THE THEORY OF SALES: A SIMPLE MODEL OF
EQUILIBRIUM PRICE DISPERSION WITH IDENTICAL AGENTS*

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1. Introduction

A fundamental tenet of neoclassical theory is the law of the single price. The parable of the Walrasian auctioneer is intended to provide us with some insight into the determination of that single price. The object of this paper is to examine equilibrium in a competitive market in which the mythical auctioneer is absent and information is costly to gather. As a result, individuals may not be perfectly informed about the prices (or qualities) of what is being sold. Equilibrium in such markets may differ markedly from the one conventionally studied by neoclassical theory. In particular, the only market equilibrium may be characterized by price dispersion for a homogeneous commodity; the law of the single price does not obtain. We illustrate this with a model in which all individuals are (ex ante) identical and in which there is no exogenous source of noise, no external disturbances to the market which have to be equilibrated. Instead noise is introduced solely by the internal functioning of the market. Thus, the information imperfection is created by the market itself.

In our model, although all individuals have identical preferences and incomes and all firms have identical technologies, some firms charge high prices and others charge low prices. Those customers who (unluckily) arrive at a high price firm purchase only for their immediate needs and

re-enter the market later. Those who (luckily) arrive at a low price store "economize" by purchasing more than is required for immediate consumption and storing the excess for future consumption. High price stores earn a larger profit per sale, but make fewer sales. Equilibrium entails equal profits for the two kinds of stores, that is, the lower volume of the high price stores exactly compensates for the higher profit per sale.

The model we develop is thus of interest not only for the insight which it provides into the nature of price dispersion in the economy, but also because it provides at least a partial explanation of some aspects of retailing which otherwise would be difficult to explain.

The model of price dispersion we analyze is only one of a wide class of such models. In Section 2, we outline the basic structure of these models, while in Section 3, we analyze in detail the simple model which we call "the theory of sales." We establish for this model that for a wide range of parameter values the only possible equilibrium entails price dispersion. We show, moreover, that search subsidies have general equilibrium impacts in reducing the average price on the market; as a result, the gain to consumers far exceeds the direct cost of these subsidies.

A simplifying assumption employed in the analysis of Section 3 is that there are no costs to entering the market. When there are costs to entering the market we show that the only possible equilibria in the market involve price dispersion; in those situations where there does not exist an equilibrium price distribution, there does not exist any equilibrium to the market.

In Section 5, we show that the results obtained from the simple model analyzed in the preceding two sections are in fact robust; the model may be

extended in a number of directions without affecting the qualitative results.

One direction in which the model cannot be extended is the introduction of non-linear pricing schedules (Section 6). When firms are allowed to employ such schedules, equilibria with price dispersion cannot exist, and, if search is costly, no equilibrium exists. The reason for this is simple: non-linear price schedules allow firms to differentiate between different groups in the population -- in our model between the young and the old. It is the inability to differentiate among these groups which leads to the price dispersion (some firms offering a low price to induce the young to purchase for future consumption), and it is the price dispersion which enables equilibrium to exist with costly search. For in the absence of noise, firms will seek to increase prices to the point where there is no consumer surplus left given that consumers are already in the market. The existence of some low price stores provides sufficient (expected) consumer surplus to induce consumers to pay the fixed costs associated with entering the market. As in the earlier models of Grossman and Stiglitz (1976, 1980), the presence of some noise seems necessary for the existence of equilibrium.

The analysis of this paper (and most of the other related literature) assumes rational expectations: consumers' beliefs about the distribution of prices, which affect their search behavior, correspond to the price distribution which emerges in equilibrium. In Section 7, we show that price distributions will emerge even in the absence of rational expectations; indeed, there is a sense in which price distributions are even more likely than with rational expectations.

The final section of this paper relates our model to other recent models of equilibrium price distribution.

2. The Basic Structure of Models with Price Dispersion

The fundamental difference between markets with perfect information and markets with imperfect information is that individuals do not all purchase from the lowest price store, because they find it uneconomical to search out better buys. The basic formal structure of such markets is the following: there are N (possibly infinite) potential stores, n of which actually exist. As a convention, we say stores which do not exist charge $p_i = \infty$. The sales Q_i of the ith store are a function of its price and the prices charged by all other stores,[1] or

$$(2.1) \qquad Q_i = Q_i(p_1, \ldots, p_N).$$

Its revenue R_i is given by

$$(2.2) \qquad R_i = p_i Q_i(p_1, \ldots, p_N) \equiv R_i(p_1, \ldots, p_N).$$

Letting costs C_i be given by

$$(2.3) \qquad C_i = C_i(Q_i), \quad C_i' > 0, \quad C_i'' \geq 0$$

then profits π_i are given by

$$(2.4) \qquad \pi_i = R_i - C_i \equiv \pi_i(p_1, \ldots, p_N).$$

A Nash-Cournot equilibrium is characterized by the following two sets of conditions:

(a) All existing firms choose a price to maximize profits.

(b) For all existing firms, profits are non-negative, and for any

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[1] This reduced form demand function depends on the precise assumptions concerning information, search, etc. In the model presented below, for instance, the "demand function" appears as in Figure 1a.

potential non-existing firm, profits are non-positive

at all potential prices.

If all firms are identical, and the number of firms is large,

then if equilibrium is characterized by price dispersion, the profit

function (as a function of price) must peak at every price charged.

For instance, suppose that the price distribution were concentrated at

two prices, p_L and p_H. Any low price firm can become a high price

firm by raising prices, and similarly, any high price firm can become a

low price firm. At the limit, if the number of firms is sufficiently

large that a price change by any one firm has only a negligible effect

on the demand of each other store then the profit function must not only

have two peaks but the level of profits at p_L and p_H must be identical

at each.[1]

There are two broad categories of mechanisms which give rise to this.

First, stores which charge a low price have larger sales, just compensating

for the lower price. There are a number of reasons why this might occur.

Sales might be larger because more individuals shop there, because more

individuals who arrive there are willing to purchase, or because those

individuals who arrive purchase more units. In this paper, this latter

possibility is explored in some detail.

The second set of reasons that profits may be the same at distinctly

different prices is related to the costs of acquiring customers. If firms

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[1] If the number of firms is small, then the nature of the equilibrium depends
on the beliefs of the potential entrant concerning the consequences of his
entry (Salop (1979)). Similarly, any producing firm must form conjectures
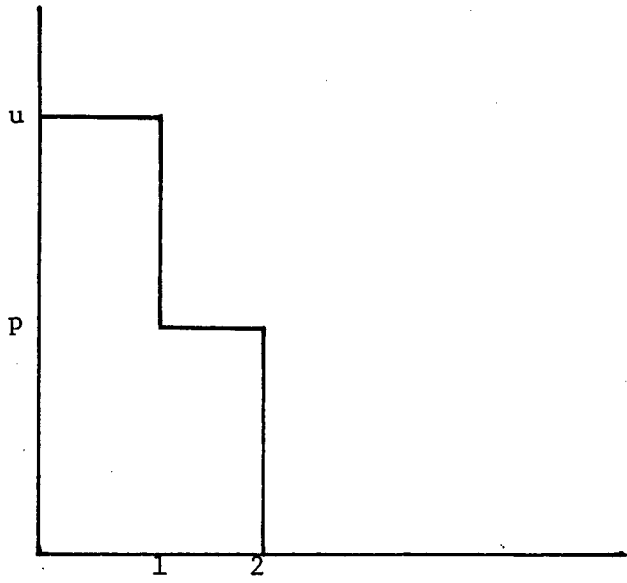about the consequences of changing his price (Salop-Stiglitz (1977a)).
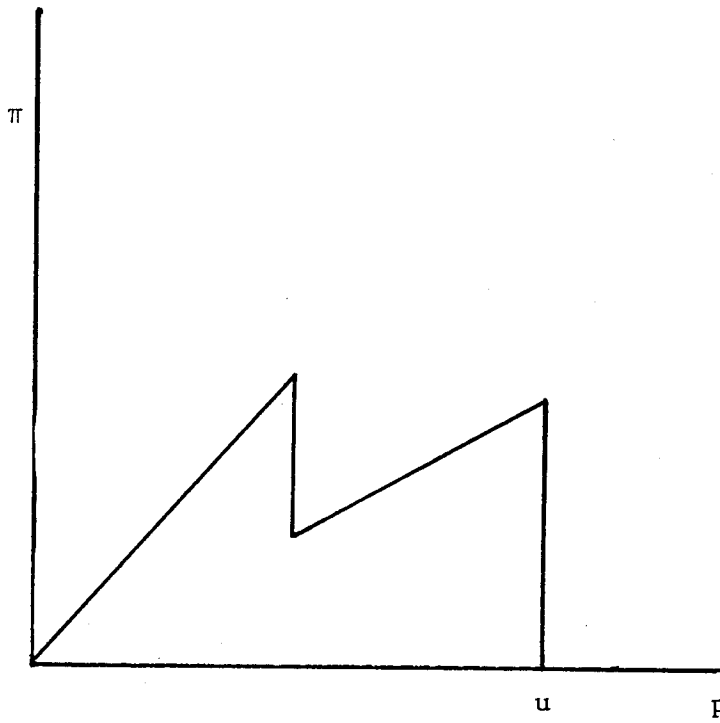
Figure la. Demand Curve Facing Firm

Figure 1b. Profit function corresponding to demand
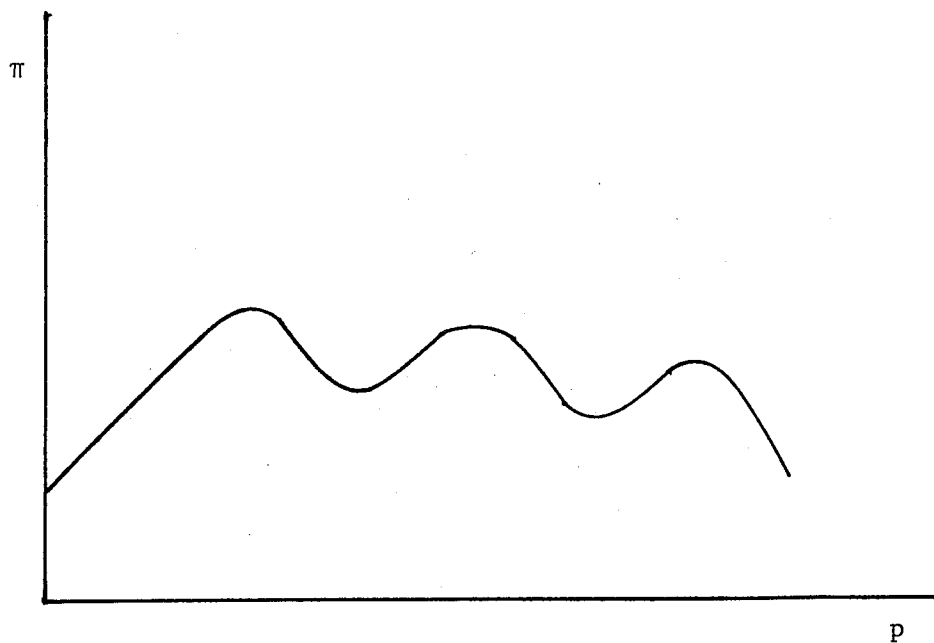curve depicted in la.

Figure 1c. Profit function has multiple peaks.

must recruit customers, for example, by advertising, then high price

stores have higher recruitment costs per customer, exactly offsetting the

higher revenue. See Butters (1977).[1]

3. The Theory of Sales

Although the formal model that follows is highly simplified and

unrealistic, it captures a general class of phenomena. The price

dispersion may be across stores, across brands of the same product, or at

a single store over time. The dispersion may be in quality as well as

price. One phenomenon is particularly interesting. Steiner (1978) observes

that most nationally advertised brands have a common "everyday" price

across stores, but different stores offer temporary discounts from time

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[1] Similar arguments apply to variations in quality. High quality stores
will have more repeat customers, and have a lower consumer recruitment cost per
customer.

to time, either passing on manufacturer discounts or holding their own
advertised and unadvertised sales. Shilony (1976) analyzes equilibrium
when discounts are advertised.[1] The model to follow here analyzes
unadvertised specials. It is important to emphasize that our model is
a highly idealized one, designed to illustrate one aspect of retailing with
imperfect information. In particular, individuals may acquire information
in a variety of ways, besides actually going to the store.[2,3]

We assume every consumer lives two periods. Each consumer demands
one unit of the commodity each period at any price no greater than some
demand price u. Thus, a monopolistic producer would choose $p^m = u$.
We refer to this as the "monopoly price." In the presence of price
dispersion, a consumer who enters the market may either purchase one unit
each period, or purchase two units in period 1, consume one unit and
store the rest for consumption in period 2. If the consumer purchases
for storage, the additional transaction cost c of re-entering the market
is saved. However, a storage cost δ must be incurred.[4] The decision to
buy-and-store or shop again balances these two considerations.

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[1] That model contains the additional assumption that each consumer has a
location-based preference to one particular store.

[2] Information may be gathered from personal inspection, experience, product
sellers (including advertising), and information sellers. Reputation,
trademarks and market signals represent the interaction of two or more
of these general search modes.

[3] The assumptions of identical firms and customers and no exogenous noise
are made to show clearly that the kind of price dispersion analyzed here
is distinctly different from that analyzed in the earlier models of
price dispersion discussed below.

[4] This should be contrasted with most of the search literature which assumes
constant costs per   search. Our assumption simplifies the analysis,
but is not essential. See Appendix D below.

Suppose consumers know <u>a priori</u> the distribution of prices f(p) charged in the market. (Conditions under which f(p) is not degenerate will be derived below.) In the absence of more detailed information, the consumer randomly selects a store in period 1. Suppose that store quotes a price p. Let $\hat{p}$ denote that "reservation price" which leaves the consumer indifferent between purchasing for storage and purchasing only for present consumption with the intention of re-entering the market next period. In order to focus on these interperiod transactions costs, we assume the consumer is not permitted to reject the price p and select a new store in period 1. We assume, in other words, that the cost of a second search in any period is so great that the individual will never undertake it.

Noting that the consumer will obtain the average price $\bar{p}$ next period and must also pay transactions cost c, $\hat{p}$ is given by[1]

(3.1) $\hat{p} + \delta = \bar{p} + c$

3.1 Characterization of Equilibrium

We wish to find an equilibrium price distribution f*(p) with the following properties:

(1) <u>Profit-maximization</u>: Each small firm chooses a price p to maximize profits given the prices of the other firms.

(2) <u>Equal Profits</u>: Each firm earns identical (to "normal") profits, and

(3) <u>Search Equilibrium</u>: Each consumer   optimally given f*(p).

We shall derive conditions under which a nondegenerate equilibrium price distribution exists. To do this, we shall first characterize such an equilibrium (if it exists); we show that it can have only two prices, that the higher price is the monopoly price and the lower price the reservation price. We then derive conditions under which profits at the reservation price and at the monopoly price can be the same.

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[1] This assumes risk neutrality on the part of the consumer. In Salop and Stiglitz (1976b) we show that this is not a critical assumption.

We first prove

Lemma 1:   In the equilibrium price distribution, there are at most

two prices. Suppose there were more than one price below $\hat{p}$, say p_m and

p_ℓ and $p_m > p_\ell$. The p_ℓ and p_m firms would obtain an identical

number of customers who purchase for two periods. The profits of the

p_m firm would be higher, and no equal profit equilibrium can obtain.

Similar reasoning eliminates the possibility of two prices at least as

great as $\hat{p}$.

3.2 Two-Price Equilibria

Denoting the two prices by p_ℓ and p_h and the respective fractions

of firms charging each in each period by $1 - \lambda$ and λ respectively,

we have

(3.2) $\bar{p} = \lambda p_h + (1 - \lambda)p_\ell$.

Equation (3.1) may be rewritten:

$$\hat{p} = \lambda p_h + (1 - \lambda)p_\ell + c - \delta .$$

If $\hat{p}$ exceeds $u - \delta$, the individual would not purchase for future

consumption. Hence

(3.3) $\hat{p} = \min [u - \delta, \bar{p} + c - \delta]$

Lemma 2: Any two-price equilibrium must have the property that the low

price store charges exactly the reservation price:

(3.4) $p_\ell = \hat{p}$.

If $p_\ell > \hat{p}$, consumers at both stores purchase for one period only,

but since $p_h > p_\ell$, profits cannot be the same. If $p_\ell < \hat{p}$, consumers at

the low price store purchase for two periods. But by raising the price slightly, a low price store would not lose any customers, but would increase profits; hence the price p_ℓ cannot be profit maximizing.

Lemma 3: In any two-price equilibrium, the high price must be the monopoly price, or

(3.5) $p_h = u$.

Since $\hat{p} < p_h$, the p_h firms sell for only one period. If a p_h firm were to raise its price, it would lose no customers and its profits would rise.

Lemmas 2 and 3 together imply that the market equilibrium price distribution is characterized as follows:

(3.6) $p_h = u$;

(3.7) $p_\ell = \min[\overline{p} + c - \delta, u - \delta]$.

The equal profit condition allows calculation of the fraction of p_h firms, λ as follows.

There are L consumers and n firms. Each firm attracts L/n young customers and $\lambda L/n$ old customers who were unlucky and selected a p_h store when they were young.[1] Since both unlucky young and old customers purchase one unit each, the sales X_h of each p_h firm is given by

(3.8) $X_h = [1 + \lambda]L/n$.

The sales X_ℓ of the p_ℓ firms are higher. Each sells 2 units to their young customers and one unit to their old customers, or

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[1] We assume all of them re-enter the market the second period. See Section 4 and Appendix B below.

(3.9) $X_\ell = [2 + \lambda]L/n$.

For simplicity, we assume that the marginal costs of production are zero. Thus equal profits implies

(3.10) $P_h X_h = P_\ell X_\ell$

or, using (3.6), (3.8), and (3.9), we obtain

(3.11) $(1 + \lambda)u = p_\ell (2 + \lambda)$.

(3.7) and (3.11) (together with the definition of $\bar{p}$, (3.2)), give us two equations in the unknowns, $p_\ell + \lambda$, which can be solved for the market equilibrium.

For the remainder of this section we focus here on the special case of $c = 0$. This greatly simplifies the calculations, and allows us to ignore the question of whether the individual enters the market. (Recall that the individual purchased at a low price store for storage for _two_ reasons: it reduced his total transactions cost and he was uncertain about whether the store he sampled next period would have a low price. In the case of $c = 0$, only the latter effect is relevant, but this, by itself, is sufficient to give rise to an equilibrium price distribution.) The more general case is discussed in Section 4.

We can now establish: If $c = 0$,

Theorem 1. If there exists an equilibrium price distribution, it consists of two prices, with

(3.12) $p_\ell = \dfrac{u + \delta}{2} \qquad p_h = u$

and the fraction of firms charging the high price

(3.13) $\lambda = \dfrac{2\delta}{u - \delta}$.

To see this, we substitute (3.2) into (3.7) and observe that $\bar{p} \overset{<}{=} u$. Hence

$$(3.14) \qquad p_\ell = \lambda p_h + (1 - \lambda)p_\ell - \delta$$

$$= u - \delta/\lambda.$$

(3.11) and (3.14) are two equations in the unknowns, p_ℓ and λ, the solution of which is given by (3.12) and (3.13).

We will now establish that, if $c = 0$,

<u>Theorem 2.</u> A necessary and sufficient condition for the existence of a two price equilibrium **(TPE)** is that

$$\underline{\delta < u/3}.$$

For (3.12) – (3.13) to constitute a two price equilibrium, clearly

$$(3.15a) \qquad 0 < \lambda < 1$$

and

$$(3.15b) \qquad p_\ell + \delta \leq u$$

The second constraint is required to ensure that an individual who arrives at a low price still purchases for storage. The constraint $\lambda < 1$ implies $\dfrac{2\delta}{u - \delta} < 1$ or

$$(3.16) \qquad \delta < u/3 .$$

The constraint $\lambda > 0$ implies that $\delta < u$ which is clearly implied by (3.16). The condition

$$p_\ell + \delta = \frac{u + 3\delta}{2} < u$$

is equivalent to (3.16).

3.3 Single-Price Equilibria

Lemma 1 established that there were at most two prices in equilibrium. In the previous subsection we considered two-price equilibria. We now consider single-price equilibria (SPE). We shall show

<u>Theorem 3</u>.   There are two possible single price equilibria:

   (a) $p = u$, the price is the monopoly price with no storage;

   (b) $p = u - \delta$, the price is the highest price which allows for storage.

   (c) A necessary and sufficient condition for $p = u$ to be a single price equilibrium is

(3.17) $\delta > u/3$.

   (d) A necessary and sufficient condition for $p = u - \delta$ to be a single price equilibrium is

(3.18) $\delta < \min[u/2, c]$

Note that if $c = 0$, (3.18) is never satisfied. Hence from theorems 1 and 2, there is a S.P.E. with $p = u$ if $\delta > u/3$, a T.P.E. if $\delta < u/3$. Whether there exists a price dispersion or a single price equilibrium depends simply on the magnitude of storage costs. If they are low (relative to the reservation price u), the only equilibrium in the market entails price dispersion. (See Figure 2.)

If it pays firms to have "sales" to induce individuals to purchase for future consumption, storage costs cannot be too great. And if storage costs are not too great, it always pays to have sales.

The proof of Theorem 3 is straightforward. $p = u$ is an equilibrium if and only if it does not pay a firm to lower its price to the "reservation level" and sell for storage. The reservation price when all firms charge $p = u$ is
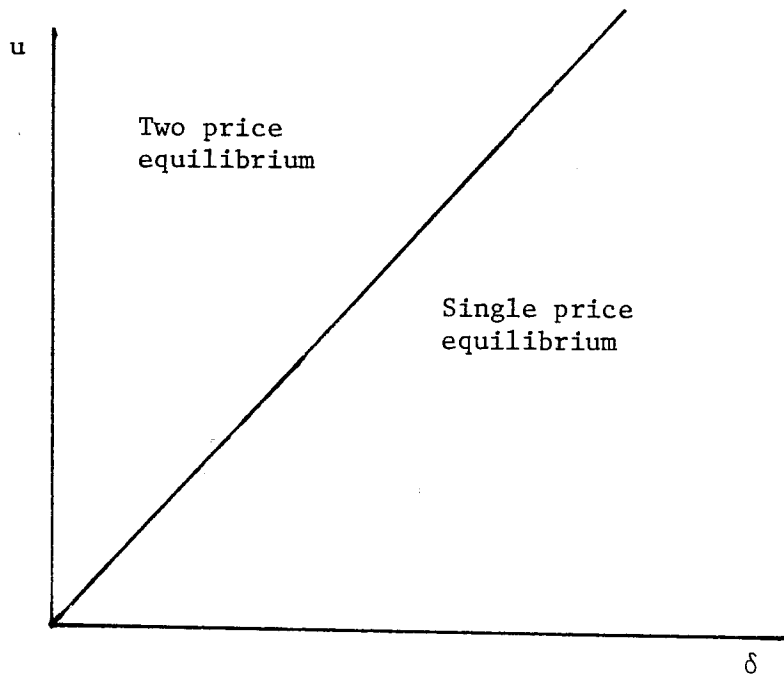
u

Two price
equilibrium

Single price
equilibrium

δ

Figure 2.   Patterns of Equilibrium: $c = 0$.

(3.18) $p = u - \delta$.

If this is an equilibrium, profits must fall if the firm lowers its price, even though sales are increased, i.e., since half of its customers are young and half old,[1] sales are increased by 50% and we require

$$\frac{3}{2}(u - \delta) < u$$

or

(3.19) $\delta > \dfrac{u}{3}$

For $p = u - \delta$ to be a single price equilibrium requires

(i)   It cannot pay any store to raise its price. If it increased price, clearly it would increase price to $p = u$. Thus we require

$2(u - \delta) > u$

or

(3.20) $\delta < u/2$

(ii) It pays all individuals not to search but to store; this simply requires

(3.21) $c > \delta$

3.3 Comparative States and Welfare

In the two price equilibrium the relative magnitude of price dispersion is simply a function of u/δ . Letting $u/\delta = \nu$, we have (from (3.12) and (3.13),

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[1] If the cost of shopping the second period is positive, no one will shop the second period if $c > 0$. Hence sales are increased by 100% if prices are lowered to $u - \delta$. Thus, a single price equilibrium at $p = u$ requires $2(u - \delta) < u$, or $\delta > u/2$.

(3.22) $\lambda = \dfrac{2}{\nu - 1}$

(3.23) $p_\ell = \dfrac{\delta(\nu + 1)}{2}$, $p_h = u.$

From (3.2),

$$\bar{p} = \frac{u(\nu + 3)}{2\nu} .$$

$\cdot$ Hence

$$\sigma_p^2 = \lambda(u - \bar{p})^2 + (1 - \lambda)(p_\ell - \bar{p})^2$$

$$= (u - p_\ell)^2\lambda(1 - \lambda)^2 + (u - p_\ell)^2 (1 - \lambda)\lambda^2$$

$$= \frac{2(\nu-3)}{(\nu-1)^2} u^2(1 - \frac{\nu+1}{2\nu})^2$$

$$= \frac{(\nu-3)}{2\nu^2} u^2$$

$$\frac{\sigma_p^2}{\bar{p}^2} = \frac{2(\nu-3)}{(\nu+3)^2}$$

Figure 3 depicts the relationship of the equilibrium to values of δ. Notice that an increase in storage costs always raises mean price -- sales become less attractive -- but it may either increase or decrease price dispersion, as measured by the coefficient of variation.

Our consumers have been modeled as being risk neutral. Their welfare is thus simply measured by the average price plus average storage and search costs:[1]

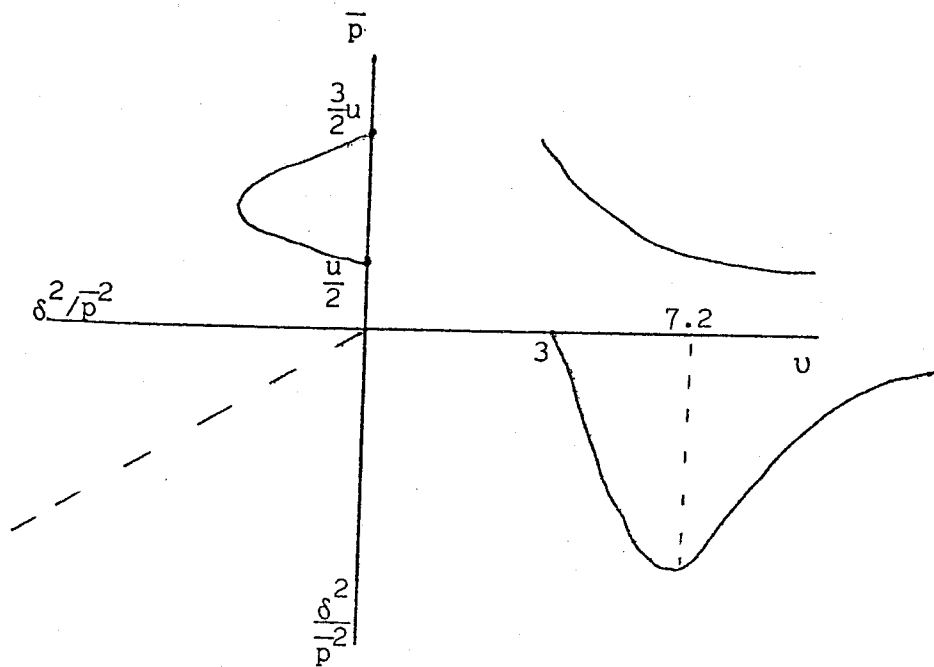[1] Here $c = 0$, so search costs do not enter.

Figure 3. Mean and variance of price distribution as function of $u/\delta \equiv \nu$.

$$- W = \lambda[u + \bar{p}] + (1 - \lambda)[2p_{\ell} + \delta]$$

$$= \frac{u}{\nu-1} \left[2(1 + \frac{\nu+3}{2\nu}) + \frac{(\nu-3)}{\nu}(\nu + 2)\right]$$

$$= \frac{u(\nu + 3)}{\nu} = 3\delta + u = 2\bar{p}.$$

Thus an increase in storage costs always makes them worse off, but the general equilibrium effect -- through the effect on mean price -- is <u>twice</u> as large as the partial equilibrium effect.

Thus, lowering storage costs (e.g. by lowering interest rates for purchases of consumer goods) may thus have significant beneficial welfare effects.

4. Costly Search, "Noise," and the Existence of Equilibrium

In Section 3 we analyzed the conditions under which the unique equilibrium is characterized by a price distribution. When consumers arrive at a low price store, the noise in the market induces them to purchase for their future consumption as well as for immediate consumption. In a different context, Grossman and Stiglitz (1980) have analyzed the role of "noise" in ensuring the existence of a competitive equilibrium when information is costly. It had previously been noted that if prices correctly reflected all the information available, then it would not pay any arbitrageur to acquire costly information; thus only costless information would be reflected in equilibrium prices. Yet if no costly information were collected, then it would pay a small speculator to acquire information. Hence, no equilibrium could exist. Grossman and Stiglitz (1976, 1980) showed that if there is noise in the market, so that prices only partially reflect the available information, then speculators would maintain an incentive to acquire information and equilibrium would exist.

A similar problem arises in the conventional search model. Consider a variant of the previous example in which storage is prohibitively expensive ($\delta = \infty$). This eliminates any intertemporal linkage present and allows a static view of the market. Assume that consumers must bear a positive transactions cost c to enter the market each period but as before, they may search only once per period. Under these circumstances, each firm is obviously a pure monopolist; once a consumer arrives at a store, he is perfectly captive, since he may not search again. Hence, the only possible equilibrium is the single price equilibrium at the monopoly price u. However, at that price, no consumer will enter the market, correctly realizing that he would attain a negative consumer surplus from doing so. Formally, although a price equal to u - c is required to induce the consumer to enter, any small firm -- believing it has a negligible effect on average price -- has an incentive to raise price to u; for once the consumer (unluckily) arrives at the store, he is captive and will pay up to u (by-gones are by-gones: he regrets having gone to the store, but he puts the regret aside as he makes the best of a bad situation). Hence no equilibrium exists. <u>Ruthless competition with a small degree of monopoly power destroys the market equilbriium.</u>

This problem is again alleviated by the existence of noise. Whereas Grossman and Stiglitz introduced exogenous noise to resolve the problem of non-existence in their model of the capital market, in the model presented here, the noise is created by the market itself. Noise ensures that there is some chance that the individual will get a good buy; it is this hope which induces him to enter the market. The remainder of this section is devoted to discussing in greater detail the implications of costly search.

The analysis proceeds in two stages.

(i) First, we ask, if the individual enters the market the first period, how will the equilibrium be characterized. The possible solutions are summarized in Table 1 and the values of the parameters associated with each are depicted in Figure 4. The calculations are presented in Appendix A.

(ii) Next, we ask, given the pattern of equilibrium thus described, would it in fact pay any individual to enter that market.

We now establish

Theorem 4.   A necessary condition for the existence of a single price equilibrium is that

$$c_1 < \delta < c_2$$

where c_i is the search cost the ith period.

We noted earlier that there were only two possible single-price equilibria, $p = u$, or $p = u - \delta$. If $p = u$, there is clearly no consumer surplus. If $p = u - \delta$, the individual will store. Clearly there is no consumer surplus associated with second period consumption. There is, however, some associated with first period consumption. We require

$$u - p > c_1$$

if the individual is to enter the market. But since $p = u - \delta$, this implies that

$$(4.1) \quad \delta > c_1,$$
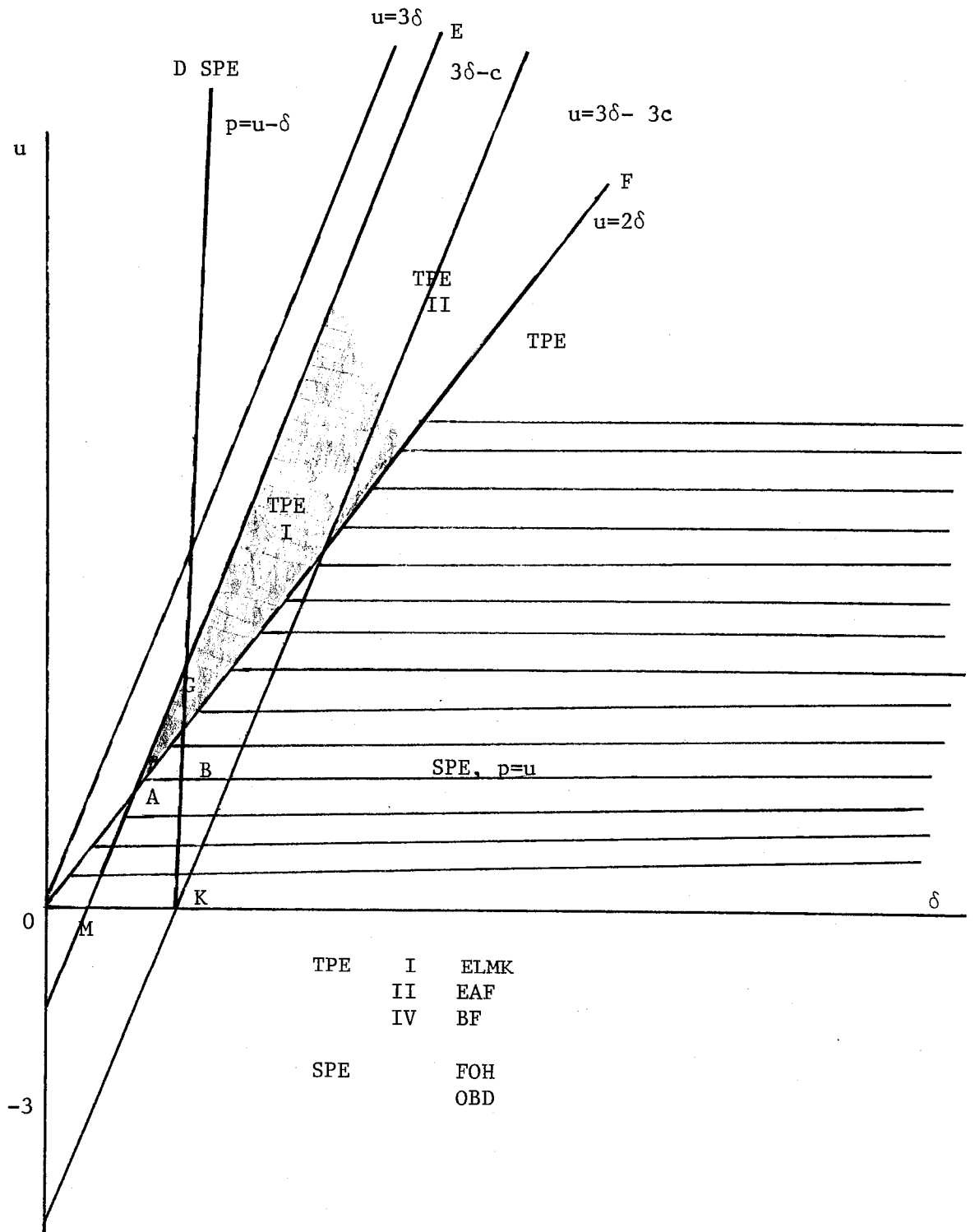
storage costs exceed search costs.

Figure 4. Patterns of Equilibrium Free Search First Period.

TABLE 1

Patterns of Equilibrium

(Free Search First Period)

Two Price Equilibria

I   Unlucky individuals shop second period;

$p_\ell < u - \delta$

$p_\ell = \dfrac{u+\delta-c}{2}$ $\dfrac{u+c}{3} < \delta < u/3 + c$

$\lambda = \dfrac{2(\delta - c)}{u -(\delta - c)}$

II   Unlucky individuals shop second period;

$p_\ell = u - \delta$

$\lambda = u/\delta - 2$ $\dfrac{c + u}{3} < \delta < u/2$

III Unlucky individuals do not shop second period; $p_\ell < u - \delta$

Impossible

IV   Unlucky individuals do not shop second period; $p_\ell = u - \delta$

$\lambda > 1 - c/\delta$ $\delta = u/2$

$c < \delta$

Single Price Equilibria

$p = u$ if $c=0$, $\delta > u/3$

if $c > 0$, $\delta > u/2$

$p = u - \delta$ $\delta < \min(c,u/2)$

But earlier, we established that for a single-price equilibrium at the price $u - \delta$,

$$\delta < c_2 .$$

For the two-price equilibrium it turns out that the constraint that it must pay individuals to enter the market the first period is always satisfied (Appendix A) if $c_1 \leq c_2$ and if it paid them to enter the market the second period. We have thus established that, if $c_1 = c_2 = c$

Theorem 5. (a) For high values of δ/u relative to c/u, no equilibrium exists.

(b) For low values of δ/u relative to c/u, the unique equilibrium entails price dispersion.

5. Robustness of the Model

The model formulated in the preceding section has several special features to it which we have not commented on so far:

(a) The demand (utility) function has a special form;

(b) There are only two periods;

(c) Storage costs are proportional to the number of units purchased;

(d) All individuals are identical;

(e) Firms use only a linear price system;

(f) Individuals know the probability distribution of prices.

All of these may be generalized without altering the basic results, the possibility of the existence of an equilibrium with price dispersion and the possibility that the only equilibrium involves price dispersion.

Altering the first four assumptions complicates the analysis but does not change it in a fundamental way. In Salop and Stiglitz (1976b), we showed how the model can be extended to the more usual case of a downward sloping demand curve. In Stiglitz (1979), the results were extended to a model in which individuals' search costs differ. The extension to the case where individuals live (purchase) for more than two periods is straightforward; this adds the possibility of an equilibrium with more than two prices. In Appendix C, we show how the results can be extended to other storage cost functions.

The last two assumptions require a more extended discussion.

6. Non-linear Price Schedules

In the simple model presented there are two groups of individuals in the population at any moment, the young and the old. We assumed that the firm could not discriminate between them. Any low price store would, if it could, discriminate between the young and the old. While there is no value to charging the old anything less than the monopoly price, since they will purchase one unit regardless, it may pay to charge a lower price to the young.

Even if direct discrimination is not possible, firms may attempt to discriminate indirectly. We shall now establish

Theorem 6. The use of a non-linear price schedule allows perfect price discrimination[1] between the young and the old. If firms can give quantity discounts, no equilibrium exists, if search is costly. Again, we observe that, in their ruthless attempt to exploit the individual who has entered the market, competitive firms lead the market to close down.

_____

[1] The use of non-linear price schedules as devices by which a monopolist can partially discriminate among various customers is discussed in Salop (1977), Stiglitz (1979), and Katz (1981).
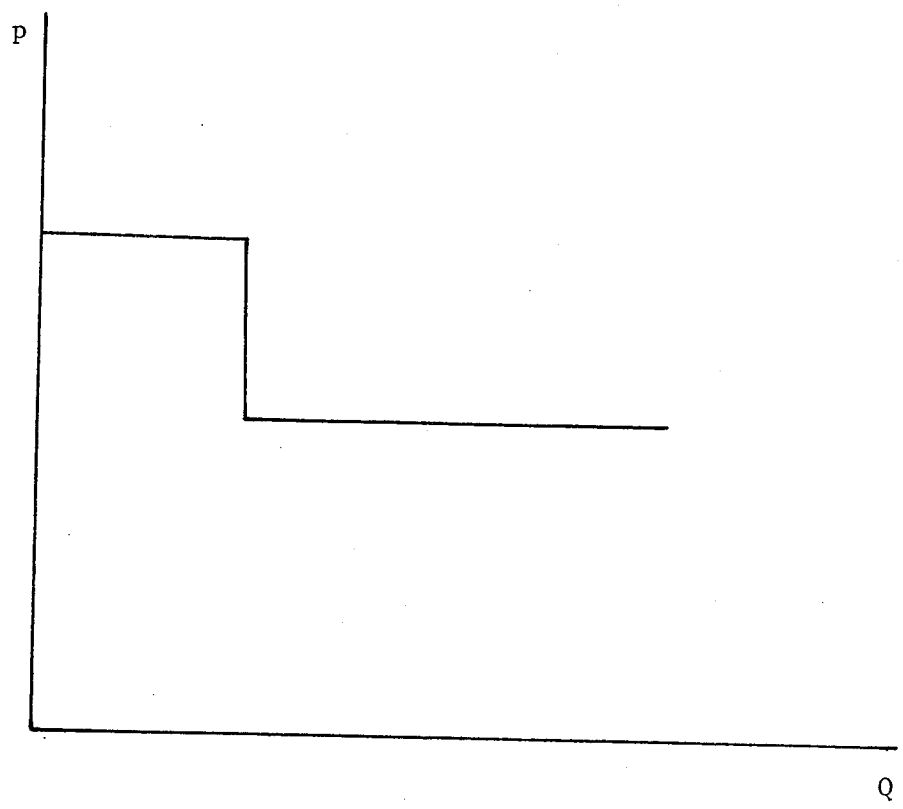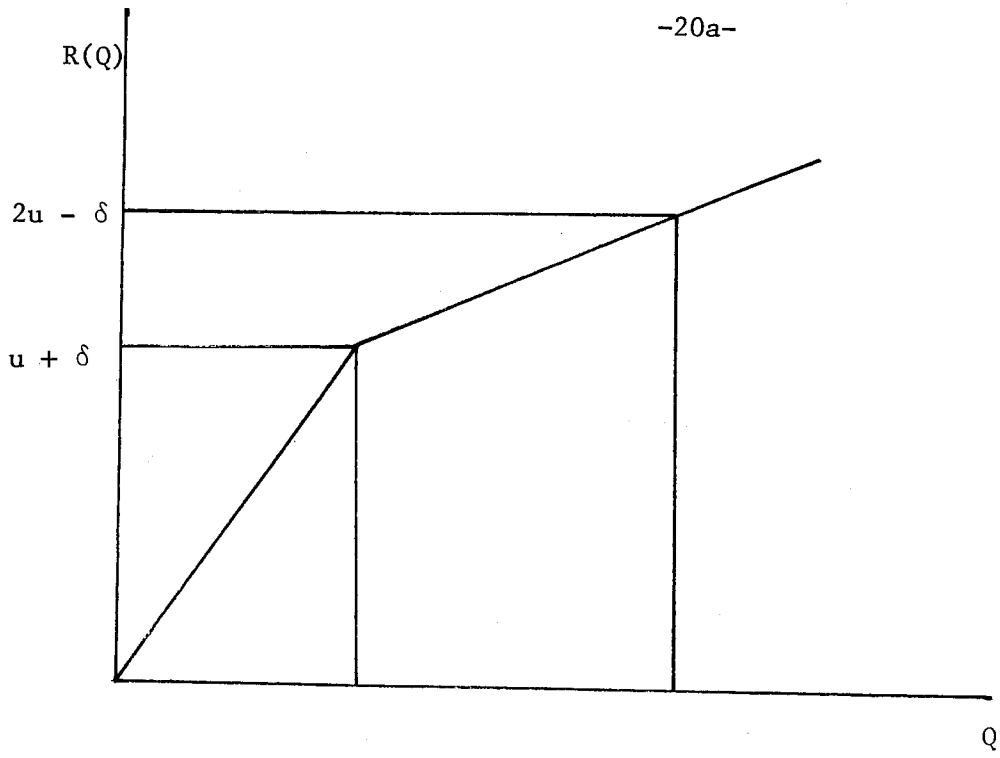
Figure 5.   Non-linear pricing schedule.

The first part of the theorem follows immediately from observing that the old will never purchase more than one unit. Thus by charging for instance, a price of u on the first unit and p on the second, the firm can perfectly discriminate.

To establish the second part of the theorem, we first prove

Lemma 4. <u>Individuals never enter the market the second period, if firms can give quantity discounts.</u>

Assume an individual re-entered the market the second period. He would have had to have purchased only one unit the first period. Assume $\hat{p}$ is the reservation price for purchasing a second unit. Clearly, provided $\hat{p} > 0$, it would have paid the store to have offered to sell the individual a second unit at a price $\hat{p}$. Thus, the first store could not have been maximizing profits.

Lemma 5. Let $R(Q)$ be the required payments by an individual to obtain quantity Q of the output. Then, if all individuals arriving at a store are young, the <u>only</u> possible price schedules entail

$$R(2) = 2u - \delta$$
$$R(1) = u + \delta$$

Such a revenue schedule is generated by a price schedule of the form

$$p = u + \xi \qquad \text{for } Q \leq 1$$
$$p = u - \delta - \xi \qquad \text{for } Q > 1$$

where

$$\xi \geq 0$$

Any individual arriving at such a store will purchase two units, and the firm's revenue will be $2u - \delta$. (See Figure 5.)

We need to show that there does not exist any other price schedule which generates higher profits. Such a price schedule must have a higher price in at least one of the two segments, $Q < 1$ or $1 \leq Q < 2$. If it is higher in both, clearly the individual will not purchase at all, and profits are zero.

If $R(2) > 2u - \delta$, then clearly $Q < 2$. But then the maximum revenue that could be generated is $u < 2u - \delta$, and hence that could not be a profit maximizing revenue function. Similarly, if $R(2) < 2u - \delta$, the profits must be less than $2u - \delta$.

Theorem 6 follows now directly from Lemma 5. With this revenue function, there is no consumer surplus, and hence no one would enter the market, if it is costly to do so.

Of course, such quantity discounts may not be operational; if two individuals, as they enter the store, observe that there are quantity discounts, one of them will make the purchase on behalf of both of them. The enforcement of non-linear price schedules is, under many conditions, not possible.

Even when non-linear pricing is possible, market equilibrium can be re-established in two different ways: (a) The markets may be for hetero-geneous commodities, about which different individuals have different attitudes; (b) There may be a set of individuals who obtain information costlessly either as a result of advertising or other sales efforts by firms or through word of mouth (see Salop and Stiglitz (1977), Butters (1977)).

There is an important moral to be drawn from this story: With costly search unbridled competition may result not in more efficient markets but in lowering of economic welfare as a result of markets not being viable. We return to this theme in our concluding remarks.

7. Rational Expectations

We have assumed that although individuals do not know the price charged at any store, they do know the price distribution. This is a kind of "rational expectations" assumption, and it is employed not so much because of its realism as because it is difficult to know what alternative expectations hypothesis to use.

It should be apparent, however, that expectations are critical. For individuals' expectations about what prices they will have to pay next period if they re-enter the market determine whether they buy for storage or only for current consumption. There is no persuasive reason to believe that individuals' perceptions of the probability distribution of prices corresponds to the actual probability distribution; indeed, there is a considerable body of literature suggesting that there may be systematic biases in individuals' perceptions of probability distributions, particularly of events (like sales) which occur infrequently. (See Tversky and Kahneman (1974).

What induces firms to charge prices below the monopoly level is that, if they lower them enough, consumers will purchase for future consumption. In an economy with price dispersion, in a two price equilibrium we require

$$(7.1) \qquad \hat{p}(2 + \lambda) = u(1 + \lambda)$$

Consumers' reservation price, $\hat{p}$, depends on their beliefs concerning the price distribution:

$$(7.2) \qquad \hat{p} = \bar{p}^e + c - \delta$$

where the superscript e on $\bar{p}$ is to remind us that what is critical is the consumers' perceptions of the mean price.

These perceptions affect the profitability of sales, and hence the frequency with which sales occur. But this, in turn, affects the mean price, which in turn may affect the perceived price.

Assume, for example, that instances where individuals have been ripped off are more memorable than those instances where individuals have found a bargain. Individuals believe that the mean price is larger than it really is:

$$\overline{p}^e = \beta(\overline{p})\overline{p}, \quad \beta > 1 .$$

Then substituting into (7.1) and (7.2) and solving, we obtain

$$\lambda = \frac{2\delta - (\beta - 1)u}{u - \delta + 2u(\beta - 1)} < 1.$$

provided

$$\delta < \frac{u}{3} + (\beta - 1)u.$$

Thus, there exists an equilibrium price distribution even in situations where with correct perceptions ($\beta = 1$) there would not.

On the other hand, if individuals believe there are bargains, even when there are not any, then again the only equilibrium may entail a price distribution. Assume, in particular, that as before

$$\overline{p}^e = \beta(\overline{p})\overline{p}$$

but now

$\frac{1}{2}\overline{p} < \beta(\overline{p})\overline{p} < \overline{p} - c$ for $\overline{p}$ near u (or $u - \delta$). Now, there cannot exist a single price equilibrium. For assume there did; as before, either $p = u$

or $p = u - \delta$. Under the postulated conditions, it would never pay anyone to store. Hence any store that reduced its price to

$$\beta(\overline{p})\overline{p}$$

would double its sales. But since $\beta > \frac{1}{2}$, profits would have to increase.

Thus, so long as individuals believe that there are some small bargains, there must, in equilibrium in fact be some bargains. In this loose sense, beliefs about the existence of price distributions are self-supporting.

8. <u>Other Models of Price Dispersion</u>

In the past decade, several related models of price dispersion have been studied. Mortensen (1973) and Grossman-Stiglitz (1976, 1980) have studied economies consisting of a number of interrelated markets, which are subject to perpetual exogenous shocks at different times (random shifts in demand or supply functions). Because information is costly, the markets are only imperfectly arbitraged, and prices vary across markets. The magnitude of this price dispersion is just sufficient to compensate the arbitrageurs for (imperfectly) equilibrating the market.

In contrast to the models of "perpetual disequilibrium," which argue that price dispersion arises because markets imperfectly arbitrate <u>exogenous</u> noise, there is a widespread view that unregulated markets may actually create noise. We have attempted to test the validity of this view in a series of related papers.

Salop (1977) showed that a monopolist might find it profitable to randomize his prices to create imperfect information and uncertainty. He showed that this allowed the monopolist to engage in discriminatory pricing, for the price dispersion provided a method of differentiating among buyers

with different search costs.[1] Salop and Stiglitz (1977) showed that

when information is imperfect, then even in competitive markets

characterized by free entry and a large number of firms, equilibrium is

characterized by price dispersion, with high search cost individuals paying

a higher price (on average) than low search cost consumers.[2] However,

if individuals were identical, no price dispersion obtains. This seemed to

confirm the link between price dispersion and discriminatory behavior,

discovered in the monopoly model. However, this paper demonstrates that

equilibrium is characterized by price dispersion, even if all individuals

are identical. Thus, price dispersion is not just a peculiar form of

price discrimination against consumers with higher search costs. Instead

the price discrimination inherent in price dispersion is directed against

that random group of consumers whose search only turns up high price

firms.

One of the more interesting recent analyses of price dispersion is

that of Diamond (1971). He was able to show that with a model of sequential

search, there could not exist an equilibrium price distribution. Diamond's

model can be viewed as a special case of the model under examination here,

where $\delta = \infty$, i.e. individuals may not store for future consumption.

This is a critical, and for many commodities, an unreasonable assumption.

It has two important implications. As noted earlier, equilibrium will be

a single price. Moreover, if there are costs to entering the market

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[1] In a related model, Heal (1976) showed that a monopolist might create
quality uncertainty in order to exploit better risk-averse consumers.

   Stiglitz (1981, 1982) has derived general conditions under which
randomization serves as an effective self-selection device.

[2] Although Salop and Stiglitz focused on the case where there was a
finite number of firms, the model extends to the case where there is an
infinite number, and each firm believes it has no effect on search behavior.
See Diamond and Rothschild (1978).

(the first search has strictly positive cost), then there will not exist an equilibrium.[1,2]

Note that there are two alternative interpretations of the model we have formulated here: Either different stores can charge different prices (λ is then the fraction of stores which persistently charge high prices) or each store can pursue a mixed strategy, charging a high price a fraction λ of the time. We have emphasized in our discussion the latter interpretation ("sales"), because when all firms pursue such strategies, there is no way that a consumer can learn the price being charged by a particular firm without searching. (If some firm persistently charges a high price, there is some presumption that eventually, this becomes known.[3,4]

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[1] Diamond, in his analysis, failed to check whether it would in fact, pay an individual to enter the market. Our analysis (with the admittedly special utility function) has shown that it never will. Elsewhere, we have established that if firms can use non-linear price schedules, then regardless of the utility functions of consumers, so long as they have strictly positive search costs, there will never exist an equilibrium (Salop and Stiglitz (1976b).)

[2] This survey of models of price dispersion is not meant to be exhaustive, but rather to illustrate the relationship between our study and several of the major strands in this fast growing literature. Perloff and Salop and Wilde and Schwartz (1979) have analyzed models of price dispersion involving non-sequential search. Hanessian (1981) and Salop and Stiglitz (1976a) have analyzed the consequences of other methods of information dissemination (e.g. personal contacts, announcements, etc.)

[3] In markets with rapid turnover of customers (such as tourism), persistence of price differences is not only plausible, but observed.

[4] Other studies generating price distributions through mixed strategies include Shilony's (in which consumers are not identical) and Varian's development of our earlier model of "Bargains and Rip-offs." Varian's assertion that our paper is concerned with "spatial" price dispersion rather than temporal price dispersion is incorrect.

9. <u>Concluding Remarks</u>

For certain markets, the mythical auctioneer of traditional economic analysis may provide the basis of a good description of the working of the market; but for others, the equilibrium which is attained is vastly different from that which would obtain were there an auctioneer. This paper has shown exactly how different the two may be in the simplest of possible contexts: there is costly information gathering. Three situations could obtain, all very different from the equilibrium of traditional competitive analysis (even though we allow free entry):

(a) There may exist equilibria with price dispersion; the price one pays depends simply on the luck of the draw; the model has an immediate application not only in terms of the dispersion in prices one sees at any one time (there being high-price and low-price stores), but also in terms of "sales" (unannounced specials). The variations in price with which we are concerned relate also to variations in quality, including durability; that is, what is important is price per effective unit of the commodity;

(b) There may exist equilibrium with a single price, but the price is above the competitive equilibrium price;

(c) There may exist no equilibrium; this will always be the case if stores are allowed to give quantity discounts.

Competition is usually thought to be in the consumer's interest; although we would not disagree with this general presumption, there is another side to this that must be borne in mind: with costly search, competition may take the form of attempting to find better ways of exploiting the small but finite degree of monopoly power associated with costly search and information. (More successful firms may not be more efficient firms,

but more effective discriminators.) Although a perfectly discriminating monopolist without transactions costs is known to be Pareto optimal, perfect discrimination with transaction costs will result in the non-existence of competitive equilibrium markets; for no one will have the desire to enter the market.

The simple model of this paper has another important moral: the kinds of comparisons that have been made in comparative systems, comparing mythical market economies with mythical socialist economies are likely, with costly information, to be of only limited relevance; and the results obtained so far, e.g., the basic equivalence theorems, are likely to be seriously misleading. We have constructed a model in which the market economy creates the "noise" which, because of costly information, it is unable to eliminate; although we have not presented a fully articulated model of either the competitive economy or a planned economy, it is clear that the two might look markedly different.

APPENDIX A

Derivation of Two-Price Equilibrium for Costly Second Period Search

There are four different cases we have to examine depending on whether individuals search second period and on whether $p_\ell < u - \delta$ (so they enjoy some consumer surplus out of storage) or $p_\ell = u - \delta$.

Case I. $p_\ell < u - \delta$, individuals re-enter the market second period.

Equilibrium is described by the simultaneous solution to the reservation price equation (3.7), which now is of the form[1]

(A.1) $\qquad p_\ell = u - \dfrac{\delta - c}{\lambda}$

and the equal profits equation, which is now of the form,

(A.2) $\qquad p_\ell (2 + \lambda) = u(1 + \lambda)$

The simultaneous solution to these two equations is given by

(A.3a) $\qquad \lambda = \dfrac{2(\delta - c)}{u - (\delta - c)} = \dfrac{2\alpha}{u - \alpha}$

(A.3b) $\qquad p_\ell = \dfrac{u + \delta - c}{2} = \dfrac{u + \alpha}{2}$

where $\alpha = \delta - c$.

The following constraints have to be satisfied:

(a) $p_\ell < u$

This implies $\alpha < u$.

---

[1] Throughout Appendix A, c is the search cost the second period.

(b) $\lambda > 0$

This implies $0 < \alpha < u$.

(c) $1 - \lambda = \dfrac{u - 3\alpha}{u - \alpha} > 0$

This implies

$u/3 > \alpha$.

(d) The individual re-enters the market if

$u - \bar{p} > c$

or

$$u - \lambda u - (1 - \lambda)p_\ell = (1 - \lambda)(u - p_\ell)$$
$$= \frac{1 - \lambda}{\lambda} \alpha > c$$

or $\alpha > u - 2c/3.$

Thus, the necessary and sufficient condition for this type of equilibrium is that

(A.4) $0 < \alpha < u/3$

Case II. $p_\ell = u - \delta$; search second period. Equal profits implies

$(u - \delta)(2 + \lambda) = u(1 + \lambda)$

or

$\lambda = (u/\delta) - 2$

$\lambda > 0$ implies

(A.4a) $\delta < u/2$

$\lambda < 1$ implies

(A.4b) $\delta > u/3$

Search second period implies

$$\lambda u + (1 - \lambda)(u - \delta) + c < u$$

or

$$c < (1 - \lambda) \, \delta = 3\delta - u$$

Thus, the necessary and sufficient condition for this type of equilibrium is

(A.5) $u/2 > \delta > u/3 + c/3$

Case III. $p_\ell < u - \delta$; no search second period.

This is not possible. For we established earlier that p_ℓ must be the price which makes individuals indifferent between purchasing for storage and not. But since $p_\ell < u - \delta$ this can only be the case if individuals are indifferent between purchasing for storage and searching next period; but by assumption, we have assumed that the expected return from search is negative; hence the expected return from storage is negative, a contradiction.

Case IV. $p_\ell = u - \delta$; no search second period.

When there is no search second period, the low price stores sell precisely twice as much as the high price stores, since there are no old people on the market. Equal profits implies

$$2(u - \delta) = u$$

or

$$\delta = u/2$$

No search implies

$$c > (u - \bar{p}) = (1 - \lambda)(u - p_\ell) \equiv (1 - \lambda)\delta$$

or

(A.6) $\lambda > \dfrac{\delta - c}{\delta} = 1 - c/\delta$

APPENDIX B

Decision to Enter the Market

<u>Case I.</u> Letting c_1 be the first period search costs, c_2 the second period search costs, we require expected consumer surplus from entering the market to exceed the cost:

(B.1) $(1 - \lambda)[2(u - p_\ell) - \delta] + \lambda[(1 - \lambda)(u - p_\ell) - c_2] > c_1$

or, rearranging terms,

(B.2) $(1 - \lambda)(2 + \lambda)(u - p_\ell) - (\delta(1 - \lambda) + \lambda c_2) > c_1$

i.e., using (3.11) and (3.13)

(B.3) $(1 - \lambda)[(2 + \lambda)u - u(1 + \lambda)] = \dfrac{u(u - 3(\delta - c_2))}{u - (\delta - c_2)} > c_1 + \delta - \dfrac{2(c_2 - \delta)^2}{u - (\delta - c_2)}$

or, multiplying through by $u - \delta + c_2$,

(B.4) $u(u - 3(\delta - c_2)) > (c_1 + \delta)(u - (\delta - c_2)) - 2(c_2 - \delta)^2$

If $c_1 = c_2 = c$, we require

(B.5) $\phi(c, \delta) = c^2 + 2c(u - 2\delta) + (u - \delta)(u - 3\delta) > 0$

$\phi(c, \delta) = 0$ when

$$c = \dfrac{-2(u - 2\delta) + 2\sqrt{u^2 - 4\delta u + 4\delta^2 - (u^2 - 4\delta u + 3\delta^2)}}{2}$$

$- (u - 2\delta) \pm \delta$

$\phi(c, \delta) > 0$ for

(B.6) $c > (3\delta - u)$

Case II. For expected consumer surplus to be positive,

$$(1 - \lambda)\delta = (3\delta - u) > c_1$$

If $c_1 = c_2 = c$, this constraint is always satisfied (constraint A.5).

Case IV. Expected consumer surplus is now

$$(1 - \lambda)\delta > c_1$$

or

$$(B.8) \qquad \lambda < 1 - c_1/\delta$$

If $c_1 = c_2 = c$, this constraint is never satisfied (cf. constraint A.6).

Note that if the inequalities (B.6) - (B.8) are not satisfied, no equilibrium exists. (Recall from the text that no S.P.E. exists if $c_1 = c_2$.)

APPENDIX C

Equilibrium with Proportional Storage Costs

Assume that storage costs are proportional to the purchase price, e.g., as a result of interest. Then, instead of (3.1), we obtain, as our reservation price,

$$(C.1) \qquad \hat{p}(1 + \delta) = \bar{p} + c$$

or since $p_\ell = \hat{p}$,

$$(C.2) \qquad \hat{p} = \min \left[\frac{u}{1 + \delta} , \frac{\lambda p_h + (1 - \lambda)p_\ell + c}{1 + \delta} \right] .$$

Since, as before, $p_h = u$,

$$(C.3) \qquad p_\ell = \frac{\lambda u + c}{\lambda + \delta} .$$

Equal profits entail (assuming individuals search second period)

$$(C.4) \qquad (1 + \lambda)u = p_\ell(2 + \lambda)$$

or

$$(1 + \lambda)u = \frac{(\lambda u + c)}{\lambda + \delta} (2 + \lambda)$$

or

$$u(\lambda + \delta + \lambda\delta) = 2c + 2\lambda u + \lambda c$$

$$(C.5) \qquad \lambda = (2c - u\delta)/(u\delta - u - c).$$

Hence, rewriting (C.3)

$$p_\ell = u - \frac{(u\delta - c)}{\lambda + \delta} ;$$

Substituting (C.5), we obtain

$$p_\ell = u - \frac{(u\delta - c)(u\delta - u - c)}{2c - 2u\delta + \delta(u\delta - c)} = u - \frac{u\delta - u - c}{\delta - 2} \quad .$$

Upon rearrangement of terms, we obtain

(C.6) $\qquad p_\ell = \dfrac{u - c}{2 - \delta}$.

For this to be an equilibrium, we require

(a) $\quad p_\ell(1 + \delta) < u \quad$ (individuals purchase for storage.)

Substituting (C.6), we have

$$(u - c)(1 + \delta) < u(2 - \delta)$$

which can be rewritten as

(C.7) $\qquad c > \dfrac{u(2\delta - 1)}{1 + \delta} \quad$.

(b) $\quad \lambda > 0$: From (C.5) we obtain directly

(C.8) $\qquad c < \dfrac{u\delta}{2}$

(c) $\quad \lambda < 1$: From (C.5), we have

$$1 - \lambda = \frac{-2(c - u\delta) - (u + c)}{u\delta - u - c} = \frac{3c + u - 2u\delta}{c + u - u\delta} \quad .$$

Hence

(C.9) $\qquad c > u(2\delta - 1)/3$.

(d) It pays to search the second period

$$c < (u - p_\ell)(1 - \lambda) .$$

Using (C.5) and (C.6), we obtain

$$(u - p_\ell)(1 - \lambda) = (u - \frac{u - c}{2 - \delta})(\frac{3c + u - 2u\delta}{c + u - u\delta})$$

$$= (\frac{u - u\delta + c}{2 - \delta})(\frac{3c + u - 2u\delta}{c + u - u\delta})$$
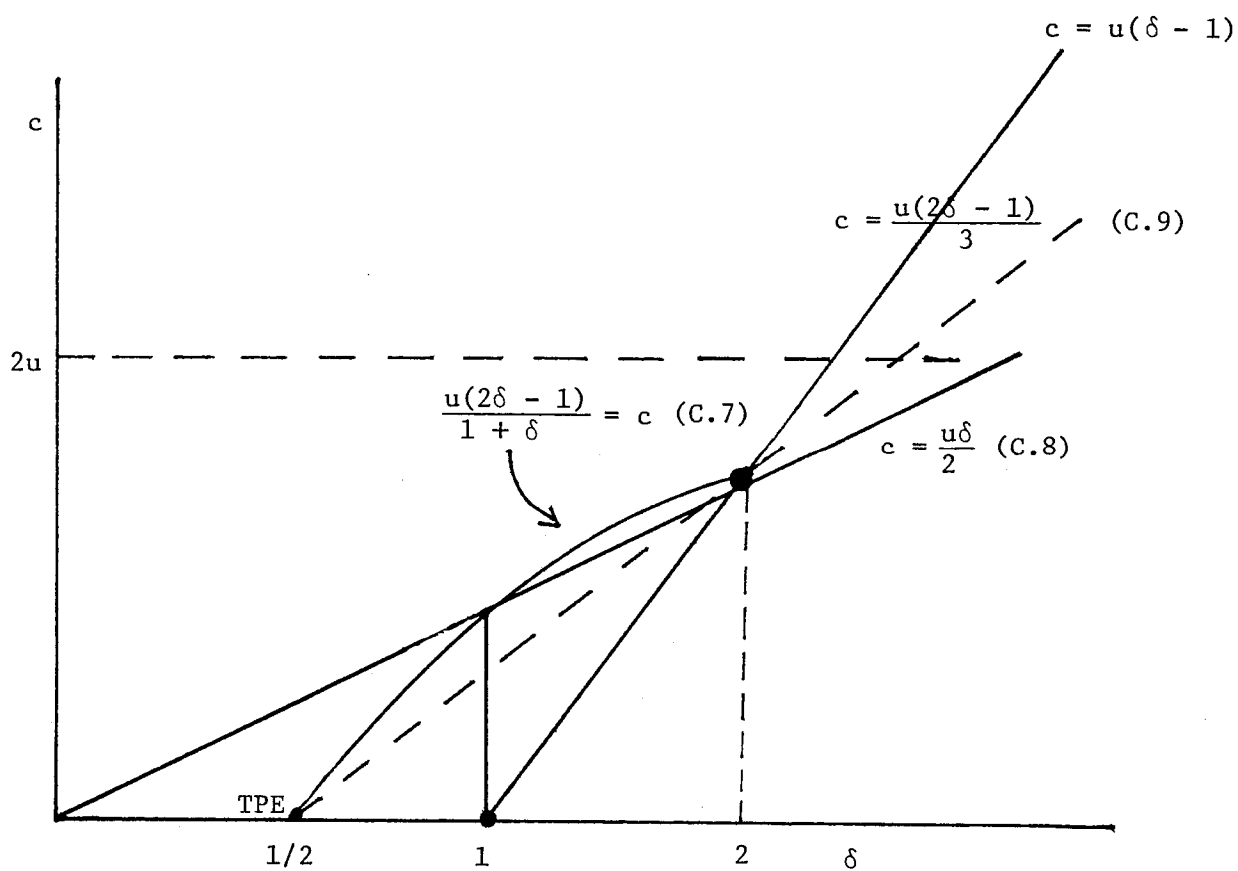
$$= \frac{3c + u - 2u\delta}{2 - \delta}$$

Figure 6.

i.e.

(C.10) $c > \dfrac{u(2\delta - 1)}{1 + \delta}$

which is identical to (C.7).

   (e) It pays to enter the market. As before, let c_1 = search costs first period, c_2 = search costs second period. Then we require:

(C.11) $(1 - \lambda)(2(u - p_\ell) - \delta p_\ell) + \lambda[(u - p_\ell)(1 - \lambda) - c_2] > c_1$.

   If $c = c_1 = c_2$, this can be written as

(C.12) $(1 - \lambda)((u - p_\ell)(2 + \lambda) - \delta p_\ell) > (1 + \lambda)c$

or

   $(2u(1 - \delta) + \delta c)(3c + u - 2u\delta) > c(u - c)(2 - \delta)$

or

   $2c^2(1 + \delta) + uc[(6 - 6\delta) + \delta(1 - 2\delta) - (2 - \delta)] + 2u(1 - \delta)\,u(1 - 2\delta) > 0$

Collecting terms, we obtain

(C.13) $c^2(1 + \delta) + uc(2 - 2\delta - \delta^2) + u^2(1 - \delta)(1 - 2\delta) > 0$.

At the boundary,

$$c = \frac{u(-2+2\delta+\delta^2) \pm u\sqrt{4(1-\delta)^2 - 4\delta^2(1-\delta) + \delta^4 - 4(1-\delta)(1-2\delta)(1+\delta)}}{2(1 + \delta)}$$

$$= \frac{u[-2 + 2\delta + \delta^2 \pm \delta(\delta - 2)]}{2(1 + \delta)}$$

$$= \begin{cases} u(\delta - 1) \\[2mm] \dfrac{u(2\delta - 1)}{1 + \delta} \end{cases}$$

In Figure 6, we have depicted the boundary curves in (δ, c) space in which there exists a two-price equilibrium.

The other cases may be handled similarly and are left as exercises for the reader.

APPENDIX D

Constant Search Costs

A. <u>Two-Price Equilibrium</u>

As before, there are several possible patterns. We focus on the case where $p_\ell < u - \delta$ leaving the others as exercises for the reader. We shall further assume that c_1 and c_2 are such that the individual enters the market the first period, but does not search, but does search the second period, if he is unlucky enough to find a high price store the first period. Reservation price is given by (since expected search costs are $c_2/(1-\lambda)$

$$(D.1) \qquad \hat{p} + \delta = p_\ell + \frac{c_2}{1-\lambda}$$

so equilibrium requires

$$(D.2) \qquad \lambda = 1 - c_2/\delta .$$

Equal profits entail (since the high-price stores only sell to the "unlucky young")

$$(D.3) \qquad (2 + \frac{\lambda}{1-\lambda}) p_\ell = u$$

or

$$(D.4) \qquad p_\ell = \frac{u}{3 - c_2/\delta}$$

Equilibrium requires

(a) $0 < \lambda < 1$

i.e.

(D.5) $\delta > c_2 > 0;$

(b) it does not pay to search the first period, i.e., if one arrives at a high price store, the return to seeking out a low price store is

$$(\frac{c_2}{1-\lambda} - \delta) + 2(u - p_\ell)$$

and this must be less than the expected search costs,

$$\frac{c_1}{1-\lambda} ,$$

i.e.

$$c_1 > 2u \frac{2\delta - c_2}{3\delta - c_2} \frac{c_2}{\delta}$$

(c) it does pay to enter the market the first period:

(D.7) $(1 - \lambda)(2(u - p_\ell) - \delta) > c_1$

i.e. using (D.2):

$$c_1 < c_2[\frac{2(2\delta - c_2)u}{(3\delta - c_2)} - 1]$$

(d) Finally, for $p_\ell < u - \delta$

$$\delta < u - \frac{u}{3 - c_2/\delta}$$

or

$$\delta < u(\frac{2 - c_2/\delta}{3 - c_2/\delta})$$

(D.10) $c_2 < \dfrac{\delta(2u - 3\delta)}{u - \delta}$

For (D.10) to be binding when (D.5) is satisfied requires

$$2u - 3\delta < u - \delta$$

i.e.

(D.11) $u < 2\delta$

   (e)   It pays to search the second period, i.e.

$$u - p_\ell > \dfrac{c_2}{1-\lambda}$$

which is identical to (D.9).

   We have plotted the constraints in Figure 7 . Clearly, there exists a two-price equilibrium provided[1]

(D.12) $c_2 < \delta < \dfrac{u}{2}$

and

$$\left(\dfrac{u(2\delta - c_2)}{3\delta - c_2}\right)\dfrac{c_2}{\delta} < c_1 < \dfrac{c_2}{\delta}\left[\dfrac{2(2\delta - c_2)u}{3\delta - c_2} - \delta\right]$$

----

[1] At $c_2 = 0$,

$$\dfrac{d\left[\dfrac{2(2\delta - c_2)u}{\delta(3\delta - c_2)} - 1\right]c_2}{dc_2} = \dfrac{4}{3}(u/\delta - 1)$$

while

$$\dfrac{du\left(\dfrac{2\delta - c_2}{3\delta - c_2}\right)\dfrac{c_2}{\delta}}{dc_2} = \dfrac{2}{3}\; u/\delta \;\; .$$
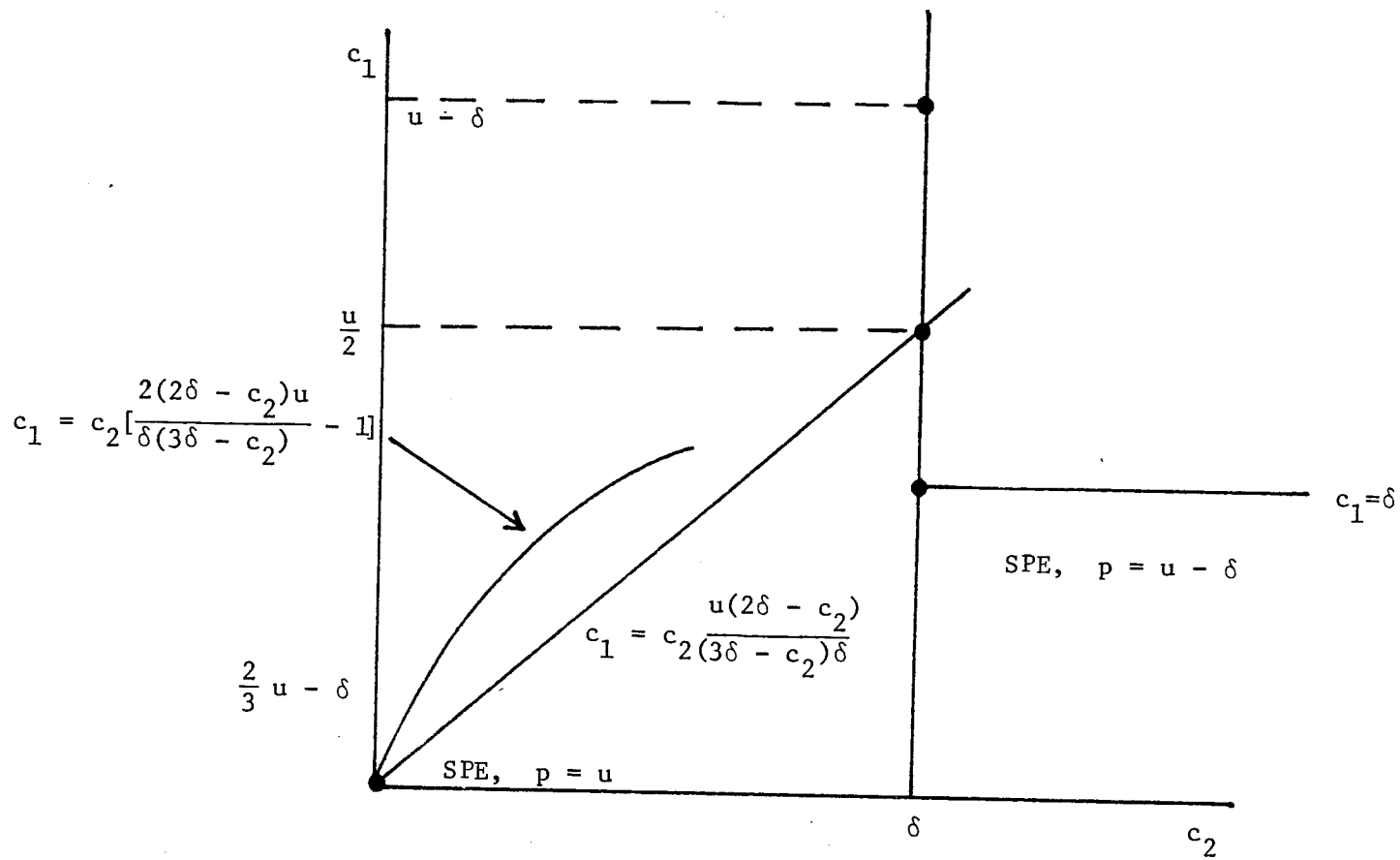
Figure 7.

B. Single Price Equilibrium

We prove if $c_i > 0$

(i) The only possible single price equilibrium is at $p = u - \delta$

(ii) There exists a single price equilibrium if and only if

$$c_1 < \delta < \min[c_2, u/2]$$

It immediately follows that if $c_1 = c_2$, there never exists a single price equilibrium.

Proof. We first establish that the only possible equilibria entail $p = u$ or $p = u - \delta$ if $c_i > 0$. $p < u - \delta$ cannot be an equilibrium if $c_2 > \delta$; for clearly all firms sell customers two units; if a firm raised its price by a small amount, it would loose no sales; if $c_2 < \delta$, there is no storage, and again, the firm loses no sales by raising its price to u. Similarly $u - \delta < p < u$ cannot be an equilibrium; for clearly, firms sell each customer only a single unit; hence if a firm raised its price by a small amount, it would lose no sales.
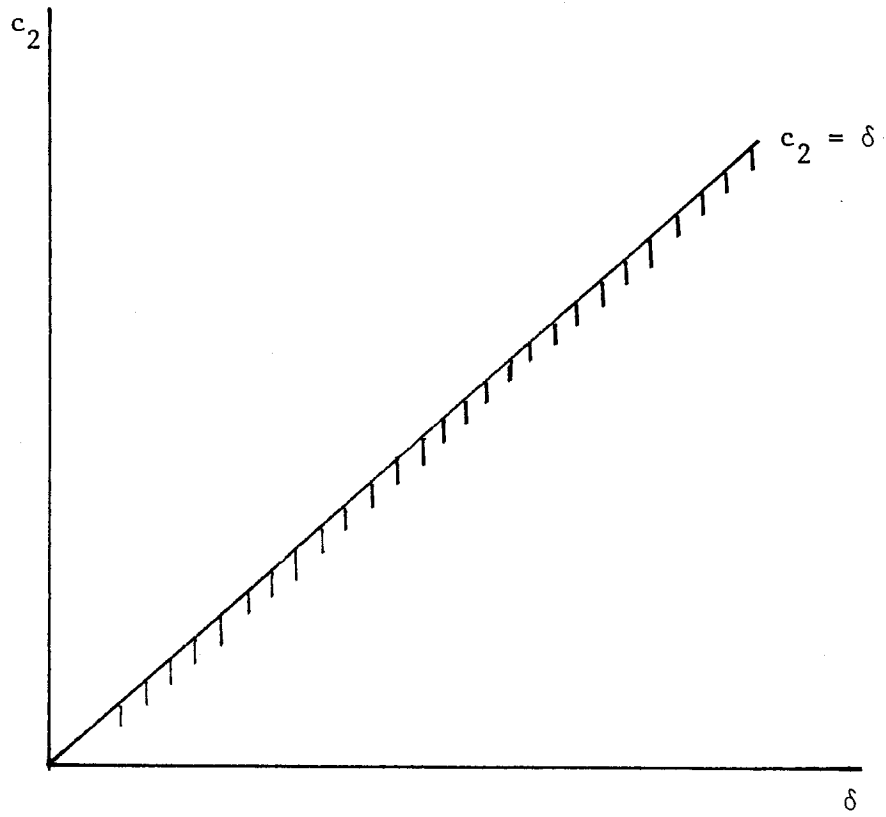
On the other hand, if $p = u$, individuals enter the market only if $c_1 = 0$.

For $p = u - \delta$ to be an equilibrium, it must pay individuals to enter the market ($c_1 < \delta$); it must not pay individuals to re-enter the market the second period rather than store ($c_2 > \delta$); and it must not pay any store to raise its price to the monopoly level, i.e.

$$u < 2(u - \delta)$$

or

$$u > 2\delta.$$

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