MODELS AND ESTIMATION OF DISEQUILIBRIUM

FOR CENTRALLY PLANNED ECONOMIES

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1. Introduction

Since the seminal paper by Fair and Jaffee (1972), numerous theoretical and applied contributions have been made to the econometrics of disequilibrium. The bulk of the applied literature deals with free-market economies and relatively little is known about suitable disequilibrium formulations for centrally planned economies (CPEs) (Hartley (1976)). In analyses of CPEs special importance is usually attached to the planned level of output as a factor in influencing market relationships (Kornai (1971), Portes (1981), Portes and Winter (1977)). In spite of this, empirically oriented disequilibrium models for CPEs often ignore the planned values of output (Howard (1976), Nissanke (1979), Portes and Winter (1980)).

The purpose of this paper is to discuss several disequilibrium models in which the planned level of output plays an important part. Section 2 introduces a simple disequilibrium model and discusses its estimation. Section 3 extends this model and examines the coherency of the extension. Section 4 considers some alternative models and their estimation. Section 5 contains a discussion of models in which prices are allowed to adjust and Section 6 contains some conclusions and suggestions for further research

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2. A Basic Model

In order to specify an aggregate disequilibrium model for a planned economy, we require aggregate demand and supply functions

$$D_t = \beta_1' x_{1t} + u_{1t} \qquad (2\text{-}1)$$

$$S_t = \beta_1' x_{2t} + u_{2t} \qquad (2\text{-}2)$$

where D_t and S_t are the effective demand and supply of goods respectively, β_1 and β_2 are parameters, x_{1t} and x_{2t} are exogenous variables and u_{1t} and u_{2t} are error terms. The exogenous variables presumably include the prices of goods. Since prices do not clear markets, the observed quantity of goods traded, Q_t , is given by

$$Q_t = \min(D_t, S_t) \qquad (2\text{-}3)$$

In a free-market context Equs. (2-1) to (2-3) are usually complemented by a price-adjustment equation (Rosen and Quandt (1978)). In the present context prices are taken to be fully rigid. However, planned output levels do adjust in response to nonzero excess demands. In the present section the plan adjustment equation is taken to be[1]

$$Q_t^* = Q_t + \gamma(D_t - S_t) \qquad (2\text{-}4)$$

where Q_t^* represents the planned level of transactions as determined by the Central Planning Authority, and where $\gamma > 0$.

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1. It is straightforward but notationally somewhat cumbersome to allow for asymmetrical adjustment by letting $\delta = \delta_1$ if $D_t < S_t$ and $\delta = \delta_2$ otherwise. This refinement is ignored in what follows.

D_t and S_t are customarily assumed to be unobserved. Without Equ. (2-4) the model is that of Portes and Winter (1980) and can be estimated by maximizing the appropriate likelihood function (Maddala and Nelson (1974)). Since both Q_t and Q_t^* are observed, the inclusion of (2-4) provides an excess demand indicator which permits a partition of a sample of observations into periods when $D_t > S_t$ and periods when $D_t < S_t$. This allows a somewhat simpler method of estimation than maximum likelihood which is similar to the two-stage least squares method of Amemiya (1974) designed for a disequilibrium model with a non-stochastic price adjustment equation. Defining $\gamma = 1/\delta$, it is easy to verify by combining (2-1) to (2-4) that

$$Q_t = \beta_1' x_{1t} + u_{1t} \tag{2-5}$$

$$Q_t = \frac{\beta_2' x_{2t}}{1+\delta} + \frac{\delta}{1+\delta} Q_t^* + \frac{u_{2t}}{1+\delta} \tag{2-6}$$

when $Q_t^* < Q_t$ and

$$Q_t = \frac{\beta_1' x_{1t}}{1-\delta} - \frac{\delta}{1-\delta} Q_t^* + \frac{u_t}{1-\delta} \tag{2-7}$$

$$Q_t = \beta_2' x_{2t} + u_{2t} \tag{2-8}$$

when $Q_t^* > Q_t$. Hence we can write

$$Q_t = \tilde{\beta}_1' x_{1t} + \tilde{\delta}_1 Q_t^{*-} + \tilde{u}_{1t} \tag{2-9}$$

$$Q_t = \tilde{\beta}_2' x_{2t} + \tilde{\delta}_2 Q_t^{*+} + \tilde{u}_{2t} \tag{2-10}$$

where

$$Q_t^{*-} = \begin{cases} -Q_t^* & \text{if} \quad Q_t^* > Q_t \\ 0 & \text{otherwise} \end{cases} \qquad (2\text{-}11)$$

$$Q^{*+} = \begin{cases} 0 & \text{if} \quad Q_t^* > Q_t \\ Q_t^* & \text{otherwise} \end{cases} \qquad (2\text{-}12)$$

and where

$$\tilde{\beta}_1 = \begin{cases} \beta_1 & \text{if} \quad Q_t^* < Q_t \\ \beta_1/(1-\delta) & \text{otherwise} \end{cases}$$

$$\tilde{\delta}_1 = \begin{cases} \delta & \text{if} \quad Q_t^* < Q_t \\ \delta/(1-\delta) & \text{otherwise} \end{cases}$$

$$\tilde{\beta}_2 = \begin{cases} \beta_2/(1+\delta) & \text{if} \quad Q_t^* < Q_t \\ \beta_2 & \text{otherwise} \end{cases}$$

$$\tilde{\delta}_2 = \begin{cases} \delta/(1+\delta) & \text{if} \quad Q_t^* < Q_t \\ \delta & \text{otherwise} \end{cases}$$

$$\tilde{u}_{1t} = \begin{cases} u_{1t} & \text{if} \quad Q_t^* < Q_t \\ u_{1t}/(1-\delta) & \text{otherwise} \end{cases}$$

$$\tilde{u}_{2t} = \begin{cases} u_{2t}/(1+\delta) & \text{if} \quad Q_t^* < Q_t \\ u_{2t} & \text{otherwise} \end{cases}$$

Whereas the equations corresponding to (2-9) and (2-10) in the case of the Amemiya estimator are nonlinear in variables but linear in parameters (taking $1/\gamma$ as the parameter of interest), (2-9) and (2-10) are nonlinear in both and also involve heteroscedasticity. Now let X_1 and X_2 denote the matrices of observations on x_{1t}, x_{2t} $(t = 1, \ldots, T)$ respectively, Q, Q^{*-} and Q^{*+} the vectors of observations on Q_t, Q_t^{*-} and Q_t^{*+} and U_1, U_2 the vectors with elements u_{1t}, u_{2t}. Define Δ_1 and Δ_2 as $T \times T$ diagonal matrices with t^{th} diagonal elements given by

$$\Delta_{1tt} = \begin{cases} 1 & \text{if } Q_t^* < Q_t \\ 1/(1-\delta) & \text{otherwise} \end{cases}$$

$$\Delta_{2tt} = \begin{cases} 1/(1+\delta) & \text{if } Q_t^* < Q_t \\ 1 & \text{otherwise} \end{cases}$$

Then (2-9) and (2-10) can be written as

$$\Delta_1^{-1} Q = X_1 \beta_1 + \delta Q^{*-} + U_1 \tag{2-13}$$

$$\Delta_2^{-1} Q = X_2 \beta_2 + \delta Q^{*+} + U_2 \tag{2-14}$$

For either (2-13) or (2-14) the following iterative procedure becomes possible.

1. Choose an initial value of δ and substitute on the left of (2-13) (or (2-14)).

2. Estimate β_1 and δ by Amemiya's two-stage least squares. This

involves regressing Q^{*-} on all the exogenous variables, replacing Q^{*-} by $\hat{Q}^{*-} = Z(Z'Z)^{-1}Z'Q^{*-}$ (where $Z = [X_1\ X_2]$), and computing

$$\begin{bmatrix} \hat{\beta}_1 \\ \hat{\delta} \end{bmatrix} = [(X_1\ \hat{Q}^{*-})'(X_1\ \hat{Q}^{*-})]^{-1}(X_1\ \hat{Q}^{*-})'\ ^{-1}\Delta_1^{-1}Q$$

and similarly for (2-14).

3. The new value of γ computed in step 2 can now be substituted on the left in (2-13) and (2-14) and the iterations continued until convergence occurs.

This produces a nonlinear two-stage least squares estimator which is consistent but not efficient since it exploits neither the across-equations restrictions nor the covariance structure of error terms. To remedy this, a nonlinear three-stage least squares estimator may be defined as in Jorgenson and Laffont (1974).

3. Extension and Coherency

There are two principal directions in which the model given by (2-1) through (2-4) needs to be extended. (1) The plan level Q_t^* may be understood to refer to the stated plan for either the current or subsequent periods (Charemza (1980)). To the extent that it refers to the current period, which is the case considered in the present section, Q_t^* should enter the supply and the demand functions. Stated plan levels will directly affect capacities and production schedules and it is therefore plausible that an increase in planned transactions will have a positive effect on effective supply. Somewhat less immediate but not unlikely is that Q_t^* will also have a positive effect on demand.

A higher plan level signals to consumers that they are less likely to be rationed. It has been shown that, if a utility cost is associated with attempting to make a purchase, a reduction in the probability of rationing has a positive effect on demand (Eaton and Quandt (1979)). In addition, higher plan levels may directly affect the demand for inputs. (2) Equ. (2-4) requires that planners adjust the plan in a precise proportion to excess demand. Since neither demand nor supply is observed, this requires heroic assumptions concerning the planners' knowledge.

In order to remedy these problems we reformulate the model as follows:

$$D_t = \beta_1' x_{1t} + \alpha_1 Q_t^* + u_{1t} \tag{3-1}$$

$$S_t = \beta_2' x_{2t} + \alpha_2 Q_t^* + u_{2t} \tag{3-2}$$

$$Q_t = \min(D_t, S_t) \tag{3-3}$$

$$Q_t^* = Q_t + \gamma(D_t - S_t) + u_{3t} \tag{3-4}$$

where u_{3t} is an additional error term and where α_1 , α_2 are positive parameters.

Although this is superficially similar to a classical single-market disequilibrium model with price adjustment, it is in practice radically different. The reason is that the variable Q_t^* which enters (3-1) and (3-2) is not determined on the basis of lagged Q but current Q. The consequence is that model (3-1) through (3-4) may fail to be coherent, even though we are dealing with a single market model.[2] We now examine the question of coherency.

Coherency will hold if there is a one-to-one and onto mapping from u's to endogenous variables; in other words, if there exists a well-defined reduced

2. Coherency conditions are stated in numerous forms. See Ito (1980), Gourieroux, Laffont, Montfort (1980), Amemiya (1977).

form. Denoting by $y_t' = (D_t \; S_t \; Q_t^*)$ and by $V_t' = (\beta_1' x_{1t} + u_{1t} \, , \, \beta_2' x_{2t} + u_{2t} \, , \, u_{3t})$, we can write the two regimes as follows.

Regime 1: $D_t < S_t$

$$A_1 y_t = V_t \tag{3-5}$$

and

Regime 2: $D_t \geqq S_t$

$$A_2 y_t = V_t \tag{3-6}$$

It is easy to verify that

$$A_1 = \begin{bmatrix} 1 & 0 & -\alpha_1 \\ 0 & 1 & -\alpha_2 \\ -(1+\gamma) & \gamma & 1 \end{bmatrix}$$

$$A_2 = \begin{bmatrix} 1 & 0 & -\alpha_1 \\ 0 & 1 & -\alpha_2 \\ -\gamma & -(1-\gamma) & 1 \end{bmatrix}$$

We can then write the reduced forms as

$$y_t = A_i^{-1} V_t \qquad i = 1,2$$

and we obtain for Regime 1

$$D_t = \frac{1}{\bar{A}_1}\left[(1+\alpha_2\gamma)V_{1t} - \alpha_1\gamma V_{2t} + \alpha_1 V_{3t}\right] \tag{3-7}$$

$$S_t = \frac{1}{\bar{A}_1}\left[\alpha_2(1+\gamma)V_{1t} + (1-\alpha_1(1+\gamma))V_{2t} + \alpha_2 V_{3t}\right] \tag{3-8}$$

and for Regime 2

$$D_t = \frac{1}{\bar{A}_2}\left[(1-\alpha_2(1-\gamma))V_{1t} + \alpha_1(1-\gamma)V_{2t} + \alpha_1 V_{3t}\right] \tag{3-9}$$

$$S_t = \frac{1}{\bar{A}_2}\left[\alpha_2\gamma V_{1t} + (1-\alpha_1\gamma)V_{2t} + \alpha_2 V_{3t}\right] \tag{3-10}$$

where $\bar{A}_1$ and $\bar{A}_2$ are the determinants of A_1 and A_2 respectively. The regime-indicator criterion $D_t - S_t$ is found by subtracting (3-8) from (3-7) and (3-10) from (3-9) and is

$$D_t - S_t = \begin{cases} \frac{1}{\bar{A}_1}\left[(1-\alpha_2)V_{1t} + (\alpha_1-1)V_{2t} + (\alpha_1-\alpha_2)V_{3t}\right] & \text{if } D_t < S_t \\ \frac{1}{\bar{A}_2}\left[(1-\alpha_2)V_{1t} + (\alpha_1-1)V_{2t} + (\alpha_1-\alpha_2)V_{3t}\right] & \text{if } D_t > S_t \end{cases} \qquad (3\text{-}11)$$

Lack of coherency would occur if there existed V_{1t} , V_{2t} , V_{3t} such that the first right hand side of (3-11) were negative and the second simultaneously positive. This will not occur if and only if the determinants $\bar{A}_1$ and $\bar{A}_2$ have the same signs. These determinants are

$$\bar{A}_1 = 1 + \gamma(\alpha_2-\alpha_1) - \alpha_1$$

$$\bar{A}_2 = 1 + \gamma(\alpha_2-\alpha_1) - \alpha_2$$

It is clear that values of α_1 , α_2 , γ satisfying the a priori restrictions $\alpha_1 > 0$, $\alpha_2 > 0$, $\gamma > 0$ exist for which $\bar{A}_1\bar{A}_2 < 0$. Such occurrence will be relatively more plausible, other things being equal, if $\alpha_1 > \alpha_2$, i.e. if the demand-encouragement aspect of a plan increase substantially exceeds the supply effect. Intuitively this can be best seen from Equs. (3-1) to (3-4). Imagine that these equations have a solution characterized by excess supply. If $\alpha_1 > \alpha_2$, an increase in the plan stimulates demand more than supply and may, therefore, create precisely the level of excess demand which, by (3-4), justifies the higher plan level.

In practice, the necessary and sufficient conditions for coherency are likely to be fulfilled. The announced plan has a direct impact on supply and α_2 is likely to be in the neighborhood of but not greater than unity. On the other hand, consumers in CPEs often have only a hazy knowledge of the plan. Plan targets are often changed and may have low credibility. Under these circumstances α_1 is likely to be in the neighborhood of zero. Coherency will then be attained for any positive value of γ.

4. An Alternative Model

The coherency problem discussed above is due to the fact that Q^* depends on Q which, in turn, depends on Q^* through the min condition. The formulation in (2-4) or (3-4) avoids these difficulties if Q_t^* is taken to be the plan formulated at time t for period $t+1$. Then the model becomes

$$D_t = \beta_1' x_{1t} + \alpha_1 Q_{t-1}^* + u_{1t} \tag{4-1}$$

$$S_t = \beta_2' x_{2t} + \alpha_2 Q_{t-1}^* + u_{2t} \tag{4-2}$$

$$Q_t = \min(D_t, S_t) \tag{4-3}$$

$$Q_t^* = Q_{t-1}^* + \gamma(D_t - S_t) + u_{3t} \tag{4-4a}$$

$$Q_t^* = Q_t + \gamma(D_t - S_t) + u_{3t} \tag{4-4b}$$

The demand and supply effects of the plan now depend on the plan formulated last period for the current period. The plan adjustment equation may have either form (4-4a) or (4-4b). The difference between the two is whether the base over which plan adjustment occurs is the lagged plan value or the current actual level of transactions. In the former case we are dealing with a model which is very similar to one in which prices adjust (except that Q^* plays the role of prices and that current demand and supply depend on lagged Q^*).[4] The likelihood function can be derived in straightforward manner (see Quandt (1978)). The latter case yields a model which is a reasonable formulation of actual practice in certain cases (Manove (1971)). In this model the question of coherency arises in principle, but it is easy to verify that this model is always coherent. Intuitively, for any Q^*_{t-1}, (4-1) and (4-2) determine a unique D_t, S_t; (4-3) determines a unique Q_t; finally (4-4b) determines a unique new Q^*_t.

We now derive the likelihood function for the model consisting of (4-1), (4-2), (4-3) and (4-4b). Assume that u_{1t}, u_{2t}, u_{3t} are normally distributed with $E(u_{it}) = 0$, $E(u^2_{it}) = \sigma^2_i$, $E(u_{it}u_{i\tau}) = 0$ for $i = 1,2,3$, and $t \neq \tau$ $E(u_{it}u_{j\tau}) = 0$ for $i \neq j$ and for all t and τ. When $D_t < S_t$, the joint pdf of D_t, S_t, Q^*_t conditional on Q^*_{t-1} is

4. Assume that plan adjustment is such as to minimize a quadratic cost function $C = \theta_1(S_t-S_{t-1})^2 + \theta_2(D^e_t-S_t)^2$ where D^e_t is expected demand and where the first and second terms measure "supply adjustment costs" and "disequilibrium costs" respectively. If we make the assumption as a first approximation that $D^e_t = D_{t-1}$, then cost minimization leads to (4-4a). See Upcher (1980).

$$f_1(D_t, S_t, Q_t^*|Q_{t-1}^*) = \frac{1}{(2\pi)^{3/2}\sigma_1\sigma_2\sigma_3} \exp\left\{-\frac{1}{2}\left[\frac{(D_t - \beta_1' x_{1t} - \alpha_1 Q_{t-1}^*)^2}{\sigma_1^2} + \frac{(S_t - \beta_2' x_{2t} - \alpha_2 Q_{t-1}^*)^2}{\sigma_2^2} + \frac{(Q_t^* - D_t - \gamma(D_t - S_t))^2}{\sigma_3^2}\right]\right\} \quad (4\text{-}5)$$

and when $D_t \geqq S_t$, the joint pdf is

$$f_2(D_t, S_t, Q_t^*|Q_{t-1}^*) = \frac{1}{(2\pi)^{3/2}\sigma_1\sigma_2\sigma_3} \exp\left\{-\frac{1}{2}\left[\frac{(D_t - \beta_1' x_{1t} - \alpha_1 Q_{t-1}^*)^2}{\sigma_1^2} + \frac{(S_t - \beta_2' x_{2t} - \alpha_2 Q_{t-1}^*)^2}{\sigma_2^2} + \frac{(Q_t^* - S_t - \gamma(D_t - S_t))^2}{\sigma_3^2}\right]\right\} \quad (4\text{-}6)$$

since the Jacobian of the transformation is unity. The joint pdf of Q_t and Q_t^* then is formally

$$h(Q_t, Q_t^*|Q_{t-1}^*) = g(Q_t, Q_t^*|Q_{t-1}^*, D_t < S_t)P_r\{D_t < S_t\} + g(Q_t, Q_t^*|Q_{t-1}^*, D_t \geqq S_t)P_r\{D_t \geqq S_t\}$$

$$= \int_{Q_t}^{\infty} f_1(Q_t, S_t, Q_t^*|Q_{t-1}^*)\,dS_t + \int_{Q_t}^{\infty} f_2(D_t, Q_t, Q_t^*|Q_{t-1}^*)\,dD_t \quad (4\text{-}7)$$

which can be obtained from (4-5) and (4-6) by integration. The likelihood function then is

$$L = \prod_{t=1}^{T} h(Q_t, Q_t^*|Q_{t-1}^*) \quad (4\text{-}8)$$

and may be maximized by numerical methods.

5. Price Adjustment in Centrally Planned Economies

The previous models all assumed that prices are fully rigid. This assumption is not completely realistic and in the present section we suggest some approaches to making prices endogenous.

For this purpose it is now desirable to rewrite Equ. (4-1) and (4-2) so that p_t appears in them explicity:

$$D_t = \psi_1 p_t + \beta_1' x_{1t} + \alpha_1 Q^*_{t-1} + u_{1t} \qquad (5\text{-}1)$$

$$S_t = \psi_2 p_t + \beta_2' x_{2t} + \alpha_2 Q^*_{t-1} + u_{2t} \qquad (5\text{-}2)$$

For completeness we restate the min condition

$$Q_t = \min(D_t, S_t) \qquad (5\text{-}3)$$

and the plan-adjustment equation (4-4b)

$$Q^*_t = Q_t + \gamma(D_t - S_t) + u_{3t} \qquad (5\text{-}4)$$

The simplest extension supplements these equations with a conventional price adjustment equation

$$p_t = p_{t-1} + \lambda(D_t - S_t) + u_{4t} \qquad (5\text{-}5)$$

It is straightforward to verify that the model given by (5-1) to (5-5) is always coherent. If u_{it} are independent of one another $(i=1, \ldots, 4)$ and iid as $N(0, \sigma_i^2)$, the application of the technique of Section 4 also permits the calculation of the joint pdf $h(Q_t, Q^*_t, p_t | Q^*_{t-1}, p_{t-1})$ as

$$h(Q_t, Q_t^*, p_t | Q_{t-1}^*, p_{t-1}) = \int_{Q_t}^{\infty} f_1(Q_t, S_t, Q_t^*, p_t | Q_{t-1}^*, p_{t-1}) dS_t +$$

$$\int_{Q_t}^{\infty} f_2(D_t, Q_t, Q_t^*, p_t | Q_{t-1}^*, p_{t-1}) dD_t \qquad (5\text{-}6)$$

where

$$f_1(D_t, S_t, Q_t^*, p_t | Q_{t-1}^*, p_{t-1}) = \frac{|1 + \lambda(\psi_2 - \psi_1)|}{(2\pi)^2 \sigma_1 \sigma_2 \sigma_3 \sigma_4} \exp\left\{ -\frac{1}{2} \left[\frac{(D_t - \psi_1 p_t - \beta_1' x_{1t} - \alpha_1 Q_{t-1}^*)^2}{\sigma_1^2} + \frac{(S_t - \psi_2 p_t - \beta_2' x_{2t} - \alpha_2 Q_{t-1}^*)^2}{\sigma_2^2} + \frac{(Q_t^* - D_t - \gamma(D_t - S_t))^2}{\sigma_3^2} + \frac{(p_t - p_{t-1} - \lambda(D_t - S_t))^2}{\sigma_4^2} \right] \right\}$$

if $D_t < S_t$, and

$$f_2(D_t, S_t, Q_t^*, p_t | Q_{t-1}^*, p_{t-1}) = \frac{|1 + \lambda(\psi_2 - \psi_1)|}{(2\pi)^2 \sigma_1 \sigma_2 \sigma_3 \sigma_4} \exp\left\{ -\frac{1}{2} \left[\frac{(D_t - \psi_1 p_t - \beta_1' x_{1t} - \alpha_1 Q_{t-1}^*)^2}{\sigma_1^2} + \frac{(S_t - \psi_2 p_t - \beta_2' x_{2t} - \alpha_2 Q_{t-1}^*)^2}{\sigma_2^2} + \frac{(Q_t^* - S_t - \gamma(D_t - S_t))^2}{\sigma_3^2} + \frac{(p_t - p_{t-1} - \lambda(D_t - S_t))^2}{\sigma_4^2} \right] \right\}$$

if $D_t \geqq S_t$.

Equ. (5-5) posits a price-sensitivity to excess demand on the part of the Central Planning Authority which may be unrealistic. There are at least two ways of coping with this. First, it may be argued that planners are not at all sensitive to excess demand but rather to shortfalls of the plan compared to current transactions. Accordingly, (5-5) might be replaced by

$$p_t - p_{t-1} = \lambda(Q_t^*-Q_t) + u_{4t} \qquad (5\text{-}6)$$

which explicity assigns a rôle in inflation to the planning process. An alternative view argues that small excess demands (or plan shortfalls) are disregarded altogether in the price setting process. Accordingly, price setting is exogenous if, say, excess deand is less than some threshold in absolute value. Then

$$p_t = p_{t-1} + \lambda(D_t-S_t) + \beta_3'x_{3t} + u_{4t} \quad \text{if} \quad |D_t-S_t| \geqq \kappa$$
$$p_t = p_{t-1} + \beta_3'x_{3t} + u_{4t} \quad \text{if} \quad |D_t-S_t| < \kappa$$

where κ and x_{3t} are exogenous and where κ itself may either be given or may be estimated. The pdf $h(Q_t,Q_t^*,p_t|Q_{t-1}^*,p_{t-1})$ is straightforward to to derive and it may be noted that in this case it consists of four "pieces" corresponding to the twofold partiion $D_t \lesseqgtr S_t$ and $|D_t-S_t| \lesseqgtr \kappa$. In each case numerical maximization of the corresponding likelihood function is required.

6. Conclusions

A basic model and various extensions were considered and computational methods suggested. Some of these, based on the interpretation of Q_t^* as the stated plan for the current period, lack in principle the property of coherency, although in practice they may be coherent. Others, based on the interpretation of Q_t^* as the plan stated in period t for period $t + 1$, avoid this difficulty and have a well-defined likelihood function. In any event, it appears preferable to specify a plan adjustment equation which includes an error term in it; otherwise planners must be assumed to have unrealistic amounts of knowledge. Although disequilibrium models with an error term in the adjustment equation are generally more difficult to estimate than those without for various numerical reasons, the increase in computational complexity may be justified by enhanced realism in the specification of the model. Finally, various models may be specified in which it is recognized that prices are not fully rigid and are adjusted by the Central Planning Authority in response to excess demand or plan short-falls.

Further work remains to be done. Models of the type discussed above will have to be estimated from actual data. Equally importantly, two-sector models will have to be formulated which explicitly recognize that CPEs have private sectors which are linked to the planned sectors by various spillovers. Even though data on private sector variables may be of inferior quality, it would be of great interest to determine econometrically whether excess demands behave comparably in the two sectors. These models are likely to be very much more complicated. If both private and planned sectors in such models may exhibit disequilibrium, the coherency properties of the models may be difficult to establish. The properties of these models remain to be explored.

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