

CREDIT RATIONING IN MARKETS WITH
IMPERFECT INFORMATION, PART II:
CONSTRAINTS AS INCENTIVE DEVICES

Joseph E. Stiglitz*
Andrew Weiss**

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*Bell Telephone Laboratories, Inc.
and Princeton University

**Bell Telephone Laboratories, Inc.
Murray Hill, New Jersey 07974

Econometric Research Program
Princeton University
207 Dickinson Hall
Princeton, New Jersey

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1. Introduction

In an earlier study, (Stiglitz-Weiss [1979]) we established that, in the presence of imperfect information, a competitive loan market may be characterized by credit rationing. That is, among observationally identical borrowers some receive loans, others do not, and the disappointed potential borrowers are willing to pay more than the market interest rate. We presented two reasons why banks would not respond to the excess demand for loanable funds by raising the interest rate for borrowers:

- 1) At a higher interest rate the "safer" borrowers could be discouraged from borrowing, thus leading to a mix of applicants with a higher risk of defaulting on loans;
- 2) a higher interest rate would lead borrowers to invest in riskier projects.

The conventional argument for why competitive markets must be characterized by demand equalling supply is that, were there any excess demand for credit, some unsatisfied borrower would offer to borrow at a higher interest rate. In our earlier study we showed that the bank might not lend to such an individual for the bank would know that the individual would, when facing the higher interest rate, undertake a riskier project,

* This is the second of a two-part study on the theory of credit rationing. An earlier version of this paper was presented to a conference on Monopolistic Competition and Imperfect Information, Bell Labs, February, 1978. We would like to thank Bruce Greenwald, Henry Landau and Rob Porter for fruitful comments and suggestions.

or that the mix of individuals applying for the loan would change adversely. For either reason the expected return to the bank might actually be lower at the higher interest rate.¹

Credit rationing was thus an outcome of the sorting and incentive effects of interest rates. Since the interest rate affects the quality of loans, it does not necessarily also clear markets.

The model we presented there was a one-period model; we pointed out that a "more realistic" model would entail the potential borrower being in the market for a number of periods. Bad behavior (undertaking risky projects) could then presumably be punished, either by charging such individuals a higher interest rate or by withholding credit. The objective of this paper is to explore the nature of the contractual arrangements which will arise in the simplest of such models.

¹ Similar results for the incentive effects of interest rates (but not the sorting effects) were independently arrived at by Keeton (1979).

We show that equilibrium will, in fact, entail contingency contracts: the interest rates that the individual pays in future dates will depend on his performance in earlier dates. More strikingly, we show that, under not implausible circumstances, the bank will make the availability of credit at future dates conditional on the performance of the borrower. Borrowers may be denied future loans even though the expected return from the projects which would be financed by such loans exceeds the opportunity cost of capital. In short, prices and constraints (the availability of credit in the future) are both used as incentive devices.

Thus, in this model credit rationing itself is one of the instruments affecting the average quality of loans made by the bank. This is very different from the form of credit rationing discussed in our earlier paper, which was solely a consequence of the interest rates affecting the quality of loans. The intuition behind our result can be simply put: because banks charge interest rates below the market clearing rate as a means of inducing safe investments, borrowers make positive profits on their loans; the bank can exploit the desire of firms to obtain loans by conditioning those future loans on the repayment of past debts. These conditional contracts provide a non-price incentive for firms to engage in safe projects (and thus enable banks to charge higher interest rates in the early periods). The rationing of credit implicit in these contingency contracts we will call type 2 rationing, to distinguish it from the kind of credit rationing discussed earlier, which we call type 1 rationing. Both kinds of credit rationing are very

different from the kind of credit rationing which has been extensively discussed elsewhere in the literature. Those models show that if, as individuals borrow more, the riskiness of the loan increases, banks will not be willing to lend an unlimited amount at a fixed interest rate. Since, under not implausible conditions, the expected return from a sufficiently large loan can be negative, there may be no interest rate at which a bank will be willing to supply loans beyond a certain size.¹ In our analysis, applications for even small loans are turned down, although other applications for loans of precisely the same size are accepted.

Constructing a simple model in which constraints serve as an important incentive device turned out to be a more complicated task than we first envisioned. There are a number of restrictions on the operation of constraints, which, within any market equilibrium model, need to be taken into account.

In particular, in a conventional competitive equilibrium model, if one person refuses to sell to an individual, it makes no difference: someone else will. We need to show that the refusal of the first bank to make an additional loan to the individual does not simply divert the individual to an alternative source of funds. We shall argue that, in equilibrium, the terms of the original contract are such that

¹ Jaffee and Russell (1976) have made an important contribution in showing the role that adverse selection can play in determining this upper bound.

changing banks after a default is difficult or impossible. (In addition, banks often have better information about their borrowers than is available to competing banks. This private information is not formally part of our analysis but would provide an additional deterrence to banks making loans to the disappointed borrowers of their competitors.) Thus a bank's own customers and the customers of its competitors are not perfect substitutes. Ex post the market lacks the property of anonymity that we usually associate with competitive markets. On the other hand, although the market is not fully competitive in the conventional sense, we shall argue that the terms of the contract are competitively determined. There is no ex ante exploitation of the hapless borrowers, who are perfectly informed about the terms of the contract; moreover, there is active competition among lenders for these borrowers. This distinction between the nature of ex ante and ex post competition is important not only in credit markets, but in a variety of other market situations where there is an important element of imperfect information.

Another difficulty is that the borrower must believe that the constraints will in fact be operative in the way the bank announces that they will be. The terms of the contingency contract are enforced either because of reputation effects or because it is in the interest of at least one of the parties to enforce that particular contractual provision. The implications of enforcability are discussed in Section 7.

Thus we provide here the foundations for studying credit markets with imperfect information. Readers of the literature on markets with asymmetric information will not find too surprising that, once again, the market equilibrium is not optimal. However, the nature of the inefficiency is very different from those cited elsewhere. Under certain circumstances rationing would be optimal, but the form rationing takes is not optimal. Random rationing may be Pareto superior to the type of rationing dictated by competitive market forces.

It should be emphasized that the objective of this study is not to construct a complete model of the capital market.¹ Rather, our concern here is simply to focus on the manner in which a competitive market economy deals with the incentive problems associated with imperfect information concerning the actions of the borrower.

2. The Model

The basic points we wish to raise are best made through a simple, two period model. In later sections we generalize the analysis.

a) Borrowers. All borrowers are identical and live for two periods. Each borrower has only two projects available to him: a safe project which succeeds with probability p^S and gives a return if successful per dollar loaned of R_1^S if undertaken in period one, and R_2^S if undertaken in period two; and a risky project which succeeds with probability $p^R < p^S$ and gives a return (if it succeeds) per dollar loaned of R_1^R if undertaken

¹ There are a number of additional questions which a more general model of the capital market would have to deal with; in practice, many of the contractual arrangements which seem to correspond to the predictions of this model are implicit rather than explicit. We have not addressed those questions, nor have we dealt with the interaction between equity and loan markets, or the cost of monitoring the behavior of borrowers. In addition the sorting effects of interest rates, which played a key role in our earlier study, are ignored here.

in the first period and R_2^r if undertaken in the second period. Each project generates its return one period after the loan is made. We assume

$$(2.1) \quad R_i^r > R_i^s, \quad i = 1, 2$$

The return to the risky project, if successful, is greater than that to the safe project. (Otherwise, the safe project simply dominates the risky project.)

$$(2.2) \quad p^r R_i^r < p^s R_i^s, \quad i = 1, 2$$

The expected return to the safe project exceeds that for the risky project.

Moreover, we shall assume that

$$(2.3) \quad R_1^j \leq R_2^j, \quad j = r, s,$$

borrowers learn from experience.¹

For simplicity, we also assume that both borrowers and lenders are risk neutral.

b) The loan contract. The loan contract is described by a rate of interest the first period, r_1 ; if the project is successful (we assume success and failure are observable) the borrower receives credit for a second period loan at a rate r_2 with probability γ_s ; if the project is a failure the borrower receives credit for a second period project at a rate r_2^f with probability γ_f . A loan contract is thus described by

¹ This assumption is made to make it more difficult to obtain our results. Clearly if $R_1^j > R_2^j$ it would be trivial to obtain circumstances under which the availability of second period debt is limited.

$\{r_1, r_2, r_2^f, \gamma_s, \gamma_f\}$.¹ In section 3

we show that the optimal contract is always characterized by $\gamma_s = 1$;
for notational simplicity we will assume $\gamma_s = 1$ and drop the subscript on
 γ_f .²

The bank chooses the parameters of the loan contract to maximize its
return, being aware of the effect of parameters on the actions of
borrowers (this is a standard indirect control problem.)

We examine this maximization problem in two different economic
environments. In one, the demand for loan applications exceeds the
supply of loanable funds at the competitively determined interest rate
for deposits. In the second, the bank must compete for loan applications.

¹ A bank can also choose to offer loans to those borrowers who were
denied additional credit by their own bank. We will discuss the
viability of those contracts in Section 6.

² Although under some conditions it may be in the bank's interest ex post
not to make any second period loans, the contract terms ensuring $\gamma^s = 1$
are easily enforced by the borrower (and it is in his interest to enforce
those terms).

3. Excess Demand for Loanable Funds

Our basic problem here is to find that choice of the terms of the banks contract $\{r_1, r_2^f, r_2, \gamma\}$ which maximizes the expected return to the bank, where the bank recognizes that the actions that the firms take are functions of the terms of the contract. We posit that at the optimal contract, the expected return per dollar loaned, ρ^* , is sufficiently low that the demand for funds exceeds the supply. As in our earlier analysis, however, banks will not respond to this excess supply simply by increasing the rate of interest charged, because to do so would lower the expected return, by inducing riskier actions. The object of the model is to show that the optimal contract may entail $\gamma < 1$, i.e. some firms who are "unlucky" the first period are refused second period loans. We wish to show not only that there is a penalty for failure, but that the penalty may take the form of credit restrictions. (We are careful in designing the model to avoid the confusion between adverse selection effects -- which we discussed in our earlier paper, and which are undoubtedly of great importance in practice -- from the incentive effects upon which we focus here.)

The analysis of the problem is divided into four steps:

i) Optimal behavior of the borrower the second period. Since, by assumption, the borrower lives only two periods, in the second period he is only concerned with his expected returns that period: there are no future consequences of a failure to repay a loan. (All individuals, those who repay their loans and those who do not, are assumed to be equal before the Lord on the Day of Judgment.)

A borrower will invest in safe second period projects if and only if the return to the safe project equals or exceeds that to the risky project:

$$(3.1) \quad p^s(R_2^s - (1+r_2)) \geq p^r(R_2^r - (1+r_2))$$

or

$$(3.2) \quad 1+r_2 \leq \frac{p^s R_2^s - p^r R_2^r}{p^s - p^r} \equiv (1 + r_2^*).$$

This is a familiar result from our earlier paper. r_2^* is the maximum second period interest rate which will induce safe second period investments.

We let $\pi_2(r_2)$ denote the maximized value of (expected) second period profits when the interest rate charged is r_2 ; then,

$$(3.3) \quad \pi_2' \equiv \begin{cases} -p^s & \text{if } r_2 < r_2^* \\ -p^r & \text{if } r_2 > r_2^* \end{cases}$$

ii) Optimal behavior of borrower first period. Firms are assumed to maximize present discounted value (PDV) of profits Π . We let δ be the discount factor, and let Π^s be the expected PDV of profits if the firm invests in the safe project the first period, Π^r if it invests in the risky project the first period. Clearly,¹

$$(3.4) \quad \Pi^i = p^i(R_1^i - (1+r_1)) + [p^i\pi_2(r_2) + \gamma\pi_2(r_2^f)(1-p^i)]\delta \quad i=s,r$$

The project chosen the first period will depend on whether

$\Pi^s \geq \Pi^r$, i.e. on whether

$$(3.5) \quad 1+r_1 \leq \frac{(p^s R_1^s - p^r R_1^r)}{p^s - p^r} + [\pi_2(r_2) - \gamma\pi_2(r_2^f)]\delta \equiv 1 + \hat{r}_1(r_2, r_2^f, \gamma)$$

Note that the critical r_1 which induces safe behavior depends not only on the characteristics of the first period technology (p^s, R_1^s, p^r, R_1^r) but also on the differential profits associated with success the first period reaped from second period investments.

These, in turn, are a function of the probability of obtaining a loan if not successful, and the interest rate differential between those who are and are not successful. Note that if $\gamma = 1$ and $r_2 = r_2^f$, the bracketed term would be zero.

Let

$$(3.6) \quad 1+r_i^* \equiv \frac{p^s R_i^s - p^r R_i^r}{p^s - p^r}$$

¹ We assume that an individual who is denied credit by this bank cannot borrow elsewhere. Later, we show that this "assumption" can be derived from the necessary conditions for type 2 rationing.

Then

$$1 + \hat{r}_1 = 1 + r_1^* + \delta[\pi_2(r_2) - \gamma\pi_2(r_2^f)]$$

iii) The return to the bank. The bank is assumed to maximize its expected profits per borrower, v , where it takes the cost of obtaining funds from depositors, ρ^* , as given:

$$(3.7)v = p_1(1+r_1) - \rho^* + \delta[p_1[p_2(1+r_2) - \rho^*] + \gamma(1-p_1)[p_2^f(1+r_2^f) - \rho^*]]$$

where

- p_2 = probability of success second period (a function of r_2)
if the borrower is successful the first period.
- p_2^f = probability of success second period (a function of r_2^f)
if the firm is not successful the first period.
- p_1 = probability of success first period (a function of r_1 ,
 r_2 , r_2^f , γ).

The bank thus chooses the terms of the contract, taking into account explicitly the fact that the probabilities of success both periods are a function of the terms of the contract.

This formulation implicitly assumes that the bank has access to an unlimited supply of funds at the interest rate ρ^* , and the bank is constrained by the number of customers; the bank thus maximizes its profits per customer. We could, alternatively, have assumed that the bank maximizes return per (discounted) dollar loaned, i.e.

$$\max \hat{v} \equiv \frac{v}{1 + \delta(p_1 + (1 - p_1)\gamma)}$$

In the context of the market situation we are analyzing here, where there is credit rationing, this formulation might seem the more natural. Fortunately,

in the central case with which we are concerned here, where competition among the banks for depositors drives the value of ρ^* to the point where the maximized value of v is zero; maximizing v and maximizing \hat{v} are perfectly equivalent.¹ The reader with a low value of δ can redo the calculations for the case of the bank maximizing \hat{v} rather than v , confirming that all our results obtain for that case.

iv) Formal analysis of bank's maximization problem. There are a large number of possible patterns of investment by the firm induced by various values of the terms of the contract (See Figure 4). For each technique, we can analyze the maximum return to the bank. We then compare the value of profit obtained for the different regimes, to establish conditions under which (a) the bank prefers that the firm engage in safe projects both periods and (b) to do this, it rations credit to borrowers who are unsuccessful the first period.

We focus our analysis on the calculation of the maximal return obtainable if the bank wishes to ensure that only the safe project is undertaken. Thus $p_1 = p_2^f = p_2^s = p^s$. From our earlier analysis, we know that

¹ Total profits of the bank are Lv , the profits per borrower, v , times the number of borrowers, L , and it is this which the bank would like to maximize. Formally, the analysis of this section can be viewed in two different ways: it describes the behavior of a bank with a fixed set of customers (e.g. it has a local monopoly) which however has access to a competitive supply of funds (depositors); alternatively, in the general equilibrium competitive analysis, the value of ρ^* will be such as to make $v = 0$, in which case firms are indifferent about the number of customers.

$$(3.8a) \quad r_2 \leq r_2^*$$

$$(3.8b) \quad r_2^f \leq r_2^*$$

$$(3.8c) \quad r_1 \leq \hat{r}_1(r_2, r_2^f, \gamma)$$

It is immediate from (3.7) that the bank always charges $r_1 = \hat{r}_1$: It always charges the maximal rate consistent with the safe project being adopted.¹ Moreover, from (3.5), since $\frac{d\hat{r}_1}{dr_2^f} > 0$, $r_2^f = r_2^*$. The intuition behind this result is clear. The bank has no incentive to sacrifice its own profits by charging first period defaulters an interest rate below r_2^* . A higher interest rate to defaulters both increases the bank's second period profits and allows the bank to charge a higher first period interest rate with the borrower still undertaking the safe project.

¹ We need to show that it is feasible for the bank to charge $(1 + \hat{r}_1)$, i.e.

$$R_1^S - (1 + \hat{r}_1) = \frac{p^r(R_1^r - R_1^S)}{p^S - p^r} - \delta(\pi_2(r_2) - \gamma\pi_2(r_2^f)) > 0$$

When $\gamma = 0$, if we let $r_2 = r_2^*$, we require

$$R_1^r - R_1^S > \delta p^S (R_2^r - R_2^S)$$

This condition can easily be satisfied.

Next, since

$$(3.9) \quad \frac{d\hat{r}_1}{dr_2} = \delta\pi_2' = -\delta p^s,$$

$$\frac{dv}{dr_2} = \frac{\partial v}{\partial r_2} + \frac{\partial v}{\partial \hat{r}_1} \frac{d\hat{r}_1}{dr_2} = 0$$

The bank is indifferent among all values of r_2 for which $r_2 \leq r_2^*$ when r_1 varies with r_2 to keep borrowers investing in safe first period projects.

For $r_1 = \hat{r}_1$,

$$(3.10) \quad \frac{dv}{d\gamma} = \frac{\partial v}{\partial \gamma} + \frac{\partial v}{\partial \hat{r}_1} \frac{d\hat{r}_1}{d\gamma}$$

Substituting for $\frac{d\hat{r}_1}{d\gamma}$ from equation (3.5), and $r_2^f = r_2^*$,

$$\begin{aligned} \frac{dv}{d\gamma} &= (1 - p^s)[p^s(1 + r_2^*) - \rho^*] \delta - p^s \delta \pi_2(r_2^*) \\ &= \delta[p^s(1 + r_2^*) - \rho^* - p^s(p^s R_2^s - \rho^*)] \end{aligned}$$

It is immediate that

$$(3.11) \quad \gamma = 0 \quad \text{if } \rho^* > \hat{\rho} \equiv p^s \left[\frac{(1 + r_2^*) - p^s R_2^s}{1 - p^s} \right]$$

$$\gamma = 1 \quad \text{if } \rho^* < \hat{\rho}$$

and γ is indeterminate if $\rho^* = \hat{\rho}$.

Therefore, without loss of generality we can assume γ is either 0 or 1.

In Section 5 we show that this result also holds when borrowers choose from a continuum of projects in each period.

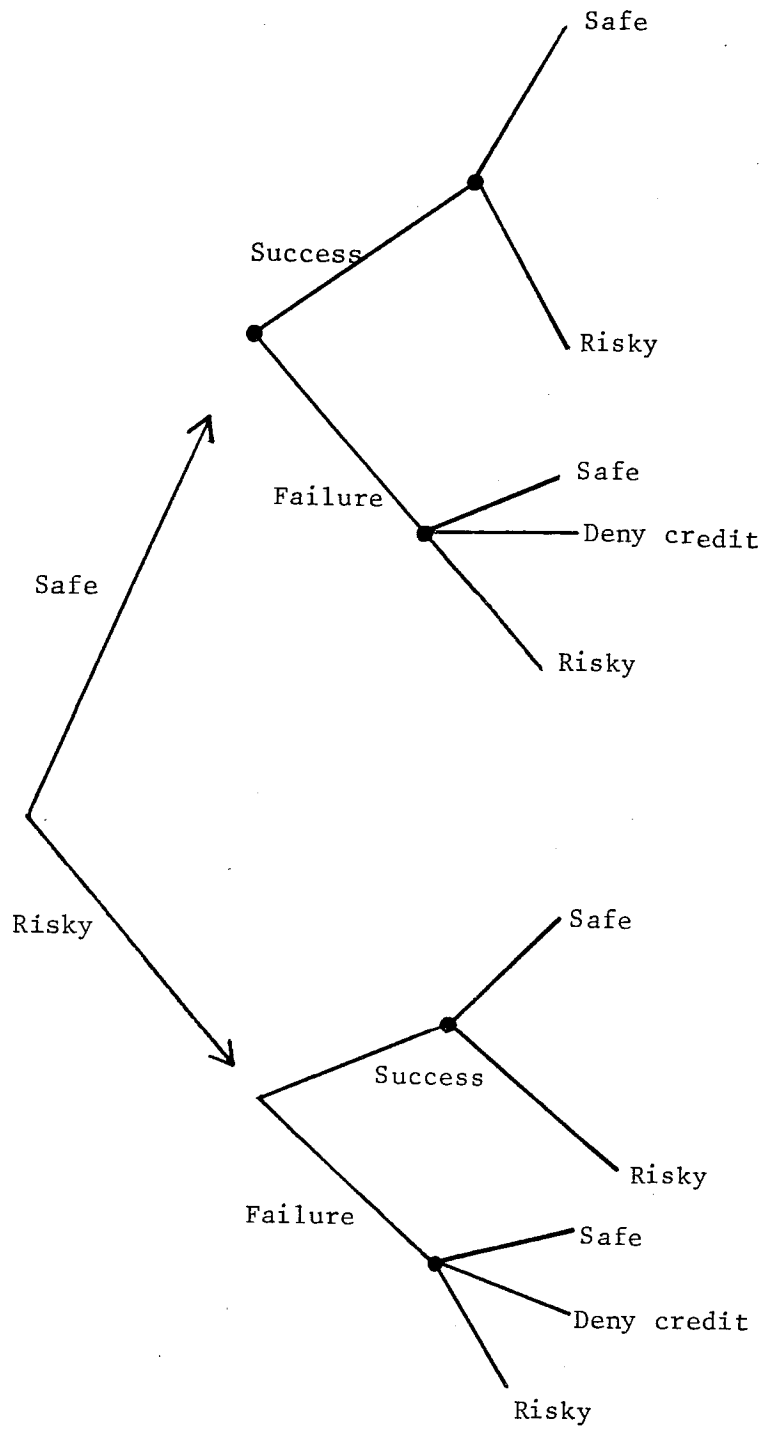


Figure 1

If the cost of funds, ρ^* , is large enough, the bank employs type 2 rationing. For any particular bank, ρ^* can be thought of as simply given exogeneously; there is no particular reason to believe that $\rho^* \geq \hat{\rho}$.

To verify that there can in fact be type 2 rationing, we need to demonstrate two things:

(a) ρ^* can exceed $\hat{\rho}$ when $v \geq 0$.

(b) v is maximized when a contract which induces a safe project to be undertaken both periods is offered.

Both conditions can be satisfied for some (but not all) values of the parameters. The remainder of this section is devoted to establishing this. Readers not interested in the details of the analysis may proceed directly to the next section.

To show that ρ^* can exceed $\hat{\rho}$ when $v \geq 0$, we observe that that v is monotonically declining in ρ^* . Then evaluating v at $\rho^* = \hat{\rho}$, $\gamma = 0$, (using (3.5) and (3.7)), we see that

$$(3.12) \quad v = p^S(1 + r_1^*) - \rho^* + \delta p^S[p^S(1 + r_2^*) - \rho^*]$$

$$= p^S[(1 + r_1^*) + \delta p^S R_2^S] - (1 + \delta p^S)\rho^* \geq 0$$

provided

$$(3.13) \quad \rho^* \leq \frac{p^S[(1 + r_1^*) + \delta p^S R_2^S]}{1 + \delta p^S} \equiv \bar{\rho}$$

Thus we require

$$\bar{\rho} > \hat{\rho}$$

or

$$\frac{(1 + r_1^*) + \delta p^s R_2^s}{1 + \delta p^s} > \frac{(1 + r_2^*) - p^s R_2^s}{1 - p^s}$$

or

$$(3.14) \quad r_1^* - r_2^* > \frac{p^s(1+\delta)[(1+r_2^*)-R_2^s]}{1-p^s} = \frac{p^s p^r (R_2^s - R_2^r)(1+\delta)}{(1-p^s)(p^s - p^r)}$$

(3.14) provides us with a necessary condition for there to be type 2 credit rationing.¹ If we confine ourselves to a fully competitive credit market in which competition drives v to zero,

$$\bar{\rho} = \rho^*$$

and (3.14) provides a necessary and sufficient condition for credit rationing.²

(3.14) is a complicated condition in the seven parameters of the problem, p^s , p^r , R_1^s , R_2^s , R_1^r , R_2^r , and δ . It is easy to verify that there are values of the parameters satisfying the basic constraints (2.1)-(2.3)

¹ Clearly if, at $\rho^* = \hat{\rho}$, $\gamma = 0$, $v > 0$, then there exists values of ρ^* for which $v \geq 0$ and $\rho^* \geq \hat{\rho}$ (so v is maximized at $\gamma = 0$).

² Assuming, of course, that the conditions (to be described below) which ensure that the bank wishes the firm to undertake the safe project both periods are satisfied.

for which (3.14) is satisfied. To illustrate this, we consider the special case which we shall refer to as "balanced learning" where

$$(3.15) \quad \begin{aligned} \lambda R_1^s &= R_2^s \\ \lambda R_1^r &= R_2^r \quad \lambda > 1. \end{aligned}$$

Then

$$\lambda r_1^* = r_2^*$$

and (3.14) can be rewritten as

$$(3.16) \quad \begin{aligned} \frac{1 - \lambda}{\lambda} &> \frac{p^s p^r (1 + \delta)}{p^s - p^r} \frac{R_2^s - R_2^r}{r_2^*} \frac{1}{1 - p^s} \\ &= p^s p^r (1 + \delta) \frac{R_2^s - R_2^r}{p^s R_2^s - p^r R_2^r} \frac{1}{1 - p^s} \end{aligned}$$

Since we have assumed that $R_2^s < R_2^r$, for all values of p^s, p^r, δ, R_2^s and R_2^r there is a value of λ sufficiently close to 1 that satisfies equation 3.16.

The second set of conditions which we need to ensure are satisfied is that it is, in fact, optimal for the bank to induce individual borrowers to undertake safe projects both periods. The bank, it will be recalled, has to pay a price to ensure that borrowers undertake the safe project: in the second period, it can only charge an interest rate of r_2^* , and in the first period, it can only charge an interest rate of \hat{r}_1 . The contract we have depicted is a local optimum, but there exist other contracts, with much higher rates of interest, in which in one or both of the periods the borrower undertakes risky projects. These contracts also are locally optimal and could conceivably yield a higher return. We now present a set of sufficient conditions which ensures that the local optimum we have just described is in fact the global optimum.

We assume

$$(3.17a) \quad p^S(1 + r_1^*) > p^R R_1^R$$

and

$$(3.17b) \quad p^S(1 + r_2^*) > p^R R_2^R$$

$$(3.17c) \quad p^R R_2^R < \rho^*$$

The first condition says that the bank prefers to lend at r_1^* , inducing the firm to undertake the safe project, than at $R_1^R - 1$, the maximum rate at which it can lend, which leads the firm to undertake the risky project. The second condition says the same thing for the second period. The third condition states that loans to risky second period projects are unprofitable.

These conditions together ensure that none of the other feasible possibilities yields a higher value of v than the policy we have just analyzed.

Let us denote by v_{SRR} the return on a contract which leads to the safe project being chosen the first period and the risky project being chosen the second period (whether the first period project is safe or risky, a success or failure); v_{RSR} the return on a contract which leads the risky project to be chosen in the first period, a safe project chosen in the second period if the first period project succeeds, and a risky second period project otherwise; v_{SS} the return on a contract which leads the safe project to be undertaken the first period, and the second period if the first period project is successful, but provides no loans if the first period project is unsuccessful; and so on.

Our earlier analysis established that if $\rho^* > \hat{\rho}$, $v_{SSS} < v_{SS}$.

We now need to show that, under the stipulated conditions, all of the remaining possibilities are inferior to v_{SS} . We can directly calculate from (3.7)

$$(3.18a) \quad v_{RRR} = p^r(R_1^r + \delta R_2^r) - (1 + \delta)\rho^* < p^s[(1 + r_1^*) + \delta(1 + r_2^*)] - (1 + \delta)\rho^* = v_{SSS}$$

(using 3.17a and 3.17b)

$$(3.18b) \quad v_{RSS} < v_{SSS} \quad (\text{using 3.17a})$$

$$(3.18c) \quad v_{RSR} < v_{RSS} \quad (\text{using 3.17b})$$

$$(3.18d) \quad v_{RRS} < v_{SSS} \quad (\text{using 3.17b})$$

$$(3.18e) \quad v_{SRR} = p^s(1 + r_1^*) + \delta p^r R_2^r - (1 + \delta)\rho^* < v_{SSS} \quad (\text{using 3.17b})$$

$$(3.18f) \quad v_{SSR} = p^S(1 + r_1^* + \delta p^S R_2^S) + \delta(1 - p^S)p^R R_2^R - (1 + \delta)\rho^*$$

$$< p^S(1 + r_1^* + \delta p^S R_2^S) - (1 + \delta p^S)\rho^* = v_{SS}.$$

(using 3.17c)

$$(3.18g) \quad v_{SRS} < v_{SSS} \quad (\text{using 3.17b})$$

$$(3.18h) \quad v_{SR} = p^S(1 + r_1^* + \delta p^R R_2^R) - (1 + \delta p^S)\rho^*$$

$$< p^S(1 + r_1^* + \delta p^S R_2^S) - (1 + \delta p^S)\rho^* = v_{SS}. \quad (\text{using 3.17b})$$

$$(3.18i) \quad v_{RS} = p^R[R_1^R + \delta p^S(1 + r_2^*)] - (1 + \delta p^R)\rho^*$$

$$< p^S[1 + r_1^* + \delta p^S R_2^S] - (1 + \delta p^S)\rho^* = v_{SS}.$$

Using 3.17a, $p^S > p^R$, the fact that $R_2^S > 1 + r_2^*$, and that $v \geq 0$ implies (with 2.2 and 2.3) $\rho^* < p^S R_2^S$

$$(3.18j) \quad v_{RR} < v_{RS}. \quad (\text{using 3.17b})$$

There is one additional possibility that we need to ensure against: the bank could only loan to individuals the first period.

This would yield a return per customer

$$(3.18k) \quad v_{s^*} = p^s(1 + r_1^*) - \rho^* < p^s(1 + r_1^*) - \rho^* + \delta p^s [p^s R_2^s - \rho^*] = v_{ss}.$$

where we have again made use of the fact that $p^s R_2^s > \rho^*$. Recalling that on page 8 we asserted that the assumption of $\gamma^s = 1$ was made for notational simplicity, (3.18k) proves that in equilibrium γ^s will always equal 1.

In section 6 we will show that banks will not choose to make second period loans to the customers of other banks.

Thus, (3.17) ensures that the local optimum analyzed earlier is in fact a global optimum. The conditions (3.17) are sufficient, but not necessary. Necessary and sufficient conditions may be derived in a straightforward manner, from the conditions (3.18). It should also be clear that (3.17) is consistent with the other constraints on the parameters ((3.14) and (2.1-2.3)).¹

In short, we have established that there is a range of parameter values for which credit rationing of type 2 occurs when there is credit rationing of type 1: that is, in the analysis of this section, we have assumed that the bank maximizes its profits without competing for customers. In the next section, we consider whether it is possible to have credit rationing of type 2 without credit rationing of type 1.

¹ We need, of course, to consider the possibility of $0 < \gamma < 1$; under the stipulated conditions, the bank can always do just as well by setting $\gamma = 0$ or 1.

4. Competition for Customers

In the previous section we showed that, when there was an excess demand for funds, the bank would, under not particularly restrictive conditions, use the allocation of funds as an incentive device; this was so even though second period projects were more productive than first period projects.

We now ask, can there be "incentive rationing" even in the absence of an excess demand for funds? We show that the answer is yes.

The bank's problem is identical to that posited in the previous section: it faces an exogenous cost of loanable funds ρ^* and wishes borrowers to invest in safe projects. However, the bank faces an additional constraint: the loan contract must yield an expected level of profits Π^* to borrowers. Any bank that offers borrowers a contract which will yield an expected return less than Π^* will not find any borrowers applying for loans. We will now show that in those cases where, in the absence of competition for customers, there was incentive rationing, there will still be incentive rationing, provided there is not "too much" competition for customers. The effect of competition is to increase γ (and reduce r_1), but provided the relative supply of funds is not "too great," γ remains less than unity.

Formally, the problem of the bank is to maximize its profits,

$$(4.1) \quad \text{maximize} \quad v = p^S [1 + r_1 + \delta p^S (1 + r_2) + \delta \gamma (1 - p^S) (1 + r_2^f)] \\ - \rho^* [1 + \delta p^S + \delta \gamma (1 - p^S)]$$

subject to two constraints:

(a) it wishes its customers to undertake the safe project; from (3.5), we know that this implies that

$$(4.2) \quad r_1 \leq \hat{r}(r_2, r_2^f, \gamma);$$

(b) its customers must receive a level of expected profits at least equal to $\bar{\Pi}$,

$$(4.3) \quad \bar{\Pi} \leq p^S [R^S - (1 + r_1) + \delta \pi(r_2)] + (1 - p^S) \gamma \pi(r_2^f) \delta$$

It is easy to see that

$$r_2^f = r_2^*.$$

As before, lowering r_2^f not only lowers returns second period, but also may lower returns first period, since r_1 may have to be lowered to induce the safe project to be undertaken.

Notice moreover that if the $\Pi \geq \bar{\Pi}$ constraint is binding (from (4.1) and (4.3))

$$\begin{aligned} \frac{dv}{dr_2} &= \frac{\partial v}{\partial r_2} + \frac{\partial v}{\partial r_1} \frac{\partial r_1}{\partial r_2} \Big|_{\Pi = \bar{\Pi}} \\ &= \delta(p^s)^2 - (p^s) (\delta p^s) = 0; \end{aligned}$$

the choice of $r_2 \leq r_2^*$ makes no difference, provided r_1 is adjusted accordingly. Thus, without loss of generality, we can let $r_2 = r_2^*$.

We now examine in detail the structure of the constraints (4.2) and (4.3). In Figure 2 we have drawn the constraint curves (4.2) and (4.3), in terms of the remaining two variables (r_1, γ) . Note that they are both linear. We denote by $\hat{\gamma}$ the value of γ at which both constraints are just binding. (If the $r_1 \leq \hat{r}_1$ constraint always lies below the $\Pi \geq \bar{\Pi}$ constraint for $0 \leq \gamma \leq 1$, $\hat{\gamma} = 0$; if above, $\hat{\gamma} = 1$.) We have also drawn a representative iso-bank profit curve, which is linear, too.¹ It is immediate that either $\gamma = \hat{\gamma}$ or $\gamma = 1$; which depends simply on the relative slopes of the $r_1 = \hat{r}_1$ curve and the iso-bank profit line, i.e. on whether (using (4.1) and (3.5))

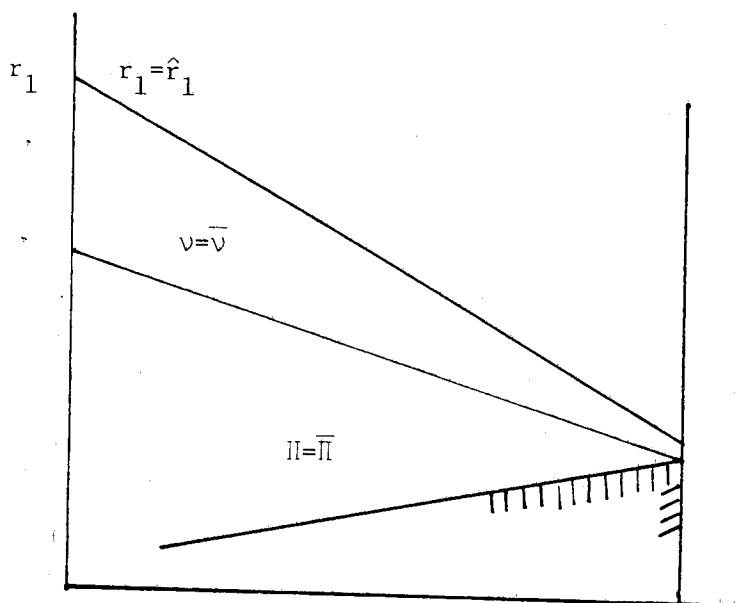
$$\pi_2(r_2^f) \geq \frac{(1-p^s)}{p^s} ((1+r_2^f) - \rho^*)$$

i.e., if

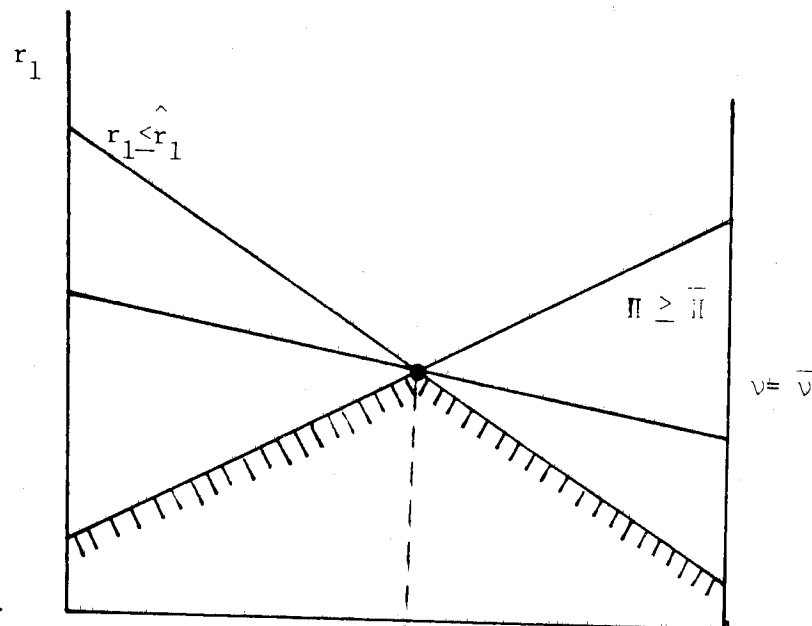
¹ The iso-bank profit curve is defined by (4.1):

$$\begin{aligned} \bar{v} &= p^s [(1+r_1) + \delta p^s (1+r_2) + \delta \gamma (1-p^s) (1+r_2^f)] - \\ &\quad \rho^* [1 + \delta p^s + \delta \gamma (1-p^s)] \end{aligned}$$

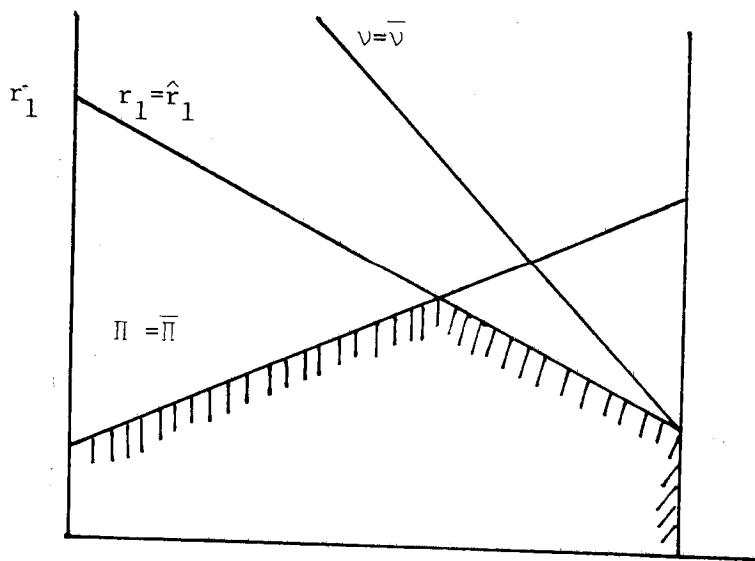
This is the bank's profit assuming that the firm undertakes the safe project and that it obtains the customer. (Obviously, outside the constraint set these assumptions are not satisfied. As in the previous section, we need to check that the local optimum is in fact a global optimum.)



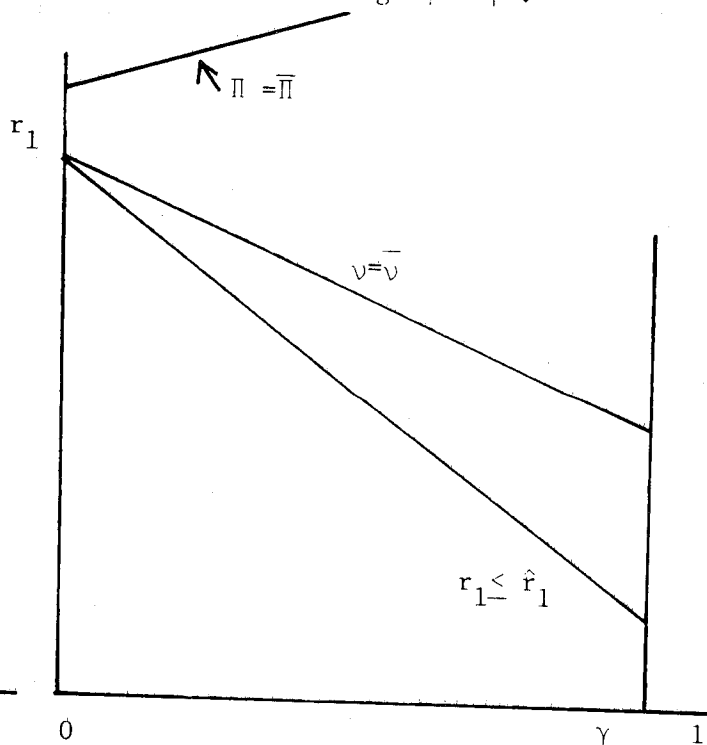
0 $\pi \geq \bar{\pi}$ is everywhere the binding constraint, thus no incentive credit rationing: $\gamma = 1$.



$\rho^* > \hat{\rho}$, therefore incentive credit rationing: $\gamma = \gamma$.



0 $\rho^* < \hat{\rho}$, therefore no incentive credit rationing: $\gamma = 1$.



$r_1 < \hat{r}_1$ is everywhere the binding constraint and $\rho^* > \hat{\rho}$, thus incentive credit rationing: $\gamma = 0$. (If $\rho^* < \hat{\rho}$ then π curve would be steeper than the $r_1 = \hat{r}_1$ curve so $\gamma = 1$.)

Figure 2

$$\rho^* > \hat{\rho} = \frac{p^S(1 + r_2^*) - p^S R_2^S}{1 - p^S}, \quad \gamma = \hat{\gamma}$$

$$< \hat{\rho}, \quad \gamma = 1.$$

This is the same as the **result** we derived on page 15 except now if $\rho^* > \hat{\rho}$ the bank chooses $\gamma = \hat{\gamma}$ rather than $\gamma = 0$.

Obviously, γ can take any value between (and including) zero and one.

In Figure 2 we have drawn in three different levels of $\bar{\pi}$ curve. If $\rho^* < \hat{\rho}$ then, at very low levels of $\bar{\pi}$, r_1 is high and γ is zero. As $\bar{\pi}$ is increased, γ increases and r_1 falls. This continues until γ reaches unity. Then competition for customers takes the form of lowering r_1 still further, so that the $r_1 = \hat{r}_1$ constraint is no longer binding. Thus, for random rationing ($\gamma < 1$) we require that $\bar{\pi}$ be less than the value of profits generated by $\gamma = 1$, $r_1 = r_1^*$.

Here, as earlier, we are treating the cost of capital, ρ^* , as an exogenous parameter as well as $\bar{\pi}$. In a full market equilibrium analysis, of course, ρ^* and $\bar{\pi}$ need to be determined as a result of the interaction of the supply and demand for funds, each of which, in turn, depends on the credit rationing policies of the various banks. Increases in supply will result in more competition for customers, raising $\bar{\pi}$, and, as we have noted, raising γ and lowering r_1 (as long as we are at an interior solution).

5. A Continuous Choice of Projects

In sections 2 and 3 we assumed that borrowers could only choose from 2 possible projects in each period. We will now show that the results obtained in those sections are not significantly affected if there is a continuum of projects available to borrowers in each period.

Let $p_i(R)$ represent the probability that a project with a gross return of R (if successful) in period i will actually succeed. We can now rewrite equations (3.7) and (3.4).

$$(5.1) \quad v = p_1(R_1)(1 + r_1) - \rho^* + \delta p_1(R_1)[p_2(R_2)(1 + r_2) - \rho^*] \\ + \delta \gamma (1 - p_1(R_1)) [p(R_2^f)(1 + r_2^f) - \rho^*]$$

$$(5.2) \quad \Pi = p_1(R_1)[R_1 - (1 + r_1)] + \delta p_1(R_1)p_2(R_2)[R_2 - (1 + r_2)] \\ + \delta \gamma (1 - p_1(R_1)) [p_2(R_2^f)[R_2^f - (1 + r_2^f)]]$$

Except for a degenerate case, the value of γ which maximizes v will be at the boundary. That is, either no defaulters get second period loans or they all do.

We show that when v is maximized, the choice variables $\{r_1, r_2, r_2^f$ and $\gamma\}$ cannot all be interior. Since r_1, r_2 and r_2^f are all unbounded (they can be negative if, for example, the bank chooses to pay borrowers to open an account) it must be the case that γ lies on its boundary. The method of proof is by contradiction; we will assume an interior maximum, and show that except for singular cases the first order conditions for profit maximization by the bank imply $\gamma = 0$ or $\gamma = 1$.

Since the borrower's choice of second period project is a function solely of the interest rate he faces, we have the behavioral equation $R_2 = q(r_2)$. Let $v_2(r_2) \equiv p_2(q(r_2))(1 + r_2) - \rho^*$, and $\pi(r_2) = \max_R p_2(R)[R - (1 + r_2)]$. Also, let $\mu = \gamma\pi(r_2^f)$. Then

$$(5.3) \quad v = p_1(R_1)[1 + r_1 + \delta v_2(r_2)] - \rho^* + \delta[1 - p_1(R_1)] \frac{v_2(r_2^f)}{\pi(r_2^f)} \mu$$

$$(5.4) \quad \Pi = p_1(R_1)[R_1 - (1 + r_1) + \delta\pi(r_2) - \delta\mu] + \delta\mu$$

If $0 < \gamma < 1$, γ and r_2^f can be changed in a way that holds

μ fixed but varies $\frac{v_2(r_2^f)}{\pi(r_2^f)}$. Since R_1 is a function of μ , r_1 , and

r_2 , if $0 < \gamma < 1$, r_2^f is uniquely determined as the interest rate

that maximizes $\frac{v_2(r_2)}{\pi(r_2)}$ subject to the constraint that r_2^f is no greater than the interest rate which maximizes $v_2(r_2)$. This latter restriction is required to restrict r_2 to enforceable interest rates.

If r_2^f were greater than the interest rate which maximizes $v_2(r_2)$ then neither party would have any incentive to enforce that term of the

contract.¹ We denote the interest rate (from the restricted range) which maximizes $\frac{v_2(r_2)}{\pi(r_2)}$ as r_2^{*f} .

Now if we keep $1 + r_1 + \delta\mu$ fixed but vary r_1 and γ (keeping r_2^f fixed) the borrower's choice of R_1 is not affected; (5.3) can be rewritten as

¹ If we were to drop the equilibrium condition of enforceability none of the qualitative results derived below would be changed.

$$(5.5) \quad v = p_1(R_1)(1 + r_1) + [1 - p_1(R_1)] \frac{v_2(r_2^f)}{\pi(r_2^f)} \delta\mu + T$$

where T is independent of r_1 and γ . Therefore, unless R_1 is such that in equilibrium

$$(5.6) \quad \frac{1 - p_1(R_1)}{p_1(R_1)} \frac{v_2(r_2^{*f})}{\pi(r_2^{*f})} = 1,$$

the bank's profits can be improved by changes in γ with compensated changes in r_1 . Thus $\gamma = 0$ or $\gamma = 1$.¹

To show that maximizing behavior does not imply (5.6) being satisfied, we define

$$(5.7) \quad x \equiv 1 + r_1 - \delta\pi(r_2) + \delta\mu$$

Thus, from (5.4), the borrower chooses R_1 such that

$$(5.8) \quad p'(R_1)(R_1 - x) + p_1(R_1) = 0$$

¹ Notice that even if the parameters of the problem change to generate a value of R_1 which also satisfies (5.6), the bank will be indifferent among all values of $\gamma \in [0,1]$. We can see this result by substituting equation (5.6) into equation (5.3); then

$$v = p_1(R_1)[1 + r_1 + \delta v(r_2) + \delta\mu] - \rho^*.$$

Recalling that R_1 is a function of $1 + r_1 + \delta\mu$ it is clear that any values of r_1 and γ generating the same sum $1 + r + \delta\mu$ will generate the same profits to the bank. Thus, if the value of R_1 described by equation (5.6) prevails so that an interior value of γ can maximize bank profits, the bank can do equally well by choosing $\gamma = 0$ or $\gamma = 1$.

Implicitly differentiating (5.8), we obtain

$$(5.9) \quad \frac{dR_1}{dr_1} = \frac{p'(R_1)}{p''(R_1)(R_1 - x) + 2p'(R_1)}$$

The bank sets

$$(5.10) \quad \frac{dv}{dr_1} = p_1(R_1) + [1 + r_1 + \delta\mu + \delta v_2(r_2)]p_1'(R_1) \frac{dR_1}{dr_1} = 0.$$

Substituting (5.9) into (5.10), we have the additional restriction, this time involving $p''(R)$, that at the value of R satisfying (5.6)

$$(5.11) \quad \frac{-p_1(R_1)}{p_1'(R_1)[1 + r_1 + \delta\mu + \delta v_2(r_2)]} = \frac{p_1'(R_1)}{p_1''(R_1)(R_1 - x) + 2p_1'(R_1)}$$

We can now show that all the terms in (5.11) are determined from the assumption of an interior maximum.

From equation (5.4) we can see that if $-(1 + r_1) + \delta\pi(r_2)$ is held constant while r_1 and r_2 are allowed to vary, R_1 is unchanged.

Letting $1 + r_1 = \delta\pi(r_2) + c$ where c is some constant, and substituting into equation (5.3), r_2 is chosen to maximize $v_2(r_2) + \pi(r_2) = p_2(R_2)R_2 - \rho^*$.

Holding this value of r_2 fixed, we see from equation (5.4) that

$1 + r_1 + \delta\mu$ uniquely determines R_1 ; since we know p_1 from equation

(5.6), we can derive the corresponding value(s) of $1 + r_1 + \delta\mu$.

Therefore, the assumption that γ is interior is sufficient to determine all of the terms in (5.11); however, those values of r_1 , r_2 , r_2^f , and γ must also satisfy equation (5.11). There is no reason to expect that additional restriction to hold.¹

To see that $\gamma = 0$ may be an equilibrium totally differentiate v with respect to γ .

$$(5.12) \quad \frac{dv}{d\gamma} = [1 + r_1 + \delta(v_2(r_2) - v_2(r_2^f)\gamma)]p_1'(R_1) \frac{dR_1}{d\gamma} + (1 - p_1) \delta v_2(r_2^f)$$

From equation (5.8) (recalling the definition of x and μ)

$$\frac{dR_1}{d\gamma} = \frac{p_1'(R_1) \delta \pi_2(r_2^f)}{\frac{d^2 \Pi}{dR_1^2}} > 0$$

From (5.1) and the equilibrium requirement that $v = 0$ we can see that

$$(5.13) \quad p_1(R_1)[1 + r_1 + \delta(p_2(1 + r_2) - \rho^*) - \delta\gamma(p_2^f(1 + r_2^f) - \rho^*)] \\ = \rho^* - \delta\gamma[p_2^f(1 + r_2^f) - \rho^*]$$

¹ In particular, none of the calculations made use of the value of p'' . Thus if (5.11) happened to hold, a slight perturbation in the curvature of $\rho_1(p_1'')$, which leaves p' and p unchanged will leave r_1 , r_2 , r_2^f and γ unchanged and (5.11) would no longer not be satisfied.

where p_2^f is the (endogenously determined) probability of success the second period if the project is unsuccessful the first.

For $\gamma = 0$ the right hand side of (5.13) is positive. Therefore, $\frac{dR_1}{d\gamma}$ in equation (5.12) is multiplied by a negative number and for sufficiently large values of $\frac{dR_1}{d\gamma}, \frac{dv}{d\gamma} \Big|_{\gamma=0} < 0$.¹

On the other hand, for $\gamma = 1$, equation (5.12) can be rewritten as,

$$(5.14) \quad \frac{dv}{d\gamma} = p_1' (R_1) \frac{dR_1}{d\gamma} [1+r_1 + \delta(v_2(r_2) - v_2(r_2^f))] + \delta(1-p_1)v_2(r_2^f)$$

Since r_2 and r_1 are chosen to maximize the banks profits,

$$p_1 [1 + r_1 + \delta v_2(r_2)] - \rho^* + (1 - p_1) \delta v_2(r_2^f) > p_1 (1 + r_1) - \rho^* + \delta v_2(r_2^f)$$

so that $v_2(r_2) > v_2(r_2^f)$.

Again, for a large value of $\frac{dR_1}{d\gamma}, \frac{dv}{d\gamma} < 0$. Since we have already eliminated interior maxima it is clear that all that is required for $\gamma = 0$ to be an equilibrium is that $\frac{d^2 \Pi}{dR_1^2}$ be sufficiently small.

¹ The magnitude of $\frac{dR_1}{d\gamma}$ depends on the value of p'' for which there are no natural restrictions.

We now examine the choice of r_2^f .

$$\frac{dv}{dr_2^f} = \delta\gamma(1 - p_1(R_1)) v'(r_2^f) + \frac{dv}{dR_1} \frac{dR_1}{dr_2^f} = 0$$

Since $\frac{dv}{dR_1} < 0$ and $\frac{dR_1}{dr_2^f} < 0$, $v'(r_2^f) < 0$ and $r_2^f \geq r_2^*$ where r_2^* maximizes $v_2(r_2)$. However, as we have previously pointed out, contract terms mandating $r_2^f > r_2^*$ will not be enforced; therefore r_2^f may reasonably be constrained to equal r_2^* .

On the other hand,

$$\frac{dv}{dr_2} = \delta p_1(R_1) v'(r_2) + \frac{dv}{dR_1} \frac{dR_1}{dr_2} = 0$$

Since $\frac{dR_1}{dr_2} > 0$, $v'(r_2) > 0$ and $r_2 < r_2^*$.

Therefore, even when second period borrowers receive loans, defaulters will be charged higher interest rates than are borrowers who repaid their loans.

6. Competition Among Banks

In the analysis of the preceding sections, we assumed that whenever the bank refused a loan to a firm, the firm would not be able to find an alternative source of funds. However, since we also assumed that second period projects were more profitable than first period projects, at first sight those assumptions appear contradictory: surely the individuals whose applications are rejected would apply elsewhere and obtain financing.

There is, however, a simple mechanism by which banks ensure that this will not occur: by putting in a provision in its first period loan contract that the first period loan has seniority over subsequent loans, the bank can lower the expected return to the second period borrower. To see this, assume that the borrower owes $1 + r_1$ on an unpaid first period loan. Then, the borrower will choose the technique which will maximize his expected return:

$$p_2(R_2) - (1 + r_1) - (1 + r_2)$$

Thus, in the model of sections 2-4, where the firm had only two techniques to choose among, the safe technique will be chosen if and only if

$$1 + r_2 \leq r_2^* - r_1$$

Thus, the maximum expected return to a new lender offering loans to borrowers who failed to repay first period loans is

$$p^S(r_2^* - r_1).$$

Since

$$r_1 = \hat{r}_1$$

and by setting r_2 arbitrarily small \hat{r}_1 can be made arbitrarily large, the first period lender can always ensure that competing banks lending to defaulters earn negative returns.¹ Hence, with seniority provisions in debt, if the bank refused to lend to the firm the second period, so would all other other banks.

Assume, on the other hand, that the firm can declare bankruptcy after the first period. Then, in the case where there is credit rationing of type I and II, there still may exist an equilibrium in which the successful project is turned down by all other banks. To see this, we note that the maximum return to the second period loan is

$$p^s(1 + r_2^*)$$

¹ We are implicitly assuming that r_2 can be negative. If the bank choose a first period interest rate greater than $R_1^s - 1$ it would have to refinance part of the loans of successful borrowers. However the large first period interest rate would still serve to deter other lenders. We could restrict interest rates to be non-negative. In that case the expected return to a competing bank lending to first period defaulters would be $p^s(r_2^* - r_1^* - p^s R_2^s \delta)$.

which may be less than ρ^* if there is credit rationing of both types. Since, if all banks are identical, competition will drive v to zero, or

in the two project case
 (6.1)
$$\rho^* = [1 + r_1^* + \delta p^s R_2^s] \frac{p^s}{1 + \delta p^s}$$

we require simply that

(6.2)
$$\frac{(1 + r_1^* + \delta p^s R_2^s)}{1 + \delta p^s} \geq 1 + r_2^*$$

or $r_1^* - r_2^* > \delta p^s (1 + r_2^* - R_2^s)$; which is not inconsistent with $\frac{\partial v}{\partial \gamma} < 0$.

Clearly, if $r_1^* > r_2^*$, this condition is satisfied, but it can also be satisfied for $r_1^* < r_2^*$ (in either case, if eq. (6.2) holds it is

sufficient for $\frac{\partial v}{\partial \gamma} < 0$ (see eq. (3.14)).

The intuition behind this result is that although the expected gross return on second period projects is greater than on first period projects, the bank has additional policy instruments if it gives both second and first period loans. It is the availability of the contingency contract as an incentive mechanism, providing an additional (nonprice) inducement for less risk in first period behavior, that makes first period loans profitable even if $r_1^* < r_2^*$.

We have thus established the possibility of credit rationing in the presence of competition from other banks even if there is a possibility that borrowers who are denied credit can apply at other banks. This possibility exists whether or not there is bankruptcy.

7. Enforcability of Contracts

The rationing equilibria described in this paper require binding contracts.

It will (almost) never be in the bank's interest (acting myopically), in the second period, to give second period loans to borrowers who repaid their first period debts while denying loans to defaulters, (remembering that the two types are identical). One obvious way of avoiding this problem is to model the credit market as a repeated game with reputation effects; the analysis would then follow that for "experience goods" in product markets. Banks would know that if they gave loans to defaulters even though they say they will not, new borrowers would come to know this, and hence would act as if $\gamma = 1$.

A more interesting way of resolving this is to allow explicit contracts to be written but to require that each term of the contract be such that at least one of the parties has an interest in enforcing it. (Notice that a contract **which** states that one player will "blow up to world" if the other player charges some price below ρ^* is clearly not enforceable.) In the context of our model with two projects in each period, enforcability entails,¹

$$(7.1) \quad r_2 \leq r_2^* \quad \text{and} \quad r_2^f \leq r_2^*$$

and

$$(7.2) \quad p^S(1 + r_2^*) \leq \rho^*$$

¹ Where ρ^* is the cost of capital and lenders face a horizontal supply schedule of funds.

Condition (7.1) holds, because if either second period interest rate were set in the contract above r_2^* the bank would prefer to charge r_2^* in the second period, and the firm would not resist this lower interest rate. Condition (7.2) holds because otherwise the bank would lend to second period defaulters despite any contract provisions setting $\gamma = 0$. The bank makes profits on second period loans and the defaulters wish to receive second period loans. This condition is identical to that discussed in Section 6 when firms are permitted to go bankrupt. Obviously, if, when bankruptcy is permitted, it is not profitable for a second firm to lend to a bank's defaulters, it will not be profitable for the bank to lend to its own defaulters.

Of course, from (7.2) the bank would also wish to renege on its commitment to give second period loans to borrowers who repay their first period debts. However, borrowers would enforce those contract terms.

If we were to impose the stricter requirement that it is not in the myopic interest of the bank acting alone to violate any terms of the contract then there is (almost) never an equilibrium set of contracts with type II rationing. This point can be seen most clearly by noticing that in the second period a myopic bank will treat borrowers who repaid, or failed to repay, their previous loans identically. Thus unless $\rho^* = p^S(1 + r_2^*)$, in which case the bank is indifferent between making, or not making, second period loans, either all second period borrowers will receive credit (in the case where $\rho^* < p^S(1 + r_2^*)$, competition for these borrowers results in the second period interest rate for borrowers who repaid prior loans falling to $\frac{\rho^*}{p^S} - 1$ ¹ or none will (when $\rho^* > p^S(1 + r_2^*)$).

¹ Because of subordinated debt banks would have some monopoly power over defaulters who would still be forced, because of the myopic assumption, to pay an interest rate above $\rho^*/p^S - 1$. This higher interest rate would be reflected in lower first period interest rates if banks were competing for loanable funds.

8. Market Equilibrium

The analysis so far has not explicitly considered the full determination of market equilibrium. This depends on the supply of funds as well as the demand for funds, and on the provisions for bankruptcy. For simplicity, we focus our analysis on the case where there is no bankruptcy. We limit our analysis to a simple economy in which all banks face the same borrowers and lenders, and we revert to the two project model of sections 2-4, **but the analysis can be extended to other situations as well.** Our analysis will make extensive use of three functions: (a) The loan supply function: the supply of funds to the banking system as a function of the return to depositors, ρ^* , is depicted as $L^S(\rho^*)$ in Figure 3

(b) The loan demand function: if all banks set $\gamma = 0$, then the demand for funds is equal to $\bar{L}(1 + p^S)$, where \bar{L} is the number of borrowers; if all banks set $\gamma = 1$, then the demand for funds is equal to $2\bar{L}$. Since we know banks will wish to set $\gamma = 1$ if $\rho^* < \hat{\rho}$ and $\gamma = 0$ if $\rho^* > \hat{\rho}$ the demand function for loanable funds when banks take ρ^* as given appears in figure 3. (c) The zero profit-safe investment function. In market equilibrium, all banks must earn zero profits. Thus, we can solve for the value of ρ^* which makes $v = 0$ for each value of γ and which induces the safe project to be undertaken. From our earlier calculations, we know

¹ Although we have depicted the loan supply function as upward sloping none of the results which follow depend on this characterization.

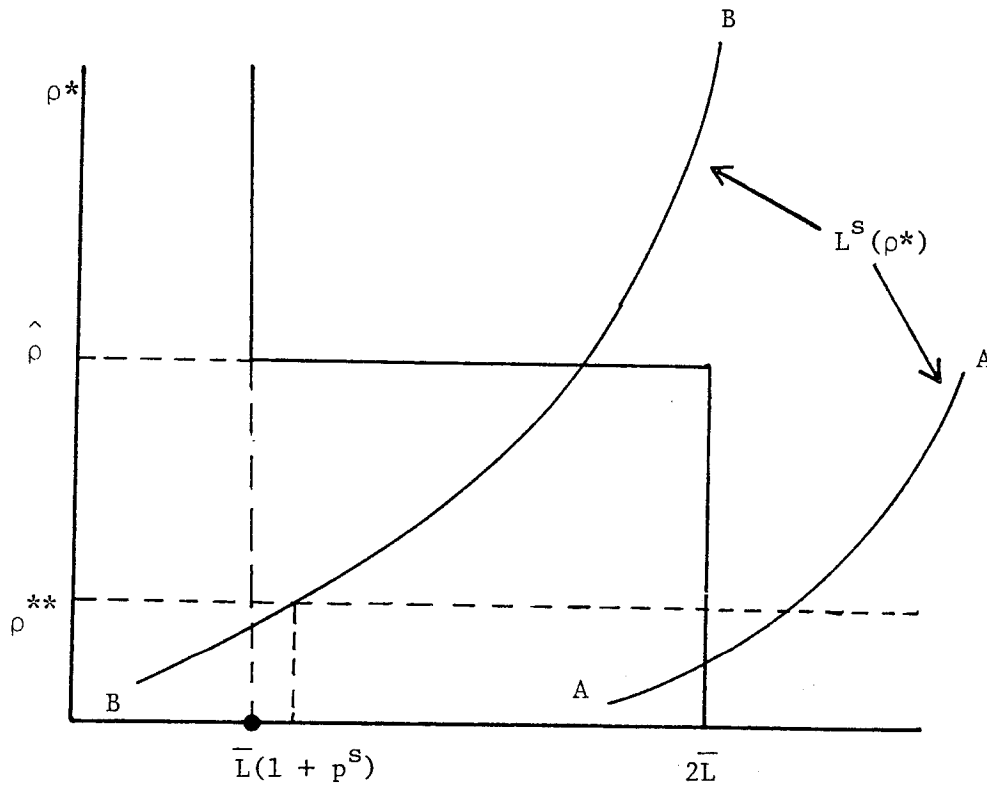


Figure 3

$$(8.1) \quad \rho^*(\gamma) \equiv \frac{p^s[(1 + r_1^*) + \delta[(1 - \gamma)p^s R_2^s + \gamma(1 + r_2^*)]]}{1 + \delta[p^s + (1 - p^s)\gamma]}$$

In particular, let

$$(8.2) \quad \rho^*(0) \equiv \frac{p^s[(1 + r_1^*) + \delta p^s R_2^s]}{1 + \delta p^s}$$

$$(8.3) \quad \rho^*(1) = \frac{p^s[(1 + r_1^*) + \delta(1 + r_2^*)]}{1 + \delta}$$

Let us define

$$(8.4) \quad \rho^{**} \equiv \max_{\gamma} \{ \rho^*(\gamma) \}$$

From the analysis in section we know that if banks are faced with an excess demand for loanable funds at ρ^{**} then,

$$(8.5) \quad \begin{aligned} \gamma &= 1 & \text{if } \rho^{**} < \hat{\rho} \\ \gamma &= 0 & \text{if } \rho^{**} > \hat{\rho} \\ 0 \leq \gamma \leq 1 & & \text{if } \rho^{**} = \hat{\rho} \end{aligned}$$

There are now two broad cases to examine.

If $\hat{\rho} > \rho^{**}$, as in Figure 3, there are only **two** possible patterns of equilibria.

If there is a large supply relative to demand, as illustrated by the supply function AA, then there is no rationing. Market equilibrium entails $\gamma = 1$; competition for customers lowers r_1 to a value below \hat{r}_1 .

On the other hand, if the supply of funds is small, as with the supply function BB, there is type I rationing but there is no incentive rationing (γ still equals 1). All individuals who get loans in the first period also get loans in the second, but not all individuals obtain loans.

However, the central concern of this paper has been with situations where $\rho^{**} > \hat{\rho}$, depicted in Figure 4, so that bank profits per borrower are maximized at $\gamma = 0$, if there is excess demand for loans at $\rho = \rho^{**}$ and $\gamma = 0$.

Again, if there is a very large supply, as in curve AA in Figure 4, there is no rationing ($\gamma = 1$).

On the other hand, if there is a very small supply, such as BB, there is rationing of both type I and type II (the case discussed in Section 2). Then $\gamma = 0$, $r_1 = \hat{r}_1$, $r_2 = r_2^*$; at these terms, not all applicants for loans in the first period obtain them.

The supply functions CC and DD correspond to the situations analyzed in Section 4 of the paper. The supply of funds is sufficiently large that at $\gamma = 0$ and $\rho = \rho^{**}$ there is an excess supply of funds. As we argued earlier, competition takes the form of lowering r_1 and raising γ , so that $r_1 = \hat{r}_1$. This lowers ρ , and thus reduces the supply of funds. Since γ is being increased, the demand for funds is increased. There is a critical value of γ , again denoted by $\hat{\gamma}$, at which demand equals supply.

As the supply function shifts to the right, $\hat{\gamma}$ increases and ρ falls, until eventually, with the supply function EE, $\gamma = 1$. We can calculate the equilibrium value of ρ and γ in the following way. In Figure 5 we plot the value of ρ , which for a given value of γ will cause banks to earn zero profits. This function is denoted $\rho^*(\gamma)$ from equation (8.1). Since $\rho^{**} > \hat{\rho}$, $\rho^*(\gamma)$ is a declining function. Similarly, we define $\rho^S(\gamma)$ as that value of ρ which would equate demand to supply for the particular value of γ , i.e.

$$L^S(\rho^S(\gamma)) \equiv \bar{L}(1 + p^S + (1 - p^S)\gamma)$$

We have also plotted $\rho^S(\gamma)$ in Figure 4; it is upward sloping. There is at most one intersection between the two, which determines the equilibrium values of ρ and γ . Notice that as we have drawn Figure 5, $\rho^*(1) > \hat{\rho}$, which ensures that the equilibrium value of ρ exceeds $\hat{\rho}$ if $\gamma < 1$. This follows from observing that

$$(7.6) \quad \rho^*(0) - \hat{\rho} = \frac{p^S[(1+r_1^*) + \delta p^S R_2^S]}{1 + \delta p^S} - p^S \left[\frac{1+r_2^* - p^S R_2^S}{1-p^S} \right] \geq 0$$

$$(7.7) \quad \text{as } (1+r_1^*)(1-p^S) - (1+r_2^*)(1+\delta p^S) + p^S R_2^S(1+\delta) \geq 0$$

while

$$\rho^*(1) - \hat{\rho} = \frac{p^S[(1+r_1^*) + \delta(1+r_2^*)]}{1 + \delta} - p^S \left[\frac{1+r_2^* - p^S R_2^S}{1-p^S} \right] \geq 0$$

as

$$(1+r_1^*)(1-p^S) - (1+r_2^*)(1+p^S\delta) + p^S R_2^S(1+\delta) \geq 0$$

Since we are considering the case where $\rho^{**} > \hat{\rho}$, this implies $\rho^*(0) > \hat{\rho}$, which as the above argument shows in turn implies $\rho^*(1) > \hat{\rho}$.

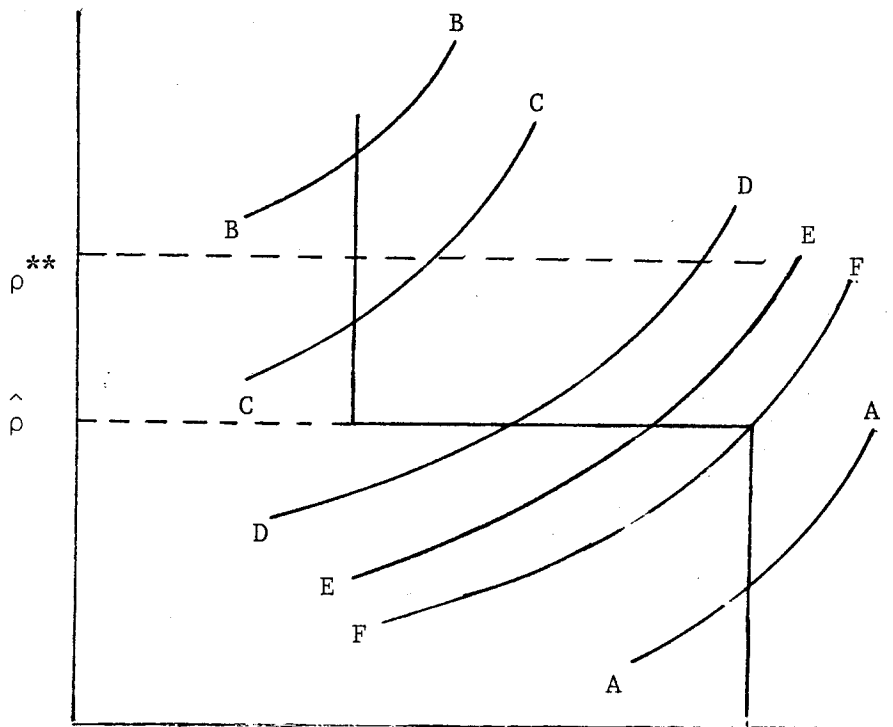


Figure 4

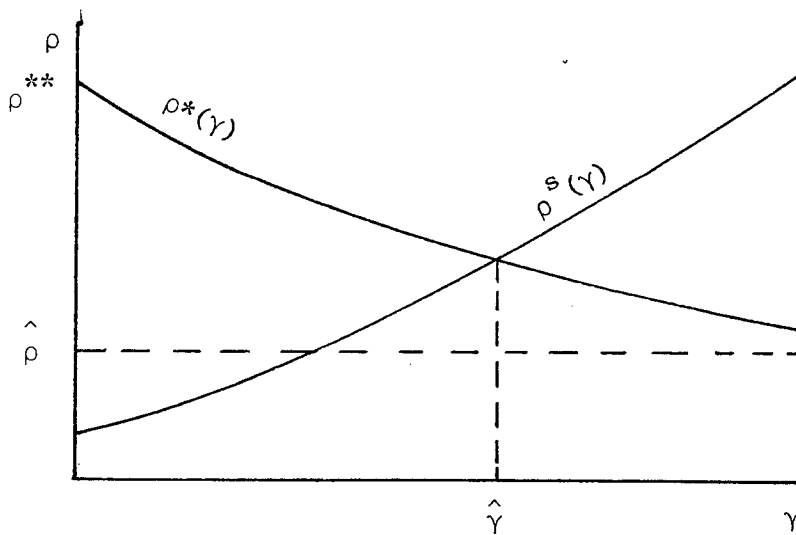


Figure 5

This establishes that there exists a supply function such as EE, to the left of FF, such that for all supply curves between the two $\gamma = 1$ (there is no incentive rationing). As the supply of funds increases beyond EE, further competition takes the form of lowering r_1 until, for supply curve EE, ρ is eventually lowered to $\hat{\rho}$.

The main difference in the market equilibrium between the 2 project case and the multiple project case is that in the former case increases in the supply of loanable funds lead to simultaneous reductions in r_1 and increases in γ . In the more general case, reductions in interest rates will precede any decrease in type 2 rationing. Thus, it is consistent with the general model for banks to compete for loanable funds by lowering interest rates while still rationing second period credit ($\gamma = 0$).

The intuition behind this result is that when there is a continuum of projects available in each period the contract which maximizes the banks profits, when there is an excess demand for loans at the relevant contract terms, is characterized by $\frac{dv}{dr_1} = 0$, $\frac{dv}{dr_2} = 0$. Thus if there is type 2 rationing, so that $\frac{dv}{d\gamma} < 0$, banks will respond to small falls in the demand for funds by decreasing r_1 and r_2^* ; the cost (to the bank) of those changes are of second order magnitude, while the cost of changing γ is a first order effect.¹

¹ A formal proof of these results follows the same line of argument as for the case of two projects in each period, but makes use of the conclusions of section 5 in which we proved that banks facing an excess demand for loanable funds will choose a contract such that $\frac{dv}{d\gamma} \neq 0$.

Using the results of section 6, we can extend the analysis of market equilibrium to the case where there is bankruptcy. Again, it can be shown that there can be a market equilibrium, even with bankruptcy, in which credit rationing is used as an incentive device.

9. Efficiency of Market Equilibrium

In preceding sections we attempted to analyze the patterns of equilibrium which emerge under alternative institutional arrangements concerning bankruptcy, debt seniority, etc. We now turn to the question of evaluating the efficiency of the economy and of these alternative institutional arrangements.

As in the analyses of other information related problems, one must be extremely careful to correctly specify the relevant welfare criterion. Clearly, if the bank (or the government) could directly observe the project being undertaken, then it could specify the action of the borrower, and welfare could be improved: all second period projects would be financed (by assumption if $p_1(\hat{R}_1) = p_2(\hat{R}_2)$, $\hat{R}_1 < \hat{R}_2$). If there is a shortage of funds, it is first period projects which are rationed.

The relevant comparisons, however, must explicitly take into account the availability (cost) of information. We will assume that the government has no more information than do the banks and thus cannot directly control firms' choices of projects.

There is a second difficulty in evaluating the efficiency of markets: do we use an ex ante criterion or an ex post criterion? We take the position here of evaluating alternatives in terms of their effect on the

ex ante expected utility (income) of the participants, but it should be noted that even when one allocation leads all individuals to have a higher ex ante expected utility, some individuals may end up having a lower level of income (utility).

It would appear that the equilibrium described above is inefficient since some second period projects are denied funds despite $R_2^S > R_1^S$; the maximum expected gross return per dollar loaned would be achieved if all first period borrowers received second period loans.

However, policies which prevent contingency contract rationing may also cause a shift in investment from safe to risky projects. That is, while the optimal contingent contract from the bank's point of view involves the safe project being undertaken, the optimal non-contingent contract entails the risky project being undertaken. For example, it is possible for $p^S R_i^S > p^R R_i^R$ and a loan contract with $\gamma = 0$ to yield higher profits to the firm than does any other contract, but for $p^S(1 + r_i^*) < p^R R_i^R$. Thus, if banks were prevented from using contingency contracts they would charge interest rate $r_1 = R_1^R - 1$ in period 1 causing investment in risky projects in that period. Therefore, given the informational asymmetries, it may not be possible both to induce safe investments in both periods and increase the proportion of second period investments.

Even if the equilibrium interest rates when contingency contracts are precluded are such that they induce safe investments, measures which eliminate or reduce the use of contingency contracts (such as

permitting bankruptcy, forbidding subordinated debt, or imposing interest rate ceilings) will -- if they are binding constraints -- reduce the rate of return to depositors and hence will change the level of aggregate saving and investment.

It is thus apparent that national output may be lower in the market equilibrium than in an equilibrium in which, for instance, there was no incentive rationing, but interest rates were kept sufficiently low that the safe project was undertaken both periods. But such a change would also entail a lowering of the rate of return to depositors and an increase in the profits of entrepreneurs. Thus, a pure depositor would be worse off, a capital-less entrepreneur better off: the change would not be a Pareto improvement.

Despite all these qualifications, we can show that the market equilibrium may well not be Pareto optimal: everyone can be made better off by restricting incentive rationing. All we require is that the redistributive effects be "relatively small"; all that this, in turn, requires is that the difference in factor ownership (capital, wages, entrepreneurship) be sufficiently small so that the changes in "shares" are small relative to the increase in the size of the national output.

The remainder of this section is devoted to constructing a simple, life cycle model within which this can be established. We make no attempt at generality; rather, our objective is to construct the

simplest life cycle model consistent with our earlier analysis in which the non-optimality of the market equilibrium can be demonstrated.

In each period, N individuals are born. They live for four periods. In the first, they work and earn a wage of w . In the second and third they can become entrepreneurs, but they can do no physical labor in the fourth they consume their remaining wealth. In keeping with our risk neutrality assumption, the utility function is

$$C_1 + \delta C_2 + \delta^2 C_3 + \delta^3 C_4$$

where C_i is consumption in the i th period. Thus, if the return to depositors, ρ^* , exceeds δ , individuals save everything the first three periods, and consume in the last period. To simplify the exposition we assume $\delta = 1$. Thus, for interest rates below zero, there will be no savings. All income is saved at interest rates exceeding zero.¹ We focus on the case where the demand for funds equals or exceeds the supply of funds at all interest rates. Then, clearly, the bank will provide the optimal contract described in Sections 2-5 which yields a return ρ^{**} which we assume exceeds $\hat{\rho}$. Thus $\gamma = 0$. We also assume $\rho^{**} > 0$.

Because the scale of a project in period i is greater than the funds available to the putative entrepreneurs, not all workers can be entrepreneurs in the second and third periods. The entrepreneurs are chosen randomly. Because individuals are risk neutral, all entrepreneurs will have invested their own previous period wages in the project they undertake.

¹ Although the inelastic supply of saving is the consequence of the consumption function we posited, we will show below that the government can maintain the return to depositors at the level received in the market equilibrium, and thus we can ignore the effect of government policies on the savings rate.

We need to show that there exists a Pareto improvement.¹ We will consider a random rationing scheme. Since individuals are risk neutral, we can evaluate such a scheme simply in terms of the average level of consumption enjoyed by each generation (which equals their expected ex ante utility). The particular set of interventions which we are about to describe may result in some individuals being, ex post, better off, and others being worse off ex post. More complicated examples in which everyone is made better off, both in an ex ante and an ex post sense, may be derived in a similar manner.

Assume the government at some date announces that it is (i) nationalizing the banking sector; (ii) switching to a policy of random rationing, in which $\gamma = 1$; all of those who **receive loans the first period have their loans renewed, regardless** of success the first period; (iii) lowering r_1 to r_1^* (iv) instituting a tax on borrowers, with the **proceeds** of the tax used to subsidize depositors.

The sequence in which this policy is implemented is as follows.

Period 1. The government lowers the interest rate charged first period borrowers to r_1^* , and taxes **net** profits (redistributing the revenues to depositors) so that the returns to depositors from each first period loan is unaffected. The required tax rate on successful projects, τ , as a

¹ In constructing an example of a Pareto improvement, we have to be careful not to compare steady state levels of utility; it is well known from growth theory that the market economy may not maximize steady state utility (in our example whenever the rate of growth of population $\neq \rho^*$) even when it is Pareto efficient.

fraction of net profits is such that

$$\text{Tax Revenues} \equiv p^S \tau_1 [R_1^S - (1 + r_1^*)] = p^S [\hat{r}_1 - r_1^*] \equiv \text{Reduction in bank revenues from lowering interest rate from } \hat{r}_1 \text{ to } r_1^*.$$

or

$$\tau_1 = \frac{\pi(r_2^*) (p^S - p^R)}{p^R (R_1^R - R_1^S)}$$

(For later reference, we note that, using (3.5), $p^S (\hat{r}_1 - r_1^*) = p^S \pi(r_2)$.)

From equation (3.1) we see that the tax will not affect the borrower's choice of a project. The government maintains the number of first and second period loans at the level prior to this policy. Thus in the first period there is no change in welfare.

Period 2 on: Increase the number of second period loans and decrease the number of first period loans to the same number; but do not change any interest rates from their period 1 level. Furthermore, announce that from then on, the probability of having a loan renewed is 1, independent of outcome.¹ In the absence of taxation, the return to lenders

$$\rho^0 \equiv \frac{p^S}{2} (1 + r^* + 1 + r_2^*)$$

¹ Two questions may be raised concerning this example. First, since the individual has funds of his own, cannot he invest them himself, without having recourse to the banking system? To eliminate this possibility, we assumed that all projects must be carried on at a minimum size considerably in excess of the funds available to any single individual from his own savings, a not unreasonable assumption.

The second question concerns the possibility of the banking system "replicating" the actions just described. To do so would require that the bank take an equity interest in the loans, which, under present banking regulations, they are precluded from doing. We hope later to provide a fuller analysis of the supply of funds, taking account of the interactions between the equity and credit markets. See the discussion in Section 9.

Note, in the market equilibrium (when $\gamma = 0$, using (3.5)),

$$\begin{aligned} \rho^* &= \frac{p^s(1 + r_1^* + \pi(r_2)) + p^s(1 + r_2^*)}{1 + p^s} \\ &= \frac{p^s(1 + r_1^* + p^s R_2^s)}{1 + p^s} \end{aligned}$$

The expected aggregate income of a "cohort" of borrowers is (in the absence of taxation)

$$\begin{aligned} \bar{Y}_e &= \frac{\bar{L}}{2} p^s [R_1^s + R_2^s - (1 + r_1^*) - (1 + r_2^*)] \\ &= \bar{L} [\bar{R}^0 - \rho^0] \end{aligned}$$

where \bar{L} is the aggregate loan supply, so each cohort receives $\frac{\bar{L}}{2}$ in loans each period and \bar{R}^0 is the average (gross) return on investments, $p^s \frac{R_1^s + R_2^s}{2}$. In contrast, previously the expected income of new borrowers was¹

$$\begin{aligned} \bar{Y}_e^m &= \frac{\bar{L} p^s}{1 + p^s} (R_1^s - (1 + \hat{r}_1) + \pi(r_2)) \\ &= \frac{\bar{L} p^s}{1 + p^s} [R_1^s + p^s R_2^s - ((1 + \hat{r}_1) + p^s(1 + r_2^*))] \\ &= \bar{L} [\bar{R}^m - \rho^*] \end{aligned}$$

¹ If m is the amount of loans to a cohort during the first period of the entrepreneurship, mp^s is the amount during the second period, and the total equals \bar{L} :

$$m(1 + p^s) = \bar{L}$$

so

$$m = \frac{\bar{L}}{1 + p^s}$$

where $\bar{R}^m = p^s \left[\frac{R_1^s + p^s R_2^s}{1 + p^s} \right]$, mean return to investments in the market

equilibrium. If we levy a tax on first period net profits at the rate τ with proceeds distributed to depositors so that

$$\text{Tax Revenues} = \frac{\tau \bar{L}}{2} (R_1^s - (1 + r_1^*)) = \bar{L}(\rho^* - \rho^0) = \text{difference in returns on loans}$$

or

$$\tau = \frac{2(\rho^* - \rho^0)(p^s - p^r)}{p^r(R^r - R^s)\bar{L}}$$

then returns to depositors will be unchanged, while, since output has increased, expected income of entrepreneurs must have increased.

All we need to check now is that the expected income of the entrepreneurs who entered the market in period one has increased. The probability of such an individual's loan application being approved second period is $\frac{1 + p^s}{2}$, while previously it was p^s . The tax he pays first period has an expected value of $p^s \pi(r_2)$. Hence their expected income (as a group) is increased if and only if

$$\frac{\bar{L} p^s}{1 + p^s} [R_1^s - (1 + r_1^*)] + \left(\frac{1 + p^s}{2 p^s} \right) \pi(r_2) - \pi(r_2) > \bar{Y}_e^m$$

This establishes that the expected utility of all individuals, in all generations, has increased.

¹ From footnote 1 on page 50 above, the number of new loans in period 1 was $m = \frac{\bar{L}}{1 + p^s}$ while the number of second period loans is $\bar{L}/2$. Hence the probability of approval of the second period loan is

$$\frac{\bar{L}/2}{\bar{L}/1 + p^s}$$

(The above exposition assumed that \bar{L} is fixed. In addition, since the income of all generations, except those in period 4, is invested, the increase in per capita income will increase aggregate investment. To simplify the exposition we have ignored the welfare gain from the future returns from these investments. Thus the argument in favor of Pareto improving government intervention could be made even stronger than the one presented.)

10. The General Principal-Agent Problem

This paper has been concerned with the problem of incentives: the bank cannot monitor the actions of the borrower directly; it can only tell whether, at the end of each period, the project undertaken was or was not successful. It thus had to rely on two critical terms of the contract, the availability of credit in future periods, and the interest rates charged, to induce the firm to undertake safe projects.

This problem is typical of a wide variety of situations where incentive problems arise. The essential ingredient of these problems is that there is one individual (the "principal") who does not directly control the action of another individual (the "agent") but has a stake in the outcome; accordingly, indirect methods of affecting the action must be resorted to.¹

¹ The problems discussed here involve only the agent taking an action. In many contexts, of course, the welfare of the agent depends on some action of the principal (even the distinction between principal and agent becomes obfuscated in many economic interactions.) Most of the remarks we have made here extend to these more general circumstances.

The problem we have posed here is equivalent (or closely related to) other principal agent problems in which fixed fee contracts are chosen as a means of payment. For example, firms base wage differentials in future periods on performance this period as a means of inducing workers to perform well this period. But they may also use quantity constraints: an individual may be fired next period if his performance this period is unsatisfactory. Similarly, a landlord may refuse to rent to a tenant in future periods at any fee if that tenant failed to pay his rent in some previous period.

Although we have explored many of the consequences of these fixed fee contracts, we have not explained why, given the obvious difficulties (inefficiencies) which they give rise to, we would see their use in practice. This omission was intentional. There is in fact a substantial literature (dating at least back to Marshall) showing the drawbacks of revenue sharing contracts (share-cropping, equity finance) which are the alternatives to the contracts we have discussed. The principal drawback of these contractual arrangements is that the borrower is not induced to equate the marginal disutility of effort to the value of the marginal product of his effort. Since he doesn't gain all the (marginal) returns from additional expenditures of effort (or other inputs), these arrangements will elicit too little effort from agents.

Fixed fee contracts, it has been argued, provide a resolution for these incentive problems (as well as the corresponding adverse selection problem) when agents are risk neutral. Since the agent making the decision receives all residual returns, his marginal return equals the marginal return of his

action. However, because these contracts force agents to bear a considerable amount of risk it is well known that they are not optimal if agents are risk averse (see Stiglitz (1974) or Stiglitz (1975)).

In this paper we show that the presumed optimality of fixed fee contracts also depends either on agents having no choice over the riskiness of their actions or, regardless of the terms of the contract, actions are never taken which result in a positive probability that the fixed fee is not paid.¹ To reduce the probability of default on the fixed fee, the principal may require that the fixed fee be paid in advance, but this may not be possible if agents have a limited supply of capital. (Indeed, individuals wish to borrow precisely because they have insufficient capital.) So long as there is a positive probability of bankruptcy, the full consequences of his actions are not borne by the agent.

It is sometimes suggested that the incentive problem can be eliminated if the "agent" is required to post a bond, or in the case of borrowing, provide partial collateral. Our analysis shows why these schemes often will not suffice. First, many individuals do not have the collateral (or the funds to purchase the bond). To restrict the provision of funds to those with collateral may be inefficient: they may be ineffectual managers of funds, or individuals with large amount of liquid funds may

¹ This motivation for non-rental contractual arrangements has been discussed in the context of sharecropping by F. Allen (1980). He focuses his analysis on the other principal problem arising from imperfect information -- that of adverse selection. Rental contracts, it should be noted, provide a resolution for these problems as well.

be those who in previous periods undertook risky projects and are likely to undertake risky projects in the future. In a previous paper (Stiglitz-Weiss, 1979) we showed that increasing collateral requirements as a means of allocating credit is not, in general, a profit maximizing strategy for banks when borrowers have different degrees of risk aversion.

In this paper, we have considered (in the context of the capital market) the set of all feasible contracts in which the payments are functions of the observables, "success" and "failure." (Similar results would be obtained in the labor market.) Introducing "bonding" thus does not increase the set of instruments available and hence cannot have any effect on the outcome. At most, it shifts the center of risk bearing. (Of course, if banks, or employers, were risk averse, this would be important.)

It is apparent then that even with risk neutral agents, we would expect equilibrium contracts to involve both revenue sharing and fixed fee components: firms will acquire capital by means of a mixture of equity and loan capital. Similarly, we should expect to see sharecropping and rental arrangements, and piece work and wage rates used concomitantly. The exact nature of the mix will be determined by the preferences (risk aversion) of principal and agent and the responsiveness of the returns on the project (including the probability of bankruptcy) to the actions (including effort and supply of other inputs) of the agent.

Thus, in the simple model presented in the text, where effort played no role, the optimal contract would be a pure equity contract; while if the probability of success was not under the control of the agent (he had no choice of technique) but the return, if successful, was a function of

his effort, the optimal contract would be a pure loan contract.

If firms face both a choice of effort and a choice of technique, we can solve for the optimal debt equity ratio. Assume, for instance, that the return for each of the two projects considered in Section 2 is a function of effort e , but the probability of success or failure is a function solely of the choice of technique (it is independent of effort).

$$(10.1) \quad R^j = R^j(e) \quad j = s, r, \quad R^{j'} > 0$$

entrepreneurs face a disutility of effort of $V(e)$. Then, if the j th project is undertaken, e will be chosen to

$$(10.2) \quad \text{maximize } p^j [(1 - \tau)(R^j - x) - x] - V(e)$$

where τ is the share of net returns received by the suppliers of equity and, $x = (1 + r)B$ is the amount to be repaid to lenders. Hence

$$(10.3) \quad p^j(1 - \tau)R^{j'} = V'(e)$$

We write the solution to (10.3) as $e^j(\tau)$. (Effort does not depend on the fixed fee component of the loan contract.) For simplicity, we assume as before that the suppliers of capital are risk neutral and we focus on the "rationing" equilibrium when one unit of capital is required by an entrepreneur for any project. A supplier of capital chooses τ and x to

$$(10.4) \quad \text{maximize } p[(R - x)\tau + x].$$

If suppliers of capital wish entrepreneurs to undertake safe projects, they choose $x = 1 + r^*$, where, now, r^* is a function of the equity rate, τ . Thus, the optimal equity rate is given by the solution to

$$R^S(e) - x^* = -\tau R^j \frac{de^j}{d\tau} - (1-\tau) \frac{dr^*}{d\tau}$$

More generally, a non-linear payments schedule, $x + \tau(R)$, where $\tau(R)$ is some function of R will be employed (in both the market equilibrium and in the optimum allocation).¹

The analysis of this paper thus provides the foundations for understanding a wide variety of principal agent problems, not only that of the supplier of capital/entrepreneur, but also the employer/employee, farmer/tenant, insurance firm/insured relationships. In each of these cases, we can analyze the equilibrium (optimal) contract: the optimal financial structure of the firm (its debt-equity ratio); the optimal tenancy (wage-sharecropping-rental mix); the optimal compensation scheme for workers; the optimal insurance policy. The detailed analysis will, of course, depend on the peculiar features of each of these markets.

In all of these markets the principal will link the terms of the contract to behavior in previous periods. Although we have focused

¹ In our simple example, for instance, by choosing $\tau = 0$ for $R \leq R^{S*}$,
 $\tau = R - R^{S*}$ for $R > R^{S*}$, the bank can ensure that the safe project is undertaken and that the correct level of effort is employed. But this particular payoff function does not extend to more general situations.

on the case where linkage takes place by denying loans to defaulters, in many instances the linkage may take the form of making prices (rents, wages) contingent on previous behavior. Linkage will only take the form of terminating the principal-agent relationship when one of the following conditions are satisfied.

- (a) The expected payoff to the principal can decrease with the fee. For example, if at lower wages workers work less or if the better workers are more likely to quit if the wages are lowered, then threatening to cut the wages of workers caught shirking may be less profitable than firing those workers.
- (b) Agents whose returns are low (who fail) in early periods are more likely to have low returns (fail) in subsequent periods. Experienced (older) agents are less able than inexperienced (younger) agents.

11. Conclusion

Although the analysis of this paper was framed in the context of borrowers and lenders, as we showed in Section (10) the results we derived are applicable to a broad class of principal agent problems such as employer-employee, and landlord-tenant.

Perhaps the most striking result of this paper has been to show that the market equilibrium may be characterized by contingency contracts in which under some contingencies the principal-agent relationship is terminated (i.e. the borrower is denied credit, the tenant land, or the worker employment) even though the return function for those terminated

agents may stochastically dominate the return function for agents who are still participating. As in our previous one period model (Stiglitz-Weiss, 1979), agents denied loans would not get credit regardless of the interest rate they offer to pay. Thus the conventional characterization of competitive equilibrium as necessarily implying supply equals demand is, we would argue, incorrect. Prices will not necessarily clear markets when firms (banks) compete, and when prices are used both as instruments to attract customers and to affect either who the customers are, or how they behave. (We use the term "competitive" in the sense that firms are small and have no monopoly power; it should be obvious that we are not using it in the specialized way it has sometimes been used to refer to equilibrium where no agent can charge a price other than the market clearing price. In that case, it is trivial that if an equilibrium exists, it is market clearing. In our model, there are market clearing prices (interest rates) but we have shown that if those rates are too high, any bank would increase its profits per borrower by lowering them. Since any bank would thus have an incentive to deviate from the market clearing prices (interest rates) it seems unreasonable to describe those prices as being determined by competitive forces.)

In Section (7) we showed that there exists an equilibrium contingency contract all the terms of which will be enforced either by the bank or the borrowers. We would argue that in principal-agent relationships in which explicit contracts are used, it is reasonable to demand that these contracts satisfy our criterion of enforceability.

We also showed that introducing competition for both borrowers and depositors does not change the qualitative nature of our results. Although the borrowers who are denied loans are "abler" than those who get loans, other banks will not lend to them. Although in an ex ante sense, there is competition for borrowers by lenders, after the initial contract has been signed, the individual loses his anonymity: his ability to turn to another banker if his present banker turns him down or increases the interest rate charged is severely limited. Note that we have obtained this result in a model in which all borrowers are identical. The arguments would be further strengthened if we took into account differences among borrowers. Then, the fact that one's present banker has refused a loan may convey information about one's desirability as a borrower. As Greenwald (1979) has shown, one can extend Akerlof's argument showing that used car markets may be thin or non-existent, to "used labor markets" -- or equivalently, in this context, to "used borrowers." There is thus a critical distinction between competition ex ante and competition ex post: there is only limited ex post competition; the characteristic of "anonymity" which is essential to the usual stories of competitive markets is not valid ex post. Of course, borrowers know that the market will not be competitive ex post, and this effects the nature of the contracts which are signed.

Moreover, our analysis has shown that fixed fee contracts (such as loan contracts) are not, in general socially optimal and will not, in general, be employed (exclusively) in markets even when both the lender and the borrower (the principal and the agent) are risk neutral, so long

as (i) it is possible to employ output-contingent (or profit-contingent) contracts, e.g. equity finance; and (ii) there is some action of the borrower which can affect the probability of default. The contracts which do emerge may not, however, be Pareto optimal.

Thus, although our original objective was simply to analyze the optimal loan contract and its implications for market equilibrium, we have, in fact, succeeded in providing a framework for the analysis of the optimal (equilibrium) financial structure of firms; and even more generally, the optimal (equilibrium) set of contingency contractual arrangements for a wide variety of related principal agent problems.

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