 The Competitive Market

3.3a Introduction

In this and the next sub-section, we analyse equilibrium in market economies with more than one firm. Since we are concerned with imperfectly competitive environments, not surprisingly a number of different equilibrium concepts present themselves; some of these seem more persuasive under one set of conditions, others seem more persuasive under other conditions. The purpose of our discussion is not to present a definite analysis, but rather to explore some of what seems to us as the more interesting possibilities and the circumstances in which they might arise. Other equilibrium concepts are explored in Gilbert-Stiglitz (1979).

Among the determinants of the structure of the equilibrium, the following appear to be important:

- a) The actions which are available to the firms. In (3.3.b) and (3.3.c) we assume that the firm has a single decision variable, "x", while in section (3.3.d) we view firms as having a research program, a sequence of expenditures at different dates. The actions in later dates may be dependent on earlier actions of rivals. Assuming only a single variable greatly simplifies the analysis; this assumption would seem most appropriate in situations where a large initial commitment is required in the research program or for very short research programs.
- b) The timing of the actions and the information available to each firm about the actions taken by other firms. In the model of (3.3.b), we assume the firms must take actions at the same time; in the model of section 3.4, we assume that some potential entrants can wait until after the monopolist has committed himself to a research strategy (and they observe his action) before they commit themselves.

The information structure available to each firm is particularly critical in those cases where there is a research program (a sequence of expenditures), since then if the rivals actions in previous dates are observable, the firm can predicate his action on his rival's actions (while, obviously, if they are not, he cannot).

c) Beliefs about the actions of rivals. The conventional Nash equilibrium assumes that each firm believes that the actions of the other firms will be unchanged as a result of his action; we explore the Nash equilibrium in section 3.3.b; it turns out that there does not exist an equilibrium in pure strategies, although elsewhere, it is shown that there do exist mixed strategy Nash equilibria.

A quite different -- and we think more plausible -- set of expectations is explored in section 3.3.c, and 3.3.d. Both of these may be viewed as reaction function equilibria; the particular form explored in 3.4 is essentially a variant of the familiar Stackleberg equilibrium; in this particular context, however, where there already is a monopolist within the industry, this equilibrium concept is somewhat more persuasive than it is in those situations where there appears to be more symmetry.

3.3.b Nash Equilibrium in Pure Strategies

We turn first to the case where no one owns the patent on the \underline{c} technology. The commodity is therefore assumed to be marketed currently at the price \underline{c} . There is free entry into R & D activity. There are assumed to be a large number of firms with the same R & D function. (This assumption is critical, particularly for the results of the next subsection; it implies there are no infra-marginal firms, and no profits (rents)). The first firm to make the invention is awarded a patent of duration T^* . If there are joint winners they are expected to share the

market for the duration of the patent (perhaps by playing a Cournot game with one another).^{11/}

In this section, we assume firms must make a single decision, the level of x , simultaneously at time 0.

To have an interesting problem let us rule out the possibility of an equilibrium in which no firm engages in R & D activity, i.e. we assume there exists a positive x such that $V_e^{-rT(x)} - x > 0$. (See figure 3).

We first analyse the competitive market on the supposition that all firms (whether or not undertaking R & D) entertain Cournot conjectures regarding the level of R & D expenditures of one another. Note first that no more than one firm will be engaged in R & D. To confirm this last suppose more than one firm were to undertake R & D activity at a potential equilibrium. Clearly any firm which does not win a patent is making a loss, and is thus not profit maximizing: a firm is allowed to shut down. Assume two firms tied; then on the assumption that none of the others alters its R & D expenditure any one of the firms could choose to increase its R & D expenditure marginally, bring forward its date of invention marginally, thereby ensure that it does not have to share the patent with any other firm and so increase the present value of its profits by a discrete amount. We conclude that if an equilibrium exists it must sustain precisely one firm.^{12/}

We next note that at an equilibrium (if it exists) this firm cannot enjoy a positive present value of profits. For, with free entry, a potential entrant, entertaining Cournot conjectures about this firm, could choose a marginally higher level of R & D expenditure and thereby ensure the patent solely for itself and earn a positive present value of profit flow. This suggests that an equilibrium is characterized by a

single active firm incurring R & D expenditure, x_e , which is the largest solution of the zero-profit condition

$$V_e e^{-rT(x)} = x, \quad \underline{13/} \tag{3.3}$$

(see figure 3). But if all firms entertain Cournot conjectures regarding the level of R & D activity of all other firms this cannot be; for this single firm can argue that on the assumption that all other firms refrain from spending on R & D it could increase the present value of its profit by reducing its R & D expenditure from the level x_e . We have therefore proved that were all firms to entertain Cournot conjectures regarding the level of R & D expenditure of all others, a Nash-Cournot equilibrium involving pure strategies does not exist. There may, of course, exist an equilibrium involving mixed strategies; we do not pursue this line of development here (see Gilbert and Stiglitz (1979)).

3.3.c An Alternative Solution

The implication of the foregoing result appears to be that the idea that active firms entertain Cournot conjectures about potential entrants is not consistent with the Schumpeterian idea of competition. This latter has to do with the threat of entry. The way to capture this is to suppose that active firms work on the reaction function of potential entrants; i.e. entertain Stackelberg conjectures regarding their behavior. With this reformulation it is clear that a Nash-Cournot equilibrium exists, that it is unique, and that it is characterized by a single firm undertaking R & D expenditure at the level x_e , which is the largest solution of the zero-profit condition (3.3).^{14/}

This solution is particularly plausible if we postulate that potential entrants can make their R & D commitment sequentially (i.e. that they do

not have to make their decisions simultaneously), and any firm's decision on x (for $x > 0$) is unalterable (alterable only at great cost). Moreover, any firm's actions are observable by all other firms. Then, since we assume that there are a large number of firms who have access to the same R & D technology, it is implausible that there be significant positive profits and some firm not take advantage of the opportunity. Moreover, it is implausible that firms do not recognize this, and hence, if they engage in R & D, they will do it at an intensity which deters anyone else from entering and undercutting them.^{15/}

An immediate corollary of this analysis combined with (2.7) is that competition may either lead to more or less R & D expenditures than in the socially optimal or monopoly allocations.

There are three factors determining the relative magnitudes of x_e , x_s , and x_m :

- i) While the socially optimal and monopoly allocations require equating marginal returns to marginal costs, in the competition for entry allocation, average revenue is equated to average costs. Since average revenues exceed marginal revenues, this leads to a bias for excess expenditures on R & D.
- ii) The longer the life of the patent, the greater is x_e . Obviously, as $T^* \rightarrow 0$, $x_e \rightarrow 0$.
- iii) The greater the ratio of $V_s/V_e (V_e/V_m)$ the more likely is $x_e < x_s$ ($x_e > x_m$). The ratio of π_s/π_e depends on the shape of the demand function. For simplicity, we focus on constant elasticity demand functions, of the form

$$p = Q^{-1/\epsilon} \tag{3.4}$$

which is derived from the utility function

$$u = \frac{Q^{1-1/\epsilon}}{1-1/\epsilon} \quad (3.5)$$

Then, without loss of generality we let $\underline{c} = 1$, and

$$\pi_s = \frac{c^{*1-\epsilon} - 1}{\epsilon - 1} \quad (3.6a)$$

$$\pi_e = (1-c^*) \text{ provided } \epsilon < 1 \text{ or } \epsilon > 1 \text{ and } c^* \text{ not too small}$$

$$= \frac{c^{*1-\epsilon}}{\epsilon^\epsilon (\epsilon - 1)^{1-\epsilon}} \text{ if } \epsilon > 1 \text{ and } c^* \text{ small} \quad (3.6b)$$

$$\pi_{in} = \frac{c^{*1-\epsilon} - 1}{\epsilon^\epsilon (\epsilon-1)^{1-\epsilon}} \quad (\text{defined only for } \epsilon > 1). \quad (3.6c)$$

Hence, provided $\epsilon < 1$ or $\epsilon > 1$ and c^* is not too small,

$$\frac{\pi_e}{\pi_s} = \frac{(1-c^*)(\epsilon-1)}{c^{*1-\epsilon} - 1} \quad (3.7)$$

while

$$\frac{\pi_e}{\pi_m} = \frac{(1-c^*)}{c^{*1-\epsilon} - 1} (\epsilon-1) \left(\frac{\epsilon}{\epsilon-1}\right)^\epsilon \quad (3.8)$$

Thus, for $\epsilon < 1$, for any given invention, $\frac{\pi_e}{\pi_s}$ decreases with ϵ , while $\frac{\pi_e}{\pi_s}$ is smaller for large inventions (small c^*).

It thus appears that, ceteris paribus, very inelastic demand curves are likely to be associated with excessive research in the competitive market while there will be too little research for "big innovations." The excessive R & D expenditure in the market economy is not due to a

duplication of research effort, but rather arises from the pressure of competition. Competition forces the single firm to innovate earlier than is socially desirable.

The foregoing characterization of a Cournot-Nash equilibrium with free entry -- only a single firm engages in R & D and it makes zero (expected) profits -- survives if we introduce uncertainty in the date of invention in the special form where there is a single research strategy available to all firms (so that all firms are obliged to follow the same decision tree) and where the uncertainties are all perfectly correlated. This means that given the pace of R & D effort of the remaining firms a given firm can guarantee that it is the first to invent by choosing a sufficiently high pace of research, even though it is unable to say at what date the sequence of tasks will be completed by its research unit. With this form of uncertainty it is immediate that precisely one firm will incur R & D expenditure at an equilibrium and, if firms are risk neutral, it is the expected present value of its profits which is nil.

The foregoing result is of importance because it tells us that the number of firms actually engaged in a particular R & D activity is not an index for the degree of competition. For the model at hand, competition is intense even though only one firm is active, but it is engaged at a level high enough to forestall entry.

3.3.d Dynamic Strategies

The previous subsection allowed each firm only a single decision variable, "x", to which the firm is committed. As we remarked earlier,

however, most R & D programs extend over a number of periods. At time 0, the firm does not have to commit itself for its entire research program. This means that if the actions of its rivals are observable, it can predicate its actions on the prior actions of its rivals.

Assume firms are allowed to "move" sequentially. Suppose that a single firm were chosen randomly to make the first move. We suppose moreover that if it were a monopolist it would spend a small amount of resources on R & D the first period.

That is, if $x(t)$ is the firm's research intensity at time t , $c(x)$ is the cost (in present discounted value terms) of the research program $\{x(t)\}$, and we now view the date of discovery as a function of the entire program, the firm's optimal R & D expenditure is that $\{x\}$ which maximizes

$$V_e e^{-rT(x)} - c(x) \quad (3.9)$$

We postulate that the optimal solution involves a strictly positive x the first period. Suppose now the firm does this and then announces that if any other firm attempts to compete it will choose a research strategy which would guarantee for itself the patent (if the rival restricts himself to research strategies involving non-negative profits.) Clearly, if all firms face the same innovation cost function then any policy which would break even for a potential rival would break even for the firm in question; and indeed such a strategy would be optimal for the firm. For note that such a strategy serves to deter entry completely. But if no other firm attempts to compete, the firm in question quite naturally chooses the monopolist's policy.

Let $T(x_{mc})$ (for "monopoly-competition") be the solution of (3.9). A comparison of (3.1) and (3.9) shows at once if T^* is large enough, $T(x_{mc}) > T(x_s)$. It is in such environments that one is entitled to claim that the incentive to invent under competitive conditions is less than is socially desirable; (see Arrow (1962), p. 152). For what we have shown is that if firms are not compelled to commit their R & D expenditure a given firm may be able to effectively deter entry and still behave like a monopolist in R & D, even when there is freedom of entry.^{16/}

This equilibrium can also be viewed as a Nash equilibrium: given the first firm's announced strategy, it does not pay any firm to enter; and given that no other firm enters, it is optimal for it to pursue a monopolist research program.^{17/}

3.4 Duopoly and Persistent Monopoly

In the previous subsections we have analysed two market situations:

- a) The present technology is controlled by a single firm; and the same firm is the only one allowed (or capable) of engaging in research.
- b) The present technology is freely available, and there are many firms capable of engaging in research. The "winner" of the research contest will, of course, enjoy a (perhaps temporary) monopoly on the new technology.

This section is concerned with a third market environment:

The present technology (c) is controlled by a patent, but there is free competition in R & D.

Assume first that the present monopolist cannot (for legal or technological reasons) engage in research. As we noted in section 2, after the invention, the market will be a duopoly. Let π_e^d and π_m^d denote the entrant's and the (original) monopolist's flow of profits after the invention, and V_e^d and V_m^d the present discounted flow of profits (valued as of the date of invention). The profits of the successful entrant in the duopoly situation will normally exceed those where there are a large number of competitors. (If the post-entry market is described by a quantity-setting Nash equilibrium, this is clearly true.)

Thus, normally,

$$V_e^d > V_e \tag{3.10}$$

Free entry into the R & D activity, however, entails zero profits:

$$V_e^d e^{-rT(x_d)} = x_d \tag{3.11}$$

Comparing (3.3) and (3.11) and using (3.10), we immediately see that

$$x_d > x_e$$

The amount of research is higher if the original market is controlled by a monopoly than if it is competitively controlled. (It is in this sense that monopoly does encourage R & D.)

It follows that, since x_e may be greater than x_s , there may well be excessive expenditures on R & D: effectively, all potential monopoly profits are dissipated in the race to be first.

We now turn to the case where the monopolist is allowed to engage in R & D. We now show that the existing monopolist will prevent potential competitors from entering by spending a sufficiently large amount on R & D to thereby guarantee the new patent for itself.^{18/} Nevertheless, it can be shown, the monopolist earns super-normal profits; moreover, it may engage in a speed of research greater than what is socially optimal. To see this, denote by

$$\pi_m(c^*) = \max_Q p(Q)Q - c^*Q,$$

the (maximal) flow of profits, post-invention, to the monopolist, and let $V_m(c^*)$ be its present discounted value (valued from the date of invention). Then, since the duopolists will not, in general, engage in joint profit maximizing activity,

$$V_m(c^*) > V_e^d + V_m^d \quad (3.12)$$

Since no competitor will choose a level of R & D in excess of x_d (defined above, equation 3.11), by spending even marginally in excess of x_d , it can guarantee the patent for itself. The present value of profits of the monopolist, if it does this, are

$$V_m(c^*)e^{-rT} + \frac{\pi_m(\underline{c})(1-e^{-rT})}{r} > (V_e^d + V_m^d)e^{-rT} + \frac{(1-e^{-rT})\pi_m(\underline{c})}{r} = V_m^d e^{-rT} + \frac{\pi_m(\underline{c})(1-e^{-rT})}{r} - x_d$$

the present value of profits if it does not pre-empt. (The first inequality follows from (3.12), the next follows by substituting (3.11)).^{19/}

Therefore that the monopolist will always forestall entry by competitors by spending in excess of x_d (perhaps only marginally so).

In the notation of section 3.2 let x_m be the profit maximizing R & D expenditure for a monopolist protected by entry barriers in R & D, (i.e. the solution of problem (3.2)). What we have proved, therefore, is that if $x_m \geq x_d$, then the fact that there is free entry into the research sector makes no difference to the conclusions we arrived at in section 3.2 regarding the incentives to innovate on the part of the monopolist protected by entry barriers into R & D. But if $x_m < x_d$, the fact that the monopolist faces R & D competition is of great importance to the outcome. The monopolist will spend in excess of x_d (perhaps only marginally) in order to forestall entry. The present value of the flow of profits that it earns is, in this case, reduced because it has to spend more on R & D than it would ideally have liked to spend. But it nevertheless earns super-normal profits. Finally, it will be noted that if the socially optimal level of R & D expenditure, x_s (see section 3.1), is less than x_d , the monopolist, in the face of R & D competition, engages in excessive research.

The reason why the monopolist will always pre-empt potential competitors is intuitively clear enough. If any other firm were to win the patent, the industry would be characterized by a duopoly structure. And with free entry this duopolist, having recognized that it will have to share the market with the existing monopolist, would be forced to spend so much as to drive down the present value of the flow of its profits to zero, (i.e. set $x = x_d$). However, the existing monopolist can always ensure that it remains a monopolist by spending a little more on R & D than any potential competitor would find profitable. The point to note is that

it is always in the monopolist's interest to do so, because by remaining a monopolist it can earn a flow of profit in excess of the sum of the duopolists' profits. The argument is of course re-enforced if in fact the existing monopolist is more efficient than its competitors in R & D activity. The implication of these conclusions is that even if there is competition in R & D activity there are strong tendencies for a monopolized industry to remain a monopoly. The fact that the monopolist is threatened by potential competitors at most spurs the monopolist to spend more on R & D than it would otherwise. But for the model at hand, the industry remains a monopoly.

The solution concept we have used here, parallel to that of section 3.2, involved the monopolist acting as a Stackleberg leader with a commitment to a level of research input x , and zero profits for research entrants. The use of a Stackleberg solution is perhaps persuasive in this context,^{20/} but the "commitment" and "zero profit" assumptions are not so persuasive. When these assumptions are dropped, the nature of the solution may be changed markedly, e.g. pre-emption may on the one hand no longer be desirable (see Gilbert and Stiglitz (1979))^{21/} or, on the other, may be possible without significantly altering its research plan from what it would be as a pure monopolist with no threat of competition (as above, section 3.3.d).

4. Speed of Research Under Uncertainty

4.1 Introduction^{22/}

One of the surprising results we observed in the previous section was that with a patent system and free-entry into the research sector there

would, in the absence of uncertainty, be at most one firm engaged in research. Moreover, we noted that if firms are compelled to commit their R & D expenditure, and if the existing technology is freely available to all, the pace of innovation would be such as to make the present value of profits precisely zero, and thus deter further entry. This result continues to hold if there is uncertainty in the date of success, but if the uncertainties faced by all firms are perfectly correlated in the sense made clear earlier. But so long as they are not, matters are different. No firm can be sure of winning the patent, and hence entry and speed of research are jointly determined to make expected profits zero.

We now assume the date of discovery is random. More precisely, if the firm spends at $t = 0$ an amount x , then the probability that it would succeed in making the invention at or before t is $1 - e^{-\lambda(x_i)t}$, i.e. the probability of making the invention in the interval $(t, t + \Delta t)$ is just $\lambda(x_i)\Delta t$. We postulate that $\lambda(x_i)$ is characterized by an initial range of increasing returns, followed by decreasing returns, as depicted in figure 4.

In order to simplify the analysis suppose all research units follow different research strategies, so that the uncertainties faced by different units are independent of one another. But we suppose that they are all equally effective. Thus, if x_i is the amount invested in the i^{th} unit, and if there are n such units, then the probability density function of the discovery being made at t (looked at from date $t = 0$) is given by the exponential form

$$\sum_{i=1}^n \lambda(x_i) \exp \left[- \sum_{i=1}^n \lambda(x_i) \right] t.$$

The expected date of discovery is $\left[\sum_{j=1}^n \lambda(x_j) \right]^{-1}$.

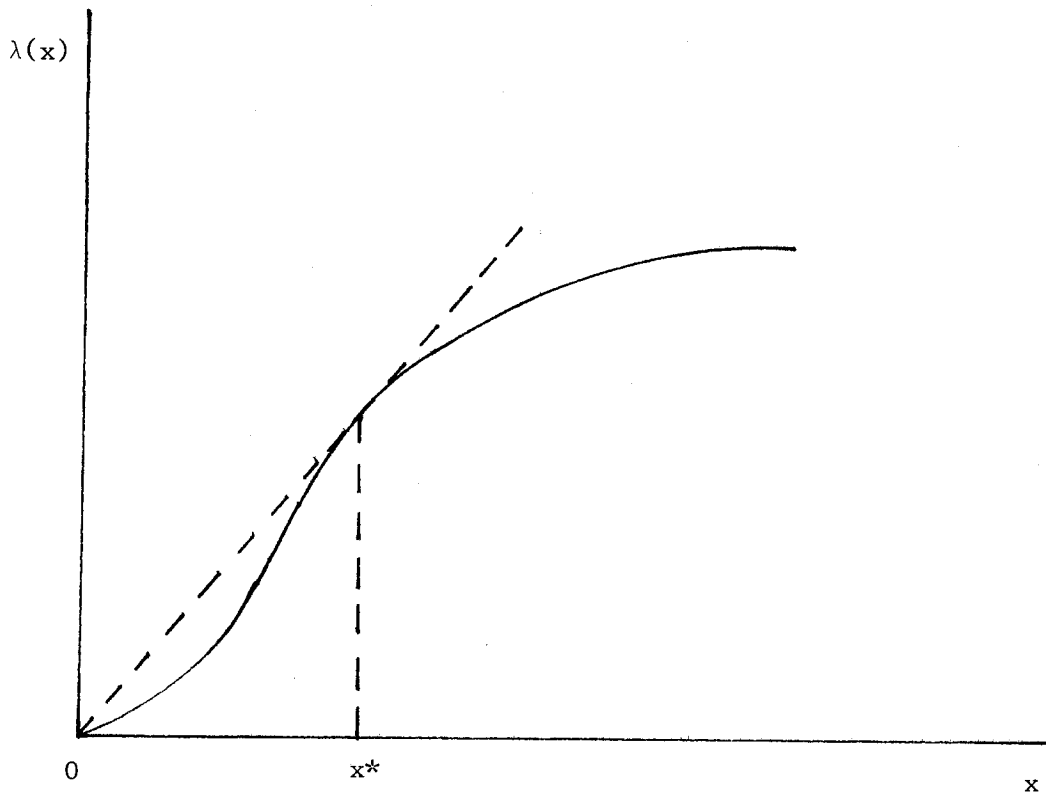


Figure 4

In the following two subsections we analyse the cases where R & D is socially managed and where it is undertaken by a monopolist protected by barriers to entry. In section 4.4 we study the competitive case and a comparison is made.

4.2 The Socially Managed Economy

As research units are identical they will be identically treated at the optimum. As before, let V_s be the present discounted value of the flow of net social benefits from the date the invention is made (discounted to the date of discovery). The objective is to maximize the expected present value of net social surplus. The planning problem is therefore to choose n and x so as to

$$\text{maximize } V_s \int_0^{\infty} n\lambda(x)e^{-(n\lambda(x) + r)t} dt - nx. \quad (4.1)$$

Assume that n is a continuous variable.^{23/} Choosing n optimally yields the first-order condition that

$$\frac{V_s \lambda(x)r}{(n\lambda(x) + r)^2} = x \quad (4.2)$$

and choosing x optimally results in^{24/}

$$\frac{V_s \lambda'(x)r}{(n\lambda(x) + r)^2} = 1 \quad (4.3)$$

Equations (4.2) and (4.3) allow us to determine n and x . Now note that they imply immediately that at the optimum

$$\lambda'(x) = \frac{\lambda(x)}{x} . \quad (4.4)$$

Let x^* be the solution to (4.4). Hence the optimal level of investment in each laboratory, (figure 4), is independent of the number of firms. Denote by n_s the optimal number of laboratories. It can be obtained as the solution of (4.2) if we use the value of x^* for x in the equation.

4.3 Pure Monopoly

The analysis is identical for the pure monopolist, (i.e. one protected by entry barriers), with V_m replacing V_s in (4.1) through to (4.4).

Thus, the monopolist operates each laboratory at the efficient level, but the number of laboratories will not be optimal. If n_m is the expected profit maximizing number of research units for the monopolist it is the solution of

$$\frac{V_m \lambda'(x^*)}{(n\lambda(x^*) + r)^2} = 1 \quad (4.5)$$

Since $V_s > V_m$, a comparison of equations (4.3) and (4.5) tells us that $n_s > n_m$. The monopolist therefore delays innovation; the expected date of success is farther away in the future. This result parallels that obtained earlier (section 3).

4.4 The Fully Competitive Market

We turn now to the economy in which competitive firms undertake R & D expenditure. Firms (laboratories) are identical, and work independently of one another. Let firm i invest x_i at $t = 0$ and let n be the

number of firms investing. i knows this number and the choice of x_j ($j \neq i$). Then, the probability density (viewed at $t = 0$) that firm i will be the winner and that it will invent at t is given as

$$\lambda(x_i) e^{-\sum_{j=1}^n \lambda(x_j) t} \quad (4.6)$$

As earlier we postulate free entry. Thus n is endogenous. As before, we let V_e be the present discounted value of the flow of profits (assumed positive for T^* years, the life of the patent) accruing to the first discoverer from the date of discovery. Firms are assumed risk-neutral. Hence i wishes to choose x_i with a view to maximizing

$$V_e \int_0^{\infty} \lambda(x_i) e^{-(\sum_{j=1}^n \lambda(x_j) + r)t} dt - x_i. \quad (4.7)$$

There are two routes that one can follow here. One can assume that firms are sophisticated calculators and take into account the fact that x_i influences $\sum_{j=1}^n \lambda(x_j)$. Alternatively, one can assume that each firm ignores its effects on $\sum_{j=1}^n \lambda(x_j)$; which is to say that it takes the expected date of the invention as given and varies x_i with a view solely to altering the probability that it is the first to make the invention and win the patent. This is plausible if the number of firms is "large", which we shall assume to be the case. Therefore, the i^{th} firm chooses x_i so that^{25/}

$$V_e \frac{\lambda'(x_i)}{n (\sum_{j=1}^n \lambda(x_j) + r)} = 1 \quad (4.8)$$

As we are supposing identical firms, let us look for the symmetric Cournot-Nash equilibrium. Thus (4.8) becomes

$$V_e \frac{\lambda'(x)}{(n\lambda(x) + r)} = 1 \quad (4.9)$$

Moreover, with free-entry, expected present value of profits are nil for each firm. Hence, from (4.7) we get

$$V_e \frac{\lambda(x)}{n\lambda(x) + r} = x \quad (4.10)$$

Equations (4.9) and (4.10) represent the free-entry Cournot-Nash equilibrium, and n and x are determined from them.

There are two immediate implications of (4.9) and (4.10). First,

$$\lambda'(x) = \frac{\lambda(x)}{x} ,$$

and hence, that at this equilibrium each firm operates its laboratory efficiently, i.e. $x = x^*$ (see Figure 4).

Since the laboratories in each of the three different institutions we are studying here are operated at the efficient level, in comparing the speed of research in these different economic environments we need only to compare the number of research units. In particular, the model predicts that the expected date of invention $\frac{1}{n\lambda}$ is simply inversely proportional to the aggregate R & D expenditures nx with proportionality constant $\frac{1}{\lambda'(x^*)}$ for all three market structures examined. Total R & D expenditure nx times the expected date of invention is independent of market structure. 26/

Moreover, since, if any research is undertaken in the social optimum, it is clear that social profits must be positive,

$$\frac{V_s e^{-(n\lambda + r)}}{n\lambda + r} > nx$$

With free entry, were $V_s \approx V_e$, there would be an excessive number of firms engaged in research. If the receiver of the patent appropriates the entire social gain from the research, with free entry, average returns must equal average costs, and hence all the potential social gains will be dissipated in the form of excessive entry. (This is just another example of a common pool problem; each entrant receives either the entire benefit or nothing, but his expected gain is just $1/n^{\text{th}}$ the value of the patent.)^{27/} The ratio of marginal to average returns is just $\frac{r}{n\lambda + r} < 1$.

In general, however, as we have argued, $V_s > V_e$, and thus whether there is too much or too little research is ambiguous. But at least within the confines of our simple model, we can ascertain conditions under which it is likely that there will be too little or too much research.

From equations (4.2) and (4.10) one obtains

$$\frac{n_c \lambda(x^*) + r}{n_s \lambda(x^*) + r} = \frac{V_e (n_s \lambda(x^*) + r)}{V_s r} \quad (4.11)$$

Using equation (4.2) in the RHS of (4.11) we can express the latter equation as

$$\frac{n_c \lambda(x^*) + r}{n_s \lambda(x^*) + r} = \frac{V_e \sqrt{\lambda(x^*)}}{\sqrt{V_s} \sqrt{x^*} \sqrt{r}} \quad (4.12)$$

We conclude:

$$n_c > n_s \text{ as } V_e \sqrt{\frac{\lambda(x^*)}{x^*}} > \sqrt{V_s r} \quad (4.13)$$

(4.13) is extremely useful. On the one hand, it allows us to compute the optimal patent life,

$$-\ln \left\{ 1 - r \frac{\sqrt{\pi_s / \lambda'(x^*)}}{\pi_e} \right\} / r \quad (4.14)$$

provided

$$r \sqrt{\pi_s / \lambda'(x^*)} < \pi_e . \quad (4.15)$$

(If (4.15) does not hold, then, for an infinite life patent, the market engages in too little research.)

On the other hand, for particular parameterizations it allows us easily to calculate whether the market (with a particular patent life) involves too much research. To illustrate this, we focus here on constant elasticity demand curves, with elasticity less the unity (so the winner of the patent will engage in limit pricing) and an infinite life of a patent. Recall from section 3 that, if we let $\underline{c} = 1$,

$$\pi_s = \frac{1-c}{1-\epsilon} 1^{-\epsilon}$$

and

$$\pi_e = 1 - c^*$$

Substituting into (4.15), and differentiating with respect to c , we see (figure 5) that for small inventions the market always provides too little research. (Contrast this with the corresponding result in section 3.) On the other hand, at $c^* = 0$, $\pi_s = \frac{1}{1-\epsilon}$, while $\pi_e = 1$. Hence, if T^e is the expected date of discovery ($\frac{1}{\lambda N}$) and R are aggregate R & D expenditures (nx), provided

$$r < \sqrt{\frac{1-\epsilon}{R \cdot T^e}}$$

for sufficiently long patent lives, the market spends too much on research. (Recall that $R T^e$ is a constant, independent of market structure.) Thus, for sufficiently low interest rates and for sufficiently small elasticities of demand, the market will, for long lived patents, have too many research units. Note that the social waste here does not arise from duplication of research effort (as in the earlier study of Dasgupta and Stiglitz (1980)).

In the case where all firms follow exactly the same research strategy, all expenditures by more than one firm are wasteful. Here, since each firm has a research strategy which is uncorrelated with the others there is a positive social benefit from each additional research unit. The social waste is more subtle: it arises from the fact that the marginal social benefit is, under the circumstances specified, less than the marginal private benefit -- the increased probability of winning the patent.

Note that industries in which the elasticity of demand is low are, **ceteris paribus**, more likely to be characterized by excessive expenditures on R & D.

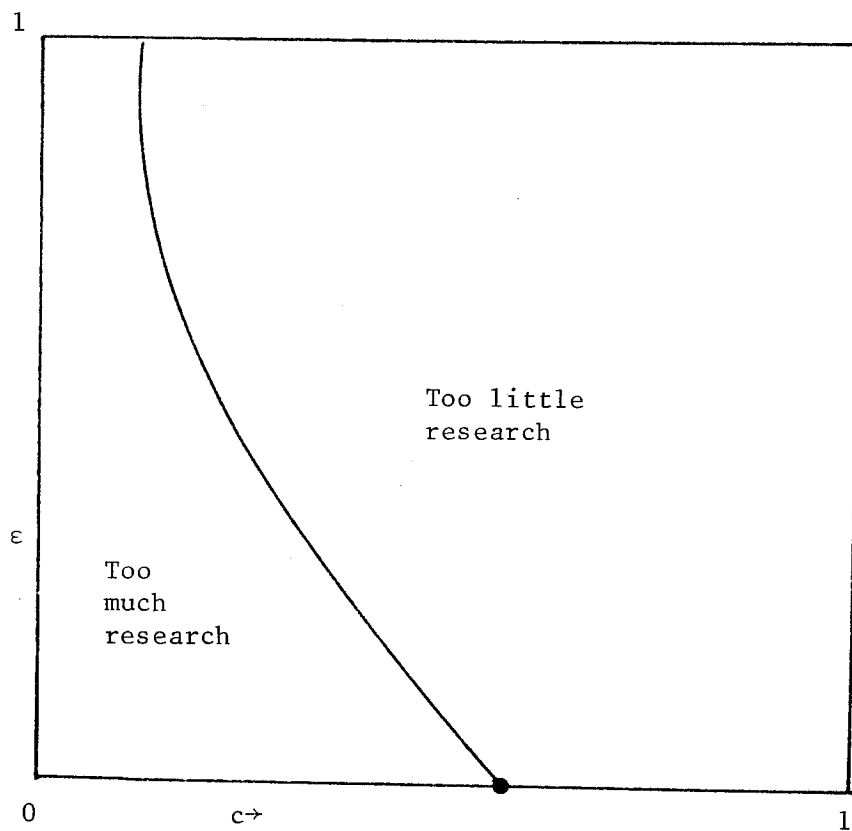


Figure 5

Our simple model also allows us to make simple comparative statics propositions. We know that an industry in which there is a greater payoff to success (i.e. greater value of v_e) will, ceteris paribus, be characterized by a larger number of firms attempting to invent cost cutting techniques of production.^{28/}

If the firm takes cognizance of its effect on the aggregate probability distribution of discovery dates, we obtain

$$\begin{aligned} \frac{\lambda'v_e}{n\lambda+r} \{1 - [\lambda \int_0^{\infty} te^{-(n\lambda+r)t} dt](n\lambda+r)\} &= 1 \\ &= \frac{\lambda'v_e}{n\lambda+r} [1 - \frac{\lambda}{n\lambda+r}] = 1 \end{aligned} \quad (4.16)$$

It immediately follows in this case that each research laboratory will be operated at a slower than optimal speed. Note that as n gets large, the scale at which each research laboratory operates converges to the efficient level, and the analysis above becomes directly applicable.

Moreover, it also follows from (4.16) in conjunction with the zero profit constraint (4.10) that, in the market equilibrium where firms take into account their affect on the aggregate probability distribution of discovery date, the number of firms is higher (than in the equilibrium where they do not); the aggregate expenditure on R & D is **lower** and the expected date of discovery is larger. The loss from the inefficiency in the scale of operation of each research laboratory more than offsets the gains from the increased number of research laboratories.^{29/}

It is then straightforward to combine these results with those obtained earlier to contrast the market equilibrium with the socially optimal number of research firms.

The results of this section have confirmed the ambiguity noted in the previous section: there is no clear presumption whether, with an infinite lived patent in markets with free entry into R & D, there will be too much or too little research. In those cases where there is excessive expenditure on R & D, with an infinite lived patent, although there is an optimal patent life for each industry and for each invention which will guarantee that the market will undertake the correct amount of research, the life of the patent will vary depending on the size of invention and the elasticity of demand in the industry. Thus, there is no simple intervention into the market allocation -- no uniform rule applicable for all inventions and industries -- which will attain the social optimum. Moreover, if firms take into account their affect on the aggregate probability distribution of discovery dates (whether it is plausible that they will presumably depend on the number of researchers), then, even for a particular invention for a particular industry, an optimal patent life will not ensure the attainment of the social optimum, since each research laboratory will be operated at too small a scale. It may be possible, by the introduction of franchise taxes or other tax instruments, in conjunction with an optimum patent life, to attain a first best optimum.

5. Speed of Research Under Uncertainty. A Generalization

The stochastic specification of the research process in the previous section had one extremely unattractive feature about it: the Poisson process does not allow one to vary the riskiness of the research process without at the same time varying the mean time of arrival. One would like to know, for instance, whether, keeping the mean time of arrival constant among a set of research projects, the market has a bias for riskier projects.

In this section we formulate a simple model allowing us to address ourselves to that and a number of other questions. We characterize the "common" state of knowledge at time 0 as having a value of, say, A_0 .

A_0 can be thought of as the magnitude of the size of hurdles that have to be overcome in order to make commercially viable whatever it is which is desired. Research consists of solving a large number of small problems, each of which when solved reduces the value of A .^{30/} Not infrequently, however, something that was not thought to be a problem turns out to be a problem, a "set back" as it is commonly described. This can be thought of as an increase in A , in the set of hurdles yet to be overcome. Finally, if enough set backs occur (A becomes large enough) one views this line of research as being unfruitful, and one starts over again.

A simple way of specifying the above process stochastically is as a diffusion process. 0 is the absorbing barrier, interpreted as "discovery" and \bar{A} is a reflecting barrier, leading to the firm undertaking a new line of attack.^{31/}

Analytically, then, our problem lies in characterizing, for any particular diffusion process, the distribution of first passage times from A to 0 . We shall begin by treating the number of research units as exogenously given. Subsequently we shall allow the number of research units to be endogenously determined.

To begin with consider a competitive market in R & D. Suppose that there are $N(\geq 2)$ laboratories (firms) working independently of one another. To simplify assume them to be identical. Let $h(t, \alpha)$ be the density function of first passages for research program α , and

let $H(t, \alpha) \equiv \int_0^t h(\tau, \alpha) d\tau$ be the cumulative function. As before we denote by V_e the capitalized value of a successful research program to the firm. It follows that the expected discounted value of the research program α to the representative firm is

$$V_e \int_0^{\infty} e^{-rt} h(t, \alpha) (1-H(t, \alpha))^{N-1} dt.$$

(h is the probability he discovers it at t , $(1-H)^{N-1}$ is the probability that none of his rivals discovers it before t .) Therefore at a market equilibrium the representative firm chooses α so that

$$\int_0^{\infty} e^{-rt} h_{\alpha}(t, \alpha) (1-H(t, \alpha))^{N-1} dt = 0. \quad (5.1)$$

Suppose instead that these N firms are socially managed. Let V_s be the capitalized social value of a successful research program.

The probability that, if there are N independent research laboratories, the invention will be discovered at date t is

$$N h (1-H)^{N-1}$$

so the present discounted value of the "research program" is

$$W_s = V_s N \int_0^{\infty} e^{-rt} h (1-H)^{N-1} dt. \quad (5.2)$$

Thus, the planner will choose α so that

$$\begin{aligned} \frac{\partial W_s}{\partial \alpha} = \int_0^{\infty} e^{-rt} [h_{\alpha}(t, \alpha) (1-H(t, \alpha))^{N-1} - h(t, \alpha) (N-1) \\ H_{\alpha}(t, \alpha) (1-H(t, \alpha))^{N-2}] dt = 0 \end{aligned} \quad (5.3)$$

As we are attempting to locate the extent to which the market selects risky research projects, suppose that choice is being made from among a family of stochastic processes with the same expected date of discovery, i.e.

$$\int_0^{\infty} t h(t, \alpha) dt = \text{Constant.}$$

Let an increase in α represent a mean preserving spread in the distribution $h(t, \alpha)$; i.e.

$$\int_0^t H_{\alpha}(\tau, \alpha) d\tau \geq 0 \quad \text{for all } t > 0. \quad \underline{32/} \quad (5.4)$$

$$\int_0^{\infty} H_{\alpha}(\tau, \alpha) d\tau = 0 \quad (5.4')$$

We assume, moreover, for all α in the relevant region,

$$H(0, \alpha) = 0, \quad H(\infty, \alpha) = 1 \quad (5.5)$$

We now prove that the market equilibrium choice of α is optimal only if $r = 0$; if $r > 0$, welfare can always be increased by increasing α from the market equilibrium.

To see this, we integrate (5.1) by parts, and using (5.4')

$$\int_0^{\infty} e^{-rt} [(N-1)h + r(1-H)] H_{\alpha} (1-H)^{N-2} dt = 0 \quad (5.1')$$

Integrating the second term in (5.1') by parts again and using (5.4'), we obtain

$$\int_0^{\infty} e^{-rt} H_{\alpha} (1-H)^{N-1} dt = \int_0^{\infty} \left[\int_0^t H_{\alpha} d\tau \right] [(N-1)h + r(1-H)] e^{-rt} (1-H)^{N-2} dt \quad (5.6)$$

Substituting (5.1), (5.1'), and (5.6) into (5.3) we obtain (letting

$\alpha = \alpha_e$ denote the market value of α)

$$\frac{\partial W_s}{\partial \alpha} \Big|_{\alpha=\alpha_e} = r \int_0^\infty \left[\int_0^t H_\alpha d\tau \right] [(N-1)h + r(1-H)] (1-H)^{N-2} e^{-rt} dt \quad (5.7)$$

$$\geq 0 \quad \text{as } r \geq 0.$$

Thus, if $r = 0$, the market equilibrium is (at least locally) optimal, while if $r > 0$, an increase in α (representing an increase in risk taking) will improve social welfare.^{33/}

It is important to observe that although we have limited each firm to a choice of research strategies with the same mean time of discovery, the mean time of discovery for the aggregate is not invariant to the choice of α ; for simplicity, we consider only the case of $N=2$.

$$\begin{aligned} \bar{T} &= 2 \int_0^\infty t h (1-H) dt \\ \frac{d\bar{T}}{d\alpha} &= 2 \int_0^\infty t [h_\alpha (1-H) - h H_\alpha] dt \\ &= -2 \int_0^\infty \left[H_\alpha - \int_0^t (H h_\alpha + h H_\alpha) d\tau \right] dt \\ &= 2 \int_0^\infty H H_\alpha dt \\ &= -2 \int_0^\infty h \left[\int_0^t H_\alpha d\tau \right] dt < 0. \end{aligned}$$

(Where we have successively integrated by parts and made use of (5.4).)

Thus, mean time to discovery is slower in the market economy than is socially optimal.

The foregoing analysis was designed to study the nature of risk-taking when both the competitive market and the socially managed one have the same exogenously given number of research laboratories. We now allow N to be endogenous. Suppose then F denotes the "fixed cost" in establishing a laboratory.

Clearly, now

$$W_s = N V_s \int_0^{\infty} e^{-rt} h(1-H)^{N-1} dt - FN \quad (5.10)$$

Differentiating with respect to N , we obtain

$$\frac{\partial W_s}{\partial N} = \frac{W_s}{N} + N V_s \int_0^{\infty} e^{-rt} h(1-H)^{N-1} \ln(1-H) dt \quad (5.11)$$

With free entry, the market equilibrium is characterized by the number of firms, N_c , being a solution of

$$V_e \int_0^{\infty} e^{-rt} h(t, \alpha)(1-H(t, \alpha))^{N-1} dt = F. \quad (5.12)$$

Comparing (5.11) and (5.12) it is apparent that the number of firms in market equilibrium may be too large or too small. There are three effects:

- a) The market does not usually appropriate all the returns to research, i.e. $V_e < V_s$; this leads to the market having too few firms.
- b) $\int_0^{\infty} e^{-rt} h(1-H)^{N-1} \ln(1-H) dt < 0$. (Since $H \leq 1$, $\ln 1-H \leq 0$.)

The social planner takes into account the reduction in returns to other firms' research from the addition of an additional research laboratory; the market ignores this effect. (This is just the application to this model of the observation, made in the previous section, that in market equilibrium with free entry average returns are equated to average costs; social optimality requires marginal returns equal marginal costs. Since marginal returns are less the average returns, this leads to a tendency in the market economy for an excessive number of research firms.)

- c) The uncoordinated choice of research strategies (here reflected in the fact that $\alpha_e < \alpha_s$) leads to a lower value of the research program in the market equilibrium relative to the social optimum, even if there is appropriability of the returns from invention.

This effect tends to reduce the number of firms relative to the social optimum.^{34/}

In contrast to these results, it should be observed that a monopolist controlling all the research laboratories would always make the socially efficient choice of techniques ($\alpha_m = \alpha_s$) but, since $V_m < V_s$, the number of research laboratories set up by the monopolist would be unambiguously smaller than is socially optimal. Thus, the mean time for the invention to occur is longer with the monopolist than is socially optimal.

6. Concluding Remarks

In this paper we have attempted to study the nature and consequences of competition in R & D and the relationship between this form of competition and competition in the product market. Earlier discussions which were directed at ascertaining whether monopoly leads to more research than competition, or whether there would be too little or too much research in competitive environments relative to the social optimum were misguided in two critical respects: they failed to distinguish between competition in the present product market and competition in R & D, and they failed to recognize that the market structure was itself an endogenous variable.

In this paper, we have focused on four questions:

- a) If there is competition in R & D, what is the effect of the present market structure (competition at the present time in the product market) on R & D activity? Here, we have shown that if the present market is dominated by a monopolist, there is likely to be more R & D than when the present market structure is competitive; the reason is simply that in the post-invention market, there will be, as a result, less competition and therefore more profits.
- b) What is the effect of competition in R & D on the level of research? Here, we have shown that competition always leads to more research than in a pure monopoly; indeed, because with free entry, expected average returns must equal average costs, even with partial appropriation of returns, competition may result in excessive expenditures on R & D relative to the social optimum.

c) Over time, how does competition in R & D affect competition in the product market? That is, if there are no barriers to entry in the R & D activity, will not competition in R & D lead to entry of new firms into the product market, ensuring that monopolies will only be short-lived? Our analysis has shown how, under certain conditions, a monopoly may persist: that, if the R & D technology is also available to the present monopolist in the product market, he can (and will) deter entry, e.g. by engaging in sufficiently fast research that it does not pay any other entrant to engage in any R & D. Under these circumstances, monopolies will not be short-lived, although the threat of competition may lead the monopolist to engage in significantly faster research than he otherwise would.

Thus, the potentiality of competition may have important effects, even when, in any market situation, no competitor is actually observed. (This is, of course, related to the well-known observation that the number of firms in an industry or the distribution of output among those firms, may not be a good measure of the degree of competitiveness within the industry; at the extreme, in the model of section 3, where research was non-stochastic, only a single firm would ever engage in a particular R & D activity at a time.)

In the models of section 3, increased expenditures by any single firm simply translated into an earlier date of discovery -- more research and faster research are equivalent. There is in these models no uncertainty (or if there is uncertainty, it is of a particularly simple form; there

is a single research strategy available to all firms). The social return to faster research is the earlier value (in present discounted terms) of having the invention at $t - \Delta t$ rather than at t . The private return, however, is the entire value of the patent, if the individual makes the discovery. In the non-stochastic models, the social and private pay-offs of a researcher who does research at a slightly slower pace than the fastest researcher is zero: it is purely wasted duplication, and society provides no rewards. This is not so if the research outcomes are not perfectly correlated.

- d) What determines the number of firms (laboratories) engaged in R & D at any time? Here, we have noted the critical role played by uncertainty in outcomes: if there were no uncertainty, there would only be a single firm; with uncertainty, there may be several. (More precisely, what appears to be critical is the correlation between the research strategies pursued by different firms. In this paper, we have explored two polar cases, where their outcomes are perfectly correlated, and where they are independently distributed. As we have emphasized, the degree of correlation itself ought to be viewed as endogenous variable; this is a question which we hope to pursue elsewhere.)

We have, moreover, contrasted the number of firms that would emerge in a competitive market equilibrium with the socially optimal number of research units. With imperfectly correlated returns to R & D, the marginal social return to each research unit is positive, but the marginal return is less than the average return. In market equilibrium, the

(average) private returns equals the costs of a laboratory, while optimality equates marginal social returns to the marginal cost of a laboratory. Thus, whether there are too few or too many depends critically on the elasticity of demand, which determines the ratio of the private profits to social returns as well as the size of the invention.^{35/}

Moreover, we have shown that each of the research units will operate at an efficient intensity, if there are enough firms that each ignores its effect on the probability distribution of discovery dates, but that if firms do not ignore this effect, each laboratory will operate at too low an intensity.

The market equilibrium may differ from the socially optimum not only in the number of research laboratories, but also in the riskiness of projects undertaken.^{36/} Here, we have noted a systematic bias in favor of too little risk provided that the market rate of interest is greater than zero. In contrast, a monopolist always operates each research laboratory efficiently (both with respect to scale and choice of risk), but will, in general, have too few research laboratories.

Thus, the question of whether restricted competition leads to more research than free competition is far more complex than most of the simplistic discussions (including those on which much of present anti-trust policy is based) would suggest. On the one hand, it is clear that without some method of appropriating returns -- some degree of monopoly power, as represented, e.g. by patents -- no firm would engage in R & D. On the other hand, competition in R & D may stimulate R & D expenditures, but the pattern of expenditures will be less efficient than with a monopoly.

Thus, it is possible that the date of discovery may be later with competition than with monopoly.^{37/} Although there is a strong presumption that monopolies will spend too few resources on R & D, it is possible that in competitive markets (with patent rights) there may be excessive expenditures.

Finally, the belief that competition in R & D is a substitute for competition in the product market, or that it will eventually give rise to competition in the product market, has been shown to be suspect: there are conditions under which monopolies may persist even without any formal barriers to entry other than those provided by the patent system (or by the lags in dissemination of information.)

At the very least, we hope we have convinced the reader that in those sectors of the economy where technological change is important, the analysis of competitive market equilibrium within the framework provided by the traditional competitive equilibrium (e.g. Arrow-Debreu) model is of limited applicability: competition in R & D necessitates imperfect competition in product markets. This form of competition requires a fundamentally different kind of analysis.

FOOTNOTES

¹For a recent set of explorations in the latter, see e.g. Spence (1976), Dixit and Stiglitz (1977) and Salop (1977).

²See Stiglitz (forthcoming) for an extensive discussion of the characteristics of knowledge which make the conventional assumptions for the production of ordinary commodities inapplicable to R & D.

³A more complete theory would, of course, endogenise even demand conditions, as has been urged most recently by Galbraith (1973).

⁴It is important to observe, however, that even in the short run, when a single firm has a monopoly on the existing technology, the traditional model of the profit maximizing monopolist may be inapplicable in the presence of R & D competition; for his actions today (with respect to sales, capacity, etc.) may have important entry deterring effects which he will take into account. See, e.g. Dasgupta, Gilbert, and Stiglitz (1979), Gilbert and Stiglitz (1979), Spence (1977), Salop (1978).

⁵It is competition in R & D activity which is the missing item in the influential article by Arrow (1962). Arrow was concerned with comparing the incentives that firms have for undertaking R & D expenditure under different market structures. In the formal models that were developed in this article the structure of the product market was varied, but it was supposed throughout that only one firm was engaged in R & D activity and that it is protected by barriers to entry in this activity. Arrow's formal model therefore eschewed competition in R & D.

In section 3.4 we shall present a formal model of an environment just described in the text.

⁶Although we do not, in this paper, characterize the full dynamic model where each invention is succeeded by the next.

⁷The assumption that all potential research units face the same R & D technology is made for expositional ease only. See footnote 14.

⁸Alternatively, x can be thought of as the present discounted value of inputs along the optimal research strategy yielding the invention at date T . When the outcome of the research is stochastic, these two are not equivalent, since the input flow will be terminated at the date of discovery; hence input as well as output becomes stochastic.

- ⁹For highly applied research such a model is not wildly off the mark. For example, it is confidently claimed by experts that the new generation of passenger jets will get going in 1981 and 1982, that Boeing 767, Airbus A310-200 and Lockheed L1011-400 will roughly have the same capacity and flight range, and that they will each cost about one billion to develop. It is the confidence in the date of delivery which we wish to emphasize here.
- ¹⁰This, as the reader may verify, is true even when the payoff functions in (3.1) and (3.3) are not concave in x .
- ¹¹As we have postulated a single invention the assumption of the winner(s) taking all (during the lifetime of the patent) makes most sense. In practice of course firms attempt to invent round patents so that the first firm to invent is not necessarily the most advantaged. It is clear how the model can be extended to incorporate such features.
- ¹²We have already ruled out, by construction, an equilibrium devoid of any R & D activity.
- ¹³The reader can readily verify that there is a largest solution of (3.4), and also why this is the only candidate for equilibrium R & D expenditure.
- ¹⁴It is clear how the analysis would run if different firms face different technologies of research. To take only one example, suppose there is a continuum of potential firms, i , ($0 \leq i \leq 1$) such that $T_i(x)$ is the date of invention by i if x is its R & D expenditure.¹ Suppose $T_i(x) > T_j(x)$ for all $x \geq 0$ and all i and j such that $i > j$. Then clearly $i = 0$ is the most efficient firm, and it is only this firm which engages in R & D. Its present value of profits in equilibrium is nil though. There would appear to be empirical correlates of this as well. McDonnell-Douglas, for example, has only recently decided to scrap its plans for developing its DC-X-200 which would have competed directly with Boeing's 767.
- ¹⁵This argument clearly extends to the environment of (3.4), when there is a monopolist in the c technology. There is a natural assumption that this monopolist is in a position to make the first move, and hence to maintain his monopoly.

¹⁶ Consider, for instance, a two stage research program. Let $S\{\hat{x}(t)\}$ be the set of zero profit research strategies, i.e.

$$V_e^{-rt(\hat{x}(1), \hat{x}(2))} = c(\hat{x}(1), \hat{x}(2))$$

Let $x_m(1)$ be the monopoly allocation in the first period. Let $\tilde{x}_m(2)$ be the value of $x(2)$, given $x_m(1)$, which would lead the first entrant to just break even, treating $x_m(1)$ as a by-gone, i.e.

$$V_e^{-r(T(x_m(1), \tilde{x}_m(2)) - 1)} = \tilde{x}_m(2)$$

We postulate that $T(x_m(1), \tilde{x}_m(2)) < T(\hat{x}(1), \hat{x}(2))$ for all $\hat{x}(1), \hat{x}(2) \in S$ (i.e., which break even). Clearly, this need not be the case; if it is not, an effective entry deterring strategy by the first entrant will require $x(1) > x_m(1)$; but there will still be positive profits associated with the entry deterring strategy.

If, however, the rivals can take actions unobserved by the "leader," then he may not be able to pursue a policy of "matching" his rival's before they overtake him. The nature of the equilibrium in this case is analyzed in Gilbert and Stiglitz (1979).

¹⁷ Indeed, it is even a perfect Nash equilibrium, if entrants are restricted to strategies involving research programs for which

$$V_e^{-rT(x)} \geq c(x).$$

¹⁸ This has been noted in the context of natural resources by Dasgupta, Gilbert and Stiglitz (1979), and a more general version has been obtained in Gilbert and Newbery (1979). For a survey of these models, see Salop (1979).

¹⁹ The argument is somewhat more general than we have put it here; we can, for instance, allow there to be durable capital goods (in which case, the maximized value of discounted profits will not have the simple form assumed in the text) and there can be uncertainty (see Gilbert and Newbery (1979)); what is crucial, as we note below, is the zero profit assumption for all entrants.

20 Gilbert and Stiglitz (1979) have investigated the structure of Nash equilibria (involving mixed strategies) for this problem .

21 The argument is simple: assume there is uncertainty about the outcome of research, and there are a number of alternative research strategies which different firms can pursue. Moreover, assume some firm has a comparative advantage relative to the other entrant (but not necessarily relative to the monopolist), so his profits (really rents) are positive even with the threat of entry. Then, to deter entry of this competitor may require a significant increase in research expenditures.

22 The model formulated in this section was originally contained in Stiglitz (1971), and was independently developed by Loury (1979). Loury focuses on the case where $V_s = V_e$ and firms take into account the affect of their action on the aggregate distribution of discovery dates. We are concerned primarily with the normal solution when $V_s < V_e$.

23 This can be justified on the assumption that the efficient size, x^* , of each research unit, is "small".

24 (4.2) and (4.3) are valid so long as $\lambda'(0) > \frac{r^2}{\pi_s}$ which we shall assume as being true.

25 In arriving at (4.8) we suppose that $\lambda'(0) > \frac{r}{V_e}$.

26 In a recent study Schwartzman (1977) has suggested that in the pharmaceutical industry the average cost per new chemical entity is about \$17 million per year, and that on average the discovery period is about 4-5 years. Assuming the upper value and ignoring discounting this means a total investment of about \$85 million per new chemical entity. In the context of our model this means that

$$\bar{x}/\lambda(\bar{x}) = n_c \bar{x} E(T(\bar{x})) = 85 \times 5 \times 10^6$$

dollar years and the model says that it is this which is independent of market structure.

27 This point was originally made in Stiglitz (1971) and Barzel (1968). Loury's analysis is confined to this limiting case.

²⁸The late Jacob Schmookler, in a series of studies emphasized the influence of the growth in demand for a product on innovative activity in the industry in question. See, e.g. Schmookler (1962). The foregoing result in the text is a theoretical confirmation of the kind of relationship Schmookler was concerned with.

²⁹Substituting the zero profit condition in (4.16), we obtain

$$\frac{\lambda'x}{\lambda} \left(1 - \frac{x}{V_e}\right) = 1$$

from which we can solve for x_e . Since the zero profit constraint implies

$$n = \frac{V_e}{x} - \frac{r}{\lambda},$$

$$\frac{dn}{dx} = -\frac{V_e}{x^2} + \frac{r\lambda'}{\lambda^2} < 0,$$

for x in the relevant range.

$$\frac{dnx}{dx} = \frac{r}{\lambda} \left(\frac{\lambda'}{\lambda} x - 1\right) > 0$$

$$\frac{dn\lambda}{dx} = \frac{V_e}{x} \left(\lambda' - \frac{\lambda}{x}\right) > 0.$$

³⁰The distance A is perhaps best viewed as the "subjective" perception of the distance to discovery.

³¹For a fuller development of the mathematics underlying this diffusion model, see Brock, Rothschild and Stiglitz (1979).

- 32 More risky research projects have a greater chance of obtaining results quickly, but also a greater chance of complete failure (or of obtaining results in a very long time). The following example may help the reader to visualize this. Consider the research process as a discrete random walk. A research program is a sequence of experiments. The outcome of any experiment is either to bring one closer to one's goal, or to yield no information at all. A big experiment carries one forward "two steps", but has a higher probability of failure, while a small experiment carries one forward only "one step". Thus, the minimum time to success for the risky research program is half that for the other (if all experiments are successful). On the other hand, there is a higher probability of having a long sequence of unsuccessful experiments.
- 33 Since W_s is not, in general, a well-behaved concave function, it is possible that the global optimum involves less risk-taking.
- 34 However, if the government cannot alter the market's choice of α , whether the government would wish to increase or decrease the number of research units (say by taxing a subsidiary F) would depend only on the first two effects.
- 35 When the elasticity is constant. This is a critical assumption. C.f. Dixit-Stiglitz (1977) for a discussion, in a slightly different context, of how the analysis is altered when elasticities are not constant.
- 36 There are other dimensions in which research strategies in the market equilibrium differ from the social optimum; we have, for instance, already mentioned the degree of correlation among the research strategies undertaken by different firms.
- 37 Gilbert and Stiglitz (1979) explore a case where more competition always leads to a later date of invention.

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