

LIMITED LIABILITY CONTRACTS BETWEEN
PRINCIPAL AND AGENT*

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Abstract

The form of the optimal contract between principal and risk-neutral agent is derived in an environment where the agent is permitted (without penalty) to choose autarky rather than supply effort to the principal after observing the state of nature. The contract which is optimal for the principal is shown to induce an inefficient outcome in all states of nature except the highest, even though contracts which induce an (ex post) efficient outcome in every state of state do exist.

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The problems associated with motivating economic agents to perform various tasks have occupied the attention of economists for years. In the past decade, the Principal-Agent model has received increasing recognition as an important analytic device in the study of incentive schemes and contracts among economic agents.

The Principal-Agent literature has, for the most part, assumed the existence of institutions which, at no cost to either the principal or agent, guarantee that (after a random state of nature, θ is realized) both parties abide by the terms of any contract which they have both endorsed (prior to the realization of θ). Shavell [1979], for example, has shown that in the presence of such institutions, the optimal contract between principal and risk-neutral agent is of a particularly simple form (see Section I) and results in an efficient outcome, whatever state of nature is realized.

In the absence of such institutions, however, the optimal contract between principal and risk neutral agent differs drastically. It is the purpose of this paper to examine the form of the optimal contract in the Principal-Agent model when institutions which bind the agent to certain agreements made prior to the realization of θ either do not exist, or are too costly to employ.¹ It is shown that absent such institutions, the principal can design "limited liability" contracts which induce the agent to supply sufficient effort in each state of nature to realize an efficient outcome from the productive process in question. However, the cost of such contracts is prohibitive, and the contract which is optimal for the principal will not induce a (non-zero) efficient outcome in any state of nature except for the very highest. These results are formally demonstrated in Sections II and III.

First, though, in Section I, the basic features of the Principal-Agent model are briefly reviewed. It is also shown there that the optimal contract between the principal and risk-neutral agent will result in an efficient outcome

in each state of nature, as long as all (ex ante) agreements are binding (ex post) on both parties. The specific type of liability limitation analyzed in this paper is also formally introduced in Section I.

After the form of the optimal limited liability contract is presented (in Section II), and shown to induce inefficient outcomes in all but the highest state of nature (see Section III), an intuitive explanation of these results is offered in Section IV. Then, in Section V, it is shown that sufficient competition among identical agents can, for certain classes of information asymmetry between principal and agent, prevent the market failure described in Section III. The fact that competition is not a panacea in all circumstances, however, is also demonstrated in this section. Finally, conclusions are drawn in Section VI.

Section I. Notation and Statement of the Principal-Agent Problem

The interaction between principal and agent which is analyzed in this paper can be characterized as follows: A principal requires the effort, a , of an agent in order to carry out production.² The agent's effort, together with the realization of a random variable, θ , determines the value of output produced, x , according to the relationship $x = X(a, \theta)$. The value of output can be observed by both parties, but the principal cannot observe a , nor can he observe the realization of θ , although he does know the functional form of the production technology. Because he is assumed to lack any type of monitoring technology, the principal must induce the agent to supply effort by designing a contract in which payments to the agent, S , depend solely upon the observed value of output, i.e., $S = S(x)$.

At the point in time when the contract is agreed upon, the principal and agent possess identical beliefs about the probability distribution of θ . Both

parties are also aware that after the contract has been agreed upon, the agent (only) will observe the actual realization of θ before choosing a .³

The utility function of the agent is of the following form:⁴

$$\begin{aligned} U^A(S,a) &= S - V(a) \\ &= S - W(x,\theta) \end{aligned}$$

where $V'(a) > 0$ and the production technology is assumed to be invertible so that $a = g(x,\theta)$ is the amount of effort required to produce x in state θ , and $W(x,\theta) = V(g(x,\theta))$.

$W(x,\theta)$ is the disutility to the agent (scaled in monetary units) of producing x in state θ . Throughout the ensuing analysis, $W(x,\theta)$ is assumed to be characterized by all but the last of the following properties. $W(x,\theta)$ is also assumed to be characterized by Property (6) in the proof of Proposition 2.

Property (1): $W(x,\theta_i) \geq W(x,\theta_j) \geq 0$ for $\theta_i < \theta_j$, $\forall x \geq 0$,

with strict inequality for $x > 0$.

Property (2): $W(x_i,\theta_h) < W(x_j,\theta_h)$ for $x_i < x_j$, $\forall \theta_h$.

Property (3): $\frac{\partial W(x,\theta_j)}{\partial x} \leq \frac{\partial W(x,\theta_i)}{\partial x}$ for $\theta_i < \theta_j$, $\forall x \geq 0$,

with strict inequality for $x > 0$.

Property (4): $\frac{\partial^2 W(x,\theta)}{\partial x^2} \geq 0$ $\forall x \geq 0$,

with strict inequality for $x > 0$.

Property (5): $\left. \frac{\partial W(x,\theta_n)}{\partial x} \right|_{x=0} < 1$.

Property (6): $\frac{\partial^2 w(x, \theta_j)}{\partial x^2} \leq \frac{\partial^2 w(x, \theta_i)}{\partial x^2}$ for $\theta_i < \theta_j$, $\forall x \geq 0$,
 with strict inequality for $x > 0$,

where $0 < \theta_1 < \dots < \theta_n$.

Property (1) simply states that the disutility to the agent of producing any value of output, x , is greater the smaller is θ . Hence, θ may be interpreted as a productivity parameter, and higher values of θ correspond to states of nature in which the agent is more productive. The second property indicates that in any state of nature, the agent must incur greater disutility in order to produce a higher level of x . Property (3), along with the assumption that $\frac{\partial X(a, \theta)}{\partial a} > 0 \forall \theta$, states that the marginal disutility of effort to the agent is higher the less productive he is (i.e., the lower is θ). The agent's marginal disutility of effort is also assumed (in Proposition 2) to increase more rapidly the lower is θ , according to Property (6). Property (4), which ensures that the second order conditions of the problem considered here are met, stipulates that $W(\cdot)$ is a convex function of x . Property (5) ensures the existence of a nontrivial solution. It states that there exists some positive level of x which can be produced by the agent in his most productive state without incurring a level of disutility which exceeds the value of that x to the principal. Following most of the Principal-Agent literature, the principal's utility function is expressed as $U^P(x, S) = x - S$.

When institutions exist to ensure that (after θ is revealed to the agent) both principal and agent abide by the terms of their agreement, the principal's task of designing an optimal contract can be formally stated as:

$$\text{Maximize } E_{\theta} \{X(a, S, \theta) - S[X(\cdot)]\}$$

$S(x)$

subject to: (1) $a \langle S, \theta \rangle = \underset{a'}{\operatorname{argmax}} S[X(a', \theta)] - v(a')$

and (2) $E_{\theta} \{S[X(a, \theta)] - W(x, \theta)\} \geq 0$

where $x = X(a \langle \cdot \rangle, \theta)$ is the value of output which will result from the effort, \underline{a} , chosen by the agent after having observed the realization of θ and the terms of the contract, $S(x)$,

and $0 < \theta_1 < \dots < \theta_n$.

The "self-selection" constraint (1) states that the principal is aware that the agent will choose \underline{a} to maximize his own utility after having observed θ and $S(\cdot)$. The "expected utility" constraint (2) requires the terms of the contract to be such that the agent will consent to become party to the agreement. Implicitly, then, the "reservation utility level" of the agent is assumed to be known to both parties, and is (arbitrarily) set at zero (suggesting that the agent's most attractive alternative to working for the principal is autarky).

Harris and Raviv [1979] have demonstrated that the solution to this problem has a particularly simple form. The optimal contract is:

$$S(x) = x - k$$

where k is a constant designed to ensure that the agent's expected utility from becoming party to such a contract is identically zero. The contract requires the agent to pay this fixed sum to the principal after the state of nature is realized. In return, the agent receives the entire value of the output which he chooses to produce.

One of the most remarkable features of this contract is that it guarantees that the outcome of the productive process is always efficient. Efficiency is achieved because the agent (who knows the value of θ when he chooses \underline{a}) is induced to maximize the difference between the value of output produced and the

true costs of production.

Note, however, that it is only "on average" that the agent will break even under this contract. In some (of the lower) states of nature, he can do no better than suffer a loss in utility below the level achieved in autarky. In these states, the agent would like to breach the contract, but the institutions which are assumed to exist in much of the Principal-Agent literature prevent him from doing so.

In the remainder of this paper, the form of the contract which is optimal for the principal in the absence of such institutions is examined. More specifically, the properties of the optimal "limited liability" contract will be derived wherein the agent cannot be held liable for any commitments which result in his (ex post) utility falling below some maximum liability limit (L).⁵ For analytic and expositional convenience, L is set equal to zero, so that the class of limited liability contracts analyzed below is that in which the agent is always permitted (without penalty) to choose autarky rather than supply effort to the principal after observing the state of nature.⁶

When the principal is restricted to this class of limited liability contracts, the agent must be induced to supply effort after the state of nature w is realized rather than being compelled to do so against his will as a consequence of a previous commitment. Thus, since $W(X(0,\theta),\theta) = W(0,\theta) = 0 \quad \forall \theta$, it must be the case that, ex post, the optimal contract must satisfy the following (limited liability) constraint in each state of nature:

$$S(x) - W(x,\theta) \geq 0$$

where x is chosen by the agent (after observing θ and $S(\cdot)$) to maximize his own utility. Note that when this limited liability constraint is satisfied in each state of nature, the expected utility constraint (2) described above is automatically satisfied. Thus, the problem under consideration here can be stated as

$$\text{Maximize}_{S(x)} E_{\theta} \{ X(\langle S, \theta \rangle, \theta) - S[X(\cdot)] \}$$

subject to: (1) $a \langle S, \theta \rangle = \underset{a'}{\operatorname{argmax}} S[X(a', \theta)] - V(a')$

(PA)

and (2) $S(x) - W(x, \theta) \geq 0 \quad \forall \theta$

In order to characterize the solution to (PA), it is helpful to formulate the following problem:⁷

$$\text{Maximize}_{x_i, S_i} \sum_{i=1}^n p_i [x_i - S_i]$$

subject to: (1) $S_i - W(x_i, \theta_i) \geq S_j - W(x_j, \theta_i) \quad \forall j \neq i$
 $i = 1, \dots, n.$

(PA') (2) $S_i - W(x_i, \theta_i) \geq 0 \quad i = 1, \dots, n.$

(3) $x_i \geq 0 \quad i = 1, \dots, n.$

where $0 < \theta_1 < \dots < \theta_n$, $\sum_{i=1}^n p_i = 1$,

p_i = the principal's assessment of the probability that $\theta = \theta_i$ ($p_i > 0 \forall i$), and

x_i = the value of output produced by the agent when θ_i occurs, given $S(x)$.

It can be shown (see the Appendix) that (PA') is equivalent to (PA) in the environment described above, so that it is sufficient to characterize the solution to (PA') in order to characterize that of (PA). This is the approach adopted throughout the discussion in the following sections.

Section II. Derivation of the Optimal Contract in the Absence of Enforcement Institutions

The solution to (PA) and, therefore, (PA') is derived by a lengthy process which is outlined in the proof of Proposition 1. The most important properties of the solution are stated here.

Proposition 1. Given that Properties (1) through (5) hold, the solution to (PA) has the following features:

- (a) The agent is permitted to choose from among $k(1 \leq k \leq n)$ positive allocations (i.e., $(x'_i, S'_i) > (0,0) \ i=1, \dots, k$) and autarky, $(0,0)$.
- (b) The allocations are designed such that the higher the value of θ which is realized, the higher will be the value of output produced by the agent.⁸
- (c) If $(H)^i =$ the set of all θ_j for which (x'_i, S'_i) is selected by the agent,
 $I^i =$ the set of all subscripts on those $\theta_j \in (H)^i$,
 and $\theta_i^{\min} = \text{minimum}_{\theta} \{(H)^i\}$
 the agent is indifferent between (x'_i, S'_i) and autarky when θ_1^{\min} occurs.
- (d) When θ_i^{\min} occurs $i = 2, \dots, k$, the agent is indifferent between (x'_i, S'_i) and (x'_{i-1}, S'_{i-1}) .⁹
- (e) $x'_i \ i=1, \dots, k$ is determined by the equation:¹⁰

$$P_i - [1 - \sum_{r=0}^{i-1} P_r] \frac{\partial W(x'_i, \theta_i^{\min})}{\partial x} + [1 - \sum_{r=0}^i P_r] \frac{\partial W(x'_i, \theta_{i+1}^{\min})}{\partial x} = 0$$

where $(H)^0 =$ the set of all θ_j for which $(0,0)$ is chosen by the agent,

$$P_i = \sum_{j \in I^i} P_j, \text{ and } \frac{\partial W(x'_k, \theta_{k+1}^{\min})}{\partial x} \text{ is finite.}$$

Proof of Proposition 1.

1. The first order conditions obtained from the Lagrangian function associated with (PA') are:

$$(x_i) \quad p_i - \sum_{\substack{j=1 \\ j \neq i}}^n \lambda_{ij} \frac{\partial W(x_i, \theta_i)}{\partial x} + \sum_{\substack{h=1 \\ h \neq i}}^n \lambda_{hi} \frac{\partial W(x_i, \theta_h)}{\partial x} - \lambda_i \frac{\partial W(x_i, \theta_i)}{\partial x} \leq 0 \quad x_i[\cdot] = 0 \quad i = 1, \dots, n$$

$$(S_i) \quad -p_i + \sum_{\substack{j=1 \\ j \neq i}}^n \lambda_{ij} - \sum_{\substack{h=1 \\ h \neq i}}^n \lambda_{hi} + \lambda_i = 0 \quad i = 1, \dots, n$$

$$(\lambda_{ij}) \quad S_i - W(x_i, \theta_i) - [S_j - W(x_j, \theta_i)] \geq 0 \quad \lambda_{ij}[\cdot] = 0 \quad \forall j \neq i \quad i = 1, \dots, n$$

$$(\lambda_i) \quad S_i - W(x_i, \theta_i) \geq 0 \quad \lambda_i[\cdot] = 0 \quad i = 1, \dots, n$$

$$\lambda_{ij} \geq 0, \quad \lambda_i \geq 0 \quad \forall i, j \quad (j \neq i) \quad i = 1, \dots, n \quad j = 1, \dots, n$$

where λ_{ij} is the Lagrange multiplier associated with constraint (1), and λ_i is the multiplier associated with constraint (2).

2. $x_i \geq x_j$ for $i > j$. This result is proved as follows:

(a) $S_i - W(x_i, \theta_i) \geq S_j - W(x_j, \theta_i)$ by (λ_{ij}) .

(b) Hence, $S_j - S_i \leq W(x_j, \theta_i) - W(x_i, \theta_i)$.

(c) Assume $x_j > x_i$. Then $W(x_j, \theta_i) - W(x_i, \theta_i) < W(x_j, \theta_j) - W(x_i, \theta_j)$ by

Property (3).

(d) Hence, $S_j - S_i < W(x_j, \theta_j) - W(x_i, \theta_j)$ which can be shown to contradict (λ_{ji}) .

3. The manner in which k , the number of distinct, positive allocations is determined is discussed in detail in Section IV. For ease of exposition, the

proof of Proposition 1 is outlined here for the case in which $k = n$. The more general proof is more complicated and tedious, but employs the same techniques outlined in the proof shown here.

4. Note that since $k = n$, $(x'_i, S'_i) = (x_i, S_i)$, $P_i = p_i$, and $\theta_i^{\min} = \theta_i$, $i = 1, \dots, n$.

5. $\sum_{i=1}^n \lambda_i = \sum_{i=1}^n p_i = 1$. This result is proved by adding (S_1) through (S_n) .

6. $\lambda_1 = 1$ and $\lambda_j = 0$ $j = 2, \dots, n$. This result is proved by reaching a contradiction of (λ_{j-1}) , assuming $\lambda_j > 0$, and using $(\lambda_{j,j-1})$ and the finding in step number 5 above along with Property (1).

7. $\lambda_{ij} = 0$ for $i > j+1$ $i = 3, \dots, n$. This result is proved as follows:

(a) If $\lambda_{ij} > 0$ for $i > j + 1$, $S_i - W(x_i, \theta_i) = S_j - W(x_j, \theta_i)$ by (λ_{ij}) .

(b) Hence, $S_j - W(x_j, \theta_i) \geq S_{i-1} - W(x_{i-1}, \theta_i)$ using $(\lambda_{i,i-1})$ and substituting from (a).

(c) By Property (3), $W(x_{i-1}, \theta_i) - W(x_j, \theta_i) < W(x_{i-1}, \theta_{i-1}) - W(x_j, \theta_{i-1})$.

(d) Combining the relationships in (b) and (c),

$$S_j - W(x_j, \theta_{i-1}) > S_{i-1} - W(x_{i-1}, \theta_{i-1}) \text{ which contradicts } (\lambda_{i-1,j}) .$$

8. $\lambda_{21} > 0$ and $\lambda_{n,n-1} > 0$. These results are obtained directly from (S_1) and (S_n) , respectively, using the finding in step number 7 above.

9. If $\lambda_{n-h,n-h-1} > 0$, $\lambda_{n-h-1,n-j} = 0$ for $h \geq 0$, and $0 \leq j < h + 1$. This result is proved as follows:

- (a) If $\lambda_{n-h-1, n-j} > 0$, $S_{n-h-1} - W(x_{n-h-1}, \theta_{n-h-1}) = S_{n-j} - W(x_{n-j}, \theta_{n-h-1})$.
- (b) By Property (3), $W(x_{n-j}, \theta_{n-h}) - W(x_{n-h-1}, \theta_{n-h}) < W(x_{n-j}, \theta_{n-h-1}) - W(x_{n-h-1}, \theta_{n-h-1})$.
- (c) Combining the relations in (a) and (b),
 $S_{n-h-1} - W(x_{n-h-1}, \theta_{n-h}) < S_{n-j} - W(x_{n-j}, \theta_{n-h})$.
- (d) Hence $S_{n-h} - W(x_{n-h}, \theta_{n-h}) < S_{n-j} - W(x_{n-j}, \theta_{n-h})$ since
 $S_{n-h} - W(x_{n-h}, \theta_{n-h}) = S_{n-h-1} - W(x_{n-h-1}, \theta_{n-h})$ by the hypothesis.
 This result violates $(\lambda_{n-h, n-j})$.
10. If $\lambda_{n-h, n-h-1} > 0$, $\lambda_{n-h-j, n-h} = 0$ for $h \geq 0$, $j \geq 1$. The proof proceeds by contradiction, where the hypothesis and Property (3) are employed to reach a contradiction of $(\lambda_{n-h-j, n-h-1})$, much as in step number 9 above.
11. $\lambda_{21} = 1 - p_1$. This relationship is derived from (S_1) , using the findings in steps number 6, 7, 8 and 9 above.
12. $\lambda_{21} = 1 - p_1$ implies $\lambda_{32} = 1 - p_1 - p_2$. This result is derived using (S_2) and the findings in steps number 6, 7, 9 and 10.
13. $\lambda_{i, i-1} = 1 - \sum_{r=1}^{i-1} p_r$ $i = 2, \dots, n$. This result is proved by induction, using the finding in step number 12 and employing the techniques outlined therein.
14. $\lambda_{ij} = 0$ for $i \neq j + 1$. This finding is a direct result of the findings in steps number 7, 9, 10 and 13.
15. Hence, from (x_i) , x_i $i = 1, \dots, n$ is determined by:

$$p_i - (1 - \sum_{j=1}^{i-1} p_j) \frac{\partial W(x_i, \theta_i)}{\partial x} + (1 - \sum_{j=1}^i p_j) \frac{\partial W(x_i, \theta_{i+1})}{\partial x} = 0$$

where $\sum_{j=1}^0 p_j = 0$ and $\frac{\partial W(x_n, \theta_{n+1})}{\partial x}$ is finite.

16. Also, from (λ_{ij}) , $S_1 - W(x_1, \theta_1) = 0$, and $S_i - W(x_i, \theta_i) = S_{i-1} - W(x_{i-1}, \theta_{i-1})$
 $i = 2, \dots, n.$ Q.E.D.

Remark: Any contract which satisfies properties (a) through (d) in Proposition 1 will, indeed, induce the agent to choose allocation (x'_i, S'_i) when any $\theta_j \in \mathbb{H}^i$ occurs.

Proof: Suppose the agent chooses (x'_r, S'_r) $r \neq i$ when $\theta_j \in \mathbb{H}^i$ occurs.

Case 1. $r < i-1$. This action violates the limited liability constraint in (PA') because, since $S'_{r+1} - W(x'_{r+1}, \theta_{r+1}^{\min}) = S'_r - W(x'_r, \theta_{r+1}^{\min})$ by (d) in Proposition 1,
 $S'_{r+1} - W(x'_{r+1}, \theta_j) > S'_r - W(x'_r, \theta_j)$ for $\theta_j \in \mathbb{H}^i$ using Property (3).

Case 2. $r = i-1$. This action violates the assumption in footnote number 9 because it can be shown that the principal's utility from allocation (x'_j, S'_j) is strictly increasing in j . (The logic required to prove this result is similar to that outlined in the proof of Theorem 1.)

Case 3. $r > i$. This action violates the limited liability constraint in (PA') because:

(a) $S'_{i+1} - W(x'_{i+1}, \theta_{i+1}^{\min}) = S'_i - W(x'_i, \theta_{i+1}^{\min})$ by (d) in Proposition 1.

(b) Hence, $S'_{i+1} - W(x'_{i+1}, \theta_j) < S'_i - W(x'_i, \theta_j)$ for $\theta_j \in (H)^i$ using Property (3),

(c) Also, the same logic explains why $S'_r - W(x'_r, \theta_j) \leq S'_{i+1} - W(x'_{i+1}, \theta_j)$
 $< S'_i - W(x'_i, \theta_j)$ for $\theta_j \in (H)^i$.

Hence, the agent is not maximizing his utility if he does not choose (x'_i, S'_i) when any $\theta_j \in (H)^i$ occurs.

This Remark demonstrates that after production has occurred, the information asymmetry between principal and agent is, at least in part, resolved. The principal can infer from the fact that the agent chooses allocation (x'_i, S'_i) that some $\theta_j \in (H)^i$ has occurred. And in cases where $(H)^i$ consists of a single element, the principal knows exactly which value of θ was realized, but only after the interaction between the two parties has been completed.¹¹

A representative solution to (PA) is illustrated in Figure 1 for the case in which $n (= k+1) = 4$. The principal's indifference mapping between x and S (which is independent of θ) is represented by the series of 45° lines, with higher levels of utility to the southeast. The agent's utility increases with movements to the northwest. Three sets of indifference curves for the agent are shown, one for each of the three highest states of nature. The steepest curves, labelled $\bar{U}^A(\cdot | \theta_2)$, are associated with the lowest state of nature for which the agent is compensated for his effort (θ_2). Property (1) ensures that the $\bar{U}^A(\cdot | \theta_1)$ indifference curve through the origin lies everywhere above the $\bar{U}^A(\cdot | \theta_2)$ locus through the origin, so the agent will choose allocation $(x_1, S_1) = (0, 0)$ if θ_1 occurs. The reasons why the agent may be forced to choose autarky in the lowest states of nature are discussed in Section IV.

Before proceeding to Section III, wherein the concept of efficient allocations is formally introduced, the reader should be convinced that the allocations

$(x'_i, S'_i) = (x_i, S_i)$ $i = 2, 3, 4$ in Figure 1 are situated such that properties (a) through (d) of Proposition 1 are satisfied. In particular, it should be noted that the allocations are situated such that the agent is indifferent between (x_i, S_i) and (x_{i-1}, S_{i-1}) in state θ_i , $i = 2, 3, 4$.

Section III. "First-Best" vs. "Optimal" Limited Liability Contracts

The vertical lines labelled e_2 , e_3 , and e_4 in Figure 1 represent the loci of allocations (x_i, S_i) which are efficient when θ_2 , θ_3 and θ_4 , respectively, are realized. An allocation (x_i, S_i) is defined to be efficient if, when both the principal and agent know that θ_i has occurred, neither can be made strictly better off by moving to another feasible allocation without making the other worse off.

If the principal were to know (ex ante) which value of θ would occur, his optimal strategy would be to induce the agent to produce the value of output which is efficient in that particular state by offering him compensation in excess (by an arbitrarily small amount) of the disutility incurred through such production. Thus, in a "first-best" situation (i.e., where there is no uncertainty) an efficient value of output will always be realized. Consequently, a contract that will induce the agent to choose an allocation which is efficient for any θ_i that may be realized is termed a First-Best contract.

The definition of an efficient allocation dictates that for any θ_i such that $\frac{\partial W(x, \theta_i)}{\partial x} \Big|_{x=0} < 1$, the equation of e_i is $\frac{\partial W(x, \theta_i)}{\partial x} = 1$. This conclusion follows because the principal's and agent's marginal rates of substitution between x and S must be equal at efficient allocations which are interior (i.e., $(x_i, S_i) > (0, 0)$).¹² However, if θ_i is such that $\frac{\partial W(x, \theta_i)}{\partial x} \Big|_{x=0} \geq 1$, the principal will not be willing to compensate the agent at a rate commensurate with the disutility incurred through

production of a positive value of output. Consequently, the allocation which is efficient when such values of θ_i are realized is $(0,0)$.

It should be noted, though, that situations exist in which the principal may decide to set $(x_1, S_1) = (0,0)$ even when $\left. \frac{\partial W(x, \theta_1)}{\partial x} \right|_{x=0} < 1$ (as will be explained in the following section). Thus, in Figure 1, $(x_1, S_1) = (0,0)$ may not be the allocation which is efficient when θ_1 occurs. $(0,0)$ will only be the efficient allocation when the $\bar{U}^A(\cdot | \theta_1)$ indifference curve through the origin does not intersect the positive (x, S) -quadrant below the 45° line through the origin.

The fact that (x_1, S_1) may be efficient when θ_1 occurs does not preclude the possibility that some other allocation offered to the agent may also be efficient when another value of θ is realized. In fact, the contract $S(x) = x$ will induce the agent to produce the value of output which is efficient for whatever state of nature occurs. This result (which constitutes the basis of the analysis by Loeb and Magat [1977]) follows because when the agent is awarded the entire value of output in each state of nature, "perfect internalization" is achieved. That is to say, the agent's maximand becomes identical to that of the principal, so that the agent will always supply exactly the amount of effort which the principal would most like to have supplied in any given situation (if he knew the state of nature).

However, since this contract awards the entire surplus from production to the agent, it will not be an appealing contract form the point of view of the principal. A contract which always results in the production of an efficient value of output and is strictly preferred by the principal to the perfect internalization contract is constructed in the proof of Theorem 1.

Theorem 1. A contract which induces the agent to choose an efficient allocation whatever state of nature occurs without granting the entire surplus (> 0) to the agent in any state is always a feasible solution to (PA).

Proof of Theorem 1.

1. If $\left. \frac{\partial W(x, \theta_i)}{\partial x} \right|_{x=0} \geq 1$, the efficient outcome when $\theta = \theta_i$ is $(x_i, S_i) = (0, 0)$ which is always a feasible allocation. Thus consider only those θ_i for which $\left. \frac{\partial W(x, \theta_i)}{\partial x} \right|_{x=0} < 1 \quad i=j, \dots, n \quad 1 \leq j \leq n$.

2. Let x_i^* be defined by $\left. \frac{\partial W(x, \theta_i)}{\partial x} \right|_{x=x_i^*} = 1$.

3. Design S_j^* such that (x_j^*, S_j^*) lies on the $\bar{U}^A(\cdot | \theta_j)$ indifference curve through the origin where it crosses e_j . (Recall e_j is the line whose equation is $x = x_j^*$.) By Property (4), the slope of this indifference curve on the interval $[0, x_j^*)$ is less than unity, so that $S_j^* < x_j^*$.

4. Similarly, design $S_i^* \quad i = j+1, \dots, n$ such that (x_i^*, S_i^*) lies on the $\bar{U}^A(\cdot | \theta_i)$ indifference curve through (x_{i-1}^*, S_{i-1}^*) where it crosses e_i . Again, by Property (4), the slope of this indifference curve is less than unity on $[x_{i-1}^*, x_i^*)$, so $S_i^* < x_i^*$.

5. This contract $\{(x_i^*, S_i^*) \quad i = 1, \dots, n \text{ and } (0, 0)\}$ consists wholly of efficient allocations (and autarky). It need only be shown, then, that the contract induces the agent to choose (x_i^*, S_i^*) when θ_i occurs.

6. It is readily verified that the contract satisfies properties (a) through (d) of Proposition 1, so that, by the Remark following the Proposition, the agent will choose (x_i^*, S_i^*) when θ_i occurs.

Note also that under this contract, the principal's utility is strictly positive in every state of nature in which production is profitable because $x_i^* > S_i^* \quad i = j, \dots, n$. Hence, the agent is not awarded the entire surplus (> 0) in any state of nature under this contract.

Remark: The contract described in the proof of Theorem 1 is, among all First-Best contracts, that which minimizes the total rent awarded the agent.

Proof: Consider the following problem:

Maximize $\sum_{i=1}^n -S_i$ subject to:

$$(1) \quad S_i - W(x_i^*, \theta_i) \geq S_j - W(x_j^*, \theta_i) \quad \forall j \neq i$$

$$(2) \quad S_i - W(x_i^*, \theta_i) \geq 0 \quad i = 1, \dots, n,$$

where x_i^* is that value of output which is efficient when θ_i occurs.

If θ_m ($m \geq 1$) is the smallest value of θ for which the corresponding efficient value of output is strictly positive, the Lagrangian function associated with the problem can be written as:

$$L = \sum_{i=m}^n -S_i + \sum_{i=m}^n \lambda_i [S_i - W(x_i^*, \theta_i)] + \sum_{i=m}^n \sum_{\substack{j=m \\ j \neq i}}^n \lambda_{ij} \{S_i - W(x_i^*, \theta_i) - [S_j - W(x_j^*, \theta_i)]\}.$$

To prove the Remark, it is sufficient to show that

$$\begin{aligned} \lambda_{ij} &> 0 \quad \text{for } i = j + 1 \quad j = m, \dots, n-1 \\ &= 0 \quad \text{otherwise.} \end{aligned}$$

This fact can be proved through a direct extension of the procedures outlined in the proof of Proposition 1.

Q.E.D.

Theorem 1 indicates that even though the principal cannot observe the realization of θ before designing the contract, he can always induce the agent to choose that allocation which is efficient for the value of θ which is actually realized. This is not as comforting a finding as it might first appear, though, because Theorem 2 reveals that (except in certain trivial cases) even the First-Best contract which awards the least amount of rent to the agent (i.e.,

the contract constructed in the proof of Theorem 1) is not the optimal one for the principal, and therefore, will not be offered to the agent. Furthermore, Theorem 3 indicates that the solution to (PA) will include only a single interior allocation which is efficient, (x_n, S_n) , and the value of output (> 0) produced in any other state will fall short of the efficient level. Note that this pattern characterizes the contract illustrated in Figure 1 because x_2 and x_3 lie to the left of e_2 and e_3 , respectively, while x_4 lies on e_4 .

Before proceeding to these theorems, though, it is necessary to prove the following lemma.

Lemma 1. In an n -state world ($n \geq 2$) the solution to (PA) has $x_{n-1} < x_n$.

Remark: Recall that x_i is the value of output produced by the agent when $\theta = \theta_i$.

Proof of Lemma 1.

Case I. $x_{n-1} = 0$. From the proof of Proposition 1, $x_1 = \dots = x_{n-1} = 0$ and (x_n, S_n) can be shown to lie on the $\bar{U}^A(\cdot | \theta_n)$ indifference curve through $(0,0)$ where it crosses $e_n > 0$.

Let x_j^* be defined by $\frac{\partial W(x, \theta_j)}{\partial x} \Big|_{x=x_j^*} = 1 \quad j = 1, \dots, n$.

Case IIa. $x_{n-1} > 0$ and $x_{n-1} < x_n^*$.

1. Suppose $x_n = x_{n-1}$. Then $\frac{\partial W(x, \theta_n)}{\partial x} \Big|_{x=x_n} < \frac{\partial W(x, \theta_n)}{\partial x} \Big|_{x=x_n^*} = 1$ by Property (4).

2. Therefore, through a marginal increase in x_n coupled with a (smaller) increase in S_n such that $S_n - W(x_n, \theta_n) = S_{n-1} - W(x_{n-1}, \theta_n)$, the principal can

increase his utility in the event that θ_n occurs without lowering his utility in any other state or violating constraints (1) or (2) in (PA).

3. Hence, $x_n = x_{n-1}$ cannot constitute a solution to (PA).

Case IIb. $x_{n-1} > 0$ and $x_{n-1} > x_n^*$.

1. Suppose $x_n = x_{n-1}$. Then by Properties (3) and (4),

$$\frac{\partial W(x, \theta_i)}{\partial x} \Big|_{x=x_{n-1}} > \frac{\partial W(x, \theta_n)}{\partial x} \Big|_{x=x_n^*} = 1 \quad \forall \theta_i \in \textcircled{H}^{n-1} \cup \theta_n.$$

2. Consider the effect of a marginal decrease in x_{n-1} coupled with a (larger) decrease $S_i - W(x_i, \theta_{n-1}^{\min}) = S_j - W(x_j, \theta_{n-1}^{\min}) \quad \forall x_i$ such that $\theta_i \in \textcircled{H}^{n-1} \cup \theta_n$, where (x_j, S_j) is that allocation which, among all those offered by the principal has x_j closest in magnitude to (but is less than) x_{n-1} .

3. If the principal alters the original (x_{n-1}, S_{n-1}) allocation in this manner, his utility in the event that $\theta \in \textcircled{H}^{n-1} \cup \theta_n$ occurs will be increased without violating (1) or (2) in (PA) or decreasing his utility in any other state.

Case IIc. $x_{n-1} > 0$ and $x_{n-1} = x_n^*$.

The analysis proceeds exactly as in Case IIb except that x_n remains equal to x_n^* when x_{n-1} is reduced marginally below x_n^* . Q.E.D.

Lemma 1 proves that the value of output produced when the highest state of nature occurs will always exceed that produced in any other state of nature. This result is used in Theorem 2 to prove that whenever $x_{n-1} > 0$ in the solution to (PA), the set of allocations which are optimal for the principal never coincide with the set of efficient allocations. ¹³

Theorem 2. The solution to (PA) will never be that contract which generates an efficient allocation regardless of the state of nature that occurs, except in the trivial case where the contract provides for only a single positive allocation.

Proof of Theorem 2.

1. From condition (e) in Proposition 1 and from Lemma 1,

$$p_n = (1 - \sum_{r=0}^{k-1} p_r) \frac{\partial W(x'_k, \theta_n)}{\partial x}$$

so that $\frac{\partial W(x'_k, \theta_n)}{\partial x} = 1$ and (x'_k, S'_k) is efficient when θ_n occurs.

2. Also from condition (e) in Proposition 1,

$$p_{k-1} - (1 - \sum_{r=0}^{k-2} p_r) \frac{\partial W(x'_{k-1}, \theta_{k-1}^{\min})}{\partial x} + (1 - \sum_{r=0}^{k-1} p_r) \frac{\partial W(x'_{k-1}, \theta_n)}{\partial x} = 0.$$

Hence

$$\begin{aligned} (1 - \sum_{r=0}^{k-2} p_r) \frac{\partial W(x'_{k-1}, \theta_{k-1}^{\min})}{\partial x} &= p_{k-1} + (1 - \sum_{r=0}^{k-1} p_r) \frac{\partial W(x'_{k-1}, \theta_n)}{\partial x} \\ &< p_{k-1} + (1 - \sum_{r=0}^{k-1} p_r) = 1 - \sum_{r=0}^{k-2} p_r \end{aligned}$$

since $\frac{\partial W(x'_{k-1}, \theta_n)}{\partial x} < \frac{\partial W(x'_k, \theta_n)}{\partial x} = 1$ by Property (4).

3. Therefore $\frac{\partial W(x'_{k-1}, \theta_j)}{\partial x} \leq \frac{\partial W(x'_{k-1}, \theta_{k-1}^{\min})}{\partial x} < 1 \quad \forall j \in I^{k-1}$.

4. Similar arguments show that $\frac{\partial W(x'_{k-j}, \theta_{k-j}^{\min})}{\partial x} < 1$ implies that

$$\frac{\partial W(x'_{k-j-1}, \theta_h)}{\partial x} < 1 \quad \forall h \in I^{k-j-1}, \quad \forall j: 0 < j < k-1.$$

5. Therefore, $\frac{\partial W(x'_i, \theta_h)}{\partial x} < 1 \quad \forall h \in I^i \quad i = 1, \dots, k-1$.

Q.E.D.

According to Theorem 2, the agent will not be induced to supply sufficient effort to produce an efficient value of output in every state of nature. The value of those outputs which will be forthcoming from the agent under the optimal contract are characterized more fully in Theorem 3.

Theorem 3. The efficient value of output (> 0) will always exceed the value of output produced by the agent under the optimal contract, except in the highest state of nature, where the two coincide.

Proof of Theorem 3.

1. The agent will choose (x'_k, S'_k) when θ_n occurs according to Lemma 1 and the Remark following Proposition 1. This allocation is efficient, as shown in the proof of Theorem 2.
2. Also from the proof of Theorem 2, $\frac{\partial W(x'_i, \theta_r)}{\partial x} < 1 \quad \forall r \in I^i \quad i = 1, \dots, k-1$.
3. Hence, by Property (4), $x'_i < x_i^* \quad i = 1, \dots, k-1$.
4. The conclusion of the Theorem follows, then, from the Remark following the proof of Proposition 1.

Q.E.D.

As noted above, the principal would always induce the agent to choose an efficient allocation if he knew which value of θ would occur. In the absence of such information, however, and when the principal cannot hold the agent liable for any agreements made before θ is observed, Theorems 2 and 3 suggest that efficient production in any state $\theta_i (< \theta_n)$ may only be achieved at costs (incurred in other states of nature) which outweigh the benefits which accrue to the principal in state θ_i . A detailed study of the benefits and costs which determine the form of the optimal contract between principal and agent is presented in Section IV.

Section IV. An Explanation of the Findings

It is evident from property (e) in Proposition 1 that the extent to which x_i is exceeded by that value of output which is efficient when θ_i occurs (x_i^*) will, in general, depend upon the principal's assessment of the probability that different values of θ will occur. In order to characterize this dependence more precisely, Proposition 2 analyzes the effect that modifications of the principal's beliefs have on the Efficiency Ratio, $R(\theta_i)$, which is defined to be the ratio of x_i to x_i^* .

Proposition 2. Let $0 < x'_1 < \dots < x'_k$ be the k ($\leq n$) distinct, positive values of output which constitute the solution to (PA). If Properties (1) through (6) are satisfied, the following comparative static results hold:

- (i) $R(\theta_i)$ increases as p_j ($j \in I^i$) increases and p_h ($h \in I^z$ $i < z \leq k$) decreases by a corresponding magnitude.
- (ii) $R(\theta_i)$ increases as p_j ($j \in I^i$) increases and p_h ($h \in I^z$ $0 \leq z < i$) decreases by a corresponding magnitude.
- (iii) $R(\theta_i)$ increases as p_j ($j \in I^z$ $0 \leq z < i$) increases and p_h ($h \in I^y$ $i < y \leq k$) decreases by a corresponding magnitude.
- (iv) $R(\theta_i)$ is unaffected when p_j ($j \in I^i$) increases and p'_j ($\neq p_j$) decreases by a corresponding magnitude.

Proof of Proposition 2.

1. Rearranging the equation in property (e) of Proposition 1,

$$P_i \left[1 - \frac{\partial W(x'_i, \theta_{i+1}^{\min})}{\partial x} \right] = (1 - \sum_{r=0}^{i-1} P_r) \left[\frac{\partial W(x'_i, \theta_i^{\min})}{\partial x} - \frac{\partial W(x'_i, \theta_{i+1}^{\min})}{\partial x} \right]$$

2. This equation can be used to prove that $B(x'_i) = \frac{P_i}{i-1}$ where $B(x'_i) = \frac{1}{1 - \sum_{r=0}^P P_r}$

is defined to be the ratio of

$$\left\{ \frac{\partial W(x'_i, \theta_i^{\min})}{\partial x} - \frac{\partial W(x'_i, \theta_{i+1}^{\min})}{\partial x} \right\} \text{ to } \left\{ 1 - \frac{\partial W(x'_i, \theta_{i+1}^{\min})}{\partial x} \right\} .$$

3. The derivative of $B(\cdot)$ with respect to x'_i can be shown to be strictly positive using Properties (3), (4) and (6) and the results reported in Theorem 3.

$$4. \quad \frac{dB(x'_i)}{dp_j} = \frac{1}{i-1} \quad j \in I^i, \quad \frac{dB(x'_i)}{dp_h} = 0 \quad h \in I^z \quad \text{where } i < z \leq k,$$

$$1 - \sum_{r=0}^P P_r$$

$$\frac{dB(x'_i)}{dp_h} = \frac{P_i}{(1 - \sum_{r=0}^P P_r)^2} = B(x'_i) [dB(x'_i)/dp_j] \quad \text{for } h \in I^z \quad 0 \leq z < i, \text{ and } j \in I^i .$$

5. Since $B(x'_i) \leq 1$, and because $B'(x'_i) > 0$ implies that any increase in $B(\cdot)$ reflects an increase in x'_i for constant values of θ_i^{\min} and θ_{i+1}^{\min} , the derivatives in step number 4 can be combined to prove (i) through (iv).

Q.E.D.

The findings in Proposition 2, which help to explain those in Theorems 2 and 3, are most easily interpreted through a careful analysis of the expected benefits and costs to the principal associated with changes in the position of any allocation (x_i, S_i) . To begin the analysis, consider the situation in which (x_{i-1}, S_{i-1}) has been determined and the principal must now decide upon a value for (x_i, S_i) . If only the benefits associated with the position of any allocation are considered, the principal will design (x_i, S_i) so that when θ_i does occur, the quantity $\{x_i - S_i\}$ is as large as possible, subject to the restriction that S_i is sufficiently large to induce the agent to produce x_i when θ_i occurs rather than x_{i-1} or any other permissible value of output. Consequently,

x_i will be increased above x_{i-1} (and S_i increased accordingly) along the $\bar{U}^A(\cdot|\theta_i)$ indifference curve which passes through (x_{i-1}, S_{i-1}) as long as $\{x_i - S_i\}$ is increasing. Because the slope of this indifference curve is less than unity for values of x_i less than ε_i (by Property (4)), and, therefore, a unit increase in x_i can be produced by the agent in state θ_i at a personal disutility which is less than the value of output to the principal, $\{x_i - S_i\}$ increases as x_i is increased to its efficient level. Consequently, if the principal only considers the benefits associated with the position of (x_i, S_i) , he will design (x_i, S_i) to be efficient when θ_i occurs.

However, there are costs to the principal associated with increasing x_i above x_{i-1} whenever x_i is not the highest value of output which is ever requested from the agent (or, according to Lemma 1, whenever $\theta_i < \theta_n$). The greater is x_i , the larger is the surplus that the agent can gain by producing x_i (in return for S_i) in any state of nature which exceeds θ_i .¹⁴ Therefore, the greater is x_i , the higher will S_{i+j} ($j \geq 1$) have to be set in order to induce the agent to produce x_{i+j} when θ_{i+j} occurs rather than choose the allocation (x_i, S_i) .

Hence, although increases in x_i along the agent's $\bar{U}^A(\cdot|\theta_i)$ indifference curve above x_{i-1} (but below ε_i) increase the principal's utility in the event that θ_i occurs, such increases decrease the utility that the principal can receive when higher values of θ are realized. Consequently, the greater is p_i relative to the sum of all p_j ($j \in I^Z$, $z > i$), the greater are the expected net benefits of increasing x_i above x_{i-1} , and therefore, the closer to x_i^* will x_i be situated (i.e., the greater will be the magnitude of $R(\theta_i)$ as stated in (i) through (iii) of Proposition (2)), ceteris paribus.¹⁵

This analysis is helpful in interpreting the findings in the preceding sections. To begin with, the optimal limited liability contract will always contain an allocation which is efficient when θ_n occurs because there are no

higher states in which an increase in x_n to the level x_n^* may prove costly to the principal. Furthermore, the contract which will induce the agent to choose an efficient allocation whatever state of nature occurs (see Theorem 2) is not optimal because it only takes into account the benefits associated with increases in x_i in the event of state θ_i $i = 1, \dots, n$, and ignores the costs to the principal.

In addition, the analysis also explains why the solution to (PA) may not consist of n distinct, positive allocations even when $\left. \frac{\partial W(x, \theta_i)}{\partial x} \right|_{x=0} < 1 \quad \forall \theta_i$. There may exist situations in which for a fixed x_{i-1} , for example, p_i may be so small relative to $\sum_{j=i+1}^n p_j$ that the expected benefits from any increase in x_i above x_{i-1} may be outweighed by the expected costs of the increase. In such situations, x_i will be set at the level of x_{i-1} . Similarly, if $x_i = \dots = x_{m-1} = 0$ ($m \geq 1$) and p_m is sufficiently small relative to $\sum_{j=m+1}^n p_j$, the principal may prefer that the agent choose autarky when θ_m occurs rather than receive compensation for positive production even though $\left. \frac{\partial W(x, \theta_m)}{\partial x} \right|_{x=0} < 1$. Thus, all $(0,0)$ allocations are not necessarily efficient.

Section V. Causes of the Market Failure

The fact that the final allocation chosen by the agent (i.e., the outcome) will, in general, not be efficient under the optimal limited liability contract can be regarded as a market failure. The issue addressed in this section is to what extent the market failure is an unalterable consequence of the absence of institutions which bind the agent to certain arrangements to which he would willingly consent (ex ante). It is demonstrated that sufficient competition among identical agents can, but will not always, ensure that the optimal strategy for the principal is to induce an efficient outcome in every state of nature, even in the absence of these institutions.

The analysis in Section I through IV assumes that when the contract is signed, neither the principal nor the agent know which state of nature will occur. If, to the contrary, the particular state of nature is known to a large number of identical potential agents, it will be both feasible and optimal for the principal to induce an efficient outcome for whatever state of nature has been realized. To accomplish this end, the principal must simply ask the agents to declare how much compensation they would require in order to produce each of a wide range of output levels. The type of bidding process described by Demsetz [1968] will effectively reveal the particular value of θ which has occurred without violating the appropriate limited liability constraint so that the principal can contract with the lowest bidder to produce the efficient value of output, and retain all but an arbitrarily small amount of the surplus for himself.

The same outcome could also be achieved through a procedure which is tantamount to the perfect internalization scheme discussed in Section III. The principal could simply announce the (dollar) value that he places on each possible level of output, and ask each agent to propose a single output-compensation pair. Again, sufficient competition will ensure the winning bid (which provides the highest level of utility to the principal) to be that efficient allocation which leaves the principal with (nearly) all of the surplus from production.

There are two distinct features of this scenario which were not specifically analyzed in Sections I through IV, but whose presence can correct the market failure discovered in Section III. These features are: (1) the existence of a competitive supply of identical agents, and (2) the fact that each of these agents knows the realization of θ before the contract is agreed upon. Absent either of these features, however, the market failure is likely to persist.

If, for example, the state of nature can only be observed after specialized plant and equipment has been installed and the process of production has begun, it may be necessary (and socially optimal, in order to avoid excessive duplication

of facilities) for the principal to contract with only a single agent. And although the principal may benefit from initial bidding among agents for the right to produce, the final limited liability contract will not, in general, induce the agent to produce an efficient value of output after the state of nature is realized (as indicated in Theorem 3).

The market failure may also persist if the abilities of the agents differ markedly, even though each agent (j) can observe his particular productivity level (θ_i^j) before the contract is signed. (θ_i^j can be interpreted as the productivity of agent j in state i .) Such is the case if, for example, the productivity levels of all but one agent are identical in any state, and the productivity of the remaining agent is substantially greater in all states than that of his potential competitors. Here, although bidding may effectively reveal the productivity level of the less productive agents, it cannot force the lone "productive" agent to reveal his actual productivity level.

A situation of this nature may arise if significant economies of scope exist between the activity in question and another activity in which the productive agent enjoys a monopoly position. And in such a situation market failure is still likely (since it will be the principal's interests to contract with the productive agent), even though the existence of a competitive fringe of agents may impose limits on the bargaining strength of the productive agent.

Consequently, although the fact that the principal may be restricted to writing limited liability contracts does not preclude the possibility that an efficient outcome will be realized in the particular state of nature which occurs, even significant competition among agents who face no uncertainty about the state of nature cannot always prevent the market failure.

Section VI. Conclusions

It has been shown that even when the principal is only permitted to write limited liability contracts, he can always induce the agent to produce the efficient value of output, and at the same time retain some surplus for himself in all states of nature in which production is profitable. However, the costs to the principal of doing so have been shown to be, in general, prohibitively high. Consequently when the principal does not know in advance which value of θ will be realized, he will abandon any hopes of achieving the efficient outcome in every state of nature through the design of a limited liability contract, except in the special case where many identical agents (each of whom knows which state of nature will occur) compete to supply effort to him. Aside from this special case, though, the principal will attempt to ensure that the value of output actually produced is close to its efficient level only in those states which are deemed most likely to occur (as described in Proposition 2), and in the most productive state of nature.

One aspect of these findings which should be emphasized in closing is that the prior beliefs of the principal play a major role in determining how closely the value of output which is actually produced will approximate its efficient level (and, therefore, the level of utility actually achieved by the principal). Consequently, the process by which the principal's beliefs are formulated presents itself as an important topic for future research. Some preliminary work along these lines suggests that, without substantial revision in the form of the contract between principal and agent, it will always be in the interest of the latter to alter the beliefs of the former, regardless of the accuracy of these beliefs. Consequently, even when the agent possesses superior information about the distribution of θ , he cannot be expected to (voluntarily) serve as a reliable information source for the principal (see Sappington [1980]).

Footnotes

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1. Inasmuch, this analysis is in the spirit of the analyses of Hurwicz and Shapiro [1978], Hurwicz [1979], and in some respects, Harris and Townsend [1978, 1979].
2. The analysis is limited in Section I through IV to the interaction between the principal and a single agent. Some remarks concerning the manner in which competition among agents may alter the analysis are presented in Section V. Ross [1979] has also examined some issues associated with the existence of a competitive supply of agents.
3. This assumption follows the work of Harris and Raviv [1978, 1979] and Harris and Townsend [1978, 1979].
4. This separable form for the agent's utility function follows the analyses of Hurwicz and Shapiro [1978] and Holmstrom [1979].
5. Diamond and Maskin [1979] offer a number of reasons why economic agents may be restricted in their interactions to contracts which involve compensatory

damages. Their main reason, paternalism, can also explain why the principal may be restricted to limited liability contracts here.

6. Limited liability contracts with $L = 0$ characterize labor agreements in which the employee is permitted to terminate his tenure without penalty. When $L < 0$, the contracts are more characteristic of those which include bankruptcy clauses.

It should be emphasized here that the general form of the contract derived below is (with some modification) also characteristic of a wide range of limited liability contracts which are optimal for the principal when L differs from zero.

7. This formulation is a direct extension of the analysis introduced by Harris and Townsend [1979].

8. This is a weak relationship in that the agent may choose to produce the same value of output in different states of nature.

9. In order to induce the agent to choose (x'_i, S'_i) rather than (x'_{i-1}, S'_{i-1}) in state θ_i $i = 1, \dots, n$, the allocations can be designed such that $S'_i - W(x'_i, \theta_i) = S'_{i-1} - W(x'_{i-1}, \theta_i) + \epsilon/n$ $\epsilon > 0$. The rational principal will choose ϵ arbitrarily small, and in the limit (as ϵ approaches zero), the total cost, ϵ , to the principal of inducing the agent to select in the desired manner in every state of nature approaches zero, so that it can be assumed without loss of generality that when the agent is indifferent between two or more allocations, he will choose the one preferred by the principal.

10. For notational convenience, $\frac{\partial W(x_i, \theta)}{\partial x}$ or $\frac{\partial W(x_i, \theta)}{\partial x_i}$ will be written to denote $\left. \frac{\partial W(x, \theta)}{\partial x} \right|_{x=x_i}$ where there is no possibility of confusion.

11. Consequently, the (partial) resolution of the information asymmetry is of no value to the principal in this static model. In a dynamic setting (which is beyond the scope of this research), however, the principal could at least use the additional information to update his prior beliefs about the distribution of θ . It is also conceivable though, that the agent might act so as to prevent such inferences by the principal in an on-going relationship.

12. Properties (2) and (4) guarantee that this equation for e_i is satisfied at a unique value of x_i , regardless of the value of S_i . Hence, for interior allocations, the equation for e_i is $x_i = x_i^*$ where x_i^* is some positive constant. x_i^* will be referred to as the efficient value of output (when θ_i is known to have occurred).

13. Observations of the type formally proved in Theorems 1 and 2 were first suggested by Harris and Townsend [1979] in the context of a two-state world where $W(x, \theta)$ is assumed to be a quadratic function of x . It has also been called to my attention that a result similar in nature to Lemma 1 has been demonstrated by Stiglitz [1977] for the case in which θ has a continuous distribution. Stiglitz has also characterized the form of the optimal contract in a two-state world.

14. Note that this surplus increases with θ , since higher values of θ correspond to states in which the agent is more productive.

15. Note that a strict increase in p_i relative to $\sum_{j=i+1}^n p_j$, ceteris paribus, need not imply a strict increase in $R(\theta_i)$, as stated in part (iv) of Proposition 2.

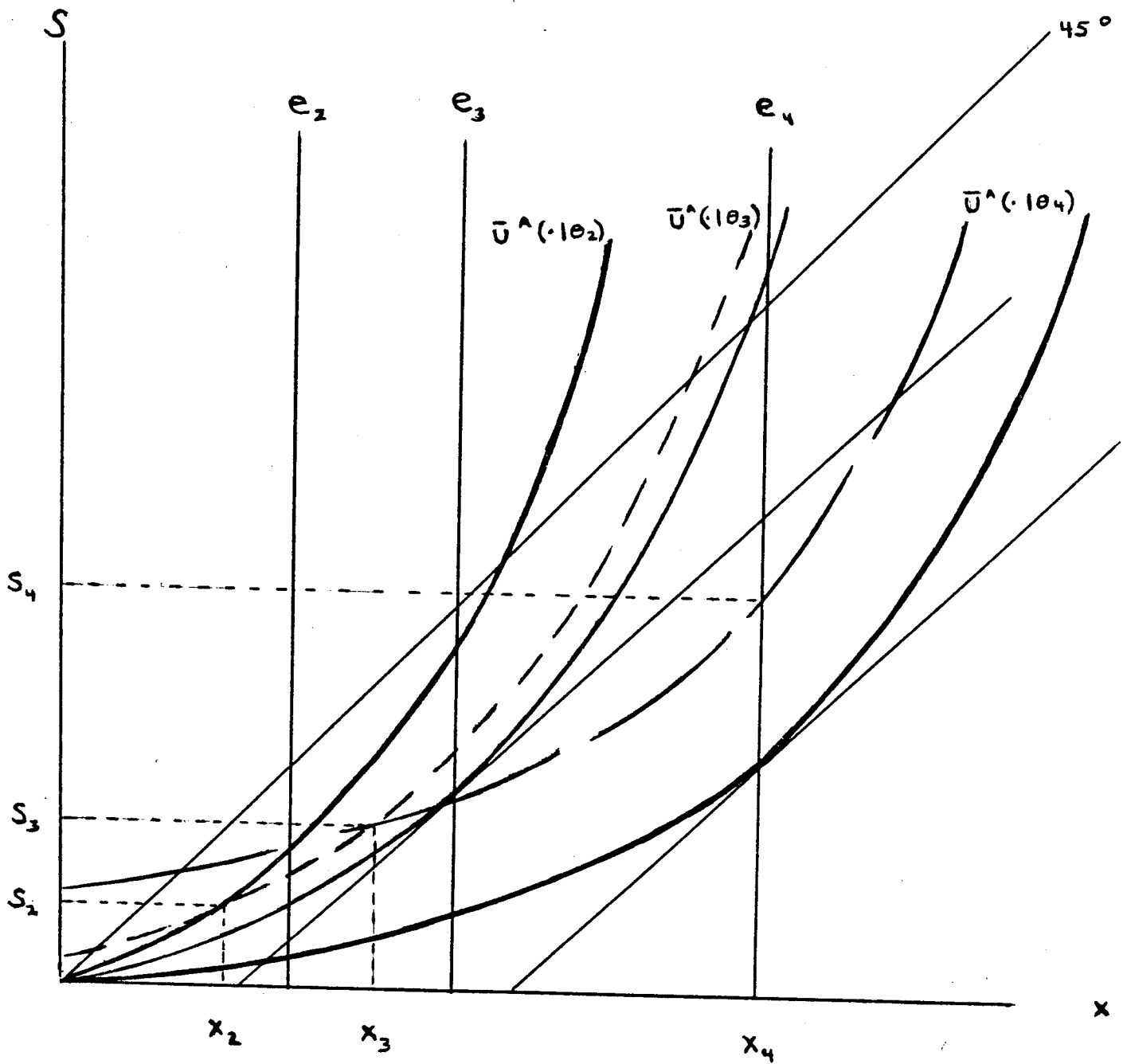


FIGURE 1

Appendix

A brief outline of the proof of the equivalence of (PA) and (PA') is presented here.

- I. Prove that any solution to (PA) is a solution to (PA').
 - A. Let $\bar{S}(x)$, $\bar{a} \langle \bar{S}, \theta \rangle$, and $\bar{x} = \bar{X}(\bar{a}, \theta)$ solve (PA).
 - B. Show that $\bar{S}(x)$ and \bar{x} satisfy the constraints in (PA').
 - C. Prove, by contradiction, that $\bar{S}(x)$ and \bar{x} maximize the objective function in (PA'), utilizing the fact that they maximize the objective function in (PA).

- II. Prove that any solution to (PA') is a solution to (PA). The proof is analogous to that outlined in I.

- III. Conclusion: Since any solution to (PA) is a solution to (PA'), and any solution to (PA') is a solution to (PA), the two problems are equivalent.

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