# EQUILIBRIUM AND DISEQUILIBRIUM: TRANSITIONAL MODELS

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#### Introduction

The determination of price and quantity in a market may be modelled either as an equilibrium process or as a disequilibrium process. In the former prices are fully flexible and clear the market whereas in the latter the price may fail to adjust fully to the market clearing level in which case the quantity transacted is often assumed to be the lesser of the quantities demanded and supplied. The customary econometric specifications are as follows:

Equilibrium Model. The demand and supply functions are

$$Q_{t} = \beta_{1}^{t} x_{1t} + \alpha_{1}^{p} t + u_{1t}$$
 (1-1)

$$Q_t = \beta_2 x_{2t} + \alpha_2 p_t + u_{2t}$$
 (1-2)

where  $Q_t$  is the quantity transacted,  $p_t$  the price,  $x_{1t}$  and  $x_{2t}$  exogenous variables,  $\beta_1$ ,  $\beta_2$ ,  $\alpha_1$ ,  $\alpha_2$  parameters and  $u_{1t}$ ,  $u_{2t}$  error terms. Equs. (1-1) and (1-2) jointly determine the equilibrium quantity and price. We denote the

<sup>\*</sup> I am indebted to Mark Plant and Harvey S. Rosen for helpful comments. Financial support from NSF Grant No. SOC77/07680 is gratefully acknoweldged.

solution value for  $p_t$  from (1-1) and (1-2) by  $p_t^*$  which is

$$p_{t}^{*} = \frac{\beta_{2}^{'} x_{2t} - \beta_{1}^{'} x_{1t} + u_{2t} - u_{1t}}{\alpha_{1} - \alpha_{2}}$$
 (1-3)

<u>Disequilibrium Model</u>. The demand and supply functions are now written as

$$D_{t} = \beta_{1}^{t} x_{1t} + \alpha_{1} p_{t} + u_{1t}$$
 (1-4)

$$s_t = \beta_2' x_{2t} + \alpha_2 p_t + u_{2t}$$
 (1-5)

where  $D_{t}$  and  $S_{t}$  are the quantities demanded and supplied and are characteristically assumed to be unobserved. The observed quantity of transactions  $Q_{t}$  is given by

$$Q_{t} = \min(D_{t}, S_{t})$$
 (1-6)

and prices adjust according to

$$p_t = p_{t-1} + \gamma(D_t - S_t) + u_{3t}$$
 (1-7)

where  $u_{3t}$  is an error term and  $\gamma$  a positive parameter. Equ. (1-7) implicity accounts for a range of alternative scenarios. When  $\gamma=0$ , price is exogenous (if also  $u_{3t}$  is independent of  $u_{1t}$  and  $u_{2t}$ ) and as  $\gamma \to \infty$ , the solution values  $Q_t$ ,  $p_t$  from (1-4) to (1-7) converge to equilibrium values. The solution for  $p_t$  from these equations, denoted  $p_t^{**}$ , is

There are several variants of this canonical model most of which will not be discussed here. See Maddala and Nelson (1974), Goldfeld and Quandt (1975), Quandt (1978), Laffont and Monfort (1976), Bowden (1978a,b).

$$p_{t}^{**} = \frac{p_{t-1} + \gamma(\beta_{1}^{*}x_{1t} - \beta_{2}^{*}x_{2t}) + \gamma(u_{1t} - u_{2t}) + u_{3t}}{1 + \gamma(\alpha_{2} - \alpha_{1})}$$
(1-8)

The two models represent discrete alternatives for an economic system. Moreover, if a system behaves at any time according to the equilibrium specification, it will always be in equilibrium; if it behaves according to the disequilibrium specification, it will always be in disequilibrium and to the same degree as indicated by the constant value of  $\gamma$ .

These features must reduce one's complacency about the satisfactoriness of the two archetypal models. It is rather natural to think of systems that are sometimes in equilibrium and sometimes not; it is equally easy to think of circumstances in which the severity of disequilibrium varies over time. There have been relatively few attempts to cope with these problems. The most common (Laffont and Monfort (1976), Ito (1980)) device for letting the severity of disequilibrium vary over time is to introduce two separate coefficients  $\gamma_1$  and  $\gamma_2$  and specify

$$p_{t} = p_{t-1} + \begin{cases} \gamma_{1}(D_{t}-S_{t}) + u_{3t} & \text{if } D_{t} < S_{t} \\ \gamma_{2}(D_{t}-S_{t}) + u_{3t} & \text{otherwise} \end{cases}$$
(1-9)

Models which may sometimes be in equilibrium and sometimes in disequilibrium appear to be limited to cases in which exogenous, observable price ceilings (floors) are imposed (Mackinnon (1978), Maddala (1979)). In these cases a set

<sup>2.</sup> That is not to say that the price determined by the disequilibrium model will always be the same distance from or the same proportion of the hypothetical equilibrium price, but it does mean that the rate at which the price would tend to approach equilibrium is the same.

of  $\bar{p}_t$ , t=1,..., T, are observed with the property that if  $p_t^* \leq \bar{p}_t$ , then  $p_t = p_t^*$  and if  $p_t^* > \bar{p}_t$ ,  $p_t = p_t^{**}$ .

The present paper formulates two models to deal with systems which

(a) are sometimes in equilibrium and sometimes in disequilibrium and (b)

exhibit different "degrees of disequilibrium" at various times. The former

is the subject of Section 2 and is based on the assumption that a system's

"choice" between equilibrium and disequilibrium is a discrete, all-or-nothing

choice. The latter, discussed in Section 3, is based on the assumption

that systems are always in disequilibrium but to varying degress, some of

which may be so mild as to make it impossible in practice to distinguish

their state from equilibrium.

## 2. Switching Between Equilibrium and Disequilibrium

Assume, as is customary, that  $(u_{1t}, u_{2t})$ ,  $t = 1, \ldots, T$ , in the equilibrium case is iid as  $N(0, \Sigma_1)$  and that  $(u_{1t}, u_{2t}, u_{3t})$  in the disequilibrium case is iid as  $N(0, \Sigma_2)$ . It is then straightforward to derive the joint pdf of  $Q_t, p_t$  in the two cases. They will be denoted by  $f_e(Q_t, p_t)$  and  $f_d(Q_t, p_t)$  respectively.

The basic assumption of the mechanism determining regime choice is that price change from period to period imposes real costs on the system as a whole. Since the observed price in period t ,  $p_t$  , can only be either  $p_t^*$  or  $p_t^{**}$  , the system acts as if it were solving each period the following optimization problem:

$$\min_{\lambda} |_{p_{t-1}} - \lambda_{p_t^*} - (1-\lambda)p_t^{**}|$$

For explicit algebraic forms see Quandt (1978).

These costs arise from consumers and producers having to adjust their optimal consumptions and productions, from increased search costs, etc.

subject to

$$\lambda = \begin{cases} 1 & \text{equilibrium model selected} \\ 0 & \text{disequilibrium model selected} \end{cases}$$
 (2-1)

Alternatively, (2-1) may be written as

Select equilibrium model if 
$$|p_{t-1} - p_t^*| < |p_{t-1} - p_t^{**}|$$

Select disequilibrium model otherwise (2-2)

We now derive the likelihood function for this model. 5

Let  $f(Q_t, P_t|M)$  be the joint pdf of the observable random variables conditional on the model selected where M = (E(quilibrium), D(isequilibrium)).

Then  $f(Q_t, p_t|E) = f_e(Q_t, p_t)$  and  $f(Q_t, p_t|D) = f_d(Q_t, p_t)$  and the pdf of  $Q_t$ ,  $p_t$  is

$$g(Q_t, P_t) = f(Q_t, P_t | E) Pr(E) + f(Q_t, P_t | D) Pr(D)$$
(2-3)

We require  $Pr\{E\}$  and  $Pr\{D\} = 1 - Pr\{E\}$ . The criteria for model selection are

$$|p_{t-1} - p_t^*| = |p_{t-1} - \frac{1}{\Delta_1} \left[ \beta_1^! x_{1t} - \beta_2^! x_{2t} + u_{1t} - u_{2t} \right]$$
 (2-4)

and

$$|p_{t-1} - p_t^{**}| = \left| p_{t-1} - \frac{1}{\Delta_2} \left[ \gamma (\beta_1^! x_{1t}^{-\beta_2^!} x_{2t}^{-\beta_2^!} x_{2t}^{-\beta_2^!} + p_{t-1}^{-\beta_2^!} x_{2t}^{-\beta_2^!} + q_{t-1}^{-\beta_2^!} + q_{t-1}^{-\beta_2^!} + q_{t-1}^{-\beta_2^!} \right] \right|$$
(2-5)

where  $\Delta_1 = \alpha_2 - \alpha_1$  and  $\Delta_2 = 1 + \gamma(\alpha_2 - \alpha_1)$ . Denote the arguments of the absolute value functions on the right hand sides of (2-4) and (2-5) by  $v_{1t}$  and  $v_{2t}$  respectively. Conditional on  $p_{t-1}$ ,  $v_{1t}$  and  $v_{2t}$  are jointly normally distributed with mean vector

<sup>5.</sup> For a more general approach see Section 5.

$$\mu_{v} = \begin{bmatrix} p_{t-1} - \frac{1}{\Delta_{1}} (\beta_{1}^{t} x_{1t} - \beta_{2}^{t} x_{2t}) \\ p_{t-1} - \frac{1}{\Delta_{2}} [\gamma(\beta_{1}^{t} x_{1t} - \beta_{1}^{t} x_{2t}) + p_{t-1}] \end{bmatrix}$$
(2-6)

and covariance matrix

$$\Sigma_{\mathbf{v}} = \begin{bmatrix} (\sigma_{1}^{2} + \sigma_{2}^{2} - 2\sigma_{12})/\Delta_{1}^{2} & (\gamma(\sigma_{1}^{2} + \sigma_{2}^{2} - 2\sigma_{12}) + \sigma_{13} - \sigma_{23})/\Delta_{1}\Delta_{2} \\ (\gamma^{2}(\sigma_{1}^{2} + \sigma_{2}^{2} - 2\sigma_{12}) + 2\gamma(\sigma_{13} - \sigma_{23}) + \sigma_{3}^{2})/\Delta_{2}^{2} \\ (2-7) \end{bmatrix}$$

The probability  $Pr\{|v_{1t}| < |v_{2t}|\}$  is

$$\begin{split} \Pr\{\left|\mathbf{v}_{1t}\right| < \left|\mathbf{v}_{2t}\right|\} &= \Pr\{\mathbf{v}_{1t} < \mathbf{v}_{2t} \middle| \mathbf{v}_{1t} \ge 0, \ \mathbf{v}_{2t} \ge 0\} + \\ &\qquad \qquad \qquad \\ \Pr\{\mathbf{v}_{1t} < -\mathbf{v}_{2t} \middle| \mathbf{v}_{1t} \ge 0, \ \mathbf{v}_{2t} < 0\} + \\ &\qquad \qquad \qquad \\ \Pr\{-\mathbf{v}_{1t} < \mathbf{v}_{2t} \middle| \mathbf{v}_{1t} < 0, \ \mathbf{v}_{2t} \ge 0\} + \\ &\qquad \qquad \qquad \\ \Pr\{-\mathbf{v}_{1t} < -\mathbf{v}_{2t} \middle| \mathbf{v}_{1t} < 0, \ \mathbf{v}_{2t} < 0\} \end{split} \tag{2-8} \end{split}$$

The required probability is the sum of the integrals of the normal pdf over two wedge-shaped areas bounded by  $|\mathbf{v}_1|=|\mathbf{v}_2|$  and extending from the origin to  $\pm \infty$ . It is evaluated most easily by rotating the coordinate system in the positive sense through an angle  $\pi/4$  according to the transformation

$$\Pr\{E\} = \Pr\{|\mathbf{v}_{1t}| < |\mathbf{v}_{2t}|\} = \int_{0}^{\infty} \int_{-\infty}^{\infty} \frac{1}{2\pi |\Sigma_{\mathbf{y}}|^{1/2}} \exp\left(-\frac{1}{2}\left[(\mathbf{y} - \mu_{\mathbf{y}}) \cdot \Sigma_{\mathbf{y}}^{-1}(\mathbf{y} - \mu_{\mathbf{y}})\right]\right) \times \frac{d\mathbf{y}_{1}d\mathbf{y}_{2}}{2\pi |\Sigma_{\mathbf{y}}|^{1/2}} \exp\left(-\frac{1}{2}\left[(\mathbf{y} - \mu_{\mathbf{y}}) \cdot \Sigma_{\mathbf{y}}^{-1}(\mathbf{y} - \mu_{\mathbf{y}})\right]\right) \times \frac{d\mathbf{y}_{1}d\mathbf{y}_{2}}{2\pi |\Sigma_{\mathbf{y}}|^{1/2}} \exp\left(-\frac{1}{2}\left[(\mathbf{y} - \mu_{\mathbf{y}}) \cdot \Sigma_{\mathbf{y}}^{-1}(\mathbf{y} - \mu_{\mathbf{y}})\right]\right) \times \frac{d\mathbf{y}_{1}d\mathbf{y}_{2}}{2\pi |\Sigma_{\mathbf{y}}|^{1/2}} \exp\left(-\frac{1}{2}\left[(\mathbf{y} - \mu_{\mathbf{y}}) \cdot \Sigma_{\mathbf{y}}^{-1}(\mathbf{y} - \mu_{\mathbf{y}})\right]\right) \times \frac{d\mathbf{y}_{1}d\mathbf{y}_{2}}{2\pi |\Sigma_{\mathbf{y}}|^{1/2}} \exp\left(-\frac{1}{2}\left[(\mathbf{y} - \mu_{\mathbf{y}}) \cdot \Sigma_{\mathbf{y}}^{-1}(\mathbf{y} - \mu_{\mathbf{y}})\right]\right) \times \frac{d\mathbf{y}_{1}d\mathbf{y}_{2}}{2\pi |\Sigma_{\mathbf{y}}|^{1/2}} \exp\left(-\frac{1}{2}\left[(\mathbf{y} - \mu_{\mathbf{y}}) \cdot \Sigma_{\mathbf{y}}^{-1}(\mathbf{y} - \mu_{\mathbf{y}})\right]\right) \times \frac{d\mathbf{y}_{1}d\mathbf{y}_{2}}{2\pi |\Sigma_{\mathbf{y}}|^{1/2}} \exp\left(-\frac{1}{2}\left[(\mathbf{y} - \mu_{\mathbf{y}}) \cdot \Sigma_{\mathbf{y}}^{-1}(\mathbf{y} - \mu_{\mathbf{y}})\right]\right) \times \frac{d\mathbf{y}_{1}d\mathbf{y}_{2}}{2\pi |\Sigma_{\mathbf{y}}|^{1/2}} \exp\left(-\frac{1}{2}\left[(\mathbf{y} - \mu_{\mathbf{y}}) \cdot \Sigma_{\mathbf{y}}^{-1}(\mathbf{y} - \mu_{\mathbf{y}})\right]\right) \times \frac{d\mathbf{y}_{1}d\mathbf{y}_{2}}{2\pi |\Sigma_{\mathbf{y}}|^{1/2}} \exp\left(-\frac{1}{2}\left[(\mathbf{y} - \mu_{\mathbf{y}}) \cdot \Sigma_{\mathbf{y}}^{-1}(\mathbf{y} - \mu_{\mathbf{y}})\right]\right) \times \frac{d\mathbf{y}_{1}d\mathbf{y}_{2}}{2\pi |\Sigma_{\mathbf{y}}|^{1/2}} \exp\left(-\frac{1}{2}\left[(\mathbf{y} - \mu_{\mathbf{y}}) \cdot \Sigma_{\mathbf{y}}^{-1}(\mathbf{y} - \mu_{\mathbf{y}})\right]\right) \times \frac{d\mathbf{y}_{1}d\mathbf{y}_{2}}{2\pi |\Sigma_{\mathbf{y}}|^{1/2}} \exp\left(-\frac{1}{2}\left[(\mathbf{y} - \mu_{\mathbf{y}}) \cdot \Sigma_{\mathbf{y}}^{-1}(\mathbf{y} - \mu_{\mathbf{y}})\right]\right) \times \frac{d\mathbf{y}_{1}d\mathbf{y}_{2}}{2\pi |\Sigma_{\mathbf{y}}|^{1/2}} \exp\left(-\frac{1}{2}\left[(\mathbf{y} - \mu_{\mathbf{y}}) \cdot \Sigma_{\mathbf{y}}^{-1}(\mathbf{y} - \mu_{\mathbf{y}})\right]\right)$$

where  $\mu_y = A\mu_v$  and  $\Sigma_y = A\Sigma_v A'$ . The integrals in (2-9) are easily evaluated. The likelihood function then is

$$L = \sum_{t=1}^{T} g(Q_t, p_t)$$
 (2-10)

It is tedious but straightforward to verify that (2-10) shares the well-known unboundedness property of disequilibrium likelihood functions in the neighborhood of certain points on the boundaries  $\sigma_1^2 = 0$  and  $\sigma_2^2 = 0$ .

## 3. Models With Variable Degrees of Disequilibrium

It is well-known that the equilibrium model is a limiting form of the disequilibrium model in the sense that, as  $\gamma \to \infty$ , the pdf of  $\mathcal{Q}_{\mathsf{t}}, \mathcal{P}_{\mathsf{t}}$  corresponding to the disequilibrium model converges to the pdf of  $\mathcal{Q}_{\mathsf{t}}, \mathcal{P}_{\mathsf{t}}$  in the equilibrium model (Quandt (1978)). It is plausible to argue that  $\gamma$  may represent the degree of disequilibrium in a model and may undergo short-term variations in response to changes in variables that affect the ease with which the system can clear markets. Although plausible, such an approach is somewhat ad hoc and will be discussed only briefly.

<sup>6.</sup> See the modification of Hausman and Wise (1978) of Owen's (1956) method.

As a first approximation  $\gamma$  may be thought to depend on variables that are exogenous in the short run. Such a variable might be the presence or absence of formal rationing constraints that might be imposed by government regulations. Other such variables may describe the extent to which prices are administered and may be measured by industrial concentration ratios, by the extent of the unionization of the labor force, and the like. In general, one might replace  $\gamma$  by

$$\gamma = \delta' z_{+} \tag{3-1}$$

where  $z_t$  is the vector of relevant variables and  $\delta$  a vector of parameters. This introduces no new conceptual problems, though the model is now more severely nonlinear and the Jacobian of the transformation from error terms to jointly dependent variables is no longer constant. Different sets of  $\delta$  parameters for positive or negative excess demands can also be easily accommodated.

A possible difficulty with the formulation given by (1-4) to (1-7) and (3-1) is that  $\gamma$  may be negative for some plausible values of the z's. A preferable formulation may be that of Bowden (1978a,b) who formulates the adjustment equation as

$$p_t = \mu p_{t-1} + (1-\mu)p_t^* + u_{4t}$$
 (3-2)

where  $p_t^*$  is the equilibrium price as before,  $u_{4t} = \mu u_{3t}$  and  $0 \le \mu \le 1$ . It is easy to verify that (3-2) is merely another version of (1-7) with  $\mu = 1/(1 + \gamma(\alpha_2 - \alpha_1))$ . When  $\mu = 0$ , the model describes equilibrium and when  $\mu = 1$  it describes disequilibrium with fully rigid prices. It is now easy to model  $\mu$  as a function of z by using, for example, the logit  $\mu = \exp(\delta z)/(1 + \exp(\delta z))$ . The corresponding likelihood function is again straightforward to derive.

### 4. An Economic Example

The Model and Estimation. The transitional model of Section 2 will be applied to the Rosen and Quandt (1978) model an aggregate labor market as modified by Romer (1980). The demand supply functions for labor are

$$\ln D_{t} = \alpha_{0} + \alpha_{1} \ln w_{t} + \alpha_{2} \ln x_{t} + \alpha_{3} t + u_{1} t$$
 (4-1)

$$\ln s_{t} = \beta_{0} + \beta_{1} \ln w_{nt} + \beta_{2} \ln P_{t} + u_{2t}$$
 (4-2)

where  $w_t$  is total wages and salaries in the U.S. private sector in 1958 dollars, divided by the number of private hours worked,  $w_{nt} = w_t (1-\theta_t)$  where  $\theta_t$  is the ratio of personal taxes to personal income,  $X_t$  is GNP in 1958 dollars and  $P_t$  is the potential number of hours worked and equals the civilian population between 16 and 64 multiplied by the average annual hours worked. All variables except the time trend were expressed in logarithms and the period covered was 1929 - 1973. The min condition is

$$lnQ_{t} = min(lnD_{t}, lnS_{t})$$
 (4-3)

where  $Q_{t}$  is total private hours worked. The wage adjustment equation is

$$lnw_t = lnw_{t-1} + \gamma_1 (lnD_t - lnS_t) + \gamma_2 + u_{3t}$$
 (4-4)

and differs from the customary adjustment equation by including the constant  $\gamma_2$  which permits wages to drift up even if excess demand is zero. The model of (4-1) to (4-4) differs from those considered by Rosen in Quandt by adapting Romer's (1980) exclusion of the nonlabor income variable from the supply equation. In addition to the straightforward disequilibrium model defined by (4-1) to (4-4) we also estimated an equilibrium counterpart consisting only of (4-1) and (4-2) in which both  $\ln D_t$  and  $\ln S_t$  are replaced by  $\ln Q_t$ . Finally, we

estimate the transitional model of Section 2. For all versions of the model the error terms were assumed to be distributed normally and independently one one another and of their own lagged values with mean zero and constant variances  $\sigma_1^2$ ,  $\sigma_2^2$ ,  $\sigma_3^2$ . Optimization was performed by the Davidon-Fletcher-Powell algorithm followed by the Quadratic Hill Climbing (GRADX) algorithm (Goldfeld and Quandt (1972)). Derivatives were evaluated numerically and asymptotic standard errors were estimated from the negative inverse of the Hessian of the loglikelihood function. The coefficient estimates are displayed in Table 1.

Table 1. Results for the Rosen-Quandt Model

	Equilibrium	Disequilibrium	Transitional
	Model	Model*	Model
a <sub>0</sub>	-1.031	455	452
	(.626)	(.066)	(.224)
$^{\alpha}$ 1	-1.125 (.209)	477 (.070)	424 (.224) .953
<sup>α</sup> 2	1.023	.948 (.010) 011	(.024) 013
α3	.005	(.002) -1.608	(.006) 1.625
β <sub>0</sub>	(1.433)	(.033) 189	(.035) 195
β <sub>1</sub>	.358	(.021)	(.055)
	(.077)	1.175	1.178
β <sub>2</sub>	004	(.006)	(.007)
	(.262)	.097	.081
Υ <sub>1</sub>		(.050) .036	.033
Y <sub>2</sub>	157.69	(.007)	(.007)
logL		193.40	189.16

\*Disequilibrium estimates from Romer (1980)

Results. The estimates from the equilibrium version are less sensible than from the disequilibrium version in that  $\alpha_3$  is positive and  $\beta_2$  of negligible magnitude. On the other hand, the equilibrium estimates of certain crucial parameters in the demand function are quite close to those of Lucas and Rapping (1970): the latter obtain  $\alpha_1 = -1.09$  and  $\alpha_2 = 1.00$  If the underlying production function giving rise to (4-1) were CES, we would have  $\alpha_1 = -\sigma$ ,  $\alpha_2 = (\sigma h + 1 - \sigma)/h$  and  $\alpha_3 = -\lambda \sigma (1 - \sigma)/h$  where  $\sigma$  is the elasticity of substitution, h measures returns to scale and  $\lambda$  is the rate of Hicks-neutral technological change. The implied values are shown in Table 2.

Table 2. Values of  $\sigma$ , h,  $\lambda$ 

	Equilibrium Model	Disequilibrium Model	Transitional Model
σ	1.125	.477	.424
h	1.225	.901	.918
λ	.043	.040	.049

Although the values of  $\lambda$  are broadly comparable for all three models, returns to scale are greater than one for the equilibrium model only. We also note that the likelihood function value for the disequilibrium model is substantially greater than for the equilibrium model and since the latter is (asymptotically) nested in the former (Quandt (1978)), we reject the equilibrium hypothesis using the critical values of the  $\chi^2$  - distribution at the .01 significance level for -2 log (likelihood ratio).

The coefficient estimates of the disequilibrium model and the transitional model are fairly similar. However, the loglikelihood is slightly smaller for the transitional model and the asympototic standard errors of the coefficents are, on the average, slightly larger. It is interesting to note that the disequilibrium and transitional models are not nested with respect to one another although, as is trivial to show, the equilibrium model is (asymptotically) nested within the transitional model. Hence, using the loglikelihood ratio we can reject equilibrium in favor of the transitional model but we cannot choose in a simple way between the disequilibrium and transitional models. The closeness of the disequilibrium and transitional model estimates is underscored by the fact that if the disequilibrium loglikelihood is evaluated at the parameter estimates that maximize the loglikelihood of the transitional model, its value declines from 193.40 to only 192.75. Another test of how close the two models' results are is to compare the implied unemployment rates. For the disequilibrium model these can be approximated from the reduced form predictions of log demand and log supply. In the transitional model we have, in effect, a probabilistic mixture of quantity predictions from the equilibrium and disequilibrium models. Denote the predictions of log demand and supply from the disequilibrium portion of the transitional model by d and s and

let the log quantity prediction from the equilibrium portion be  $\hat{q}$ . Then one might predict log demand and supply from the transitional model as  $\hat{l}_{nD} = \hat{q}_{Pr}\{E\} + \hat{d}_{(1-Pr}\{E\})$  and  $\hat{l}_{nS} = \hat{q}_{Pr}\{E\} + \hat{s}_{(1-Pr}\{E\})$  respectively. This procedure yields implicit unemployment rates  $(\hat{D}-\hat{s})/\hat{s}$ , displayed in Table 3, which are extremely similar to those found by Romer (1980) from the disequilibrium model alone and track the official unemployment rates very well.

Finally, it is interesting to examine the implied probabilities of equilibrium, Pr{E}, over the period, also in Table 3. In no year does this probability exceed .24. There is a strong relationship between the absolute value of unemployment and the probability of equilibrium. Whenever the proportionate excess supply is greater than about .08 or smaller than -.08, the probability of equilibrium is essentially zero. As the absolute value of excess supply falls toward zero, the probability of equilibrium rises nearly monotonically, being in the range of .18 to .24 when the absolute value of proportionate excess supply is less than .02.

## 5. Conclusions, Critiques and Extensions

Two approaches were examined that are capable of representing transitions from equilibrium states to disequilibrium states. One of these is based on the simple idea that the adjustment parameter  $\gamma$  (or  $\mu$  in the Bowden formulation may have different values at different times. The price-quantity observations in this approach are always determined by a disequilibrium model but at times this model may be indistinguishable from its equilibrium counterpart.

The alternative approach, which was applied to an aggregate labor market model, is based on the assumption that price change involves system costs and that the observed price and quantity are determined either from the equilibrium

Table 3. Unemployment Predictions and the Probability of Equilibrium

Year         Predicted Unemployment         of Equilibrium           1930         .142         .000           1         .221         .000           2         .334         .000           3         .336         .000           4         .230         .000           5         .224         .000           6         .171         .000           7         .178         .000           8         .218         .000           9         .193         .000           1940         .161         .000           1         .064         .028           2         .032         .239           3         .032         .239           3         .032         .239           3         .003         .001           6         .087         .004           7         .114         .001           8         .086         .004           9         .104         .005           1950         .037         .115           1         .006         .209           3         .031         .089           4			Predicted Probability
1930 1	Year	Predicted Unemployment	of Equilibrium
1	1020	.142	
2			
3			
4 .230 .000 5 .224 .000 6 .171 .000 7 .178 .000 8 .218 .000 9 .193 .000 1940 .161 .000 1 .064 .028 2 .032 .239 2 .033 .001 4 .130 .000 5 .001 6 .087 .004 7 .114 .001 8 .086 .004 9 .104 .005 1950 .037 .115 1 .004 .005 1950 .037 .115 2 .006 .209 3 .031 .089 4 .006 .209 3 .031 .089 4 .006 .209 3 .031 .089 4 .006 .220 2 .038 6 .000 .234 7 .008 .238 8 .007 7 .008 .238 8 .007 1 .008 .238 8 .037 .114 9 .006 1 .008 1 .240 1 .008 1 .009 1 .0006 .234 1 .0006 .234 1 .0006 .234 2 .012 .382 2 .031 3 .008 .238 3 .001 3 .008 .238 4 .009 1 .008 .234 5 .000 2 .234 6 .000 .234 6 .000 .234 7 .008 8 .238 8 .037 .114 9 .025 .174 1960 .041 .098 1 .098 1 .001 .023 3 .023 .233 4 .0011 .233 5 .023 .233 4 .0011 .233 5 .023 .235 9 .000 .235		.336	
5       .224       .000         6       .171       .000         7       .178       .000         8       .218       .000         9       .193       .000         1940       .161       .000         1       .064       .028         2       .032       .239         3       .001       .001         4      130       .000         5      100       .001         6       .087       .004         7       .114       .001         8       .086       .004         9       .104       .005         1950       .037       .115         1950       .037       .115         1       .004       .220         2       .006       .209         3       .031       .089         3       .031       .089         4       .005       .240         5       .012       .182         6       .000       .234         7       .008       .238         8       .037       .114         9       .025		.230	
6		.224	
7		.171	
8       .218       .000         9       .193       .000         1940       .161       .000         1       .064       .028         2       .032       .239         3      093       .001         4      130       .000         5      100       .001         6       .087       .004         7       .114       .001         8       .086       .004         9       .104       .005         1950       .037       .115         1       .004       .220         2       .006       .209         3       .031       .089         4       .006       .240         5       .012       .182         6       .002       .234         7       .008       .238         8       .037       .114         9       .023       .182         6       .001       .234         7       .008       .234         8       .025       .174         9       .023       .183         4       .011		.178	
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model or from the disequilibrium model, depending on which exhibits a smaller ex post price change. The maximum likelihood estimates from this model were quite close to those from the pure disequilibrium mode. Consistent with this result was the finding that the predicted probabilities of equilibrium were low, never exceeding .24. The following heuristic calculation may be noted. The average of the probabilities of equilibrium in the 17 years in which this probability is relatively large (which we take to be greater than .15) is .212. If one were to think of equilibrium as a binomial event, the upper 95% confidence limit for the number of "equilibrium years" is about 7. This is a heuristic upper bound for the number of equilibrium years for the entire period. Even more interesting is that the probability of equilibrium is very sensitive to the absolute value of excess supply. Whenever the excess supply, whether positive or negative, exceeds about 6-8 percent, the probability of equilibrium effectively falls to zero.

Both models may run into serious estimation difficulties. If, in fact, the equilibrium model was in force in all periods, the price-change minimizing model will have difficulty identifying the parameters  $\gamma$  and  $\sigma_3^2$ . In effect, likelihood maximization will attempt to make the value of  $\gamma$  large at which point the likelihood surface is likely to become flat with respect to  $\gamma$  and computation may break down. In the variable - $\mu$  model difficulties will arise if either pure model is in effect all the time. In these cases the likelihood function will be flat with respect to  $\delta$  and the resulting difficulties are not dissimilar to those of an ordinary logit model when the same alternative is chosen every time.

Various extensions of these models may be suggested. In the variable-  $\gamma$  or variable- $\mu$  model it would be desirable to have  $\gamma$  (or  $\mu$ ) to be a nondeterministic function, as in  $\gamma = \delta'z + \epsilon$  where  $\epsilon$  is an error term. This, unfortunately, does not appear to yield a tractable model in that the resulting error structure is hopelessly complicated and does not permit derivation of the pdf of the endogenous variables.

The model of Section 2 can be generalized in at least two ways. First, we may allow the system to be biased in favor of either pure model by the rule

Select equilibrium model if 
$$|p_{t-1}-p_t^*| < k_1|p_{t-1}-p_t^{**}| + k_2$$
  
Select disequilibrium model otherwise

where  $k_1$  and  $k_2$  are parameters to be estimated. No difficulties are introduced by this in principle. A second generalization emerges from noting that systems costs arise both from price change and from the presence of disequilibrium. If the equilibrium model is in effect, the price change cost is proportional to  $|p_{t-1}-p_t^*|$  and the disequilibrium cost is zero. If the disequilibrium model is in effect, the price change cost is proportional to  $|p_{t-1}-p_t^*|$  and the disequilibrium cost is proportional to  $|p_{t-1}-p_t^*|$  and the disequilibrium cost is proportional to  $|p_{t-1}-p_t^*|$ . Minimizing total systems cost then implies, as a first approximation,

Select equilibrium model if 
$$|p_{t-1}-p_t^*| < |p_{t-1}-p_t^{**}| + k |D_t-S_t|$$
  
Select disequilibrium model otherwise

No new principles are involved, but the necessary calculations become more difficult. In particular, Equ. (2-8) becomes more cumbersome to evaluate. Future work is needed to see if such a formulation is tractable in practice.

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