THE DEMAND AND SUPPLY FOR INVESTMENT GOODS: DOES THE MARKET CLEAR?*

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Abstract

Standard theoretical considerations suggest that the quantity and price of a good are jointly determined by supply and demand. In the literature on physical investment, however, attention has been focused almost exclusively on the demand side. This paper considers the theoretical and statistical problems that arise when the demand and supply sides of the market for investment goods are estimated simultaneously. One of the important problems is dealing with the possibility that the price may not adjust instantaneously to clear the market.

The model is estimated using data from post-war Japan. The two most important results that emerge are: (1) the long run supply curve of investment goods is virtually horizontal; and (2) the market appears to be characterized by equilibrium.

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I. Introduction

The level of real investment is a major concern in most countries, and economists have devoted substantial effort to studying its determinants. Much of the research has centered on specifying and estimating the parameters of "the" investment function. More specifically, the literature has focused on criticisms and extensions of the models developed by Jorgenson and his colleagues. $\frac{1}{}$ In such models, investment demand is generally a function of some combination of output, price, and financial variables.

Practically all the econometric literature, then, has been on the demand for investment. 2/ However, standard theoretical considerations suggest that the quantity and price of a good are jointly determined by demand and supply. The implicit theoretical assumption in studies that focus only on the demand side is that the supply curve is perfectly elastic at the going price. To the extent this is not the case, estimates from such studies will be biased. A major purpose of this study is to formulate a simple model for the supply of investment goods, and to estimate it jointly with the demand schedule. The data used are from Japan, whose large rate of investment has been the subject of much discussion.

Although investment output is determined by the interplay of supply and demand forces, it does not necessarily follow that this market is one in which price instantaneously adjusts each period to equate supply and demand. Our model explicitly allows for the possibility of disequilibrium in the market for investment goods. We can therefore explore the possibility that delays in response to investment incentives may be due in part

 $[\]frac{1}{\text{See}}$ Jorgenson [1967], [1971], and Hall and Jorgenson [1967]. Much of the literature is surveyed in Bischoff [1971] and Hall [1977].

 $[\]frac{2}{1}$ There are exceptions, such as Engle and Foley [1975], which will be discussed below.

to price rigidities, and not solely to adjustment cost and expectational considerations internal to firms. $\frac{3}{}$

In Section II we review briefly earlier work on estimating investment functions. Attention is focused on assumptions regarding the supply side of the model. A disequilibrium market model of price and output determination in the market for investment goods is developed in Section III. In Section IV the data are discussed and then econometric and computational problems explained. Section V contains the results and an analysis of their implications. We find that the equilibrium hypothesis appears to be the appropriate one for characterizing this market. A concluding section provides a summary and suggestions for future research.

II. Previous Studies

Most investigation of the impact of taxes on investment behavior has built upon the neo-classical model developed by Jorgenson and his collaborators. Jorgenson's formulation of the problem is now well known, so we summarize it only briefly. If: (i) product and factor markets are perfectly competitive; $\frac{4}{}$ (ii) the production function is Cobb-Douglas with constant return to scale; and (iii) capital is completely malleable, then the optimal capital stock at time t, K_t^* is

$$K_{t}^{*} = \alpha \left(\frac{p_{t}}{c_{t}}\right) Q_{t}$$
 (2.1)

where Q_t is output, p_t is the product price, c_t is the user cost of capital, and α is the elasticity of output with respect to capital. The user cost,

 $[\]frac{3}{\text{Lags}}$ due to adjustment costs have been studied by Craine [1975], Eisner and Nadiri [1963] and Mussa [1977], among others.

 $[\]frac{4}{\text{Hall}}$ [1977] has argued that a less stringent assumption is required, namely, that firms are cost minimizing.

 \mathbf{c}_{t} , is the price of a unit flow of capital input:

$$c_{t} = \frac{q_{t}(r_{t} + \delta)(1 - A_{t})}{1 - u_{t}}, \qquad (2.2)$$

where q_t is the purchase price of a unit of a capital good, r_t is the net real rate of return, δ is the proportional rate of actual depreciation, u_t is the tax rate of company profits, and A_t is the discounted value of the tax savings due to the investment "subsidies" which follow one dollar of investment. Assuming perfect foresight, the current values of these variables define c_t .

Changes in K_t^* are not reflected immediately in changes in net investment. Rather, due to the presence of ordering and delivery lags, net investment is a distributed lag on past changes in desired capital stocks:

$$K_{t} - K_{t-1} = \sum_{j=0}^{\infty} \omega_{j} (K_{t-j}^{*} - K_{t-j-1}^{*})$$
 (2.3)

Assuming that replacement investment is proportional to the existing capital stock and rearranging (2.3) yields

$$I_{t} = \sum_{j=0}^{\infty} \gamma_{j} K_{t-j}^{*} + \delta K_{t-1}$$
 (2.4)

where I_t is gross investment. Upon selection of a generating function for the γ_j 's and an appropriate stochastic specification, the parameters of (2.4) can be estimated. 6/

Models based upon equations like (2.1), (2.2) and (2.4) have been criticised for a number of reasons which will be discussed in Section III

 $[\]frac{5}{\text{These}}$ include investment tax credits, depreciation allowances, etc.

^{6/}We follow investigators such as Feldstein and Flemming [1971] who have estimated investment functions in the 'level' form (2.4), although Clark [1979] has suggested that in certain circumstances it might be better to use the 'difference' form (2.3).

below. Hall has argued, however, that even if the crucial malleability assumption is inappropriate, the neoclassical model is a reasonable starting point for a theory of investment [1977, p. 10]. An additional attractive feature of the model, of course, is its relative tractability for econometric purposes. For these reasons, we adopt the neoclassical model for investment demand with only minor modification.

Although basic microeconomic considerations suggest that the impact of changes in tax policy parameters will be determined by the demand and supply curves for investment goods, most of the econometric literature is concerned only with the magnitude and timing of shifts in the demand curve. Study of the supply side appears rather thin. 7/ For example, in the FRB-MIT-PENN model, although there is a sophisticated investment demand function similar in structure to (2.4), the "supply" side consists only of an equation that determines the asset price of investment goods as an exogenously given proportion of the general price level [Hickman, 1972, p. 580]. A very similar equation appears in the OBE model [Hickman, 1972, p. 109]. The condensed version of the Wharton model described by Hickman contains a rudimentary supply equation. The implicit price deflator for nonresidential fixed business investment depends on the level of investment, and the implicit price deflator for GNP [Hickman, 1972, p. 633].

There has been some discussion of econometric problems that may arise when the price of capital services is endogenous (Maddala and Kadane [1966] and Berndt [1976]). The chief goal of these papers is to assess the bias that such endogeneity may induce in estimates of the elasticity of substitution between capital and labor. Berndt estimates a simple version of

Note that since capital goods do not encompass all investment in the economy, the "supply of investment goods" is not synonomous with "saving." Considerable attention has recently been focused on the determinants of the saving rate. See, for example, Boskin [1978].

Equation (2.4) using two stage least squares, but there is no structural model of the supply side. On the other hand, Engle and Foley [1975] explicitly consider the supply side of investment goods, but assume that the demand function is completely elastic at the asset market clearing price (p. 627). Abel's [1979] model focuses mostly on the supply side, although demand considerations are brought to bear upon the problem of valuing installed capital. The model developed in the next section allows both supply and demand responses to be estimated in the context of a structural model, and also permits a test of whether or not the equilibrium hypothesis is appropriate.

III. The Model

In this section we develop a model of the supply and demand for investment goods which is both simple and based as far as possible upon choice theoretic considerations. The model consists of four equations: one each for the marginal productivity of investment goods, the supply of investment goods, the observed quantity of investment goods, and the adjustment in their price.

Marginal Productivity of Investment Goods. We follow that basic neoclassical theory of the demand for capital goods by specifying the following equation for the gross quantity of investment goods demanded,

$$I_{t}^{D} = \gamma_{c} + \sum_{j=0}^{\infty} \gamma_{j} K_{t-j}^{*} + \delta K_{t-1}$$
 (3.1)

where I_t^D is gross demand for investment goods, K_t^* is desired capital stock, K_{t-1} is last period's capital stock, δ is the rate of proportional replacement

 $[\]frac{8}{\text{More specifically, investment supply is a linear function of potential GNP, the difference between potential and actual GNP, and a measure of the price of capital (p. 637).$

investment, the γ_j 's determine the shape of the distributed lag, and γ_c is a constant. $K_t^\star,$ in turn, is given by

$$K_{t}^{*} = \alpha \left(\frac{p_{t}}{c_{t}}\right) Q_{t} , \qquad (3.2)$$

where the variables are defined at the beginning of Section II above.

Several points need to be made with respect to Equations (3.1) and (3.2):

- (a) The superscript D indicates that (3.1) is the quantity demanded given the level of output. Equation (3.1) is a proper structural relationship, and not a reduced form equation and hence not in the usual form for the demand equation due to the appearance of the endogenous variable Q_t on the right-hand side. Ideally, one would want to study a multimarket model in which output was treated econometrically as an endogenous variable. This task is not attempted here since the econometric problems involved in estimating multiple markets in disequilibrium involve a degree of complexity which is beyond our present scope. For tractability it will be assumed that output is exogenous, an assumption common to most earlier studies in this and related areas.
- (b) Equation (3.2) implicitly assumes that the underlying production function is Cobb-Douglas. Although Hall [1977] has marshalled an impressive collection of evidence that this is indeed a good approximation to reality, we regard it mainly as a useful assumption which reduces the number of parameters to be estimated. In future work it would be useful to experiment with a more flexible functional form.
- (c) The lag structure in Equation (3.1) is not derived on the basis of choice theoretic considerations. Unfortunately, it seems unlikely that a tractable estimating equation can be derived from a lag structure generated by optimizing behavior unless very severe restrictions are put on the

model. 9/ We follow Bischoff in assuming that the γ_j 's are generated by an Almon distributed lag. We experimented with both quadratic and cubic specifications. For the cubic case

$$\gamma_{j} = \theta_{1} + \theta_{2}(j-1) + \theta_{3}(j-1)^{2} + \theta_{4}(j-1)^{3}$$
, (3.3)

where the θ 's are parameters to be estimated. For the quadratic case, θ_4 is set equal to zero. In preliminary experiments we found that for the quadratic case the most plausible results were obtained when both the beginning and end points of the lag distribution were constrained to zero. For the cubic case, only the end point was constrained to zero. For all cases, the lag extends 15 quarters into the past, a figure similar to those used by Bischoff [1976, p. 25]. The process generating the γ 's is assumed to reflect both physical lags in processing, and lags due to expectations. Incorporating rational expectations into the model would be an interesting exercise, but it lies beyond the scope of this paper.

(d) Equation (3.1) contains a constant term, γ_c . There is some controversy with respect to whether or not the neoclassical logic allows for the presence of a constant (see Hall [1971]), but we follow Bischoff [1971] and Jorgenson [1971] in including it. Similarly, whether or not δ should be estimated or specified a priori has been debated. 10/ In this paper, δ is computed as part of the perpetual inventory algorithm used to generate the investment series (see Section IV.A below), and this value is imposed prior to estimation.

For an interesting attempt along these lines, see Craine [1975].

^{10/} See Hall [1971, pp. 61-62]. The proportional replacement hypothesis itself is controversial. A constant δ is supported by the following result of the economic theory of replacement: that under certain conditions a sequence of time-dependent replacement rates generated by retirement or loss of efficiency of a capital asset tends asymptotically to a constant regardless of the time-path of a decline in relative efficiency of the asset. See Jorgenson [1967]. Feldstein and Rothschild [1974], among others, have criticized the proportional replacement assumption, but to relax it would be beyond the scope of the current paper.

(e) The model assumes complete malleability of investment. As Hall [1977] notes, "An empirical investment function not based on . . . [this] crucial simplifying assumption appears hopelessly complex . . . " (p. 11). Hall further suggests that this assumption may be a better approximation to reality than is commonly suggested. Similarly, it would be interesting to allow for differential investment responses to different components of $c_{\rm t}$ (as done by Feldstein and Flemming [1971]), but this was not attempted in order to keep down the number of parameters.

Supply of Investment Goods. It is assumed that investment goods are supplied by profit maximizing firms. Thus, a given investment goods firm will attempt to produce up to the point where the real price it receives q_t/p_t , is equal to marginal cost. The firm's marginal cost schedule, in turn, depends upon the (exogenously given) factor prices that it faces and the state of technology. The aggregate supply curve for capital goods is the summation of the marginal cost curves, and hence can be written:

$$I_{t}^{S*}=f(\left(\frac{q_{t}}{p_{t}}\right), \underline{x}_{t}, t)$$
 (3.4)

where I_t^{S*} is the amount that firms would supply if their production levels could be varied instantaneously, \underline{x}_t is a vector of factor prices, and t is a time trend representing improvements in technology over time that are expected to lower marginal costs, ceteris paribus. $\underline{11}$ /

To make Equation (3.4) operational, it is necessary to assume a specific functional form for $f(\cdot)$, and to decide which prices are to be included in \underline{x}_t . For simplicity, a linear specification was chosen. $\underline{12}$ In a preliminary

 $[\]frac{11}{A}$ supply equation like (3.4) is probably less appropriate for structures than for plant and equipment, because structures are typically produced to order. An interesting possibility for future research would be to disaggregate these two components of investment.

^{12/}For purposes of estimating the model, a nonlinear specification based upon an explicit production function proved to be intractable.

experiment, both the real cost of capital, c_t/p_t , and the real gross wage, w_t/p_t , were entered in the supply equation. However, it was found that c_t/p_t had a statistically insignificant coefficient and its inclusion lead to nonsensical values for the other parameters. $\frac{13}{}$ We therefore excluded c_t/p_t , and estimated

$$I_{t}^{S*} = \beta_{0} + \beta_{1} \frac{q_{t}}{p_{t}} + \beta_{2}t + \beta_{3} \frac{w_{t}}{p_{t}}$$
 (3.5)

In any given period, the investment goods firms may not be able to achieve I_t^{S*} because it takes time to adjust production levels. We assume a simple partial adjustment model:

$$I_{t}^{S} - I_{t-1} = n(I_{t}^{S*} - I_{t-1})$$
, (3.6)

where η is a parameter between zero and unity, and all the other variables have been defined above. Taking (3.5) and (3.6) together, we find

$$I_{t}^{S} = \eta \beta_{0} + \eta \beta_{1} \frac{q_{t}}{p_{t}} + \eta \beta_{3} \frac{w_{t}}{p_{t}} + (1-\eta)I_{t-1}$$
 (3.7)

There are probably some aspects of supply behavior that are not modeled adequately by equation (3.7). For example, it fails to account for the possibility that if suppliers have been rationed historically, this may influence their current behavior. We settle upon equation (3.7) as a relation that contains the main ingredients of a supply curve as suggested by basic microeconomic theory.

Observed Quantity of Investment Goods. In an equilibrium model, the observed quantity of investment goods is given by the intersection of the supply and demand curves. As noted above, this has been the implicit

 $[\]frac{13/}{\text{We}}$ conjecture that this was due to collinearity between $\textbf{q}_{\textbf{t}}/\textbf{p}_{\textbf{t}}$ and $\textbf{c}_{\textbf{t}}/\textbf{p}_{\textbf{t}}$

assumption in earlier studies of investment functions. In a disequilibrium model, this is not the case. We will assume, in conformity with much of the recent work in disequilibrium theory [Korliras, 1975], that the quantity observed is the minimum of quantity demanded and quantity supplied at the given price:

$$I_{t} = \min(I_{t}^{S}, I_{t}^{D})$$
 (3.8)

Clearly, (3.8) is not the complete story. It does not, for example, explain how rationing takes place if q_t/p_t fails to equate supply and demand. Moreover, if aggregation is over submarkets, some of which are characterized by excess demand and some by excess supply, the observed quantity might be some combination of I_t^S and I_t^D . Averaging over markets may provide an observed I_t less than both I_t^S and I_t^D , since the expected value of the minimum of two normal variables is smaller than the expected value of either. Some of these qualifications might be captured by specifying (3.8) to have an error term on the right-hand side. The corresponding likelihood function is straightforward, but appears to be somewhat difficult to deal with computationally (see Goldfeld and Quandt, [1979]). For this reason, we shall employ (3.8) as a fairly reasonable approximation to how the observed I_t is determined.

<u>Price Adjustment</u>. The treatment of lags in both our supply and demand equations has been quite traditional. They are due to adjustment costs, delivery lags, and expectations, although no attempt is made to separate the various components. $\frac{15}{}$ In the disequilibrium model, however, a new

 $[\]frac{14}{\text{Muellbauer}}$ [1978] deals explicitly with estimating techniques which are appropriate when averaging over submarkets takes place.

 $[\]frac{15}{\text{As}}$ Hall [1977] notes, "Adjustment costs and delivery lags are probably best viewed as alternative explanations of the lagged response of investment to its determinants. A model containing both would be complex and redundant." (p. 34)

source of delays in the system becomes potentially important. The price may not adjust instantaneously to equate supply and demand. In other words, the traditional lags embodied in Equations (3.1) and (3.6) determine the extent to which supply and demand will shift in a given period, but there is no guarantee that even after the schedules have shifted, the observed quantity will be at their intersection. $\frac{16}{}$

Standard Walrasian analysis suggests that if the price of investment goods fails to clear the market during a given period, the forces of supply and demand will tend to move the price toward its equilibrium value. This construct has played an important role in theoretical and empirical work. Unfortunately, economic theory says little about why the price is sticky, or the determinants of the speed at which it moves toward equilibrium. Search costs, uncertainty, and the time needed to negotiate contracts are probably key parts of the answer. The presence of adjustment costs may provide a choice theoretic foundation (Barro [1972]), and we adopt the following simple version:

$$\frac{q_t}{p_t} - \frac{q_{t-1}}{p_{t-1}} = \xi(I_t^D - I_t^S)$$
 (3.9)

where $\xi > 0$.

IV. Data and Estimation Issues

A. Data

Our estimates are based on quarterly data for the aggregate Japanese

^{16/}There is a rough correspondence here to the constructs of 'internal' and 'external' cost developed by Mussa [1977]. Internal costs are generated by adjustments within the firm, and external costs by changes in the price of capital goods as society moves along a consumption investment production possibilities frontier. In our model, both internal and external costs generate lags.

economy over the time period 1952:1 to 1976:4. I_t is gross enterprise domestic investment in 1970:1 prices, where the term enterprise as opposed to private domestic investment signifies that we consider investment activities undertaken by both the private business sector and the government enterprise sector in Japan. $\frac{17}{}$ In Japan the production environment for the latter is essentially identical to that of the former. Q_t is gross enterprise domestic product in 1970:1 prices. Q_t measures quantity of output from the producer's point of view, and therefore includes subsidies but excludes all indirect business taxes other than those levied on ownership and utilization of factors of production. $\frac{18}{}$ P_t is the price index for gross enterprise domestic product measured from the point of view of the producer.

We estimate the end-of-quarter capital stock, K_{t} , in 1970:1 prices using the perpetual inventory method:

$$K_{t} = \sum_{s=1}^{t} (1-\delta)^{t-s} I_{s} + (1-\delta)^{t} K_{0}$$

where δ is the constant depreciation rate, and K $_{1955:4}$ and K $_{1970:4}$ are the benchmark capital stocks. We compute δ by noting that the perpetual inventory equation is a real polynomial in terms of $(1-\delta)$, and that all coefficients of the polynomial including the initial and the terminal capital stocks are observable. $\frac{19}{\delta}$ The Newton-Raphson straight-line iteration algorithm for nonlinear equations

^{17/}All data unless otherwise noted below are based on series obtained from various issues of Annual Report on National Income Statistics.

^{18/}Subsidies are considered as part of producer's income, while business taxes on capital assets, for example, are regarded as costs of production. See Christensen and Jorgenson [1969, 1970, 1973] for further details on this point.

^{19/}Benchmark capital stocks for 1955:4 and 1970:4 are detailed from the National Wealth Survey [1955, 1970].

is used for the calculation. $\frac{20}{}$ We assume that the flow of capital services per time period t is proportional to stock of capital at the beginning of t, and generate the service price of capital c_t as the price implicit in the observed value of capital services, i.e., the ratio of capital compensation to lagged capital stock. The value of compensation to capital is derived as the residual difference between the value of gross enterprise product and the value of compensation to labor. $\frac{21}{}$

 w_t is gross wage bill inclusive of all taxes and other deductions per person employed. Because of lack of information on hours worked by the self-employed, no attempt is made to compute wage rates on an hourly basis. This is not likely to be a serious problem, since it has been shown that hours worked in Japan during the post-war period have changed very little (see, for example, Nishimizu [1975]). Finally, asset price of investment goods q_t is the implicit deflator for gross enterprise domestic investment.

For the reader who is unfamiliar with the Japanese economic experience in the post-war period, it may be useful at this point to present some broad comparisons with the United States. The comparisons are based upon computations made by Jorgenson and Nishimizu [1978], whose treatment of aggregate data differs from ours in one important respect. They include purchases of consumer durables in investment, while we exclude consumer durables from investment and treat them as part of consumption goods output. Their figures are nevertheless quite informative, and indicate very different patterns of investment activity in the Japanese private

 $[\]frac{20}{\text{The algorithm is discussed by Nishimizu [1975]}}$ who also provides a proof of existence and uniqueness of the real zero of the polynomial in the open interval (0, 1).

^{21/}We impute the capital component of income of unincorporated enterprises by applying the value share of capital computed for corporate business sector and government enterprise sector.

domestic economy compared with that of the United States. In the early post-war years, the investment goods share in Japan was similar to that of the U.S., roughly one-third of total output. While the U.S. share remained relatively stable over time, Japan's share increased, and by the end of 1974 slightly more than one-half of Japan's aggregate output was devoted to investment goods.

Comparison of annual investment growth rates shows a much more rapid increase in investment for Japan, averaging 12.2 percent in real terms during 1952-74. The comparable figure for the U.S. was only 3.2 percent. Growth of output also differed between the two economies. The Japanese economy grew at an average rate of about 10 percent per year in real terms, while the U.S. growth rate was somewhat less than 4 percent per year.

B. Statistical and Computation Problems

In order to estimate the parameters of our system, a stochastic specification must be assumed. We add random errors ε_1 , ε_2 and ε_3 to the demand (3.1), supply (3.7) and price adjustment (3.9) equations, respectively. It is assumed that the ε 's are jointly normally distributed with zero means, variances σ_1^2 , σ_2^2 and σ_3^2 , respectively, and with zero covariances. $\frac{22}{3}$

Given that the error terms are normally and independently distributed, equations (3.1), (3.7) and (3.9) define the joint density function of the endogenous variables I_t^D , I_t^S and q_t/p_t . Denoting this density function by $g(I_t^D, I_t^S, q_t/p_t)$, it is straightforward to show (see Quandt [1978]) that

This convenient assumption reflects: (a) the computational difficulty of estimation with non-zero contemporaneous covariances (see Goldfeld and Quandt (1979)); and (b) the intractability of the likelihood function when there is both an error term in the price adjustment equation and serial correlation of errors.

the joint density of the observable random variables I_t , q_t/p_t is

$$h(I_{t}, q_{t}/p_{t}) = \int_{I_{t}}^{\infty} g(I_{t}, I_{t}^{S}, q_{t}/p_{t}) dI_{t}^{S} + \int_{I_{t}}^{\infty} g(I_{t}^{D}, I_{t}, q_{t}/p_{t}) dI_{t}^{D}.$$
 (4.1)

Note that since the model is non-linear in (q_t/p_t) , the Jacobian of the transformation required to compute (4.1) must be evaluated separately for each observation. $(c_t$ is treated as endogenous since it contains (q_t/p_t) .)

The likelihood function is obtained from (4.1) as

$$L = \prod_{t=1}^{T} h(I_t, q_t/p_t).$$

Maximum likelihood is the estimation technique. The numerical optimizations were performed using the Davidon-Fletcher-Powell, the Powell conjugate gradient and the GRADX quadratic hill-climbing algorithms (Goldfeld and Quandt [1972]).

The asymptotic standard errors of the estimates were computed by taking the square roots of the diagonal elements of the negative inverse Hessian matrix of the loglikelihood function. Elements of the Hessian were in turn calculated as numerical differences of various loglikelihood function values. Unfortunately, the likelihood function turned out to be extremely flat around its maximum value, and the second partials were therefore quite sensitive to the size of the intervals over which these differences were taken. Indeed, we were not able even to obtain a negative semi-definite variance-covariance matrix until after extensive experimentation with different interval sizes. We cannot, then, attach great confidence to the accuracy of the standard errors, although they are presented along with the parameter estimates in the next section.

V. Results

A. Disequilibrium Models

For convenience, we restate the basic model developed in Section III

above:

$$I_{t}^{D} = \gamma_{c} + \sum_{j=0}^{\infty} \gamma_{j} K_{t-j}^{*} + \delta K_{t-1} + \epsilon_{1}$$
(5.1)

$$I_{t}^{S} = \eta \beta_{0} + \eta \beta_{1} \frac{q_{t}}{p_{t}} + \eta \beta_{2} + \eta \beta_{3} \frac{w_{t}}{p_{t}} + (1 - \eta)I_{t-1} + \epsilon_{2}$$
 (5.2)

$$I_{t} = \min(I_{t}^{S}, I_{t}^{D})$$
 (5.3)

$$\frac{q_{t}}{p_{t}} - \frac{q_{t-1}}{p_{t-1}} = \xi(I_{t}^{D} - I_{t}^{S}) + \epsilon_{3}$$
 (5.4)

Table 5.1 contains the parameter estimates when equations (5.1), (5.2), and (5.4) are estimated under alternative economic and statistical assumptions. In the first model estimated, it is assumed that the γ 's of equation (5.1) are generated by a quadratic distributed lag with tail constraints. The results are shown in column 1. The numbers in parentheses are estimated standard errors, but for reasons discussed in the last section, these must be taken $\underline{\text{cum grano salis}}$.

In the demand equation, the main parameter of interest is θ_3 , which, given the constraints imposed, determines entirely the shape of the lag distribution. 23/ The shape is an inverted U and the implied mean lag is about 7.5 quarters. This value is strikingly close to values that have been computed on the basis of U.S. data. For example, many of Jorgenson's empirical distributed lags have a mean lag of about two years (Jorgenson [1971], pp. 1138-39).

On the supply side, the β 's have signs as expected: quantity supplied increases with asset price and time (improvements in technology), and

 $[\]frac{23}{\text{By}}$ construction, the long-run demand elasticity with respect to user cost is -1.0.

Table 5.1

	(1)	(2)	Parameter Estimates	\$	(į
	Disequilibrium	Disequilibrium	(S) Equilibrium	(4) Equilibrium	(5) 0LS	(9) S:10
	Degree = 2	Degree = 3	Degree = 2	Degree = 3	Degree = 2	Degree = 3
۲ م	.9200 (.1781)	.4410	.8187	.3162	.8579	.3234
$^{\theta}_2$	`	009153 (.3722D-4)		009153 (.2397D-2)		(.2224) 9664D-2 (.3893D-2)
θ,	2355D-5 (.1770D-5)	.1771D-2 (.1178D-6)	2275D-5 (.1678D-5)	.1771D-2 (.3679D-3)	1876D-5 (.1824D-5)	. 1832 (. 598BD-3)
7 _θ		7539D-4 (.1186D-6)		7534D-4 (.1456D-4)		7742 (.2372D-4)
β ₀	-118.2737 (90.2306)	-110.2776 (88.5856)	-153.8557 (113.2974)	-110.2846 (53.1178)		
β	195.2025 (94.5780)	18 6.6 274 (81.1831)	518.0225 (389.6014)	186.7882 (74.3305)		
β2	2.1241 (1.0090)	2.0392 (.8400)	5.4929 (4.0885)	2.0376 (.7943)		
β ₃	-71.8512 (38.2032)	-69.0602 (31.3882)	-194.3434 (150.5496)	-68.8475 (30.3197)		
E	.4653 (.4061)	.4545 (.3473)	.2288	.4530		
w °	.6965	.4340				
7 ^H °	.4426 (.1031)	.1877 (.0215)	.4596	.3063		
ν _α ς ς	7.4355 (11.2195)	6.6198 (10.1981)	12.0420 (17.7759)	6.4129 (6.1700)		
, a ,	.0355	.0706				
mean lag	7.5	22.2	7.5	23.0	7.5	22.9
log L	73.45	88.15	74.63	86.97		

decreases with the real wage. The estimated value of n suggests that firms try to close about two-fifths of the gap between actual and desired quantity supplied each period. In the first column of Table 5.2 we report supply elasticities evaluated at the means. 24/ In general, the figures tend to be large. The supply elasticity with respect to asset price is of particular interest. The value of 28 implies an essentially horizontal long-run supply curve for capital. As noted above, this has been an implicit economic assumption in most previous studies. As far as we know, this is the first piece of evidence that tends to justify this assumption.

Due to the supply equation's linear specification, the elasticities depend upon the point of evaluation. To assess the robustness of the results of Table 5.2 to different points, we evaluated the elasticities at those values of the RHS variables associated both with the highest and lowest deciles of I_t . In the high range, the elasticities were somewhat lower in absolute value, and conversely for the low range. But the general qualitative result of highly sensitive supply responses did not change.

Considering now ξ , the coefficient in the price adjustment equation, we note that it is positive as expected. However, the standard error suggests that ξ is not estimated with much accuracy. We discuss below the implications of this fact for the plausibility of the disequilibrium hypothesis.

It is useful to determine the sensitivity of these results to changes in the lag specification in the demand equation. When a cubic polynomial

 $[\]frac{24}{In}$ these computations, the derivative of I_{t}^{S} with respect to RHS variable i is $\eta\beta_{t}$. Although it would be possible to use just β_{t} in the computation, that would not make sense in the present context because it cannot be assumed that $I_{t}^{S} = I_{t-1}$, even in the long run.

 $[\]frac{25}{\text{These results are available upon request to the authors.}}$

Table 5.2

Supply Elasticities

Elasticity with Respect to	(1) Disequilibrium Degree = 2	(2) Disequilibrium Degree = 3	(3) Equilibrium Degree = 2	(4) Equilibrium Degree = 3
q/p	28.4	26.6	37.1	26.5
w/p	-7.3	-6.83	-9.7	-6.8
t	10.8	10.3	13.9	10.2

distributed lag is postulated, θ_2 , θ_3 and θ_4 must be determined by the data. The estimates are shown in column 2 of Table 5.1. The coefficients of the price adjustment and supply equations do not change very much; alternatively, a glance at column 2 of Table 5.2 indicates that the supply elasticities are quite similar. But there is a considerable difference on the demand side: the mean lag is now about 22 quarters. This figure is toward the high end of the distribution of mean lags for the U.S. discussed by Jorgenson [1971] in his survey article, although it is well within the range reported there. Nevertheless, the sensitivity of the mean lag to changes in the lag specification is somewhat disconcerting. It is a consequence of the fact that there is insufficient information in the sample to make very fine distinctions with respect to the shape of the lag distribution. Nevertheless, in likelihood ratio terms, the quadratic lag must be rejected.

Another way to judge whether or not the estimates make sense is to determine whether or not they imply stable behavior for the system. It appears unpromising to deal with this question analytically, because equations (5.1) through (5.4) lead to a difficult system of nonlinear difference equations in q_t/p_t and I_t . Indeed, given the presence of autonomous growth in notional supply induced by the time trend on the right-hand side, it is not even obvious precisely what stability means in the present context. If all right-hand side variables except time are held fixed and the system is allowed to run, a time-invariant price-output configuration clearly will not emerge. Rather, price will continually fall and output will continually rise.

It therefore seems reasonable to characterize our system as stable if, when it is perturbed at an initial starting point and then the stimulus is removed, it eventually returns to the path it would have taken in the absence of the perturbation. Simulations cannot settle the stability issue

definitively, because the answer may depend upon the starting point and the magnitude of the perturbation. Nevertheless, it is illuminating to experiment with a few cases. The following procedure is used: For the model of column 1 of Table 5.1, we find the series of predicted values $\frac{26}{}$ of q_t/p_t and I_t when all the 'exogenous' variables except time are held constant at their values for 1976:4, and the system is then allowed to run for 100 periods. The exercise is then repeated several times, with various perturbations added to the demand and/or supply equation in the initial simulation period only. We investigate four cases:

- (i) demand decreased by 10%, supply decreased by 10%;
- (ii) demand decreased by 10%, supply increased by 10%;
- (iii) demand increased by 10%, supply decreased by 10%;
- (iv) demand increased by 10%, supply increased by 10%.

It turns out that in all four cases, by the time 100 quarters elapse, the effects of the perturbations virtually vanish--price and quantity return to the paths that they otherwise would have taken. Thus, our model appears to be stable in the sense described above. Table 5.3 presents some evidence on the speed of adjustment. Time period 0 is the last quarter before the simulation begins. In panel (θ) are shown the model's predictions of I_t and $(q/p)_t$ in the absence of perturbations of supply or demand. Panels (i) through (iv) correspond to the perturbations described above. It is striking how fast quantity and price move toward the paths shown in panel (θ), although the speed appears to depend upon the nature of the perturbation. In every case, however, the model is stable.

The most obvious question at this point is whether or not the disequilibrium system is "better" than its equilibrium analogue. It is not obvious,

 $[\]frac{26}{}$ The method for generating the predictions will be described below in detail.

however, on precisely what bases the models should be compared. We first present the equilibrium results, and then discuss methodological problems in making comparisons.

B. <u>Equilibrium Models</u>

The equilibrium system consists of Equations (5.1) and (5.2) together with the market clearing conditions, $I_t^S = I_t^D$. Maximum likelihood estimation of such models is straightforward. Columns 3 and 4 of Table 5.1 show the equilibrium analogues to columns 1 and 2, respectively. On the demand side, neither equilibrium specification generates a mean lag much different from its disequilibrium counterpart. There is greater variability in the supply side parameters, however. As indicated by column 3 of Table 5.2, with a quadratic distributed lag in the demand equation, the equilibrium supply elasticities are considerably larger than those in the disequilibrium version. However, the column 4 results show that for the cubic case, they are very similar in magnitude.

As suggested above, even the equilibrium version of this investment model is somewhat novel because most earlier investment functions have ignored the supply curve and estimated the demand function, (5.1), with single equation methods. In columns 5 and 6 of Table 5.1 we exhibit ordinary least squares estimates of the demand equation with quadratic and cubic polynomial distributed lags, respectively. When the OLS coefficients are compared to their maximum likelihood counterparts, no important differences appear to be present. The implied mean lags are virtually identical. Indeed, the most striking feature of Table 5.1 is the insensitivity of the demand estimates to all maintained hypotheses except those concerning the shape of the lag distribution. It is reassuring that maintaining the equilibrium hypothesis does not lead to wildly different parameter estimates,

Table 5.3

S	im	u1	at	1	OI	18
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	(d Unperturbe		t)	1)	(ii)
Time	Ī	<u>q/p</u>	<u> I</u>	q/p	I	<u>q/p</u>
0	8.6435	.8925	8.6436	.8925	8.6436	.8925
1	8.6493	.8787	7.9112	.7923	7.6869	.9465
2	8.7686	.8646	8.6419	.8675	8.8551	.8712
3	8.9267	.8547	8.8626	.8547	8.8427	.8533
4	9.0898	.8447	9.0180	.8443	9.0092	.8443
5	9.2597	.8347	9.1895	.8344	9.1790	.8343
6	9.4300	.8247	9.3611	.8244	9.3483	.8243
7	9.6015	.8147	9.5337	.8144	9.5190	.8143
8	9.7722	.8047	9.7055	.8044	9.6890	.8043
9	9.9496	.7948	9.8838	.7944	9.8658	.7943
10	10.1242	.7848	10.0590	.7844	10.0400	.7843
11	10.2979	.7748	10.2333	.7744	10.2134	.7743
12	10.4697	.7648	10.4053	.7644	10.3850	.7643
	(ii:	<u></u> i)	(iv)		
lime	(ii: _I	1) q/p	(iv)	9/p		
Cime O		_		q/p		
	<u>I</u>	<u>q/p</u>	<u>I</u>	<u>q/p</u> .8925		
0	<u>I</u> 8.6436	<u>q/p</u> .8925	<u>I</u> 8.6436	9/P .8925 .9652		
0	8.6436 9.6119	<u>ч/р</u> .8925 .8110	8.6436 9.3976 8.8956	9/P .8925 .9652 .8616		
0 1 2	8.6436 9.6119 8.6823	9/p .8925 .8110 .8579	8.6436 9.3976	9/P .8925 .9652 .8616 .8547		
0 1 2 3	8.6436 9.6119 8.6823 9.0112	9/p .8925 .8110 .8579 .8561	8.6436 9.3976 8.8956 8.9915	9/P .8925 .9652 .8616 .8547 .8451		
0 1 2 3 4	8.6436 9.6119 8.6823 9.0112 9.1710	4/p .8925 .8110 .8579 .8561 .8451	8.6436 9.3976 8.8956 8.9915 9.1624	9/P .8925 .9652 .8616 .8547 .8451		
0 1 2 3 4 5	8.6436 9.6119 8.6823 9.0112 9.1710 9.3411	9/p .8925 .8110 .8579 .8561 .8451	8.6436 9.3976 8.8956 8.9915 9.1624 9.3310	9/P .8925 .9652 .8616 .8547 .8451 .8351		
0 1 2 3 4 5	8.6436 9.6119 8.6823 9.0112 9.1710 9.3411 9.5125	9/p .8925 .8110 .8579 .8561 .8451 .8351	8.6436 9.3976 8.8956 8.9915 9.1624 9.3310 9.5002	9/P .8925 .9652 .8616 .8547 .8451 .8351 .8251		
0 1 2 3 4 5 6	8.6436 9.6119 8.6823 9.0112 9.1710 9.3411 9.5125 9.6849	9/p .8925 .8110 .8579 .8561 .8451 .8351 .8251	8.6436 9.3976 8.8956 8.9915 9.1624 9.3310 9.5002 9.6708 9.8407	9/P .8925 .9652 .8616 .8547 .8451 .8351 .8251 .8151 .8050		
1 2 3 4 5 6 7 8	8.6436 9.6119 8.6823 9.0112 9.1710 9.3411 9.5125 9.6849 9.8565	9/p .8925 .8110 .8579 .8561 .8451 .8351 .8251 .8151	8.6436 9.3976 8.8956 8.9915 9.1624 9.3310 9.5002 9.6708	9/P .8925 .9652 .8616 .8547 .8451 .8351 .8251 .8151 .8050 .7951		
0 1 2 3 4 5 6 7 8	8.6436 9.6119 8.6823 9.0112 9.1710 9.3411 9.5125 9.6849 9.8565 10.0346	9/p .8925 .8110 .8579 .8561 .8451 .8351 .8251 .8151 .8051	8.6436 9.3976 8.8956 8.9915 9.1624 9.3310 9.5002 9.6708 9.8407 10.0174	9/P .8925 .9652 .8616 .8547 .8451 .8351 .8251 .8151 .8050		

but further investigation of the shape of investment demand lags is called for.

C. Comparing the Models

With the results of Table 5.1 in hand, we now turn to the question of how to compare a disequilibrium model to its equilibrium counterpart. A seemingly natural way would be to perform a likelihood ratio test. However, because the equilibrium model is not nested in the disequilibrium model, $\frac{27}{}$ a likelihood ratio test is not strictly appropriate. Indeed, a glance at the bottom of Table 5.1 indicates that in one case, an equilibrium model has a higher loglikelihood value than its equilibrium counterpart. If the (not strictly appropriate) critical values of the χ^2 distribution were used for the likelihood ratio test, we would be unable to reject the hypothesis of equilibrium for either specification.

Another possible test of the null hypothesis that the equilibrium model is correct is based on whether $(1/\hat{\xi})/\hat{\sigma}_{(1/\hat{\xi})}$ is significantly different from zero (where $\hat{\sigma}_{(1/\hat{\xi})}$ is the estimated standard error for $1/\hat{\xi}$). (See Quandt [1978].) The test statistics are .758 and .431 for columns 1 and 2 of Table 5.1; assuming that it is appropriate to use the critical values of the normal distribution due to asymptotic normality, we cannot reject the hypothesis of equilibrium in either case. This result, though consistent with that found above, should be interpreted with caution due to the previously noted difficulty of estimating standard errors.

Yet another possibility is to compare the models on the basis of "goodness of fit." To do this, we generate price and quantity predictions for both the equilibrium and disequilibrium models over the sample period,

^{27/}It is nested only asymptotically. See Quandt [1978].

and determine which predictions track the actual price and output observations better.

With the disequilibrium models, the following procedure is used to generate predictions for each period: Substitute the demand and supply equations (5.1) and (5.2) into the price adjustment equation (5.4). A quadratic equation in q_t/p_t is thereby obtained. When the equation is solved, in every case there is one positive root and one negative root, and the latter is discarded. The positive root, our predicted value of q_t/p_t , is then substituted into the demand and supply equations, and the minimum of I_t^D and I_t^S used for the predicted value of I_t . For the equilibrium models, we set supply equal to demand, find the implied value of q_t/p_t , and substitute back to find I_t .

Table 5.4 records for each model the R²'s for regressions of predicted prices and outputs on their actual counterparts. (The columns correspond to those of Table 5.1.) Two main conclusions emerge from the table:

(i) all the models fit the data well; and (ii) it is virtually impossible to distinguish between the equilibrium and disequilibrium models on the basis of goodness of fit. In the light of this ambiguity, considerations of simplicity suggest that the equilibrium model be retained as the appropriate specification.

All the evidence we have, then, suggests rejection of the disequilibrium hypothesis. This conclusion may at first appear surprising, particularly to those who are familiar with the rapid economic growth performance of the Japanese economy. The average annual rate of growth of real aggregate output from the early fifties to 1973 was approximately 9.5 percent. Furthermore, as we noted in Section IV above, the growth of output was accompanied by a dramatic increase in the share of investment goods in total output: investment grew at a rate of about 15 percent per year throughout the postwar

 $[\]frac{28}{The}$ quantities of interest are actually $E(I_t^D|I_t)$ and $E(I_t^S|I_t)$, and the present procedure provides only a rough approximation. See Hartley [1977].

Table 5.4

	GOOGLESS C	or fit of Predic	ted Values	
	(1)	(2)	(3)	(4)
^I t	.9227	•9399	.9249	.9460
q _t /p _t	•9375	.9403	.9373	. 9389

period. One might conjecture that rapid economic growth in Japan would provide an environment conducive to disequilibrium in the investment goods market. Our equilibrium conclusion, however, suggests the contrary for the Japanese experience. It may be that in a <u>sustained</u> rapid growth environment, with few worries about major cyclical fluctuations and with "optimistic" expectations continuously realized, prices in the investment goods market will change sufficiently rapidly to keep it in equilibrium.

A somewhat more general evaluation of the plausibility of our result might also be offered. Investment goods are additions to capital stock which generate capital input services for use in production. There are two characteristics of capital which distinguish it from labor input:

(1) capital is a produced factor of production, and (2) capital is predominantly owner-utilized. It may be the case that these characteristics give rise to a regime of relatively more perfect information and careful planning than is present, for example, in the labor market. In this context, it is interesting to note that Rosen and Quandt [1978] rejected equilibrium as an appropriate characterization of the U.S. labor market.

VI. Concluding Remarks

The purpose of this paper has been to specify and estimate a model of the market for investment goods that explicitly incorporated a supply side, and that allowed for the possibility that the market might not clear instantaneously. The model was estimated using Japanese quarterly data from the post-World War II period. The most striking results were that the long-run supply curve is virtually horizontal, and the market appears to be characterized by equilibrium.

As we have stressed throughout this paper, the supply and demand functions are both quite simple. This simplicity was due in part to our wish to facilitate comparisions with earlier studies in the field. But part was also due to computational problems. The costs of estimating additional parameters are very high, because the likelihood functions associated with disequilibrium models often are not "well-behaved," and it is therefore extremely difficult to locate global optima. We hope that as computational experience increases, it will be possible to estimate more sophisticated models.

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