PRICE DYNAMICS BASED ON THE ADJUSTMENT OF FIRMS*

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1. INTRODUCTION

In his classic Elements, Leon Walras provided the following description of the adjustment of an economic system under free competition (p. 225):

> ...if the selling price of a product exceeds the cost of the productive services for certain firms and a profit results, entrepreneurs will flow towards this branch of production or expand their output, so that the quantity of the product (on the market) will increase, its price will fall, and the difference between price and cost will be reduced; and, if (on the contrary), the cost of the productive services exceeds the selling price for certain firms, so that a loss results, entrepreneurs will leave this branch of production or curtail their output, so that the quantity of the product (on the market) will decrease, its price will rise and the difference between price and cost will again be reduced...

This description remains very much at the heart of our understanding of the workings of a capitalist economy. In the short run firms are "in place" and the returns to factors that are temporarily immobile are not necessarily equalized. The prices that at each moment clear markets reflect only the relative scarcity of factors that are instantaneously variable. Walras described the formation of these short run prices by a tatonnement, but his analysis did not stop there.

Walras and many of his followers looked further to a long run equilibrium that evolves from a sequence of short run or period equilibrium. Between periods all factors are free to vary, and they "flow towards that branch of production" in which there are profits to be realized. This is perhaps the most essential ingredient in the workings of the "invisible hand"; in addition, it is rich with empirical counterparts and should have important predictive value. Nevertheless, the descriptions of the adjustment of prices that exist in most of formal theory concentrate on the tatonnement, and with few exceptions, economists have not modeled the process by which scarce factors flow in the direction of increased profit.[1] In this paper we analyze profits as signals for the adjustment of firms in a situation of product differentiation. We inquire regarding the range of situations in which a simple entry dynamics, based on limited information regarding future prices, will lead, in the long run, to efficient allocation.

The model substantially generalizes the example put forth in [4]. Our framework is a straightforward continuous time general equilibrium model with a continuum of consumers and a continuum of firms. In addition to the numeraire there is a continuum of differentiated products. Each consumer demands one unit of one type of differentiated commodity and has a utility function that is additive in the numeraire; each firm produces exactly one type of commodity at each point in time. Given an initial density of firms, the initial equilibrium price function for commodities is defined by the condition that supply equals demand for all commodities. Up to this point the analysis follows Rosen [3], but now firms adjust the commodity which they produce by maximizing the rate of change in profit subject to a quadratic cost of adjustment. This has them move in the direction of higher profit at a velocity that is proportional to the rate of change in profit. Thus the density of firms changes over time, and prices adjust so that at each instant supply equals demand. As prices change, the gradient of profit changes, which explains further changes in the distribution of firms, and so on.

Despite the fact that firms have limited vision concerning future prices, the system converges over time to the unique equal profit equilibrium at which the resources devoted to the production of the continuum of differentiated products cannot be allocated more efficiently. This result does not require that firms calculate an optimal plan for production over time; more to the point, it does not even require that there exist markets for future commodities. Firms are only required to know prices locally and at each point in time. Short run profits provide adequate signals for effectively guiding an economy toward an efficient allocation of resources; this supports the institution of free competition.

An alternative interpretation of the model concerns the case in which competition is not free. Suppose that a planner is responsible for determining the distribution of firms and that the same quadratic cost of adjustment continues to apply. (The planner may have no information regarding tastes or the technology for producing the various commodities, but each firm requires one unit of a particular form of land, labor, or capital that must be produced by the planner.) Start from a position in which the distribution of firms is arbitrary and given. Because of the special form of the utility functions, there is a unique assignment of consumer to differentiated commodity types that is consistent with Pareto efficiency constrained by the distribution of firms. Relative to the assignment associated with an optimal distribution of firms, there is a well-defined loss that is measured in terms of the numeraire. It turns out that the procedure of adjusting the position of each firm at a directed velocity proportional to the rate of change in profit can be interpreted as a gradient process for minimizing loss. In effect, for this model profits are sufficient signals for determining the instantaneous adjustment in the distribution of firms that maximizes the rate of welfare increase.

Section II presents a single period equilibrium model. Section III introduces the adjustment process and establishes convergence to an optimal distribution of firms; the section concludes with some arguments concerning the generality of the results. Section IV proves that the dynamic process under consideration is a gradient, and finally, Section V shows that the myopic case considered here is the limit of case in which expectations are rational.

II. SHORT RUN EQUILIBRIUM

We start by describing the equilibrium with firms in place. Before the introduction of dynamics, the model is rather similar to the hedonic price model of Rosen [3]. In addition to a numeraire commodity, there is a differentiated product for each point on the circumference of the unit circle C. A generic differentiated commodity is denoted by $x \in [0, 2\pi]$ with 0 and 2π identified, and amounts of the numeraire commodity are indicated by the positive real variable y. Similarly, consumer preferences are indexed by $\beta \in C$, and the utility function $u(\cdot,\cdot,\beta)$ represents the preferences of type β: $u(x,y,\beta) > u(x',y',\beta)$ means that the agent with preference type β prefers one unit of the differentiated commodity of type x and y units of numeraire to one unit of the differentiated commodity of type x' and y' units of numeraire. The formulation on the circle is inessential, and the value of x could refer to horsepower, color (defined by wave length), size, or location. For the present we will assume that $u(x,y,\beta) = y - (\beta - x)^2$, where $\beta - x$ is sometimes $(\beta + 2\pi) - x$ and so on, and β is distributed uniformly over C with density $1/2\pi$. More generally we could consider $u(x,y,\beta) = y - g(\beta-x)$, where g is convex, $g(0) = 0$, and $g'(0) = 0$; however, the additivity in y is indispensable for the arguments. Finally, the income of the consumer with preferences β is denoted by I_β, with the dependence of I_β on the prices of the differentiated commodity suppressed.[2] If the function ϕ gives the prices of the differentiated commodities as a function of type, then consumer β can afford bundles (x,y) that satisfy $I_\beta - \phi(x) - y \geq 0$.

FIGURE 1

For each price function $\phi(\cdot)$, consumer β, defined by $u(\cdot,\cdot,\beta)$ and I_β solves:

(1) $$\max_x \; I_\beta - \phi(x) - (\beta - x)^2,$$

with the necessary conditions $\phi'(x) = 2(\beta - x)$. It is clear that the variety of differentiated product chosen by β is independent of the income I_β; implicitly, we have assumed that for each β, $I_\beta > \phi(x) + (\beta - x)^2$ for some $x \in [0, 2\pi]$. Now introduce the variable t to index time and let $T: [0, 2\pi] \times R_+ \to [0, 2\pi]$ define the type β of consumer who takes commodity variety x at prices $\phi(x,t)$ at time t. Then,

(2) $\phi_x(x, t) = 2(T(x, t) - x)$. Partial differentiation with respect to x yields

(3) $T_x(x, t) = 1 + [\phi_{xx}(x, t)/2]$, and the density of demand for x at time t at prices ϕ is

(4) $(1/2\pi)T_x(x, t) = (1/2\pi) + [\phi_{xx}(x,t)/4\pi]$.

Next we turn to the supply side. Let $f(x, t)$ denote the density of firms at x at time t; at each time t the integral of f over C is one. Over time the mass of firms that produce the differentiated commodities remains unity. The terms entry and exit refer to movement into or out of the production of a specific type of differentiated product. For the example I have chosen the simplest form of technology: for all t, the firm at x at time t can produce one unit of x from one unit of numeraire y, or it can remain out of the market. Note that no equilibrium price function is consistent with inaction for a positive mass of firms. The qualitative results remain the same if the relation between y and x has the usual neoclassical flavor.

FIGURE 2

It is important to characterize those assignments of consumers to commodity type that are optimal with respect to a given distribution of firms $f(\circ, t)$. Subsequently, we will characterize those assignments of consumers to commodity type that are equilibria relative to $f(\circ, t)$ and observe that equilibrium assignments and optimal assignments coincide. It is clear that optimality relative to the distribution of firms $f(\cdot, t)$ requires that

$$\int_0^{2\pi} (T(x, t) - x)^2 f(x, t)dx$$ is minimized, where $T(x, t)$ is the

agent assigned to the commodity of type x; and T is an assignment; that is $(1/2\pi)T_x(\circ, t) = f(\circ, t)$. It follows that $T^p(\cdot, t)$ is an optimal assignment of the differentiated commodity relative

$f(\circ, t)$ if and only if $T^p(0, t)$ minimizes

(5) $\int_0^{2\pi} (2\pi F(x,t) + T^p(0, t) - x)^2 f(x,t)\,dx$, where $F(\circ, t)$ is the cumulative distribution associated with $f(\circ,t)$, and this gives

$$T^p(0,t) = \int_0^{2\pi} (x - 2\pi F(x, t))\, f(x,t)\,dx =$$

$$\int_0^{2\pi} xf(x,t)dx - \int_0^{2\pi} 2\pi F(x,t)f(x,t)dx$$

Applying integration by parts to the first integral and substituting $f(x)dx = du$ in the second yields

(6) $$T^p(0, t) = \pi - \int_0^{2\pi} F(x,t)\,dx\ .$$

It is natural to define the maximal welfare associated with $f(\circ, t)$ by

(7) $$W(t) = -\int_0^{2\pi} (2\pi F(x,t) + T^p(0, t) - x)^2 f(x,t)\,dx.$$

Because of the special form of the utility functions, an assignment $T(\cdot,t)$ is Pareto efficient relative to the firm distribution $f(\circ,t)$ if and only if $T(\circ,t)$ realizes the maximal welfare. Observe also that each assignment $T(\circ,t)$ is determined by $T(0, t)$.

We now characterize the assignments of consumers to commodity type that is an equilibrium relative to the distribution of firms $f(\cdot, t)$. The equality of supply and demand at x requires

(8) $f(x,t) = (1/2\pi) + [\phi_{xx}(x,t)/4\pi]$; equivalently,

(9) $2\pi f(x,t) - 1 = \phi_{xx}(x,t)/2$.

Integrating both sides from 0 to x and rearranging yields

(10) $\phi_x(x,t) = 4\pi F(x,t) + \phi_x(0,t) - 2x,$

where we recall that $F(\cdot,t)$ is the cumulative distribution corresponding to $f(\cdot,t)$. Integrating both sides with respect to x from 0 to 2π gives

(11) $$0 = \int_0^{2\pi} \phi_x(x,t)\, dx = 4\pi \int_0^{2\pi} F(x,t)\, dx - 2 \int_0^{2\pi} x dx + 2\pi\phi_x(0,t),$$

and so

(12) $$\phi_x(0, t) = 2\pi - 2 \int_0^{2\pi} F(x,t)\, dx .$$

This result combines with (2) to give

(13) $$T^e(0,t) = \pi - \int_0^{2\pi} F(x,t)\, dx,$$ where $T^e(\cdot,t)$ is the equilibrium assignment relative to $f(\cdot,t)$.

Since $T^e(0, t)$ determines $T^e(\cdot, t)$ and $T^p(0, t)$ determines $T^p(\cdot, t)$, it follows from (6) and (13) that the equilibrium assignment corresponding to the density $f(\cdot,t)$ maximizes welfare relative to $f(\circ,t)$. Equilibrium is unique and the distribution of differentiated commodities is efficient.

Equations (10) and (12) yield

$$(14) \quad \phi_x(x,t) = 4\pi F(x,t) + 2\pi - 2\int_0^{2\pi} F(x,t)\,dx - 2x .$$

For each t this equation determines the price function $\phi(\circ,t)$ up to a positive constant. The fact that ϕ is not further defined is a casualty of the especially simple but non-smooth technology.[3]

III. THE ADJUSTMENT OF FIRMS

Now assume that firms move from x with a directed velocity $\phi_x(x,t)$. For our immediate purpose such action can be thought of as following from a naive behavioral rule: maximize the rate of change of profit when the cost of adjustment is a quadratic function of the velocity of adjustment w (for concreteness $(1/2)w^2$) . (Later this rule will be shown to follow from profit maximization over a short horizon.) By the chain rule, a firm that changes its product at velocity w will change its profit at a rate $w\frac{\partial}{\partial x}(\phi(x,t) - 1) - (1/2)w^2$, and the firm maximizes this expression by choosing $w = \phi_x(x,t)$. It is now possible to derive the expression that explains how prices ϕ and welfare W evolve over time. Using the definition of F, and then differentiating with respect to t under the integrals, (14) becomes

$$(15) \quad \phi_{xt}(x,t) = 4\pi\int_0^x f_t(v,t)\,dv - 2\int_0^{2\pi}\int_0^x f_t(v,t)\,dv dx .$$

It is fundamental that for a flow with density $f(\circ,t)$ at t and velocity $w(\circ,t)$ at t,

(16) $$f_t(v,t) = -\frac{\partial}{\partial v}(w(v,t)\ f(v,t)) .$$

Combining this with (15) yields

(17) $$\phi_{xt}(x,t)=4\pi\int_0^x -\frac{\partial}{\partial v}[\phi_v(v,t)f(v,t)]dv-2\int_0^{2\pi}\int_0^x -\frac{\partial}{\partial v}[\phi_v(v,t)f(v,t)]dvd.$$

which readily reduces to

(18) $$\phi_{xt}(x,t) = -4\pi\phi_x(x,t)f(x,t) + 2\int_0^{2\pi}\phi_x(x,t)f(x,t)dx,$$ for all x and t.

Equation (18) reveals quite a lot about the evolution of prices over time. The first term tells us that the price gradient has a tendency to flatten. If $\phi_x(\circ,t)$ is zero everywhere, then profit is the same everywhere and all motion ceases. In this case $T(x,t) = x$ for all x and so f is uniform; also, W is as large as possible. However, $\phi_x(x,t) = 0$ does not guarantee that the price gradient will remain zero at x. If the second term on the right hand side of (18), call it H, is positive, then ϕ_x is positive "for too many x," which indicates that too many consumers are moving counterclockwise (ϕ_x positive means, by (2), that $2(T(x,t) - x)$ is positive, which means that the agent who takes x is clockwise from x). The term H, which is independent of x adjusts $\phi_x(x,t)$ (for each x, over time) to take this into account.

Next we compute the evolution of W over time.

(19) $$\frac{d}{dt} W(t) = - \frac{\partial}{\partial t} \int_0^{2\pi} (T(x,t) - x)^2 f(x,t)\, dx$$

$$= - \frac{\partial}{\partial t} \int_0^{2\pi} \left(\frac{\phi_x(x,t)}{2}\right)^2 \left[\frac{1}{2\pi} \left(1 + \frac{\phi_{xx}(x,t)}{2}\right)\right] dx$$

$$= - \frac{1}{2\pi} \frac{\partial}{\partial t} \left[\frac{1}{4} \int_0^{2\pi} [\phi_x(x,t)]^2\, dx + \frac{1}{8} \int_0^{2\pi} [\phi_x(x,t)]^2\, \phi_{xx}(x,t)\, d\right.$$

$$\frac{d}{dt} W(t) = - \frac{1}{8\pi} \int_0^{2\pi} \frac{\partial[\phi_x(x,t)]^2}{\partial t}\, dx + \frac{1}{16\pi} \frac{\partial}{\partial t} \left[\left(\frac{\phi_x(x,t)}{3}\right)^3 \Bigg|_0^{2\pi}\right]$$

$$= - \frac{1}{8\pi} \int_0^{2\pi} \frac{\partial[\phi_x(x,t)]^2}{\partial t}\, dx \quad \text{(since } \phi_x(2\pi,t) = \phi_x(0,t)\text{, for all } t\text{)}$$

$$= - \frac{1}{4\pi} \int_0^{2\pi} \phi_x(x,t)\phi_{xt}(x,t)\, dx,$$ and using (18) yields

$$= - \frac{1}{4\pi} \int_0^{2\pi} \phi_x(x,t) \left[-4\pi\phi_x(x,t)\, f(x,t) + 2 \int_0^{2\pi} \phi_x(x,t)\, f(x,t)\, dx\right] d$$

$$= \int_0^{2\pi} [\phi_x(x,t)]^2 \; f(x,t)\; dx + Q(t) \int_0^{2\pi} \phi_x(x,t)\; dx, \text{ for some } Q(t)$$

a function of t alone,

$$= \int_0^{2\pi} [\phi_x(x,t)]^2 \; f(x,t,)\; dx$$

$$=4\int_0^{2\pi} (T(x,t) - x)^2 \; f(x,t)\; dx .$$

Thus,

(20) $\frac{dW}{dt} = - 4W(t)$, and so $W(t) = de^{-4t}$, $(d < 0)$.

This illustrates the claim made in the introduction; the reciprocal of welfare loss is exponential in time. Convergence to the Pareto efficient distribution of firms is assured, and this takes place even with extremely myopic behavior. Profits are excellent signals.

The convergence of the above profit gradient dynamics to an efficient distribution of firms is assured under rather general conditions, provided we maintain the assumption that utility functions are additive in the numeraire commodity and provided that the set of consumers is connected. For any given f, the set of assignments is convex, and if g is convex (recall, $u(x,y,\beta) = y - g(\beta - x)$), then

$$- \int_0^{2\pi} g(T(x,t) - x) f(x,t)\; dx$$ is concave and so there is a

unique optimum which is the competitive equilibrium. As long as the relation between an efficient allocation of differentiated commodity and the maximization of a welfare function such as

$$-\int_0^{2\pi} g(T,x,t) - x)\ f(x,t)\ dx$$ applies, we have a Lyapunov function for the convergence argument. In the example we have just analyzed, the first order conditions for utility maximization require that $T(x,t)$ is in a clockwise direction from x whenever $\phi_x(x,t) > 0$. Since the rate of change in profit is ϕ_x, the firm x moves clockwise towards the consumer it is serving. An increase in welfare can be achieved even if one maintains the

FIGURE 3

assignment of consumers to differentiated commodities, and in a competitive allocation relative to the new density of firms this loss can only be further reduced. The argument does not depend on the fact that x varies over the circle, and it should apply if C is replaced by an arbitrary smooth manifold M without boundary. Of course, if costs in terms of the numeraire vary over M, then ϕ_x will no longer be a profit gradient and firms may move away from the consumer they serve; to be a bit more precise, it is possible that

$g(T(x,t) - [x + w(x,t)\ dt]) > g(T(x,t) - x)$, where $w(\cdot,t)$ is the profit gradient. In this case $g(T(x,t) - x)$ increases, but a bit of reflection reveals that this increase is more than offset by the reduction in the cost of production and the price to the consumer. Also, one must verify that the process does not converge to a distribution that is short of the optimum. This is assured by the fact that there is a compact set of "relevant" firm distributions. If the limit of a subsequence does not give the same profit everywhere, then there will be more movement.

IV. A GRADIENT PROCESS FOR ADJUSTING THE DISTRIBUTION OF FIRMS

Suppose that a planner wishes to adjust the distribution of firms so as to maximize the rate of welfare increase subject to an instantaneous cost

$$\int_0^{2\pi} \psi(w(x,t))f(x,t)\,dx\ .$$

Here, $w(x,t)$ is the directed velocity of the firm at x at time t; we say that $w(\cdot,t)$ is a tangent vector field for C. The function ψ defines the rate of utilization of numeraire associated with the adjustment velocity w. Let $U(x,y,\beta) = y - g(\beta - x)$, and for simplicity assume, as in the example, that profits are measured by the price function ϕ. From utility maximization $\phi_x(x,t)$ is $T(x,t)$'s marginal utility, in terms of numeraire, of reducing $T(x,t) - x$.

We will now argue that the tangent vector field w, defined by the condition that each firm adjusts so as to maximize the rate of change in its own profit, maximizes the rate of increase in welfare W, taking into account the cost of adjustment ψ.

To see this we calculate the rate of change in welfare associated with an arbitrary vector field $w(\cdot,t)$. First,

$$(21)\quad W(t) = -\int_0^{2\pi} g(2\pi F(x,t) + T(0,t) - x)\ f(x,t)\ dx,$$

where $T(0,t)$ maximizer $W(t)$; this follows from the definition of W and the fact that equilibria are optima. Thus, the derivative of W in the direction $w(\cdot,t)$ is

$$(22)\quad -\lim_{s\to 0}\left[\int_0^{2\pi} g(2\pi F(x,t) + T(0,t) + s\frac{dT(0,t)}{dt} - [x+ sw(x,t)])f(x,t)dx - \int_0^{2\pi} g(2\ F(x,t) + T\ (0,t) - x)\ f(x,t)\ dx\right] \Big/ s$$

This is equal to

$$(23)\quad -\int_0^{2\pi} g'(2\pi F(x,t) + T(0,t) - x)\ \frac{dT(0,t)}{dt}\ f(x,t)\ dx + \int_0^{2\pi} g'(2\pi F(x,t) + T(0,t) - x)\ w(x,t)\ f(x,t)\ dx$$

The first term must be zero since $T(0,t)$ maximizes $W(t)$. Since $g' = \phi_x$, the derivative of W in the direction $w(\cdot,t)$ is

$$(24)\quad \int_0^{2\pi} \phi_x(x,t)\ w(x,t)\ f(x,t)\ dx$$

To maximize the rate of welfare increase subject to the cost $\int_0^{2\pi} \psi\,(w(x,t))\, f(x,t)dx$ the planner chooses the vector field w that maximizes

$$\int_0^{2\pi} \phi_x(x,t)\; w(x,t)dx - \int_0^{2\pi} \psi\,(w(x,t))\; f(x,t)dx$$

It follows that for the maximization it is necessary that for each x and t $\phi_x(x,t) = \psi'(w(x,t))$.
Since for a firm located at x at time t , the rate of change in profit if it moves with directed velocity $w(x,t)$ is $\phi_x(x,t)\; w(x,t) - \psi(w(x,t))$, the same condition defines both the function $w(\cdot,t)$ that leads to the maximal rate of change in W and the function that has each firm move at a directed velocity that maximizes its own rate of profit increase.

V. <u>THE EXPECTATIONS OF FIRMS</u>

Clearly, if the price function $\phi(\circ,\circ)$ was known to a firm that maximized profit (for simplicity undiscounted) over a horizon T, then that firm would not behave in the manner we have attributed to it. But suppose that expectations are rational, so that for an equilibrium price function $\phi(\circ,\circ)$ supply equals demand at each location and at each point in time when each firm solves (for example):

$$\text{choose } x(\cdot) \text{ to maximize } \int_0^T [\phi(x,t) - \frac{T}{8}\dot{x}(t)^2]dt .$$

Note that the adjustment cost is proportional to the time horizon. Elementary calculus of variation reveals that for T small, so that prices move very little, the average velocity of adjustment over the time horizon is $\phi_x(x,t)$. The equilibrium we have studied, with its naive rule for the adjustment of firms, can thus be interpreted as a limit of a sequence of rational expectations equilibria, where the planning horizons become increasingly small.

VI. CONCLUSION

A classic theme of economic analysis concerns the adequacy of the potential for profit, as a signal for entry, to guide an economy towards an efficient allocation. Just as tatonnement dynamics will not always lead to efficient allocations, one cannot expect this of the entry dynamics explored here; our inquiry concerns the range of situations in which a simple entry dynamics, based on limited information regarding future prices, will lead to efficient allocations. We have considered a framework with differentiated products and transferable utility. There are no futures markets or storage and thus no means by which a consumer can voice his time preference. Nevertheless, myopic profit seeking behavior by firms leads an economy to the unique equal profit equilibrium at which the resources devoted to the production of the differentiated products cannot be allocated more efficiently.

Furthermore, the movement toward the optimum takes place so as to maximize the rate of increase in single period welfare. This is not to say that it takes place at an optimal rate; such a statement is not even meaningful when we specify the time preference of consumers. The strength of the result is that even with myopic behavior on the part of firms, convergence to the optimum is assured. Certainly the argument that free competition of firms results in efficient allocations is much enhanced when short run profits provide adequate signals for effectively guiding the allocation of resources.

Let me close with a disclaimer. The dynamics illustrated here are very different than tatonnement dynamics. They are intended to complement and not to replace. The difference in the two dynamics is well illustrated by an example that comes from my work with William Novshek. There are simple(two consumers two commodities) economies with a continuum of firms, in which equilibrium is unique and unstable from the entry point of view. Since these are only two commodities, that equilibrium is of course stable with respect to tatonnement. This example underscores the importance of transferable utility for the stability result.

DIAGRAMS

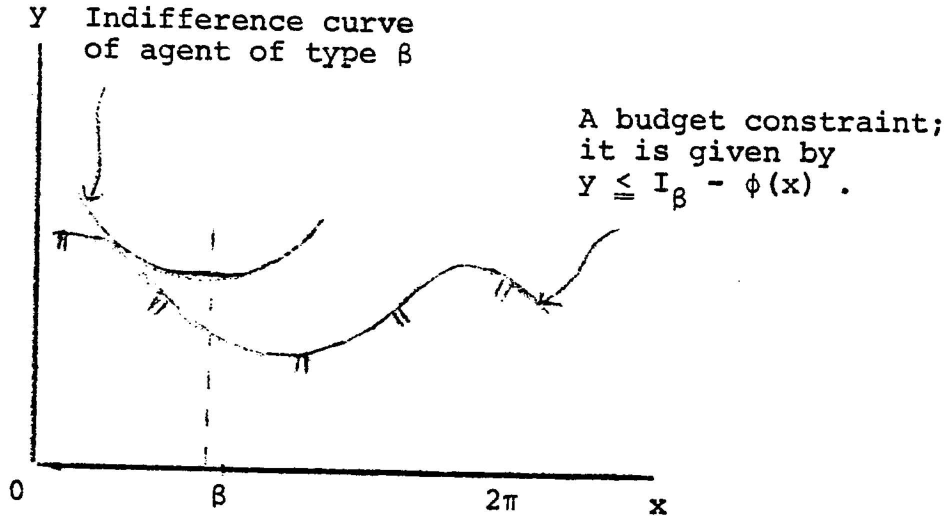

Figure 1

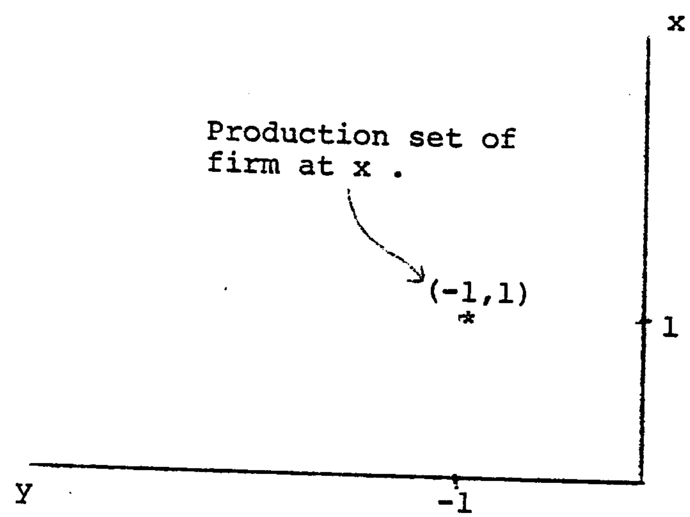

Figure 2

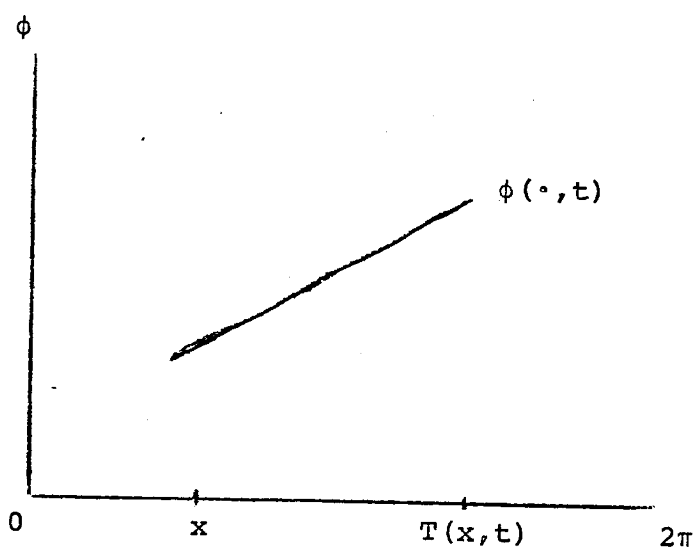

Figure 3

1. Given the great deal of literature concerned with tatonnement dynamics, it is perhaps surprising that so little work has gone into the study of entry dynamics. Arrow and Hurwicz [1] were clearly aware of the possibility of such dynamics. Taylor's proposal [5] for the organization of a socialist economy is related and Mas-Colell [2] with the work of Taylor in mind, formalized some aspects of a regime in which tatonnement and entry dynamics go on at the same time.
2. For the preferences considered here, the demand for the differentiated commodity is independent of the distribution of income.
3. With a neoclassical production function the positive constant would be determined.

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