OPTIMAL CURRENCY DIVERSIFICATION FOR A

CLASS OF RISK-AVERSE INTERNATIONAL INVESTORS

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Abstract

This paper derives the optimal consumption and portfolio rules for an international investor with constant expenditure shares on goods produced in N countries and constant relative risk aversion. The index of value obtained from the consumption rule is used to generate real returns on N different currencies in terms of their purchasing power over goods. The portfolio rule is expressed in terms of the changes in N currency prices and exchange rates, which are assumed to be Itô processes.

Real return differentials are shown to depend on consumption preferences so that the effect of a shift in these on portfolio proportions depends on the relative degree of risk aversion. Special cases like restricting the stochastic processes to Brownian motion, imposing purchasing power parity and assuming that inflation is anticipated are also discussed.

1. Applications of the theory of optimum consumption and portfolio rules in continuous time to international finance have generally assumed that real returns are Brownian motion and that purchasing power parity obtains so that there is only one good. Kouri (1975) derived the basic portfolio decomposition in a two country model with different consumption preferences in each country and exchange rate and price changes generated by Brownian motion processes. Kouri-Macedo (1978) introduced the notion of an international investor consuming N composite goods produced in N different countries and derived a time-invariant portfolio rule. This paper derives the consumption and portfolio rules for an international investor consuming $\sum_i^N M_i$ goods purchased in N different countries. It therefore retrieves the index of value of the international investor from the consumption rule when the expenditure shares and relative risk aversion are constant[1] and generalizes the portfolio rule to a dynamic context. Another contribution is the analysis of the effects of an increase in consumption demand on asset demands. While the computational difficulties implied by the departure from Brownian motion are beyond the scope of this paper, evidence gathered elsewhere is used to suggest the usefulness of the approach.[2]

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[1] Stulz (1980) has a general model where constant expenditure shares are not assumed.

[2] See Macedo (1979, 1980) and Chow (1979). Dornbusch (1980a, 1980b) has a survey and applications of the two country Brownian motion model. Ross-Walsh (1980), however, conclude their "pedagogic" survey "on a note of skepticism."

2. Consider the intertemporal consumption-investment optimization problem of an individual who consumes goods $j=1,\ldots,M_i$ produced in $i=1,\ldots,N$ countries and has a portfolio of one period bonds denominated in the currencies of these N countries, with a price of unity and a certain nominal return R_i.

If the utility function is Cobb-Douglas, real consumption expenditure is given by:

$$(1) \quad V = E_k Q_k$$

where

$$(2) \quad E_k = \sum_i^N \sum_j^{M_i} P_{ij} S_{ki} X_{ij}$$

and

$$(3) \quad Q_k = \prod_i^N \prod_j^{M_i} (P_{ij} S_{ki})^{-\alpha_i \beta_j^i} \quad.$$

Real expenditure net of real interest payments is

$$(4) \quad E = V - \sum_{i=1}^N N_i Q_i R_i$$

where E_k is given by (2) above

        N_i are holdings of currency i bonds

and Q_i are the stochastic purchasing powers of currency i. Given prices, real expenditure in a given period is equal to the change in real wealth during that period, obtained by reshuffling the asset portfolio. When the period becomes infinitesimally short, Merton (1971) has shown that the instantaneous rate of consumption expenditure is

$$(5) \quad -Edt = \sum_{i=1}^{N} dN_i dQ_i + \sum_{i=1}^{N} dN_i Q_i \quad .$$

At each instant in time, the individual's real wealth is

$$W(t) = \sum_{i=1}^{N} N_i(t) Q_i(t) \quad ,$$

so that, by Itô's lemma, the change in wealth is

$$(6) \quad dW = \sum_{i}^{N} N_i dQ_i + \sum_{i}^{N} dN_i Q_i + \sum_{i}^{N} dN_i dQ_i \quad .$$

This change has a capital gains component $(\Sigma N_i dQ_i)$ and a portfolio reshuffling component which is equal by (5) to real expenditure. Using the definition of real expenditure net of interest payments in (4), and substituting in (6) we obtain:

$$dW = \sum_{i}^{N} N_i dQ_i + (\sum_{i}^{N} N_i Q_i R_i - V)dt \quad .$$

Defining portfolio proportions x_i such that $\sum^{N} x_i = 1$ and real returns r_i we have

$$(7) \quad dW = W \sum^{N} x_i r_i dt - Vdt$$

where $\quad x_i = \dfrac{N_i Q_i}{W}$

$$r_i dt = R_i dt + \frac{dQ_i}{Q_i} \quad .$$

Denoting logs by lower case letters we use Itô's lemma to find the proportional change in the purchasing power of currency k as given by

$$(8) \quad dq_k = -ds_k - \sum_{i}^{N} \sum_{j}^{M_i} \alpha_i \beta_j^i dp_{ij}^* + \frac{1}{2}(ds_k)^2 + \frac{1}{2} \sum_{i}^{N} \sum_{j}^{M_i} \sum_{\ell}^{N} \sum_{m}^{M_e} \alpha_i \beta_j^i \alpha_\ell \beta_m^\ell dp_{ij}^* dp_{\ell m}^*$$

$$+ \sum_{k}^{M} \sum_{j}^{M_k} \alpha_k \beta_j^k ds_k dp_{kj}^*$$

where $p_{ij}^* = p_{ij} - s_i$ $\quad i=1,\ldots,N-1$; $\quad j=1,\ldots,M_i$

is the N currency price of good ij ;

and α_i and β_j^i are the same as in (3) above.

It is convenient to eliminate x_N from (7) using the constraint that portfolio proportions sum to unity. The change in wealth becomes

$$(9) \quad dW = W \sum_{i=1}^{N-1} x_i \hat{r}_i dt + Wr_N dt - Vdt$$

where $\hat{r}_i = r_i - r_N$.

We now parametrize the utility function of the individual to have constant relative risk aversion $1-\gamma$, so that the objective function of our problem is

$$(10) \quad \mathcal{U}^k = E_0 \int_0^T \frac{1}{\gamma} U^k(t)^\gamma dt$$

where E_0 is the expectations operator conditional on information available at time 0.

U^k in (10) is Cobb-Douglas as mentioned and can be written as:

$$U^k = \prod_{i=1}^{N} \prod_{j=1}^{M_i} X_{ij}^{\alpha_i \beta_j^i} \quad ,$$

where the weights are the same as in (3).

We have assumed that terminal wealth is zero but a bequest function with elasticity γ with respect to terminal wealth would not change the results. We have also assumed that the system is autonomous, so that the dependence of $U^k(t)$ on time comes only through the date of consumption X_{ijt}. It would be easy to introduce a discount factor ρ on $U^k(t)$ and write $\exp(-\rho t)U^k(t)$ in (10).

The problem of maximizing (10) subject to (9) given the stochastic processes generating the purchasing powers of the currencies or their components, prices and exchange rates, is solved by the technique of stochastic dynamic programming. Taking as state variables wealth, and the components of Q_i, namely the exchange rates, S_i, and prices of the goods in terms of currency N, denoted by P_{ij}^* [1/], we define the Bellman function as

$$J(W, S_i, P_{ij}^*) = \max E_t \int_t^T \frac{1}{\gamma} \prod_j^N \prod_i^{M_i} X_{ij}^{\alpha_i \beta_j^i \gamma}(\tau) \, d\tau$$

where the expectation at time t is conditional upon the steady-state values of the $\sum_i^N M_i + N$ state variables

$$W(t) \quad = \quad W$$

$$S_i(t) \quad = \quad S_i \qquad i=1,\ldots,N-1$$

$$P_{ij}^*(t) \quad = \quad P_{ij}^* \qquad j=1,\ldots,M_i$$

[1/] It is straightforward but tedious to present equivalent derivations in terms of bilateral rates and domestic currency prices. See Macedo (1979), Appendix I, where references to the stochastic control literature can also be found.

Since U is strictly concave in X_{ij}, by Bellman's theorem there exists optimal portfolio proportions, $\tilde{x}_j$, and rates of consumption expenditure, $\tilde{X}_{ij}$, such that $\tilde{\Phi} = 0$, where Φ is defined as

(11) $\Phi (x_j, X_{ij}, W, S_i, P_{ij}^*) = \mathcal{U} + \mathcal{L}(J)$;

where $\mathcal{U}$ is given by (10)

and $\mathcal{L}$ is the Dynkin operator, giving the "average" expected rate of change of J.

To simplify notation we will assume henceforth that national outputs are composite goods, so that in contrast with (8) above the difference in proportional changes in purchasing power between currencies k and N and the proportional change in the purchasing power of currency N are respectively given by

$$dq_k - dq_N = -ds_k + \frac{1}{2} ds_k^2 + \sum_j^N \alpha_j ds_k dp_j^* \quad ;$$

(12)

$$dq_N = -\sum_j^N \alpha_j dp_j^* + \frac{1}{2} \sum_j^N \sum_\ell^N \alpha_j \alpha_\ell dp_j^* dp_\ell^*$$

where $p_i^* = \sum_j^{M_i} \beta_j^i p_{ij}^*$.

Furthermore we will assume that ds_i and dp_j^* are generated by the following stochastic processes[1]:

(13) $ds_i = \pi_i (S_i, P_j^*) dt + \sigma_i (S_i, P_j^*) dz_i \quad ;$

(14) $dp_j^* = \mu_j (S_i, P_j^*) dt + \delta_j (S_i, P_j^*) du_j$

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[1] See Macedo (1979), Appendix 2 for the case where the price of each good is generated by a different stochastic process.

where π_i and μ_i are the instantaneous conditional expected proportional changes per unit of time;

σ_i^2 and δ_i^2 are the respective instantaneous conditional variances per unit of time;

and dz_i and du_i are Wiener processes.

Using Itô's lemma, we express the differential of J as

$$(15) \quad dJ = \frac{\partial J}{\partial W} dW + \sum_i^{N-1} \frac{\partial J}{\partial S_i} dS_i + \sum_j^N \frac{\partial J}{\partial P_j} dP_j^* + \frac{1}{2} \frac{\partial^2 J}{\partial W^2} dW^2$$

$$+ \frac{1}{2} \sum_i^{N-1} \sum_j^{N-1} \frac{\partial^2 J}{\partial S_i \partial S_j} dS_i dS_j + \frac{1}{2} \sum_i^N \sum_j^N \frac{\partial^2 J}{\partial P_i^* \partial P_j^*} dP_i^* dP_j^*$$

$$+ \sum_i^{N-1} \sum_j^N \frac{\partial^2 J}{\partial S_i \partial P_j^*} dS_i dP_j^* + \sum_i^{N-1} \frac{\partial^2 J}{\partial S_i \partial W} dS_i dW + \sum_j^N \frac{\partial^2 J}{\partial P_j^* \partial W} dP_j^* dW \quad .$$

Substituting from (9), (13) and (14) into (15), we get

$$\mathscr{L}(J) = J_W \left(\sum_i^{N-1} x_i \hat{r}_i + \hat{r}_N - V \right) + \sum_i^{N-1} J_{S_i} \pi_i + \sum_j^N J_{P_j^*} \mu_j$$

$$+ \frac{1}{2} J_{WW} \left(\sum_i^{N-1} \sum_j^{N-1} x_i x_j \sigma_{ij} + \sum_i^N \sum_j^N \alpha_i \alpha_j \delta_{ij} + 2 \sum_i^{N-1} \sum_j^N x_i \alpha_j \theta_{ij} \right)$$

$$+ \frac{1}{2} \sum_i^{N-1} \sum_j^{N-1} J_{S_i S_j} \sigma_{ij} + \frac{1}{2} \sum_i^N \sum_j^N J_{P_i^* P_j^*} \delta_{ij} + \sum_i^{N-1} \sum_j^N J_{S_i P_J^*} \theta_{ij}$$

$$+ \sum_i^{N-1} J_{S_i W} x_i \left(\sum_j^{N-1} x_j \sigma_{ij} + \sum_j^N \alpha_j \theta_{ij} \right) + \sum_i^N J_{P_i^* W} \alpha_i \left(\sum_j^N \alpha_j \delta_{ij} + \sum_j^{N-1} x_j \theta_{ij} \right) \quad .$$

Where, to ease the notational burden, we have defined semi-elasticities with respect to J as subscripts, for example

$$\frac{\partial J}{\partial W} W = J_W$$

$$\frac{\partial^2 J}{\partial P_i^* \partial W} P_i^* W = J_{P_i^* W}$$

and θ_{ij} is the covariance between exchange rate i and N currency price j, defined in terms of (13) and (14) as

$$\theta_{ij} = \rho_{ij} \delta_i \delta_j$$

where ρ_{ij} is the instantaneous correlation coefficient between the two Wiener processes dz_i and du_j.

The first order conditions are obtained by differentiating Φ in (11) with respect to good ℓm and asset m. They are:

$$(16) \quad \frac{\partial \Phi}{\partial X_{\ell m}} = \gamma \alpha_m \beta_\ell^m X_{\ell m}^{-1} - \frac{\partial J}{\partial W} P_{\ell m} S_{\ell k} Q_k = 0 \quad ;$$

$$(17) \quad \frac{\partial \Phi}{\partial x_m} = J_W \hat{r}_m + J_{WW} \left(\sum_i^{N-1} x_i \sigma_{mi} + \sum_j^N \alpha_j \theta_{mj} \right) + \sum_j^{N-1} J_{S_i W} \sigma_{im}$$

$$+ \sum_j^N J_{P_j^* W} \theta_{jm} = 0 \quad .$$

From the consumption rule in (16) demand functions are easily derived. Adding up the $\sum_i^N M_i$ conditions we have

$$\gamma = \frac{\partial J}{\partial W} E_k Q_k$$

which upon substitution in (16) yields a demand function for good $X_{\ell m}$

$$(18) \quad \tilde{X}_{\ell m} = \alpha_\ell \beta_m^\ell E_k (P_{\ell j} S_{k\ell})^{-1} \quad .$$

Substituting these back in the utility function we obtain an indirect utility function

$$V = \frac{1}{\gamma} (E_k Q_k)^\gamma \quad .$$

This function is still separable but the utility of one extra unit of currency k is given by Q_k times the marginal utility of real income, so that, instead of (4), we have

$$\frac{\partial V}{\partial E_k} = V^{\gamma-1} Q_k \quad .$$

Next we write the $N-1$ first order conditions in (17) in matrix form:

$$(19) \quad J_W \hat{r} + J_{WW}(S\underset{\sim}{x} + \Theta\alpha) + S J_{SW} + \Theta J_{P}{}^*{}_W = 0$$

where $\hat{r} = [\hat{r}_1 \qquad \hat{r}_{N-1}]'$

$\underset{\sim}{x} = [x_1 \qquad x_{N-1}]'$

$\alpha = [\alpha_1 \qquad \alpha_N]'$

$J_{SW} = [J_{S_1 W} \qquad J_{S_{N-1} W}]'$

$J_{P}{}^*{}_W = [J_{P_1^* W} \qquad J_{P_N^* W}]'$

$$
S \quad = \quad \begin{bmatrix} \sigma_{11} & & & \sigma_{1N-1} \\ & & & \\ & & & \\ \sigma_{N-11} & & & \sigma_{N-1N-1} \end{bmatrix}
$$

$$
\Theta \quad = \quad \begin{bmatrix} \theta_{11} & & & \theta_{1N} \\ & & & \\ & & & \\ \theta_{N-11} & & & \theta_{N-1N} \end{bmatrix}
$$

and 0 is a $N-1$ column vector of zeros.

Dividing each element of the vectors of the wealth cross effects of exchange rates and N currency price changes on utility by the marginal utility of wealth we define the semi elasticities:

$$
J_{S_i} = \frac{\partial^2 J}{(\partial S_i / S_i) \partial W} \quad \frac{\partial J}{\partial W} = \frac{\partial J}{\partial S_i / S_i} \quad i = 1, \ldots, N-1
$$

$$
J_{P_j^*} = \frac{\partial J}{\partial P_j^* / P_j^*} \quad j = 1, \ldots, N \quad \bullet
$$

From the definition of relative risk aversion, we obtain:

$$
1 - \gamma = - \frac{J_{WW} W}{J_W} \quad .
$$

We also decompose the Θ matrix of covariance between exchange rates and N currency prices into the difference between a $N-1$ by N matrix of covariance between exchange rates and domestic currency prices, denoted by Ψ, and the variance covariance matrix of exchange rate changes, denoted above by S, augmented by a column vector of zeros.

$$\underset{\substack{\Theta \\ \text{N-1xN}}}{\Theta} = \underset{\substack{\Psi \\ \text{N-1xN}}}{\Psi} - \underset{\substack{[S \mid 0] \\ \text{N-1xN-1} \quad \text{1xN-1}}}{[S \mid 0]}$$

Taking these definitions into account, we solve for $\underset{\sim}{x}$ in (19) and obtain the portfolio rule

$$(20) \quad \underset{\sim}{x} = (I - S^{-1}\Psi)(\alpha - \frac{1}{1-\gamma} J_p*) + \frac{1}{1-\gamma} (S^{-1}\hat{r} + J_s)$$

$$x_N = 1 - \underset{\sim}{e}'\underset{\sim}{x}$$

where $\tilde{I} = [I \quad 0]$ is the $N-1$ identity matrix with an N column of zeros;

$\underset{\sim}{e}$ is a $N-1$ column vector of ones;

$$J_p* = [J_{p*_1} \qquad J_{p*_N}]'$$

$$J_S = [J_{S_1} \qquad J_{S_{N-1}}]'$$

When risk aversion is infinite $(\gamma \to -\infty)$ the portfolio is given by

$$(21) \quad \underset{\sim}{x} = \underset{\sim}{\alpha} - S^{-1}\Psi\alpha$$

where $\underset{\sim}{\alpha} = (\alpha_1 \qquad \alpha_{N-1})'$

so that, when inflation is fully anticipated and Ψ is a zero matrix, the portfolio proportions are given by the expenditure shares and an increase in the share of goods from country i in expenditures implies an equal increase in the share of currency i in the portfolio. The larger the variance of inflation relative to the variance of exchange rate changes the less an increase in α_ℓ will be reflected in an increase in x_ℓ.

When risk aversion is less than infinite, the optimal portfolio in (20) is divided into a minimum variance portfolio x^m which depends on prices and exchange rates and on the expenditure shares, corrected for risk adjusted effects of changes in N currency prices on utility and a speculative portfolio x^s which only depends on exchange rate changes and real returns, adjusted for the effects of changes in exchange rates on utility and for risk as captured by γ . As in domestic finance, the minimum variance portfolio is the capital position and the speculative portfolio has zero net worth.

3. In these cases it is useful to make it explicit that the vector of mean real return differentials, as shown in (12), depends by Jensen's inequality on the variance of the exchange rate and on the covariance between exchange rates and N currency prices. We write, therefore,

$$(22) \quad \hat{r} = \hat{R} + \Theta\alpha$$

where $\quad \hat{R} = R - R_N \underset{\sim}{e} - \pi + \frac{1}{2} \hat{S}$

$$\pi = [\pi_1 \qquad \pi_{N-1}]'$$

and $\quad \hat{S} = \begin{bmatrix} \sigma^2 & 0 \\ 0 & \sigma^2_{N-1} \end{bmatrix}$

Substituting (22) into the first equation in (20) we obtain

$$(23) \quad \underset{\sim}{x} = \frac{1}{1-\gamma} [(S^{-1}\psi - \tilde{I})(\gamma\alpha + J_p*) + S^{-1}\hat{R} + J_S].$$

The effect is therefore to make the minimum variance portfolio depend on relative risk aversion. If inflation is anticipated and ψ is

a zero matrix we have

$$(24) \quad x = \frac{-\gamma}{1-\gamma} \underset{\sim}{\alpha} + \frac{1}{1-\gamma} (S^{-1}\hat{R} + J_S) \ .$$

If the individual is more risk averse than the Bernouilli investor ($\gamma < 0$) the minimum variance portfolio is given by less than expenditure shares. For example, if $\gamma = -1$, what might be called the "Samuelson presumption"[1], we have:

$$(25) \quad \underset{\sim}{x}^P = \frac{1}{2} \underset{\sim}{\alpha} + \frac{1}{2} (S^{-1}\hat{R} + J_S)$$

The Bernouilli investor has a portfolio independent of preferences

$$(26) \quad x^B = S^{-1}\hat{R} + J_S \ .$$

The individual less risk averse than the Bernouilli, $0 < \gamma < 1$, for example the Cramer investor ($\gamma = \frac{1}{2}$)[2], holds a minimum variance portfolio short in the goods consumed:

$$(27) \quad \underset{\sim}{x}^C = -\underset{\sim}{\alpha} + 2(S^{-1}\hat{R} + J_S)$$

Returning to (20), we use the fact that the sum of the minimum variance portfolio proportions for the N-1 currencies is equal to one minus the proportion for the Nth currency and we can write the minimum variance portfolio for the N assets as:

1/ See Macedo (1980).

2/ See Macedo (1980).

$$(28) \quad x^m = \alpha + \frac{1}{1-\gamma} \begin{pmatrix} \tilde{J}_{P^*} \\ -\underset{\sim}{e}'\tilde{J}_{P^*} \end{pmatrix} - \Phi[\alpha + \frac{1}{1-\gamma} J_{P^*}]$$

where $\quad \Phi = \begin{bmatrix} S^{-1}\psi \\ \text{-------} \\ -\underset{\sim}{e}'S^{-1}\psi \end{bmatrix}$

and $\quad \tilde{J}_{P^*} = [J_{P^*_1} \qquad J_{P^*_{N-1}}]'$.

It is easy to see that the requirement that $\sum\limits_i^N x_i^m = 1$ is met and, in particular, that

$e'\Phi = 0$

where $\quad e$ is a N column vector of ones

and $\quad 0$ is a N row vector of zeros.

The speculative portfolio for the N assets is similarly constructed by imposing the requirement that the matrix of own and cross effects is symmetric and that rows and column sum to zero. It can be written as

$$(29) \quad x^S = \Sigma r + \begin{pmatrix} J_S \\ -\underset{\sim}{e}'J_S \end{pmatrix}$$

where $\quad \Sigma = \begin{bmatrix} S^{-1} & \vdots & -S^{-1}\underset{\sim}{e} \\ \text{-------} & \vdots & \text{-------} \\ -\underset{\sim}{e}'S^{-1} & \vdots & \underset{\sim}{e}'S^{-1}\underset{\sim}{e} \end{bmatrix}$

$r = [r_1 \qquad r_N]$.

Again it is easy to see that the requirement $\sum\limits_i^N x_i^S = 0$ is met

and in particular, that

$$e'\Sigma = 0$$

$$\Sigma e = 0' \quad .$$

The total portfolio is therefore given by

$$(30) \quad \underset{\sim}{x} = (I-\Phi)\alpha + \frac{1}{1-\gamma} \left[\Sigma r - \Phi J_P* + \begin{pmatrix} \tilde{J}_P* + J_S \\ -\underset{\sim}{e}'J_P* - \underset{\sim}{e}'J_S \end{pmatrix} \right] \quad .$$

If prices and exchange rates are stationary and lognormally distributed, so that π_i, μ_j, σ_i and δ_j in (13) and (14) above are given constants, wealth becomes the only state variable in (11) so that J_P* and J_S in (30) become zero vectors and the rule can be written as:

$$(31) \quad x = (I-\Phi)\alpha + \frac{1}{1-\gamma} \Sigma r \quad .$$

Consider further that inflation rates are known. Then, as in (24) through (27) above, Ψ, and thus Φ, become zero matrices and the minimum variance portfolio is given by expenditure shares.

$$(32) \quad x^m = \alpha$$

If purchasing power parity holds, the Ψ matrix, defined above as the covariance matrix between prices and exchange rates, decomposes into the variance-covariance matrix of exchange rate changes and a matrix formed by the last column of the Ψ matrix, denoted by Ψ_N :

$$\Psi = [S \mid 0] - \Psi_N e'$$

where $-\Psi_N = (\Psi_{1N} \quad \Psi_{N-1N})'$ is minus the covariance vector between exchange rates and N country prices.

Therefore the minimum variance portfolio becomes

$$(33) \quad x^m = S^{-1} \psi_N e' \alpha = S^{-1} \psi_N \quad .$$

It is independent of preferences since there is in fact only one good. Also because of this, if inflation in the Nth country is known and $\zeta_N = 0$, then ψ_N is a zero vector and the minimum variance portfolio vanishes.[1] If inflation is only random in the Nth country all elements of ψ_N reduce to ζ_N^2 and the minimum variance portfolio can be written as

$$(34) \quad x^m = \zeta_N^2 S^{-1} e \quad .$$

4. This paper has derived the optimal consumption and portfolio rules for an international investor with constant expenditure shares α_i and β_j^i and constant relative risk aversion $1-\gamma$. The index of value obtained from the consumption rule was used to obtain real returns on different currencies in terms of their purchasing power over goods. The portfolio rule was expressed in terms of the determinants of the purchasing powers, exchange rate changes and prices and the connection with time invariant portfolios was stated (Section 2).

Recasting this framework in terms of nominal returns suggests the importance of the assumptions about the relative degree of risk aversion of the investor in assessing the effects of shifts in expenditure patterns for the stability of the foreign exchange market (Section 3).

An explicit solution of the Bellman function and the introduction of asset supplies are topics for future research.

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[1] See Kouri (1975) reproduced in (1977) and Fama-Farber (1979) for an analysis of this case with a richer menu of assets.

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