

THE INEFFICIENCY OF THE
STOCK MARKET EQUILIBRIUM

by

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This paper is one of a series attempting to explore the efficiency with which the economy allocates resources in the presence of uncertainty. Arrow's classic paper (1964) established a set of sufficient conditions under which the market equilibrium was a Pareto Optimum. He required that there be a complete set of risk markets, a condition which was obviously not satisfied in any economy. Karl Borch (1962) showed that when there was not a complete set of markets, the market equilibrium would not be a Pareto optimum. These results engendered a search for a weaker set of conditions under which the market equilibrium might satisfy a perhaps weaker sense of optimality.

Ideally, one would like to have a theory which explains incompleteness of markets; several explanations have been offered associated with transactions costs and imperfections in information .

Then an evaluation of the market economy would entail a comparison with a socialist economy facing the same transactions technology or the same information problems. In such a comparison, the market economy might be found deficient with respect to the set of markets which operate. The discussion of this paper can be viewed as a "minimal" test of the market economy: given the set of markets operating does it allocate resources efficiently; if it does, it still does not mean that one should view the market economy as being efficient, but if it does not, it clearly can be viewed as being inefficient.

This still leaves us with the problem of defining some notion of "constrained efficiency". A natural way to pose this problem is to ask, is there a set of taxes - subsidies which the government could impose which

would make some group better off without making anyone worse off? This, however, may still provide what some might regard as an unfair test of the market economy: if the taxes - subsidies are state contingent, the government may have effectively introduced an additional security (or set of securities) into the economy. To avoid this criticism, we wish to tie the government's hands even further: we require any taxes and subsidies to be not state dependent.¹ We need to emphasize that what we are proposing is a minimal test of the efficiency of the market economy; even if it passes this test, there may be grounds for government intervention, either in taking actions which affect the set of markets which operate or in introducing state-contingent taxes and subsidies.

In the case of a simple stock market economy with a single commodity the test which we are proposing is equivalent to the notion of constrained Pareto Optimality introduced by Diamond (1967). In an economy with a stock market, each individual purchases a fraction of each of the firms. In addition, the individual may borrow (or lend) at the safe rate of interest. Diamond considered an economy in which these were the only securities allowed, and contrasted the market allocation where firms maximized their stock market value, with that where the government allocated all resources, but was severely restricted in its ability to distribute the output of the economy: each individual received a linear function of the output of the different firms. He then showed that, (a) if the technology of each firm exhibited stochastic homotheticity (so that the ratio of the output in any two states was independent of scale)²; and (b) each firm believed that its market value was proportional to its scale³, then the market equilibrium would be a constrained Pareto optimum.

The objective of this paper is to show that, even under the extremely favorable conditions for the market economy which Diamond hypothesized, the stock market allocation of resources is not a constrained Pareto Optimality when there are more than two outputs. The reason for this is that the constraint on the distribution of profits involves prices; although in a competitive economy each firm will ignore its effects on prices, a socialist economy would not. Even extending the set of markets - introducing futures markets - may not lead to (constrained) optimality.⁴

The matter may be put another way. With a single output there is no distinction between stochastic homotheticity for a firm or for an industry: the ratio of output or profits in any two states of nature remains a constant. With more than one commodity, if a single firm increases its scale, it may perceive that the ratio of its profits in any two states of nature remains constant; but when the industry expands its scale, the price of the output of the industry may change differently in different states, so that the ratio of industry profits in two different states changes as the scale of the industry changes; there is private stochastic homotheticity --taking prices in each state of nature as given-- but not social stochastic homotheticity. The former assures that firms will wish to maximize stock market value and makes it reasonable for them to assume that value is proportional to their scale. But the latter is what is necessary to ensure constrained optimality of the market solution.

Although Stiglitz (1975) and Hart had earlier noted the non-optimality of the stock market allocation when there were more than one output, they provided a less complete characterization of the nature of the inefficiency than we are able to provide here.⁵

We shall attempt to provide a set of necessary and sufficient conditions for the market equilibrium to be a constrained Pareto Optimum. We have not yet succeeded in providing a complete characterization, but our basic results show that there is an extremely strong presumption that the market equilibrium will not be a constrained Pareto Optimum. Only if the structure of the economy is such that risk itself does not matter can we be sure that the economy is efficient.

Section 2 presents the basic model and two sufficient conditions for the optimality of the stock market equilibrium. Sections 3 and 4 present necessary conditions for the optimality of the stock market equilibrium. Section 5 relates our results to other results on the optimality of the stock market equilibrium.

2. The Model

The simplest model for establishing the non-optimality of the stock market allocation is one involving two commodities, denoted by subscripts 1 and 2. We let the first commodity be our numeraire, and denote the relative price of the two commodities by p . The j^{th} individual's utility is a function of the relative price, p , and his income Y^j :

$$(1) \quad V^j = V^j(p, Y^j)$$

(V is the indirect utility function.) The individual's income (wealth) next period depends on how he allocates his portfolio. For simplicity, we assume all technological risk resides in the second industry, and that it has stochastic constant returns to scale, i.e.

$$(2) \quad X_2 = g(\theta)I_2$$

where X_2 is the output of the second industry in state θ , when the investment level is I_2 . Without loss of generality, we can let $g(\theta) \equiv \theta$. (This simplifies the notation.) The first industry is assumed to be perfectly safe:

$$(3) \quad X_1 = (1 + r)I_1$$

Thus, if the j^{th} individual has an initial wealth of I^j , and he invests a fraction, α^j , in the first industry and the remainder in the second

$$(4) \quad Y^j = [\alpha^j(1 + r) + (1 - \alpha^j)p\theta]I^j$$

Because of the assumption of constant returns to scale, in market equilibrium⁶ it makes no difference whether we think of the individual as investing in the production processes directly or investing in a firm which invests in the production process.

Thus, let β^j be the j^{th} individual's ownership share in industry 2 and γ^j be his share in industry 1. (Again, because of our assumptions, all firms within an industry are identical and have perfectly correlated returns, so we need not distinguish among them.)

Then

$$(5a) \quad (1 - \alpha^j)I^j \equiv \beta^j I_2, \quad \sum \beta^j = 1$$

$$(5b) \quad \alpha^j I^j \equiv \gamma^j I_1, \quad \sum \gamma^j = 1$$

$$(6a) \quad I_2 = \sum (1 - \alpha^j)I^j$$

$$(6b) \quad I_1 = \sum \alpha^j I^j$$

The j^{th} individual takes the relationship between p and θ as given, independent of his actions; thus the optimal portfolio allocation for the j^{th} individual is given by

$$(7) \quad \max EV^j(p, (\alpha^j(1+r) + (1-\alpha^j)p\theta)I^j)$$

i.e.

$$(8) \quad EV_Y^j ((1+r) - p\theta) = 0$$

which implies that

$$(9) \quad 1 + r = \frac{EV_Y^j p\theta}{EV_Y^j},$$

the weighted expected returns from the risky asset must be equal to the safe return, where the weights are relative marginal utilities.

Market equilibrium with rational expectations in this simple model is

characterized by (9) and just one more equation: the relationship between p and θ which individual's expect is the actual market relationship. In the market equilibrium, we require that the (consumption) demand for commodity 2 (say) be equal to its supply, i.e., aggregate supply is given by

$$(10) \quad x_2(\theta, I_2) = \theta I_2 \quad .$$

The demand by the j^{th} individual is just a function of the price and his income:

$$(11) \quad c_2^j = c_2^j(p, Y^j)$$

(where Y^j is given by (4)). (The demand curve may be derived easily from the indirect utility function by Roy's formula:

$$(12) \quad c_2^j = -v_p^j / v_Y^j ;$$

we shall make use of this result later.)

Aggregate demand is thus given by

$$(13) \quad c_2 \equiv \sum_j c_2^j$$

Thus $p(\)$ is given by the solution to

$$(14) \quad c_2 = \theta I_2 \quad .$$

This market solution now needs to be contrasted with a constrained Pareto Optimum. We assume (as did Diamond) that the government can control (a) the allocation of shares of profits and (b) the levels of production of firms. This is done to maximize, say, EV^1 , given the expected utility levels of all other individuals. Thus the government maximizes, for some set of λ^j ,

$$(15) \quad \Sigma \lambda^j v^j(p, Y^j) \\ \{\hat{\beta}^j, \hat{\gamma}^j, \hat{I}_2\}$$

where now

$$\hat{Y}^j = \Sigma \hat{\beta}^j \theta p \hat{I}_2 + \Sigma \hat{\gamma}^j (1+r) (\bar{I} - \hat{I}_2)$$

where $\hat{\beta}^j$ is the fraction of the output of firm 2 assigned to the j^{th} individual, $\Sigma \hat{\beta}^j = 1$ and similarly, $\hat{\gamma}^j$ is the fraction of the output of firm 1 assigned to the j^{th} individual, $\Sigma \hat{\gamma}^j = 1$, and \bar{I} is the total level of investment, $\bar{I} = \Sigma I^j$.

This yields the first order conditions

$$(16a) \quad \lambda^j_{EV_Y^j} = \lambda^1_{EV_Y^1} \quad \text{all } j$$

$$(16b) \quad \lambda^j_{EV_Y^j p \theta} = \lambda^1_{EV_Y^1 p \theta} \quad \text{all } j$$

$$(16c) \quad E\{\Sigma \lambda^j v_Y^j (\beta^j \theta p - \gamma^j (1+r)) + \Sigma \lambda^j (v_p^j + v_Y^j \beta^j c_2) \frac{dp(\theta)}{dI_2}\} = 0$$

Dividing (16b) by (16a) we obtain

$$(17a) \quad \frac{EV_Y^j p \theta}{EV_Y^j} = \frac{EV_Y^1 p \theta}{EV_Y^1}$$

The weighted marginal return from the risky asset is the same for all individuals. Note that (from 9) this condition is always satisfied in a competitive market.

From (16a)

$$\hat{\gamma}^j \lambda^j_{EV_Y^j} = \hat{\gamma}^j \lambda^1_{EV_Y^1} \\ (18a) \quad \Sigma \hat{\gamma}^j \lambda^j_{EV_Y^j} = \lambda^1_{EV_Y^1} (\Sigma \hat{\gamma}^j) \\ = \lambda^1_{EV_Y^1}$$

Similarly,

$$(18b) \quad \sum \beta^j \lambda^j_{EV_Y} p^j_{\theta} = \lambda^1_{EV_Y} p^1_{\theta} .$$

Finally, we define

$$(19) \quad T^j = \hat{\beta}^j_{\theta I_2} - c_2^j \\ = (1 - \alpha^j) I^j_{\theta} - c_2^j$$

$\hat{\beta}^j_{\theta I_2}$ is the individual's "ownership" of X_2 (through his share in the firm in this industry). Thus T^j is the (implicit) net trade of the j^{th} individual. ($T^j > 0$ implies that the individual is selling X_2 net, $T^j < 0$ implies that he is buying X_2). We can thus rewrite (16c) to read::

$$(20) \quad E \lambda^1_{V_Y} (p_{\theta} - (1+r)) + \sum_j E (\lambda^j_{V_Y} T^j \frac{dp}{dI_2}) = 0$$

(20) should be contrasted with (9). We immediately obtain the result that the market allocation is a constrained Pareto Optimum if and only if

$$(21) \quad B \equiv E \sum_j \lambda^j_{V_Y} T^j \frac{dp}{dI_2} \equiv 0$$

Two sufficient conditions are immediate:

- (a) if all individuals are identical, the market is a constrained Pareto Optimum; for then $T^j \equiv 0$.
- (b) if for all individuals $dV_Y^j/d\theta = 0$, the market is a constrained Pareto Optimum; for then we have (from (16a))

$$\sum_j E \lambda^j_{V_Y} T^j \frac{dp}{dI_2} = \lambda^1_{V_Y} \frac{1}{E} \sum_j T^j \frac{dp}{dI_2} = \lambda^1_{V_Y} \frac{1}{E} \frac{dp}{dI_2} \sum T^j = 0 .$$

The condition $\frac{dv_Y}{d\theta} = 0$ is a very stringent condition. To see what it implies observe that

$$(22) \quad \frac{dv_Y^j}{dp} = v_{Yp}^j + v_{YY}^j (1 - \alpha^j) \theta I^j .$$

But

$$(23a) \quad v_p = -C_2^j v_Y$$

$$(23b) \quad v_{pY} = -C_2^j v_{YY} - v_Y \frac{dC_2^j}{dY} .$$

Thus

$$(24) \quad \frac{dv_Y^j}{dp} = -v_Y \left[\frac{dC_2^j}{dY} + \left(\frac{-v_{YY}}{v_Y} \right) T^j \right] .$$

Let

$$(25a) \quad \rho = \frac{-v_{YY} Y}{v_Y} = \text{measure of relative rule aversion}$$

$$(25b) \quad \eta = \frac{d \ln C^j}{d \ln Y} = \text{measure of elasticity of demand}$$

and

$$(25c) \quad -\frac{1}{\sigma} \equiv \frac{\theta}{p} \frac{d p}{d \theta} = \frac{d \ln p}{d \ln X_2/X_1} \quad (\sigma \text{ is the "aggregate"}$$

elasticity of substitution).

Then

$$(26) \quad \begin{aligned} \frac{dv_Y^j}{d\theta} &= \frac{dv_Y^j}{dp} \frac{dp}{d\theta} + v_{YY}^j (1 - \alpha^j) p I^j \\ &= \frac{v_Y C^j p}{\theta Y^j} \left\{ \left(\eta + \rho \frac{T^j}{C^j} \right) \frac{1}{\sigma} - \rho \left(1 + \frac{T^j}{C^j} \right) \right\} = 0 \end{aligned}$$

if and only if (assuming $\sigma \neq 1$)

$$(27) \quad \frac{T^j}{C^j} \equiv -\frac{\rho\sigma - \eta}{\rho(\sigma - 1)} \quad \text{for all } \theta .$$

Clearly (27) is very restrictive.

Note that possessing income risk neutrality ($V_{YY} = 0$) is not sufficient to ensure the optimality of the stock market equilibrium. Indeed, with income risk neutrality, a necessary condition is that all individual's demands for the commodity be income inelastic. More generally, with constant income elasticity, relative risk aversion and aggregate elasticity of substitution for (27) to hold identically for all p requires c_2^j/c_2^k to be constant for all j, k i.e., the proportion of the output of the second commodity consumed by each individual is the same in all states of nature.

3. Necessary Conditions for Optimality with Identical Preference

In the previous section, we found two, extremely restrictive sufficient conditions for the optimality of the stock market equilibrium. In this and the next section, we find necessary conditions for the optimality of the market equilibrium, under certain simplifying assumptions. In this section, we simplify the analysis by assuming individuals have identical homothetic preferences (indifference maps) but different degrees of risk aversion. This leads them to hold different portfolios. We shall establish that if the market equilibrium is Pareto Optimal for all probability distributions over θ , then all individuals must have indifference maps which have a very special shape.

The assumption of the identical homothetic preferences implies that prices will be independent of the distribution of income. This greatly simplifies the analysis. The j^{th} individual's indirect utility function can be written

$$(28) \quad v^j = \phi^j(yu(p))$$

with

$$(29) \quad v(p) \equiv -\frac{u'p}{1 + \frac{u'p}{u}} = \frac{pX_2}{X_1} = \frac{p\theta}{(1+r)I_1} = \frac{1-s}{s}.$$

where s = share of commodity 1.

Thus

$$(30a) \quad \frac{I_2}{p} \frac{dp}{dI_2} = \frac{v}{v'p - v} \left(1 + \frac{I_2}{I_1}\right)$$

and

$$(30b) \quad -\frac{\theta}{p} \frac{dp}{d\theta} = -\frac{v}{(v'p - v)} \equiv \frac{1}{\sigma}, \quad \text{the elasticity of substi-}$$

tution. Thus

$$\frac{v'p}{v} = (1 - \sigma)$$

Substituting (30a) and (30b) into (21), we obtain

$$(21') \quad -B = \frac{\bar{I}}{I_2 I_1} \sum E \lambda^j \phi^j u(p) T^j \frac{p}{\sigma}$$

Let us consider the case of two groups, denoted by superscripts a and b. Then $T^a = -T^b$. Assume that the government has engaged in optimal lump sum transfers, so

$$(31a) \quad E D u(p) = 0$$

where D is the difference in (weighted) marginal utilities

$$(31b) \quad D \equiv \lambda^1 \phi^a - \lambda^2 \phi^b$$

Thus

$$(32) \quad -B \equiv \frac{\bar{I}}{I_1 I_2} E \frac{Du(p) T^a p}{\sigma}$$

From this it follows that a necessary condition for the optimality of the market for all distributions of θ is that

$$\frac{T^a p}{\sigma}$$

be independent of θ , or that $D \equiv 0$.

(a) For $D \equiv 0$ for all values of the parameters requires that
(letting $\lambda^a/\lambda^b \equiv \lambda$)

$$(33) \quad \frac{d\lambda}{\lambda} \equiv \rho^a \frac{dY^a}{Y^a} - \rho^b \frac{dY^b}{Y^b}$$

Without loss of generality, we can write

$$(34) \quad Y^j = A^j + B^j v(p)$$

(Since

$$(35) \quad \frac{p\theta I_2}{(1+r)I_1} = v(p)$$

$$p\theta = kv(p))$$

Thus

$$d \ln \lambda \equiv \frac{\rho^a (dA^a + v(p)dB^a)}{A^a + v(p)B^a} - \rho^b \frac{dA^b + v(p) dB^b}{A^b + v(p)B^b}$$

Since

$$D \equiv \lambda \varphi^a(u(p)(A^a + B^a v)) - \varphi^b(u(p)(A^b + B^b v)),$$

$D' = 0$ implies

$$(36) \quad \rho^a \left(\frac{u'p}{u} + \frac{B^a v'p}{A^a + B^a v} \right) = \rho^b \left(\frac{u'p}{u} + \frac{B^b v'p}{A^b + B^b v} \right)$$

or

$$(36') \quad \rho^a \left(- (1-s) + \frac{B^a}{Y^a} \frac{(1-s)}{s} (1-\sigma) \right) = \rho^b \left(- (1-s) + \frac{B^b}{Y^b} \frac{(1-s)}{s} (1-\sigma) \right)$$

Consider now a perturbation in the parameters of the problem (keeping λ constant). Clearly

$$(37) \quad \rho^a \frac{dA^a + v dB^a}{Y^a} = \rho^b \frac{dA^b + v dB^b}{Y^b}$$

Using (36'), we obtain

$$(38) \quad \frac{dA^a + vdB^a}{B^a \frac{(1-\sigma)}{s} - Y^a} = \frac{dA^b + vdB^b}{B^b \frac{(1-\sigma)}{s} - Y^b}$$

or

$$(38') \quad dA^a + vdB^a = \frac{B^a \frac{(1-\sigma)}{s} - Y^a}{B^b \frac{(1-\sigma)}{s} - Y^b} dA^b + vB^b$$

The LHS of (38) is linear in v . Hence the RHS must be linear in v .

Hence

$$B^a(s-\sigma) - A^a = k[B^b(s-\sigma) - A^b]$$

or

$$(39) \quad (s-\sigma)(B^a - kB^b) = A^a - kA^b$$

Thus, either $s-\sigma$ is constant or $B^a - kB^b = A^a - kA^b = 0$:

$$(40a) \quad (i) \quad s - \sigma = \text{constant};$$

there is a special family of utility functions for which this is true; it is found by solving the differential equation

$$(41) \quad \frac{(1+v)d v}{v(1-c)(1+v) - v} = d \ln p$$

for some value of c ; and then solving the differential equation

$$(42) \quad \frac{d \ln u}{d \ln p} = - \frac{v^*(p)}{1+v^*(p)},$$

where

$$v^*(p) \text{ is the solution to (41).}$$

Within the class of constant elasticity functions, the only utility function satisfying (40a) is (trivially) the unitary elasticity

of substitutions, i.e. $p\theta$ is constant, so there is no risk.

$$(ii) \quad B^a = k_B^b$$

$$A^a = k_A^b$$

The two individuals have identical portfolio allocations, i.e. they must have identical (constant) relative risk aversions, or there is no risk ($\sigma = 1$).

(b) Similarly, $T^a p/\sigma$ will be constant only under special conditions,

Since $-u'p/u = \frac{v}{1+v} = \frac{pX_2^j}{Y^j}$, and using (33)

$$(40) \quad p T^a = \beta^a I_2 \theta p \left(1 - \frac{v}{1+v}\right) - \frac{v}{1+v} \gamma^a I_1 (1+r)$$

$$= [\beta^a - \gamma^a] I_1 (1+r) \frac{v}{1+v}$$

pT^a/σ must be constant, for all values of p , β^a , and γ^a .

Hence either $\beta^a \equiv \gamma^a$ (all individuals are identical) or

$$(41) \quad \frac{v}{(1+v)\sigma} = \frac{1-s}{\sigma} \equiv \text{constant}$$

where s is the share of commodity 1 in consumption. Within the family of constant elasticity functions, only

$$\sigma \equiv 1$$

will yield $s/\sigma = \text{constant}$.

Hence, we have shown that if all individuals have identical homothetic indifference maps, then, if for all distributions of θ , the

market allocation is a constrained Pareto Optimum, then either all individuals must be identical or

$$s-\sigma \quad \text{or} \quad s/\sigma \text{ must be constant.}$$

In the constant elasticity case, this implies $p\theta$ is constant.

In both of these cases, the stock market is not performing any function in sharing or spreading risks, in the first because all individuals are identical, in the second because $p\theta$ is constant, and hence there is no risk in this economy.

4. Differences in Tastes

In this section, we allow consumers to differ in tastes, but to obtain simple results we assume two groups, each with a Cobb-Douglas utility function. The j^{th} individual spends s^j of his income on commodity 1, with $0 < s^j < 1$. We also introduce an additional market - a gamble on the price of X_2 next period (a "futures" market). Let δ^j be the purchases (sales) of the j^{th} group on the futures market and q be the present price. Then

$$(43) \quad Y^j = [\beta^j p\theta I_2 + (1+r)[I^j - \delta^j q - \beta^j I_2]] + \delta^j p$$

If δ^j is chosen optimally.

$$EV_Y^j \frac{p}{q} = EV_Y^j (1+r)$$

or

$$(44) \quad \frac{EV_Y^j p/q}{EV_Y^j} = 1 + r$$

Again, the weighted average returns must be the same as those on a safe asset.

Now in equilibrium

$$(45) \quad I_1(1+r) = s^a Y^a + s^b Y^b$$

where s^i is the share of commodity 1 in the i^{th} person's budget. Thus

$$(46) \quad I_1(1+r) = (s^a \beta^a + s^b \beta^b) p \theta I_2 + (s^a - s^b) \delta^a p + \kappa I_1(1+r)$$

where

$$\kappa = (s^a \gamma^a + s^b \gamma^b)$$

By exactly the kind of analysis used earlier, we can show that the market is a constrained Pareto Optimum if and only if

$$(47) \quad B \equiv E \sum \lambda^j v_Y^j T^j \frac{dp}{dI_2} = 0$$

where now

$$(48) \quad pT^j = p\beta^j \theta I_2 - pC_2^j + p\delta^j = M + Np$$

where the constants M and N can be solved for using (46). The "trades" are a linear function of price.

With two groups, (48) can be rewritten as

$$(49) \quad B = E D T^a \frac{dp}{dI_2}$$

where now

$$(50) \quad \frac{1}{p} \frac{dp}{dI_2} = - \frac{[(s^a \beta^a + s^b \beta^b) p \theta + (1-\kappa)(1+r)]}{I_1(1-\kappa)(1+r)}$$

$$(51) \quad \frac{dp\theta}{dp} = - \frac{(s^a - s^b)\delta^a}{(s^a\beta^a + s^b\beta^b)I_2}$$

From the first order conditions for choosing δ and β optimally,

$$(53) \quad E D p = 0$$

$$(54) \quad E D p \theta = 0$$

Hence

$$(55) \quad E D \frac{pT^a}{p} \frac{dp}{dI_2} = - \frac{s^a\beta^a + s^b\beta^b}{I_1(1-\kappa)(1+r)} E D N p^2\theta = 0$$

if and only if

$$(a) D \equiv 0$$

or $(b) s^a = s^b$ (so $p\theta$ is a constant)

or $(c) N = 0$ (pT^a is independent of p)

We now see what each of these entails

$$(a) \quad D \equiv 0.$$

Since Y^j is a linear function of p , we can write

$$(56) \quad V^j = \phi^j (A^j p^{-(1-s^j)} + B^j p^{s^j})$$

For simplicity, we again focus on the case of constant relative risk aversion. Then

$$D \equiv 0$$

implies

$$(57) \quad \lambda [A^a p^{-(1-s^a)} + B^a p^{s^a}]^{-\rho^a} = [A^b p^{-(1-s^b)} + B^b p^{s^b}]^{-\rho^b} p^{s^b - s^a}$$

for all p , or

differentiating logarithmically,

$$\rho^a \left\{ -s^a + \frac{A^a}{B^a p + A^a} \right\} +$$

$$- \rho^b \left\{ -s^b + \frac{A^b}{B^b p + A^b} \right\} + s^a - s^b = 0$$

i.e.

$$(58) \quad \rho^a \{A^a(1-s^a) - s^a B^a p\} = \rho^b \{A^b(1-s^b) - s^b B^b p\} \frac{A^a + pB^a}{A^b + pB^b} + (s^b - s^a)(A^a + pB^a)$$

$$= (A^a + pB^a) \left\{ \rho^b \frac{(A^b(1-s^b) - s^b B^b p)}{A^b + pB^b} + s^b - s^a \right\}$$

Hence the RHS of (58) must be a linear function of p, i.e. either

$$\frac{A^a}{A^b} \equiv \frac{B^a}{B^b},$$

they obtain identical allocations (which will be optimal for all distributions of θ if and only if $s^a = s^b$, $\rho^a = \rho^b$)

or

$$B^b = 0,$$

but this will be optimal only if the farmer is risk neutral.

(b) $s^a = s^b$ is the case analyzed in the previous section

(c) $N = 0$

From (46) and (48)

$$N = s^a \delta^a - s^a \beta^a \frac{(s^a - s^b) \delta^a}{(s^a \beta^a + s^b \beta^b)} = \frac{s^a s^b \delta^a}{s^a \beta^a + s^b \beta^b} > 0$$

unless $\delta^a \equiv 0$. But if $\delta \equiv 0$,

$p\theta$ is constant, and there is no more risk. Moreover, for it to be optimal for $p\theta$ to be constant,

$$(59) \quad E D p = 0$$

when

$$(60) \quad E D = 0.$$

But (from (56), letting $B^a = B^b = 0$), at $D = 0$

$$pD' = \lambda \psi' \{ \rho^a (1-s^a) - \rho^b (1-s^b) + (s^a - s^b) \} p^{s^a - 1}$$

Then, for (59) and (60) both to hold,

$$(61) \quad s^a + (1-s^a)\rho^a = s^b + (1-s^b)\rho^b$$

If ρ^j is not constant, changing λ will change ρ^a and ρ^b so (61) cannot hold identically. If ρ is constant, (61) can hold, but this is obviously a very special case.

5. Concluding Comments

This paper is a contribution to the growing literature which has developed from Karl Borch's seminal work; this research has cast doubt on the efficiency of the economy in allocating resources in the presence of an incomplete set of risk markets.

This recent literature has questioned

- (a) the appropriateness of the criterion of stock market value maximization
- (b) the persuasiveness of the assumption that the value of firms will increase in proportion to their scale
- (c) the generality of the result that the market will be a constrained Pareto Optimum
- (d) the optimality of the set of markets which are in existence

This paper has considered a situation in which the technology is such that all shareholders would wish the firm to maximize the stock market value and in which the hypothesis that the value of the firm increases in proportion to its scale is plausible. Nonetheless, we have shown that there is a strong presumption that the market is not a constrained Pareto Optimum; it is not just that "examples" can be found in which the market equilibrium is a Pareto Optimum. Rather, the basic marginal conditions describing the market equilibrium and the optimum are different. Although Stiglitz (1973, 1976) had noted this difference earlier, what we have done here is to show that the set of circumstances in which the two conditions coincide are very restrictive.

Indeed, when we impose additional constraints on the set of markets which operate, the presumption that the economy is not efficient is strengthened even further.

Newbery and Stiglitz (1979) have analyzed in detail a model which may be viewed as a limiting case of the model presented here; they assume that there is no stockmarket (although there may be a futures market). In that case, again, the market equilibrium is a constrained Pareto Optimum only under very special conditions (consumers have logarithmic utility functions). In general, producers will make "incorrect" production decisions. When markets are "very incomplete" then even the assumption of identical utility functions is not sufficient to ensure the constrained Pareto Optimality of the market allocation.

In both the situation analyzed here and in Newbery-Stiglitz, prices are performing two functions: not only do they have their conventional role in allocating resources, they perform a critical function in sharing and transferring risk (insurance). When, for instance, price and quantity vary inversely, the risk of production is transferred from producers to consumers by the price system. Evidently, when too much is asked of the price system, it does not perform well, or at least, as well as it would with selective government intervention.

(It is important to realize that the kind of argument we employ here is markedly different from the naive argument that the market does not allocate resources efficiently because producers will not undertake sufficient risk. If there are incomplete risk markets, the risks which are borne by producers (which in a complete set of markets they might be able to insure against) have a real social cost; that is, the fact that the risks might, under some other institutional arrangement, be spread does not alter the fact that under the institutional arrangement under consideration, producers do in fact bear a risk which affects their expected utility. The conventional discussions of the issue seem to make

the error of comparing the allocation of resources with one market structure with the allocation which would have emerged with another market structure, a comparison which we have argued is unfair, and probably irrelevant. In our analysis, we have explicitly taken into account the costs of risk as they are actually borne in the second-best situation under consideration.)

The inefficiencies with which we have been concerned here are marginal inefficiencies; in addition, it is important to remember that there may be, in such situations, what I have referred to elsewhere as structural inefficiencies: given the set of production decisions made by other firms, each firm is making the correct decisions; yet, there exists another set of (mutually consistent) actions which leads to a Pareto Improvement. There are, in other words, multiple Nash Equilibrium, some of which Pareto Dominate others (Stiglitz, (1972a), Hart (1975).)

The question of whether the appropriate set of markets is in operation may be viewed as one of structural efficiency. Note that in the analysis of section 4, if there were no futures market, the economy would have been a constrained Pareto Optimum; but with the opening of a futures market, it no longer will be. There are clearly incentives for introducing futures markets. Whether, in our model, not allowing a futures market would lead the competitive economy (without further intervention) to a Pareto superior position to that with a futures market is a question we have not answered. In other work, (Green and Sheshinski, Newbery and Stiglitz, Hart) it has been shown that the opening of additional markets may lead to a Pareto Inferior Equilibrium.

There are two marked limitations on our analysis which should be noted. First, we have not provided a quantitative estimate of the

magnitude of the distortions (or the welfare losses to which they give rise); indeed, we have not even obtained a clear qualitative analysis of the direction of the bias. (Our analysis does shed some light on the kinds of parameters which are likely to be relevant, e.g. the magnitude of differences in tastes and risk aversion.) Although, in principle, a set of taxes-subsidies could be devised which would enable the economy to attain a constrained Pareto Optimum, the magnitude of the taxes and subsidies may vary markedly from industry to industry and depend on parameters (probability distributions of returns) which are difficult to estimate with any degree of accuracy. Thus the implementation of such tax-subsidy schemes may be extraordinarily difficult; our analysis may perhaps be better viewed as casting doubt on the validity of the claims that the market provides an efficient allocation of resources rather than as a basis of a prescriptive theory of government intervention.

Secondly, we have not provided an explanation of the limitations on the set of markets which function. The kind of analysis which we have employed here, analyzing the performance of the economy with a given set of markets, and then evaluating whether there is an optimal set of markets in operation, is clearly a second best approach to a second best problem: it would be desirable to analyze both problems simultaneously. This is the next task on our research agenda.¹

1 In research currently under way, we have gone some way towards this objective; in situations where the reason for the imperfect risk markets is moral hazard (related to the unobservability of individuals' actions), the market equilibrium is not, in general, a constrained Pareto Optimum.

NOTATION

p	price of commodity 2 (relative to commodity 1)
Y^j	income of j^{th} individual (in commodity 1 numeraire)
X_i	output of commodity i
I_i	investment in industry i
I^j	j^{th} individual's initial wealth
I	aggregate investment
α^j	proportion of j^{th} individual's initial wealth invested in industry 1
β^j	proportion of industry 2 owned by j^{th} individual
γ^j	proportion of industry 1 owned by j^{th} individual
C_i^j	j^{th} individual's demand for commodity i
C_i	aggregate demand for commodity i
θ	state of nature
$\hat{}$	over variable denotes corresponding values of variables in constrained pareto optimum
η	income elasticity of demand
ρ	measure of relative risk aversion
σ	aggregate (demand) elasticity of substitution

Footnotes

- 1 Without loss of generality, we can allow them also to be state dependent, but if so, to be linearly dependent on the outputs of the different firms in the economy.

- 2 The assumption of stochastic homotheticity (or multiplicative uncertainty as Diamond calls it) is an essential assumption in the Diamond analysis. Because of that assumption each firm can be viewed as producing a composite commodity; the theorem on the constrained Pareto Optimality of the market can be viewed as the statement that, given the set of "composite commodities" being produced, if each firm acts as a price taker with respect to the price of the composite commodity it produces, then the market is Pareto Optimal. If stockholders (consumers) take the set of composite commodities produced as given as well, then clearly the only way that an action of the firm affects their welfare is through its affect on the valuation of the firm. Thus, there is unanimity among the shareholders both that the firm should maximize its stock market value, and that a particular level of investment will lead to the maximum stock market value. (For a more general discussion of the conditions for unanimity and the conditions under which the firm should maximize value, see Grossman and Stiglitz (1977).)

In the absence of multiplicative uncertainty, there may be disagreement both with respect to the effects of any given action on the market value and on the desirability of firm market value maximization. Any given action of the firm will have an effect on the value of the firm and on the consumption (or investment) opportunity set facing the individual, even apart from the affect on the value of the firm, i.e., if there were two securities and three states of nature, the opportunity set is represented by a straight line in three dimensional space; with multiplicative uncertainty, individuals always perceive an action of the firm as moving the straight line out in a parallel manner (draw the plane defined by the line and the origin; then draw a straight

line in that plane parallel to the original line) and all agree that such a movement is an improvement; but without multiplicative uncertainty the new opportunity set is a straight line which is not parallel to the original straight line. The change is preferred by some individuals, not by others. The relative importance of the "consumption" and valuation effects depends on the plans of the individuals with respect to the sale of securities. If, as in much of the recent literature, it is assumed as a condition of equilibrium that an individual neither plans to buy nor sell shares, then the direct valuation effect is of no concern (since the price of anything which is neither bought nor sold has no effect, at the margin, on utility); but any reasonable model of the stock market involves trade, either because of life cycle effects, stochastic birth and death of firms, or differential information, and hence to some individuals but not all the valuation effect will be important; there will not be unanimity on the policy which the firm ought to pursue.

3 The market valuation of the firm will in general depend on the debt-equity ratio of the firm, if there is a finite probability of bankruptcy; thus a change in the debt-equity ratio will have an effect both on the value of the firm and on the opportunity sets facing individuals. This is true even when there is marginal multiplicative uncertainty, i.e., increasing investment has a relative marginal effect on profits in different states of nature which is independent of the level of investment. In the absence of bankruptcy, with marginal multiplicative uncertainty again all individuals might agree on the objective of the firm, but with bankruptcy they will not. See Stiglitz (1972b), Grossman-Stiglitz (1977).

4 The issue of futures markets does not arise in Diamond's analysis, since there is only one commodity.

5 Hart (1975) provides an example of multiple equilibrium in which in one of the equilibrium all individuals are better off than in another. This is an example of what I have called elsewhere a structural inefficiency (Stiglitz (1972a)); such examples can occur even with a single commodity as was shown there. One of the Nash equilibria is Pareto Optimal, but there is no way of ensuring that this is the one which will occur.

On the other hand, the inefficiency which is noted in Stiglitz (1975) and which we discuss here is a marginal inefficiency; at the margin, the private market makes incorrect investment decisions. This would be the case even were there a unique equilibrium.

6 In market equilibrium, the value of any firm in the i^{th} industry must be equal to its investment. Under the competitive hypothesis, it will believe by increasing investment it will increase its market value proportionately; the net worth of the original shareholders is thus increased so long as the value of the firm V_i , exceeds I_i . Thus, investment continues until $V_i = I_i$. See, for instance, Diamond (1967) or Stiglitz (1972a).

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