

PRIZES AND INCENTIVES:
TOWARDS A GENERAL THEORY
OF COMPENSATION AND COMPETITION*

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by

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One of the dominant characteristics of modern capitalist economies (or so it is widely alleged) is the important role played by competition: Competition, as it is usually visualized, is not the peculiar static form of pure price competition embodied in the Arrow-Debrue model, but rather a dynamic competition, more akin the kind of competition represented by sports contests and other races (including patent races). At the more micro economic level, rewards within a firm seldom take the form of the pure piece rate (equal to the value of the marginal product) implicit in the conventional competitive paradigm. More commonly, rewards are, at least partially, based on relative performance: the most productive scholars judged on a comparative basis are promoted, the salesman who sells the most gets a large bonus, and the manager who does "best" (in some sense) gets promoted to the company vice presidency. Yet, surprisingly, there have been few studies attempting to model such competitive processes and to delineate the circumstances in which they are superior to alternative reward structures.¹ This we propose to do here.

The essential problem with which we are concerned here arises

¹Two important exceptions are the recent work by Rosen and Lazear and FitzRoy who independently asked questions similar to those posed here. There are, however, some important differences which we discuss at greater length below.

from the fact that the input (effort) of workers (managers) is not directly observable, at least not without cost. Thus, firms must either monitor inputs (which can seldom be done perfectly) or they must devise reward structures, in which compensation (using the term in its broadest sense, including promotions, pensions, etc.) are functions of variables which are (a) themselves functions of inputs and (b) are less costly to observe (monitor) than input itself.

There are three critical characteristics of any reward structure:

(a) Risk. Variability in exogenous variables results in variability in individuals' income and effort. Incentive schemes linking rewards to output inevitably result in the workers bearing risks; and if individuals are risk averse, there is a loss of welfare as a result.

(b) Incentive Levels. To ameliorate the risk problem, a significant part of most individual's compensation is not directly related to output; but to the extent that this is so, incentives may be reduced. We are concerned, of course, not only with the incentive to provide the right level of effort, but also the incentive to make the correct decisions.

(c) Flexibility. The "correct" risk-incentive compensation scheme in one situation will not, in general, be correct for another situation. In principle, if all the relevant environmental variables were costlessly observable, we would simply have a different incentive structure for each set of environmental variables. Such a "contract" would, obviously, be prohibitively expensive to set up; but more to the point, many of the relevant environmental variables are not costlessly observable to all parties to the contract (here, to both the worker and his employer). Thus, a single incentive structure must do in a variety of circumstances. Indeed, the lack of flexibility of the piece rate system is widely viewed to be its critical shortcoming: the process of adapting the piece rate is costly and contentious.

It is this third characteristic of competitive compensation schemes which makes them so attractive: when a task is easy for me, it will be easy for my rivals. Assume we had two individuals assigned to identical machines making widgets. Assume that we pay the individual who makes the most widgets a bonus. The amount of effort I put out will depend, of course, on the level of effort I believe my rival will put out (and conversely). Assume we were in equilibrium, and an improvement in the machine occurs. With a piece rate system, we would now become involved in a complicated process of determining what the new piece rate ought to be. Consider, in contrast the competitive process. For each level of my effort, it pays my competitor to increase his effort, and conversely. The Nash equilibrium entails both of us putting out greater effort. Our effective compensation per widget produced has been automatically adjusted downwards.

There are a variety of circumstances in which the performance of other individuals (firms) conveys information about the environment, information which can and should be introduced into the compensation scheme. Contests are one way of doing this; rewards based on performance standards, which in turn are adjusted on the basis of the performance of the group, are another.

The problems with using an individualistic reward scheme when there is uncertainty can be compared to the disadvantages of fixed grading systems (90 = A, 80 = B, ...) with a test of unknown difficulty. The use of percentiles solves this problem and returns us to considering compensation schemes based on relative performance.

Although one can formulate the problem of the optimal compensation scheme and write down the Euler equations

such equations (except in certain limiting cases) seem to provide only limited insight. After setting up the general problem, we focus our attention on certain special cases; in particular, we ask, under what circumstances would contests (where it is only the individual's rank that counts) be preferred to individualistic compensation schemes in which compensation is based only on output.

Compensation structures also serve a second, critical function: they enable the differentiation (screening) of workers of different abilities (see Stiglitz (1975)).¹ In this paper, we focus our attention on the incentive properties of compensation schemes; hence, we shall assume that all individuals have the same abilities. There are circumstances, noted briefly below, where individual differences may significantly impair the functioning of a contest system and thus our analysis may seem to exaggerate the benefits of contests; on the other hand, there are other circumstances in which contests can be shown to provide an efficient² basis of discrimination; in these circumstances, our analysis may underestimate the virtues of contests.

Because the analysis is fairly detailed, it may prove useful at the onset to summarize our major results:

1. When there are a large number of participants;
 - (i) appropriately designed contests can often approach the first best allocation; (ii) a penalty to the lowest ranked individual will be superior to a prize to the highest ranked individual in motivating effort.

¹For an analysis within the context of capital markets, see Stiglitz and Weiss (1981a).

²In a sense to be defined below.

2. In contests with little risk and hence small prizes, no symmetric pure strategy Nash equilibrium will exist.
3. The second best solution will in general require knowledge of each agent's output. This can be simplified when the set of outputs can be characterized through the use of a sufficient statistic. For example, when there are a large number of agents, optimal compensation may be based solely on the individual's output and the average output.
4. With risk neutral agents, the first best optimum can be obtained through either an appropriately designed contest or a piece rate but not through a bonus scheme.
5. Although the general principal-agent problem arises due to the difficulty in motivating unmonitored effort, in the second best solution agents may supply more effort than they would in the first best optimum.
6. Contests may be preferred to individualistic schemes especially when the risk associated either with common environmental variables or idiosyncratic random variables is large. Relative performance schemes work best when the idiosyncratic random variable is relatively less important than the common environmental variable so that workers are competing under similar conditions.

PART IThe General Theory of Compensation2. Basic Model

In this paper, we shall consider a particularly simple economic environment: each individual's output, Q_i , (which is observable) is a random function of his input, μ_i , of the form

$$(2.1) \quad Q_i = Q(\mu_i, \theta, \varepsilon_i)$$

where θ is some common "environmental" variable, and ε_i is an "individualistic" random variable. θ and ε have the distribution $G^*(\theta, \varepsilon)$.

There are several alternative interpretations of (2.1). In an agricultural context, for instance, θ represents the general weather in the area, ε_i the weather (rainfall) on a particular farm. Alternatively, θ could represent the general level of difficulty of a set of tasks, while ε_i could represent the individual's comparative advantage (disadvantage) in performing one of the various tasks. In our analysis, two properties of the functions G^* and Q will turn out to be critical:

- (a) $Q_{\mu\theta} \neq 0$, the state of nature effects the return to effort; and
- (b) for each θ , G^* is not the improper distribution, so that from knowledge of Q_i and θ , one cannot infer precisely the value of μ .

The role of these two critical assumptions will be clarified in the subsequent discussion. Throughout, we assume the contract must be signed before θ and ε are known; the worker then observes θ and decides on μ , neither of which are observable to the firm.

For much of the analysis we focus on a special case where

$$(2.1') \quad Q_i = \mu_i \theta + \varepsilon_i.$$

where G is the distribution function of ε_i , g its density, and

$$E\varepsilon_i = 0, E\theta = 1, E\varepsilon_i \varepsilon_j = 0.$$

The linear form of (2.1') is chosen for analytical simplicity; nothing important depends on this specification.¹ What is important is that only Q_i is observable. Hence, by observing Q_i , one cannot infer perfectly what the level of effort must have been. Since only the set $\{Q_i\}$ is observable, any compensation scheme must be a function of $\{Q_i\}$ (and only of Q_i):²

$$(2.2) \quad Y_i = F(Q_1, Q_2, \dots, Q_i, \dots, Q_n) .$$

What should be noted about (2.2) is that there is no a priori reason to restrict ourselves to "individualistic" compensation schemes,³ where i 's income depends on his output (and his output alone).

$$(2.3) \quad Y_i = \hat{f}(Q_i) .$$

Indeed, there is good reason not to restrict ourselves in this manner. Some information about θ can be gleaned from observing the whole array of Q_i 's, and presumably, this information should be used to make inferences about μ_i . Often, we can find a sufficient statistic for that information, $T(Q)$, and then the optimal compensation scheme may be simplified.

¹This is not, of course, the most general form of linear structures; we could have written (2.1") $Q_i = u_i\theta + \varepsilon_i + \gamma + u_i\eta_i$, i.e., there is an idiosyncratic effect on marginal productivity and a common effect on total output. Note that Mirrlees' [1975] optimal income tax model can be viewed as a special case of (2.1') with $\sigma_{\varepsilon i}^2 = \sigma_{\theta}^2 = \sigma_{\gamma}^2 = 0$, while Varian's [1981] analysis of social insurance can be viewed as a special case when $\sigma_A^2 = \sigma_{ni}^2 = \sigma_{\gamma}^2 = 0$.

²This is not quite correct. Individuals' announcements about θ can (with the appropriate incentive structure) convey information. These revelation schemes are discussed in Stiglitz (1981).

³Of course, sometimes there is no choice. In the example of a patent race, there may only be one observable output--the time to discovery. Those who lose in a patent race do not continue working on the problem to see how long it would have taken them.

$$(2.4) \quad Y_i = F(Q_i, T(Q)).$$

Intuitively, all the information relevant to person i is contained in Q_i and $T(Q)$.¹ The use of any other variables would only add random noise to the compensation scheme and would thus be suboptimal. A special application of this theorem is the following: If θ is not variable, then the optimal compensation scheme is individualistic. This follows immediately from observing that in that case, a sufficient statistic for θ is a constant. Other applications are also immediate. If θ and ϵ_i are all normally distributed random variables, then \bar{Q} is a sufficient statistic for $\mu^*(\theta)\theta$ and hence for θ . In the classroom example, when there are a large number of students, the average test score will reveal how hard the test was. The estimate of θ determined from $\bar{Q} = \mu(\theta)\theta$ will converge to the true θ as the number of contestants becomes large. In the limit, we will have a sufficient statistic for θ since we will actually know θ . We can then apply the sufficient statistic theorem to show that the optimal second best compensation scheme will depend only on an individual's output and θ , $Y_i = Y_i(\theta, Q_i)$. Even when there are a limited number of workers, it may be advantageous to base compensation on the size of output relative to the mean as this can reduce the noise associated with θ .

One can thus view the problem we are posing, of the choice of a payment scheme, as a statistical problem: The firm does not know what the individual's level of effort is. It attempts, from knowledge of the structure of the situation and observations of individual behavior, to make the "best" estimate, and then base compensation on this estimated value. The difficulty with this approach is that the statistic used has both risk and incentive effects, and these must be taken into account in the choice of the "best" statistic.

¹We are indebted to Joe Farrell for extensive discussions on this question. For similar sufficient statistic results and proofs, see Farrell (1981), Grossman and Hart (1980), Holmstrom (1980), and Arrow (1970).

We will be concerned with contrasting the individualistic compensation scheme (2.3) with two other compensation schemes in which rewards depend on relative performance. In this analysis, we focus on relative performance compensation schemes that base rewards on rankings of performance. Thus, the winner gets a large prize, the second person gets a smaller prize, the third a still smaller prize, etc. In the simplest such scheme

$$(2.5) \quad Y_i = \begin{cases} Y_w & \text{if } Q_i = \max \{Q_j\} \\ Y_L & \text{if } Q_i < \max \{Q_j\} \end{cases}$$

In (2.5) only the winner gets a prize¹. A second scheme which plays an important role in the subsequent analysis is just the opposite of the "winner take all" scheme of (2.5). We call this the "penalty to the loser" scheme:

$$Y_i = \begin{cases} Y_w & \text{if } Q_i > \min \{Q_j\} \\ Y_L & \text{if } Q_i = \min \{Q_j\} \end{cases}$$

This analysis will focus on a competitive market in which firms are risk neutral and expected profits are driven to zero. The probability distribution of θ and ε is known to both firms and workers. Workers are identical, and maximize their expected utility

$$(2.6) \quad EU(Y) - V(\mu)$$

where $U' > 0$ (positive marginal utility of income) and $U'' \leq 0$ (individuals

¹The formulation (2.5) ignores the possibility of ties; in the formal models to be presented below, these occur with probability zero, and hence can be ignored. In general, however, the compensation scheme must specify whether, in the event of a tie, each will receive the prize Y_w , whether the prize will be divided equally, or whether it will be randomly assigned to one individual or the other.

are not risk preferrers). For most of the analysis, we shall assume $U'' < 0$: individuals are risk averse. In Section 6(d) we discuss the special case where individuals are risk neutral ($U'' = 0$). $V(\mu)$ is the disutility of work. We postulate that it increases at an increasing rate: $V' > 0$, $V'' > 0$.

We focus on Nash equilibrium, where individuals take the levels of effort of their colleagues (competitors) as fixed.¹ Given the compensation scheme, F , this results in²

$$(2.7) \quad Y_i = \tilde{Y}(\mu_i; \mu_1, \dots, \mu_n; \varepsilon_1, \dots, \varepsilon_n; F).$$

Income is a (well specified) random function of the individual's effort. The i th individual maximizes his expected utility (2.6)

$$(2.8) \quad \max_{\{\mu_i\}} EU(\tilde{Y}(\mu_i)) - V(\mu_i)$$

The solution to this gives μ_i as a function of $\{\mu_j\}$ and θ .

$$(2.9) \quad \mu_i = \mu_i(\mu_1, \dots, \mu_n; \theta)$$

A set of values $\{\mu_1^* \dots \mu_n^*\}$ which solves simultaneously the equations

$$(2.10) \quad \mu_i = \mu_i(\mu_1, \dots, \mu_{i-1}, \mu_{i+1}, \dots, \mu_n; \theta) \quad i=1, \dots, n$$

represents a Nash equilibrium. We focus on the symmetric Nash equilibrium where $\mu_i = \mu_j$ all i, j , which can be characterized simply by the value of μ for which, for all i ,

$$(2.11) \quad \mu^*(\theta) \equiv \mu_i(\mu^*, \dots, \mu^*; \theta)$$

¹The Nash equilibrium solution concept is more persuasive in the context of large groups than it is in situations where there are only a few individuals.

²There is an alternative interpretation: In a rational expectations equilibrium framework, Y_i is simply the distribution of payments as a function of the level of effort exerted.

If there are only two individuals, we can represent the equilibrium as in Figure 1. For any value of μ_2 there is an optimal value of μ_1 , and conversely.¹

The mean profits π per worker associated with any compensation scheme may then easily be calculated as

$$(2.12) \quad \pi = E\{\mu^*(\theta)\theta - E[Y(\underline{\mu}^*; \underline{\varepsilon}, \theta; F) | \theta]\}$$

where the inner expectation is taken over $\underline{\varepsilon}$ and the other expectation is taken over θ . Thus, in the competitive equilibrium

$$(2.13) \quad E\theta\mu = E\tilde{Y}$$

Any compensation scheme satisfying (2.13) we shall call feasible.² We contrast the level of expected utility under alternative feasible compensation schemes. The competitive equilibrium compensation scheme is the feasible compensation scheme for which expected utility is maximized. We would like to characterize this compensation scheme. We do not present here the Euler equations characterizing the optimum.³ Rather, our approach has been to contrast the optimum individualistic compensation scheme (of the form (2.3)) with the optimum compensation schemes of the form (2.4) in which compensation depends on relative performance, to ascertain conditions under which the latter are likely to be superior.

¹If μ_2 is a continuous, increasing function of μ_1 and if for large enough values of μ , $d\mu_2/d\mu_1 < 1$ (if "one" works harder so does "two," but his increment in work effort is smaller), then there exists a (symmetric) equilibrium. However, it is conceivable that "two" decreases work effort in response to an increase by "one," and indeed, the response may be discontinuous (as in Fig. 2). In this case there may exist no symmetric equilibrium, but an asymmetric one, or no pure strategy equilibrium, or multiple equilibrium. [see Section 4(a) and discussion in Appendix].

²This approach has assumed that the value of outputs $\{Y_1, \dots, Y_n\}$ is just $\sum Y_i$. In the context of a patent race, where only $\text{Max}\{Y_i\}$ matters, the definition of feasible would be different [Nalebuff and Varian (1981)].

³Formally, the problem can be modelled as a standard variation problem; we seek the function F which maximizes expected utility subject to the zero profit constraints.

3. The First Best Optimum: A Benchmark

In the subsequent discussion, it will prove useful to compare the results of alternative compensation schemes with the equilibrium with perfect information (where θ is observable). Under our assumptions with risk neutral firms and risk averse workers, it is clear that in equilibrium, the individual is perfectly insured so that he receives the same income in all states: $Y = \bar{Y}$.

Effort is supplied until the marginal utility in income multiplied by the increase in income with effort is just equal to the disutility of work.

$$(3.1) \quad \mu^*(\theta): \quad U'(\bar{Y}) \theta \equiv V'(\mu^*(\theta))$$

or

$$(3.1') \quad \mu^*(\theta) = V'^{-1}(U'(\bar{Y})\theta)$$

Hence

$$(3.2) \quad \bar{Y} = E\mu\theta = E\theta V'^{-1}(U'(\bar{Y})\theta)$$

PART II. The Theory of Contests

4. The Basic Analytics of Contests

For simplicity, we begin our analysis with the case where there are only two contestants, with the winner receiving Y_w , the loser Y_L . Unlike the standard marginal analysis, the winner's output is not necessarily worth Y_w . The winner is paid more than his marginal product so as to introduce a carrot to motivate greater effort among the contestants.

In considering the feasible competitive equilibrium scheme, expected profits will be zero which implies $Y_w + Y_L = 2\bar{Q}$. It is instructive to change notation. Let

$$\bar{Y} \equiv \bar{Q} = \frac{Y_w + Y_L}{2} \quad \text{and} \quad x \equiv \frac{Y_w - Y_L}{2}$$

Any two prizes can be thought of as a safe income \bar{Y} , plus or minus risk x .

(a) Individual Behavior. The individual's expected utility is a function of the probability of his winning; this in turn depends simply on his level of effort, the other individual's level of effort, and the distribution of ε . We denote (for a given distribution of ε), the probability of winning by $P(\mu_1, \mu_2, \theta)$. Then the individual's expected utility is simply

$$(4.1) \quad W = PU(\bar{Y} + x) + (1 - P)U(\bar{Y} - x) - V(\mu).$$

In paying individuals with a "contest", rather than on the basis of output, we have taken a random disturbance, ε , which had zero mean and was uncorrelated with effort, μ , and replaced it by a disturbance, x , that is correlated with effort, μ . If the contestant works harder, the chance (P) of a positive x rises as he is more likely to win. Effort will be supplied until the marginal disutility from work is just balanced by the increased chance of winning the value of the prize.

$$(4.2) \quad \frac{\partial W}{\partial \mu_1} = \frac{\partial P(\mu_1, \mu_2, \theta)}{\partial \mu_1} \cdot \Delta U - V'(\mu_1) = 0$$

where

$$\Delta U = U(\bar{Y} + x) - U(\bar{Y} - x).$$

(4.2) can be solved for μ_1 as a function of μ_2 , and a symmetric reaction

function giving μ_2 as a function of μ_1 can similarly be derived. Although it should be apparent that it is possible that there exists asymmetric equilibrium, with one individual working hard, the other taking it easy, for this part we focus on the symmetric equilibrium with $\mu_1 = \mu_2$.

(b) The Return to Effort. Equation (4.2) makes clear that the function P is critical in the analysis of the individual's behavior in a contest. Assume that the second individual chooses a level of effort μ_2 and has a realization of ε of ε_2 . Then for the first individual to win with effort level μ_1 it must be the case that

$$(4.3) \quad \mu_1 \theta + \varepsilon_1 > \mu_2 \theta + \varepsilon_2.$$

The probability of this occurring (i.e. that $\varepsilon_1 > (\mu_2 - \mu_1)\theta + \varepsilon_2$) is

$$(4.4) \quad 1 - G((\mu_2 - \mu_1)\theta + \varepsilon_2)$$

The probability that a particular value of ε occurs, say ε_2 , is just $g(\varepsilon_2)$. To calculate the total probability of winning, we simply integrate over all possible values of ε_2 . Hence

$$(4.5) \quad P \equiv \int (1 - G((\mu_2 - \mu_1)\theta + \varepsilon_2)) g(\varepsilon_2) d\varepsilon_2.$$

At the symmetric equilibrium

$$(4.6) \quad \left. \frac{\partial P}{\partial \mu_1} \right|_{\mu_1 = \mu_2} = \theta \int g(\varepsilon_2) g(\varepsilon_2) d\varepsilon_2 = \theta \bar{g}$$

where $\bar{g} \equiv E(g(\varepsilon))$.¹

¹ We may now easily see that the second order conditions are likely to be satisfied. Differentiating (4.2)

$$\frac{\partial^2 W}{\partial \mu^2} = \theta \cdot \Delta U \cdot \int g(\varepsilon) g'(\varepsilon) d\varepsilon - V''(\mu_1) < 0$$

If the disturbance is symmetric then the first term drops out and the second term is negative by assumption. More generally, one only requires $\theta \cdot \Delta U \cdot [g^2(\bar{\varepsilon}) - g^2(\underline{\varepsilon})] < V''(\mu)$ where $\bar{\varepsilon}, \underline{\varepsilon}$ are maximum and minimum values of epsilon. Hence, it is also sufficient if $\lim_{|\varepsilon| \rightarrow \infty} g(\varepsilon) = 0$ when $g(\varepsilon)$ is continuous on the support $(-\infty, \infty)$.

The following distribution does not, however, satisfy the second order conditions. Consider $g(\varepsilon) = \frac{1}{\varepsilon \bar{\varepsilon}} e^{-\varepsilon/\bar{\varepsilon}}$ for $\varepsilon > -\underline{\varepsilon}, 0$ otherwise. As $\underline{\varepsilon}$ approaches zero, the second order conditions will be violated.

(c) The fundamental equation of contests. Substituting (4.6) into the first order condition (4.2) we obtain the fundamental equations describing equilibrium in contests:

$$(4.7) \quad \boxed{\theta \Delta U \bar{g} = v'}$$

For the contest to be feasible (zero profit), we require that the expected compensation be equal to the expected output.

$$(4.8) \quad \bar{Y} = E[\mu(\theta)\theta] = E\theta v'^{-1}[\theta \Delta U \bar{g}].$$

The optimal contest is defined as a pair of $\{\bar{Y}, x\}$ in the set of feasible contests which maximizes expected utility.

(d) Central properties of contests. (4.7) has one critical property: effort varies with θ , while expected income (and indeed, the expected utility of income) does not. This is the property of flexibility which, as we suggested in the introduction, was one of the important characteristics of a good compensation scheme. Indeed, it should be noted that we can perfectly replicate the first best pattern of effort supply if \bar{g} does not vary simply by setting

$$(4.9) \quad \boxed{U'(\bar{Y}) = \bar{g} \Delta U}$$

$$(4.10) \quad \theta U'(\bar{Y}) = \theta \Delta U \bar{g} = v'(\mu^*(\theta))$$

But the level of utility attained will not be equal to that attained with perfect information, since to generate the first best level of effort requires (if \bar{g} is finite) that the individual bear some risk. Of course, if individuals are risk neutral then this variation in income is unimportant, and setting the prize according to (4.9) will achieve the first best outcome.

In general, the firm will sacrifice some "efficiency" for some level of risk reduction.

To see how effort varies with the prize, differentiate the worker's first order condition (4.7).

$$(4.11) \quad \theta \bar{g} [U'(\bar{Y}+x) \left(\frac{d\bar{Y}}{dx} + 1\right) - U'(\bar{Y}-x) \left(\frac{d\bar{Y}}{dx} - 1\right)] = V''(\mu) \frac{d\mu(\theta)}{dx}$$

We multiply through by θ and take expectations to obtain

$$(4.12) \quad \frac{d\bar{Y}}{dx} = \frac{\bar{g} S E \theta^2 / V''}{1 - \Delta U' \bar{g} E \theta^2 / V''} > 0$$

where

$$S = U'(\bar{Y}+x) + U'(\bar{Y}-x)$$

$$\Delta U' = U'(\bar{Y}+x) - U'(\bar{Y}-x) < 0$$

A larger prize motivates greater effort and thus increases mean income.

Although the agents believe that by working harder they will increase their chance of winning, P , in fact both agents will work equally harder leaving P at $1/2$. But as a result of their greater effort, the new feasible compensation scheme must commensurately increase \bar{Y} .

(e) The Optimal Contest. It is now easy to solve for the optimal contest. Using (4.7), we can write expected utility

$$(4.13) \quad W = \frac{1}{2} [U(\bar{Y}+x) + U(\bar{Y}-x)] - EV(V'^{-1}(\theta \Delta U \bar{g}))$$

(where we have made use of the fact that in the symmetric equilibrium each individual wins half the time). Differentiating (4.13) with respect to x , we obtain at the optimal prize, \tilde{x} ,

$$(4.14) \quad \frac{dW}{dx} = \left[\frac{1}{2}S - \bar{g}\Delta U \right] \bar{Y}' + \frac{1}{2}\Delta U' = 0$$

where \bar{Y}' is as given in (4.12).¹

5. Properties of the Second Best Solution Using Contests

This section considers when a prize system is likely to be an effective compensation scheme.

(a) Problem from a Nonconvexity. Our model of a prize system will in general have a symmetric pure strategy Nash equilibrium. There is one important exception. When the variance of ϵ becomes small, there is a nonconvexity in our maximand that disrupts any pure strategy solution. The second order conditions may be satisfied, but the symmetric equilibrium will only be a local maximum for both players.

The symmetric solution would, if it were an equilibrium, approach the first best optimum as $\sigma_\epsilon^2 \rightarrow 0$. By choosing x according to (4.9) we can replicate the first best level of effort. But the value of \bar{g} must become infinite as σ_ϵ^2 approaches zero. This follows from $\bar{g} = E(g(\epsilon))$.² Thus the prize, x , required to generate the first best level of effort becomes arbitrarily small. From (4.10)

$$(5.1) \quad U'(\bar{Y}) = \bar{g}\Delta U \approx \bar{g}2x U'(\bar{Y}) \rightarrow x \approx \frac{1}{2\bar{g}}$$

¹We used (4.7) and (4.8) to transform $EV' \frac{d\mu^*}{dx} = \bar{g}\Delta UE\theta \frac{d\mu^*}{dx} = \bar{g}\Delta U\bar{Y}'$

²As the variance of a mean zero distribution approaches zero, the density collapses to a peak at the origin. The mean value of the density becomes arbitrarily large. For example, when $g(\epsilon)$ is the normal density, mean zero and variance σ^2 , then

$\bar{g} = \frac{1}{[\sqrt{2\pi}\sigma]}$. \bar{g} is inversely related to the standard error, approaching infinity as σ_ϵ^2 becomes smaller.

Intuitively, as the marginal increase in the agent's chance of winning becomes arbitrarily large, the value of winning, ΔU , must become zero. If a little more effort guarantees you a prize, then the prize need only be very small.

Welfare approaches the first best

$$(5.2) \quad \lim_{\sigma_{\epsilon}^2 \rightarrow 0} W = U(\bar{Y}-x) + \frac{1}{2}\Delta U - EV(\mu(\theta)) \rightarrow U(\bar{Y}) - EV(\mu^*(\theta))$$

Yet, the symmetric solution is only a local optimum. Why work at all? An agent who decides not to supply effort μ^* and does no work will be sure to lose. He will forfeit his chance at the vanishing small amount ΔU and gain $V(\mu) - V(0)$.

$$(5.3) \quad \lim_{\sigma_{\epsilon}^2 \rightarrow 0} W_{\mu=0} \approx U(\bar{Y}) - V(0) > W_{\mu=\mu^*}$$

Doing no work is not dropping out of the game. It is more akin to cheating on the work contract. With the variance of the noise becoming arbitrarily small, there would be no doubt that this low output resulted from no effort rather than bad luck. But the prize reward scheme is too simple to deter this violation. The rewards are based only on ordinal rank and so in this case a mile is as good as a miss! Were we to establish different punishments for different low levels of output this would be a departure from the prize system and a return to general nonlinear compensation schemes. In the following comparisons, we must obviously restrict our

attention to situations in which there is sufficient noise such that the symmetric solution is indeed an equilibrium.¹

(b) Comparison with First Best Level of Effort. The claim in our introduction that it is possible for the second best optimum with a prize system to involve more effort than the first best should be greeted with scepticism. If the first best is unobtainable because we cannot monitor effort and effort is under-supplied, then it would initially seem doubtful that with a prize scheme we might desire more than the first best optimal amount of effort. There is some confusion in the literature on this point which we believe emanates from the existence of two different first best solutions. The unconstrained first best involves perfect insurance, monitoring of effort, and no prize.

$$(5.4) \quad \Theta U'(\bar{Y}) - V'(\mu^*) = 0 \text{ defines } \mu^*(\theta) \text{ first best.}$$

Given that we have a prize, x , if we could base payment on effort, there is also a constrained first best.

$$(5.5) \quad \Theta \frac{1}{2} [U'(\bar{Y}+x) + U'(\bar{Y}-x)] - V'(\hat{\mu}) = 0 \text{ defines } \hat{\mu}(x, \theta) \text{ constrained first best.}$$

For the optimal prize, \tilde{x} , as defined in (4.14), the second best solution satisfies

$$(5.6) \quad \Theta g[U(\bar{Y}+\tilde{x}) - U(\bar{Y}-\tilde{x})] - V'(\tilde{\mu}) = 0 \text{ defines } \tilde{\mu}(\tilde{x}, \theta) \text{ second best.}$$

In each of these cases, $\bar{Y} = E\mu(\theta)\theta$, where $\mu(\theta)$ is as defined in each of the equations.

¹ Alternative resolutions of the problems posed by this non-convexity are discussed in Stiglitz (1981).

Theorem 1. If U' is convex then $\hat{\mu}(\tilde{x}, \theta) > \mu^*(\theta)$.

Under the natural assumption that $U''' > 0$, we have the result that if we can monitor effort then when we must have a prize, \tilde{x} , it would be optimal for the agents to work harder than when there is no prize.

Proof: At $x=0$, the first order conditions (5.4) and (5.5) are the same

$$(5.7) \quad \hat{\mu}(0, \theta) \equiv \mu^*(\theta)$$

Hence, we only need examine how $\hat{\mu}$ changes with x . From (5.5)

$$(5.8) \quad \frac{d\hat{\mu}}{dx} = \theta \frac{1}{2} [(U''(\bar{Y}+x) - U''(\bar{Y}-x)) + (U''(\bar{Y}+x) + U''(\bar{Y}-x)) \frac{d\bar{Y}}{dx}] / V''(\hat{\mu}) > 0$$

Because the sign of $\frac{d\hat{\mu}}{dx}$ is independent of θ , $\frac{d\bar{Y}}{dx}$ and $\frac{d\hat{\mu}}{dx}$ must have the same sign. By the convexity of U' , they must both be positive. Hence, at the prize, \tilde{x} , that generates the second best solution, $\hat{\mu}(\tilde{x}, \theta) > \mu^*(\theta)$.

Our intuition that the second best level of effort, $\tilde{\mu}$, should be less than the first best is confirmed if we choose the constrained first best, $\hat{\mu}$, for comparison. Our inability to monitor effort in the second best solution leads to an undersupply to effort only in the sense that given the presence of income variability due to the prize, if payment could be based on effort, agents would choose to work harder.

Theorem 2. The second best level of effort, $\tilde{\mu}(\tilde{x}, \theta)$, is less than the constrained first best, $\hat{\mu}(\tilde{x}, \theta)$.

Proof: We know at \tilde{x} , $dW/dx = 0$. We simply evaluate this derivative at \hat{x} defined by $\tilde{\mu}(\hat{x}, \theta) \equiv \hat{\mu}(\tilde{x}, \theta)$. At \hat{x} , the prize would be large enough to generate the constrained first best level of effort in the second best solution. From (5.5) and (5.6),

$$(5.9) \quad \tilde{\mu}(\hat{x}, \theta) = \hat{\mu}(\tilde{x}, \theta) \rightarrow \frac{1}{2} S = \bar{g}\Delta U$$

$$(5.10) \quad \left. \frac{dW}{dx} \right|_{\hat{x}} = \left[\frac{1}{2}S - \bar{g}\Delta U \right] \bar{Y}' + \frac{1}{2}\Delta U' = \frac{1}{2}\Delta U' < 0 .$$

Thus, the \hat{x} prize is too large in the second best framework. From (4.12) we know that smaller prizes motivate less effort.

To compare the effort level at the second best solution with that in the first best, we choose a prize, x^* , that motivates the first best effort level and then see if this prize is too large or small. At x^* , $\tilde{\mu}(x^*, \theta) = \mu^*(\theta)$ implies (*) $\bar{g}\Delta U = U'(\bar{Y})$ and

$$(5.11) \quad \left. \frac{dW}{dx} \right|_{x^*} = \left[\frac{1}{2}S - \bar{g}\Delta U \right] \bar{Y}' + \frac{1}{2}\Delta U' = \left[\frac{1}{2}S - U'(\bar{Y}) \right] \bar{Y}' + \frac{1}{2}\Delta U' < 0 .$$

Remark: If U' is concave ($U''' \leq 0$) then a suboptimal level of effort will be provided in the second best.

If U' is concave then $\frac{1}{2}S - U'(\bar{Y}) < 0$ and both terms of dW/dx will be negative. But the normal assumption of declining absolute risk aversion ensures $U''' > 0$.

Although the constrained first best involves greater effort than either the first best or the second best, it is entirely possible that $\mu^* < \tilde{\mu} < \hat{\mu}$, more effort will be supplied in the second best than in the first best (for an example see Nalebuff (1981), p. 28).¹ This result is sensitive to the specification of the utility function. In the framework of Lazear and Rosen (1981), $W = U[Y - V(\mu)]$, the disutility of work is negatively correlated with income.

¹Let $U(\bar{Y}) = \ln \bar{Y} + \frac{1}{4}\bar{Y}$, $V(\mu) = \frac{2}{3}\mu^2$, $\bar{g} = \frac{1}{4}$, $\sigma_\theta^2 = 0$

Moreover, $\hat{\mu}(x, \mu) = \mu^*(\theta) \forall x$ so that the prize system will lead to an undersupply of effort even when compared to the first best.¹

(c) Loss from Contest for Large Variances. If the prize is chosen optimally, it is apparent from (4.13) that changes in the variance of θ affect expected utility directly and through the effect on effort (but not through an effect on x). From theorem 2 we know that if the change in effort increases mean output, then the prize system would be moving to the constrained first best and utility would be higher. The direct effect on the change in the distribution of θ is more difficult to sign and in general will depend on whether the disutility of effort, $V(\mu(\theta))$, is concave or convex as a function of θ . Consider the example with $V(\mu) = \mu^2/2$.

$$(5.12) \quad \mu(\theta) = \bar{\theta} \bar{g} \Delta U; \quad \bar{Y} = \bar{g} \Delta U E \theta^2; \quad EV(\mu) = \frac{1}{2} \bar{g} \Delta U \bar{Y}$$

$$(5.13) \quad \frac{\partial \bar{Y}}{\partial E \theta^2} = \frac{\bar{g} \Delta U}{[1 - \bar{g} \Delta U' E \theta^2]} > 0$$

$$(5.14) \quad W = 1/2 [U(\bar{Y}+x) + U(\bar{Y}-x)] - 1/2 \bar{g} \Delta U \bar{Y}$$

$$(5.15) \quad \frac{dW}{dE \theta^2} = \frac{\partial W}{\partial x} \frac{dx}{dE \theta^2} + \frac{\partial W}{\partial \bar{Y}} \frac{d\bar{Y}}{dE \theta^2} + \frac{\partial W}{\partial E \theta^2}$$

$$= \left[\frac{1}{2} S - \frac{1}{2} \bar{g} \Delta U - \frac{1}{2} \bar{g} \Delta U' \cdot \bar{Y} \right] \frac{\partial \bar{Y}}{\partial E \theta^2}$$

$$> \left[\frac{1}{2} S - \bar{g} U \right] \frac{\partial \bar{Y}}{\partial E \theta^2} > 0$$

as seen from the first order conditions determining x in (4.14). In this example mean preserving increases in the spread of θ raise welfare. Hence, the loss in welfare for zero variance in θ represents an upperbound to the loss in welfare from a contest. More generally, this result will depend on assumptions concerning V which determine how the new distribution of effort affects output and disutility of work.

¹Proof: the equation determining $\mu^*(\theta)$ is now $EU'[\bar{Y}-V(\mu)] \cdot [\theta - V'(\mu^*)] = 0$. The equation for $\hat{\mu}$ is $E \frac{1}{2} (U'[\bar{Y}+x-V(\mu)] + U'[\bar{Y}-x+V(\mu)]) \cdot [\theta - V'(\hat{\mu})] = 0$. Thus, both the constrained first best, $\hat{\mu}$; and the first best, μ^* , are determined by the same equation $\mu = V'^{-1}(\theta)$. The undersupply of effort in a prize system follows directly from Theorem 2.

Although risk is normally associated with the variance of θ , it may also be appropriate to consider the effects of changes in the higher moments of θ . Consider a mean preserving spread that increased the variance of θ^2 while holding both the mean and the variance of θ constant. Let $R = E\theta^4$. From (5.12) and (5.14) we observe that welfare is a function only of $E\theta^2$. Thus,

$$(5.16) \quad \frac{W}{dE\theta^4} = 0.$$

A contest when workers have a quadratic disutility of effort is sensitive to the distribution of θ only through its second moment.

6. Contests with Large Numbers of Players

Increasing the number of players in a contest does two things: the amount of information conveyed by output is increased (as seen in the sufficient statistic result in Section 2), and the scope for designing reward structures is increased. Indeed, in the limit, it is clear that if θ did not vary any individualist performance payments scheme can be replicated by a contest with the appropriate prize structure.¹ But if θ varies, the two are not equivalent; they necessarily induce different responses to changes in θ .

(a) Prizes. We start by considering a very simple generalization: a single prize to the one winner. When there are n agents competing for a single prize, we can think of each of them putting an amount x into a kitty and the winner collecting nx . At the symmetric equilibrium, expected utility is

$$(6.1) \quad W = U(\bar{Y} - x) + \frac{1}{n} \Delta U(n) - EV(\mu)$$

where $\Delta U \equiv U(\bar{Y} - x + nx) - U(\bar{Y} - x)$.

As before, we can calculate the expected return to increased effort. Assume everyone else supplies effort μ^* , and the i th individual supplies effort μ_i ; then if he obtains a realization ε_i , he wins only if

$$(6.3) \quad \varepsilon_j \leq \varepsilon_i + \theta(\mu_i - \mu^*) \quad \text{all } j.$$

¹In the limit, there would be an infinite number of prizes (1st, 2nd, ... ∞) and these would begin to correspond to different levels of output. After reading this section see the second appendix for a detailed discussion of this point.

This occurs with probability $G(\epsilon_i + \theta(\mu_i - \mu^*))^{n-1}$. Thus the probability of winning is

$$\int G(\mu_i + \theta(\mu_i - \mu^*))^{n-1} g(\epsilon_i) d\epsilon_i$$

or the marginal return is (at $\mu_i = \mu^*$)

$$(6.3) \quad \bar{g}(n) \equiv \theta(n-1) \int G(\epsilon)^{n-2} g(\epsilon)^2 d\epsilon$$

Although in the symmetric equilibrium, agent i 's chance of winning is (at $1/n$) inversely proportional to the number of contestants, it is not immediately obvious whether his probability of winning is more sensitive to effort when there are n contestants or just 1. To get a grasp of how the expanding effect of $(n-1)$ is counterbalanced by the shrinking effect of G^{n-2} , let ϵ be distributed uniformly on $[-T/2, T/2]$, then

$$\bar{g}(n) = (n-1) \int_{-T/2}^{T/2} T^{-2} \left(\frac{\epsilon+T/2}{T}\right)^{n-2} d\epsilon = 1/T.$$

There is no effect from n . The chance of winning may become arbitrarily small, but the marginal effect of working harder is locally constant. This points out the very important distinction between marginal and average analysis. There is a presumption that one would need a very large prize to motivate effort when there is only a very small chance of winning. But workers determine their effort levels based on marginal trade offs.

In general, when the support of ϵ is unbounded, $\bar{g}(n)$ will approach zero as n tends to infinity.¹ But the speed of convergence is very important.

We can determine a prize, $x^*(n)$, such that each agent will supply the same effort as in the first best optimum by setting

$$(6.4) \quad \bar{g}(n) \Delta U(n) = U'(\bar{Y}).$$

¹For example, the Sigmoid distribution, $G(\epsilon) = (1 + e^{-k\epsilon})^{-1}$, has $\bar{g} \rightarrow 1/n$, approaching zero with large n .

In rewriting (6.4) we note that provided $\bar{g}(n)$ goes to zero slower than $\frac{1}{n}$ then $\frac{\Delta U(n)}{n}$ must in the limit be zero.

$$(6.5) \quad \lim_{n \rightarrow \infty} n \bar{g}(n) \cdot \frac{\Delta U(n)}{n} = U'(\bar{Y}) \rightarrow [\lim_{n \rightarrow \infty} n \bar{g}(n) = \infty \rightarrow \lim_{n \rightarrow \infty} \frac{\Delta U(n)}{n} = 0]$$

If we recall the expected utility W (from 6.1) then if utility function is unbounded and $\frac{\Delta U(n)}{n}$ approaches zero

$$(6.6) \quad \lim_{n \rightarrow \infty} \frac{\Delta U(n)}{n} > \frac{nxU'(\bar{Y}+nx)}{n} = xU'(\bar{Y}+nx) = 0 \rightarrow \lim_{n \rightarrow \infty} x^*(n) = 0$$

$$(6.7) \quad \lim_{n \rightarrow \infty} W = U(\bar{Y}) - EV(\mu^*) \quad \text{first best level of utility.}$$

Although the symmetric solution would approach the first best in these circumstances, the nonconvexity constraints again disrupt the equilibrium. An agent who chooses to do no work has an expected utility.

$$\lim_{n \rightarrow \infty} W \Big|_{\mu_i=0} = U(\bar{Y}) - V(0) > U(\bar{Y}) - EV(\mu^*) .$$

Increasing the size of the tournament was initially attractive as it provided better incentives for smaller risks. Unfortunately, it also changed the rewards in a way that made breaking the equilibrium also more attractive.¹

(b) Penalties. The solution to our nonconvexity problem is suggested in Mirrlees' (1974) and Stiglitz' (1975) discussion of punishment to workers who fail to meet their quota.² Stiglitz established (in the notation of our present model) that if θ is observable and ϵ has a bounded support, a first best optimum can be obtained by threatening workers who produce less than $\mu^*(\theta)\theta + \epsilon_{\min}$ with a very large penalty. Everyone will choose μ^* and thus no one will be penalized.

¹Nalebuff (1981) develops conditions for W to be monotonically increasing in n . When these hold, the optimal number of contestants in a tournament is the largest number, n , such that a pure strategy equilibrium exists; $\Delta U/n > V(\mu^*_{\max}) - V(0)$.

²This analysis is also similar to Becker's (1976) discussion of optimal punishment.

Mirrlees showed that even if ε has an unbounded support, under certain assumptions, lowering the quota and raising the punishment will in the limit approach the first best solution.¹ In our framework, this would imply a penalty to the single worker with the lowest output. As the number of agents increases, the chance of being the loser becomes very small. But it is the marginal conditions that are important in determining the size of the penalty. Provided these stay constant, we will not need increasingly horrific punishments to the loser.

Mathematically the model is identical if we replace x by $-x$ and the chance of winning by the chance of losing

$$(6.8) \quad W(n) = U(\bar{y} + x) - \Delta U/n - EV(\mu) \quad \text{at the symmetric equilibrium.}$$

$$(6.9) \quad \Delta U = U(\bar{y} + x) - U(\bar{y} + x - nx)$$

As before with a single prize the first order condition determining effort is

$$(6.10) \quad -\theta \tilde{g} \Delta U - V'(\mu(\theta)) = 0$$

where \tilde{g} is the marginal decrease in the chance of losing by working harder.

$$(6.11) \quad \tilde{g}(n) \equiv -(n-1) \int [1-G(\varepsilon)]^{n-2} g^2(\varepsilon) d\varepsilon .$$

In general, when the error distribution is on a support $[-\infty, \infty]$, the marginal change in the probability of losing, \tilde{g} , will tend to zero as the number of

¹ Whether these reward schemes are desirable under more realistic assumptions (e.g., if the distribution of ε is unknown or varies over time) is questionable. Note that the design of these reward schemes requires knowledge about the properties of the tail of the distribution, information which is often hard to come by.

players becomes very large. But provided the approach is slower than $1/n$, that is $\lim_{n \rightarrow \infty} n\tilde{g}(n) = -\infty$, then the expected utility of the agents will approach the first optimum and a pure strategy Nash equilibrium will exist.¹ To replicate the first best effort supply choose x such that

$$U'(\bar{Y}) = -\tilde{g}\Delta U = -n\tilde{g} \cdot \frac{\Delta U}{n}.$$

As $n\tilde{g}$ approaches infinity, the prize must be such that $\Delta U/n$ approaches zero

$$\lim_{n \rightarrow \infty} W = U(\bar{Y}) - EV(\mu^*) \text{ first best optimum.}$$

But, in the penalty framework, a pure strategy symmetric Nash equilibrium will exist. A worker who considers cheating ($\mu=0$) will be sure to lose. Now, he must forfeit the winners' prize that he received $(n-1)/n$ of the time and pay the loser's penalty. In the model with a single winner, cheating only cost $\Delta U/n$ which fell with n eventually to zero. Here, cheating costs $(n-1)\Delta U/n$ which rises with n , approaching ΔU . Thus, the nonconvexities become less serious as the tournament size increases.²

7. Piece Rates

There are two simple individualistic payment schemes, the linear piece rate system and the bonus system. In this section, we review the basic proper-

¹A formal proof is presented in Nalebuff (1981).

²Actually, cheating may not cause a player to forfeit his entire chance of winning. The problem is convex because the more he cheats, the marginal epsilons on which he loses will be smaller and thus more likely, and hence the greater is his marginal cost in terms of an increased chance of losing. More formally, for most unimodal distributions with $\text{sign}(g'(\epsilon)) = -\text{sign}(\epsilon)$, the marginal decrease in an agent's chance of losing becomes increasingly negative the more he shirks.

ties of the linear piece rate system, and in the next, we use these results to compare the contests with piece rate scheme. In the following section, we analyze the bonus scheme, and compare it with contests.

The individual receives as compensation

$$(7.1) \quad Y_i = \alpha Q_i + \underline{w}.$$

\underline{w} is the guaranteed income. The zero profit condition implies that

$$(7.2) \quad EY_i = \alpha EQ_i + \underline{w} = EQ_i \rightarrow \underline{w} = (1-\alpha)EQ_i$$

Hence, we can rewrite (7.1) as

$$(7.3) \quad Y_i = \alpha Q_i + (1 - \alpha)\bar{Q}$$

The i th individual's compensation is a weighted average of his own output and the average output of the group; equivalently, the individual receives as compensation the mean output of the group plus an "incentive bonus" based on the difference between his output and the mean output of the group.

Given a compensation scheme of this form, the individual chooses to maximize expected utility, yielding the first order condition

$$(7.4) \quad \alpha\theta EU'(\alpha\mu\theta + \alpha\varepsilon + \underline{w}) = V'(\mu)$$

(Again there are some problems with second order conditions. These are discussed in Arnott and Stiglitz (1980)).

Several properties of (7.4) are worth noting. First, as in the "contest", μ adjusts to θ , but the response is less than with a contest (or in the first best optimum).

$$\frac{d\mu}{d\theta} = \alpha[E(U' + \alpha\mu\theta U'')]/[V'' - \alpha^2\theta^2 EU''']$$

Indeed, it is possible that μ moves inversely with θ . If \underline{w} and σ_ε^2 are small, and U is logarithmic, then $\frac{d\mu}{d\theta} \approx 0$.

For a fixed contract (α, \underline{w}) and $\sigma_\varepsilon^2 \approx 0$, a change in the variance of ε increases or decreases effort as $U''' < 0$, in contrast to the contest, where the effect of a change in the distribution of ε depended only on the effect on \bar{g} .

7.1 Optimal Piece Rate

We can solve for the optimal piece rate system in the standard way (see Stiglitz (1975)): For any value of α we can solve (7.4) for $\mu = \mu(\theta; \alpha)$. We choose α to

$$(7.5) \quad \max EU[\alpha(\mu\theta + \varepsilon) + (1-\alpha)E\mu\theta] - V(\mu)$$

The first order condition is

$$(7.6) \quad EU'(Y - \bar{Y}) + (1-\alpha)EU' \cdot \frac{d\bar{Y}}{d\alpha} = 0.$$

By differentiating (7.4) to determine $d\mu/d\alpha$ and integrating over θ we find

$$(7.7) \quad \frac{d\bar{Y}}{d\alpha} = \frac{E\{\theta^2 E(U'[1-\alpha A(Y-\bar{Y})]|\theta)/[V''-\alpha^2\theta^2 EU''|\theta]\}}{1-E\{\theta^2 \alpha(1-\alpha)EU''|\theta/[V''-\alpha^2\theta^2 EU''|\theta]\}}$$

where A is the measure of absolute risk aversion, $-U''/U'$. For small σ_θ^2 and σ_ε^2 we can approximate the solution to α in (7.6)

$$(7.8) \quad \frac{1-\alpha}{\alpha^2} \approx A[A + \eta] [\sigma_\theta^2 + \sigma_\varepsilon^2]$$

where $\eta = V''/V'$.

The factors determining the size of the piece rate are precisely what one would have expected: the greater the variance, the less the reliance on piece rates; the greater the degree of risk aversion, the less the reliance on piece rates; and the greater the effort supply response, the greater the reliance on piece rates. In particular, in the limiting case, as the variance goes to zero, $\alpha \rightarrow 1$: there is no fixed wage component of the compensation scheme.

We can write the maximized value of welfare as $W^* \equiv EW(\bar{Y}(\alpha), \alpha; \sigma_\theta^2, \sigma_\varepsilon^2)$. Thus, for small values of σ_ε^2 and σ_θ^2 we make use of the envelope theorem to approximate

$$\frac{1}{EU'} \frac{\partial W^*}{\partial \sigma_\theta^2} = (1 - \alpha) \frac{\partial \bar{Y}}{\partial \sigma_\theta^2} - \frac{A}{2} \alpha^2 \bar{Y}^2 \approx -\frac{A}{2} \alpha^2 \bar{Y}^2$$

$$\frac{1}{EU'} \frac{\partial W^*}{\partial \sigma_\varepsilon^2} \approx -\frac{A}{2} \alpha^2$$

Welfare is a declining function of variance. This should be contrasted with the result obtained for the contest, where we showed that welfare was an increasing function of the variance of θ (but an increasing function of \bar{g} , which is inversely related to the variance of ε). We make use of these results in the next section.

8. Comparison of Contests and Piece Rates

A prize system is likely to be better than a piece rate when the range of outputs is highly variable. This follows as a contest truncates the extreme possibilities, restricting the risk to $\pm x$, while in a piece rate the worker must accept a small chance of a very large error. Contests are also likely to be superior when there are a larger number of agents each sharing the risk and reducing the expected penalty. But as is usual, the comparison of two alternative regimes is difficult, and we have not been able to obtain general results. We have, however, been able to make clear comparisons in four situations which we consider in each of the following subsections.

- a. with risk neutrality
- b. with small degrees of risk aversion
- c. with a large number of contestants
- d. with large variances.

Envelope theorems are extremely useful in these type of complicated problems, so we will wish to start from an equilibrium in which we know the solution. The two special cases are (1) zero risk, and (2) zero risk aversion. Because of the nonconvexity problem that disrupts pure strategy equilibriums near zero risk, we will consider a utility specification that starts at risk neutrality and then proceed to introduce a small amount of risk aversion.

9.1 Risk Neutrality

We first establish the (perhaps obvious) result that both the piece rate and the prize contest are able to achieve the first best level of effort and utility when agents are risk neutral. In the piece rate system, we set $\alpha=1$; there is no loss from risk and incentives are identical to what

they would be in the first best optimum. In the contest, we can induce the first best level of effort state by state by setting x according to (4.9) $x = 1/2 \bar{g}$. Having motivated μ^* , the prize does not affect the agents' utility because its expectation is zero and he is risk neutral.

9.2 Small Degrees of Risk Aversion

Although there are many ways to introduce risk aversion to a risk neutral specification, we consider just the simple example of a quadratic utility function (risk neutral when $c = 0$)

$$W = a + bY + c(Y - Y^*)^2 - \frac{\mu^2}{2}$$

where Y^* is the expected output in the first best solution.

We start by considering changes in the contest as c is increased from 0.

$$\frac{dW^c}{dc} = \frac{dW^c}{dx} \frac{dx}{dc} + \frac{\partial W^c}{\partial \mu} \frac{\partial \mu}{\partial c} + \frac{\partial W^c}{\partial c}$$

The first term drops out as x was chosen optimally. Recalling

$$\mu = \theta \bar{g} \Delta U = \theta \bar{g} 2bx,$$

we note that effort is not a function of c and thus the second term also drops out.

$$\left. \frac{dW^c}{dc} \right|_{c=0} = \frac{\partial W^c}{\partial c} = -E(Y - Y^*)^2 = -x^2$$

The loss is proportional to the variance of the risk, x . Moreover, this result does not depend on either σ_θ^2 or σ_ε^2 being small. At $c=0$, $x=1/2\bar{g}$ and

$$\frac{dW^c}{dc} = \frac{-1}{4\bar{g}^2}$$

In the piece rate system,

$$\frac{dW^P}{dc} = \frac{dW^P}{d\alpha} \frac{d\alpha}{dc} + \frac{dW^C}{d\mu} \frac{\partial \mu}{\partial c} + \frac{\partial W^C}{\partial c}$$

As α was optimally chosen at 1, a slight change will have no effect.

From the worker's optimal choice of μ , the decrease in consumption due to less effort is just offset by the gain in leisure, $\partial W / \partial \mu = 0$. Again, we have that the only term that is important is the direct effect from c :¹

$$\left. \frac{dW^P}{dc} \right|_{c=0} = \frac{\partial W^P}{\partial c} = -(\sigma_\epsilon^2 + b^2 \sigma_{\theta^2}^2)$$

Now comparisons are very easy to make.

$$W^C > W^P$$

as

$$(\sigma_\epsilon^2 + b^2 \sigma_{\theta^2}^2) > 1/4g^2$$

Clearly, for large enough values of $\sigma_{\theta^2}^2$ the contest will always be preferred. Even when $\sigma_{\theta^2}^2$ is zero one can find distributions of epsilon such that the contest is better [Nalebuff (1981)]. However, if θ is a constant and the errors are normally distributed, then $1/4g^2 = \pi\sigma_\epsilon^2 > \sigma_\epsilon^2$ so that a linear piece rate is superior.

¹The variance of Y is just the variance of ϵ plus the variance of $\mu\theta$. At $c=0$, $\mu = b\theta$ so $\mu\theta$ has a variance $b^2\sigma_{\theta^2}^2$.

9.3 Large Number of Contestants

If there are a large number of contestants, and \tilde{g} (defined on pg. 26) approaches zero slower than $1/n$, then the contest with a penalty is preferred to the piece rate system. Indeed the contest is preferred not only to the linear piece rate system, but to the optimal non-linear piece rate system (provided individuals are not risk neutral and the variance of σ_θ^2 is positive). This follows from the observation made earlier that if there are a large number of contestants, the contest approaches the first best allocation. The optimal non-linear (individualistic) piece rate system does not.

9.4 Large Variances

Earlier we demonstrated conditions in which increases in the variance of θ improve the welfare in a contest while they diminish welfare in piece rate scheme. Intuitively, when θ is highly volatile, in order to give piece rate workers proper incentives to supply effort, α must be positive and workers must bear the risk associated with large variations in Y . In contrast, a contest can replicate the first best level of effort and thus the loss in welfare relative to the first best is strictly less than the risk associated with the prize [which is of the order of $x^2 \cdot U''(\bar{Y})$] and thus stays relatively constant as σ_θ^2 increases.

Large variances in ε worsen the incentives in a contest as luck becomes more important than effort in determining the winner so \bar{g} tends to zero. This generates a need for large prizes (and thus risk bearing) in order to motivate effort. A linear piece rate scheme is also unsatisfactory when σ_ε^2 is large as the optimal α will be small and thus very little effort will be supplied. Which system is worse is best seen in the context of specific examples.

The breakdown in a piece rate system when there are large variances in the sizes of output occurs because of the conflict between needing a small α to reduce risk and a large α to motivate work. The advantage in a prize system is that the risk does not depend on the fluctuations in Y , but is limited to the uncertainty associated with x .

The argument that for large variances in θ contests are preferred to piece rates would seem to be valid for non-linear piece rates as well. The loss associated with any piece rate system in which the marginal returns are bounded increase in an unbounded way; and if the marginal returns goes to zero, the loss from the disincentive effect must again be unbounded,¹ if the elasticity of supply becomes large as the average piece rate goes to zero.

9.5 Extensions

We have thus shown that there are a variety of circumstances in which the contest is clearly superior to the piece rate system. It should be emphasized that although we have established our results in the context of a linear piece rate system, many (but not all) of the results are valid for a non-linear piece rate system. In particular, the central result that with a large number of contestants, the contest may approach the first best optimum while the optimal non-linear piece rate does not implies that, if there are a large number of contestants, simple contests may be preferred to individualistic schemes.

On the other hand, we know from the sufficient statistic theorem (Section 2) that when θ is constant, a non-linear piece rate is as good as any second best compensation scheme and, therefore, superior to a contest. The fact that we were able to demonstrate that a prize system can dominate a linear piece rate (9.2) when θ is constant is now seen as conclusive proof that a linear piece rate is far from optimal when the utility function is quadratic with only a slight amount of risk aversion.²

In order to motivate the first best level of effort with a contest we require $\bar{g}\Delta U = U'(\bar{Y})$. However, implicit in this equation is the underlying assumption that the value of the output is independent of θ . This assumption is not always appro-

¹ Perhaps, more accurately, the loss (relative to the first best optimum) approaches $u(\bar{Y}) - E v(u) - [u(0) - v(0)]$ since the individual always has the option of not entering employment.

² The optimal non-linear reward scheme is

$$Y_i = \bar{Y} + \frac{1}{c} \left[b + \frac{A(c)g(Y_i - \bar{Y}^*)}{g(Y_i - \bar{Y}^*) + B(c)g'(Y_i - \bar{Y}^*)} \right]$$

which when g is normal takes on a hyperbolic distribution which is nothing like the linear piece rate, $Y_i = \alpha Y$.

priate as seen in the agricultural context where the value of harvest is likely to depend the total crop output which is a function of the weather, θ . Even if agents are risk neutral neither a contest nor a linear piece can replicate the first best solution when the price of the output depends on θ .¹ For example, in the contest there does not exist a ΔU such that $\bar{g}\Delta U = P(\theta)U'(\bar{P}Y)$ for all θ (unless of course if P is constant). An equally serious problem arises when the distribution of ε varies with θ , $g(\varepsilon_i, \theta)$. Here again no single ΔU can always satisfy $\bar{g}(\theta)\Delta U = U'(\bar{Y})$.

In a more general framework, we can think of an additive risk, $\hat{\varepsilon}_i$, that is comprised of both individualistic elements, ε_i , and common environmental factors, θ . Moreover, the relationship of θ and ε_i in determining $\hat{\varepsilon}_i(\theta, \varepsilon_i)$ could also vary across individuals. For a given variance of this additive risk, $\sigma^2_{\hat{\varepsilon}_i(\theta, \varepsilon_i)}$, it follows from the definition of \bar{g} that positive correlation of $\hat{\varepsilon}_i$ across workers will increase \bar{g} while negative correlation will lower \bar{g} . As the positive correlation increases, the idiosyncratic noise will be less important. The competition will become more fierce and a smaller prize will motivate the same effort. With a small enough prize, agents in a tournament can be better off than with a piece rate. Conversely, we conjecture that if there is sufficient negative correlation between the $\hat{\varepsilon}_i$ then workers will be competing in "different" environments and a linear piece rate will be superior.

¹ The problem arises when the price of the output depends on θ either directly or even indirectly through output, $Q_i(\theta)$.

In the next section, we consider a particular non-linear piece rate system.

10. Bonus Schemes

The simplest non-linear individualistic payment scheme is the bonus scheme, with

$$(10.1) \quad Y = \begin{cases} Y_1 & \text{if } Q > \hat{Q} \\ Y_2 & \text{if } Q < \hat{Q} \end{cases}$$

Thus the individual chooses μ to

$$(10.2) \quad \max U(Y_1)(1 - G(\hat{Q} - \mu\theta)) + U(Y_2)G(\hat{Q} - \mu\theta) - V(\mu)$$

which has a first order condition

$$(10.3) \quad \theta \Delta U g(\hat{Q} - \mu\theta) = v'$$

There are two immediate difficulties with the use of such a simple quota or bonus scheme. First, even when the worker is risk neutral the bonus scheme cannot replicate the first best solution (because $g' \neq 0$ unless g is uniform). Secondly, there is again a nonconvexity problem that arises when θ is small. When the quota is high and the task is difficult then the worker may decide that the bonus is not worth competing for. Moreover, if the task is easy, the worker has little incentive to produce more than the quota.

The distinctive feature of contests is that only information about rank is used in determining compensation; the magnitude of the difference between the competitors does not enter into the determination of the level of compensation.

We consider here some simple schemes in which the magnitude of relative performance enters into the compensation scheme. This will enable us to obtain some intuition concerning the kinds of situations where contests may be preferred.

Assume

$$(11.1) \quad Y_i = \beta(Q_i - Q_j) + \bar{Y}$$

The amount received by the individual is a fixed sum, \bar{Y} , plus a linear function of the difference between the two individuals' levels of output.

In the symmetric equilibrium

$$(11.2) \quad Y_i = \beta(\varepsilon_i - \varepsilon_j) + \bar{Y}$$

and hence the variance of the individual's income is just

$$2\beta^2\sigma_\varepsilon^2$$

Notice that in this scheme, the variability of θ has no effect on the variability of individual's income. Whether, for a given incentive level (β), this scheme results in higher or lower risk (and therefore higher or lower utility) than the piece rate depends simply on the relative magnitude of $E(\mu\theta)^2$ and $E\varepsilon^2$. If the variability of ε is small relative to the variability of θ then this scheme is clearly preferred to the individualistic piece rate system. On the other hand, the earlier analysis

can be modified to show, as before, that for large values of the variance of ε , the contest is preferred to this scheme.

This scheme can attain the first best level of effort, simply by setting

$$(11.3) \quad \beta EU'(\beta(\varepsilon_1 - \varepsilon_2) + \bar{Y}) = U'(\bar{Y})$$

(this may entail setting $\beta > 1$.)

This is not the case, however, if we base compensation on the ratio of the individual's output to the mean output.

$$(11.4) \quad Y_i = f(Q_i/\bar{Q})$$

One might have thought that this simple change in specification would have no significant consequences, but that is not the case, as we shall now see. In the symmetric equilibrium

$$(11.5) \quad \frac{Q_i}{\bar{Q}} = \frac{\mu_i \theta + \varepsilon_i}{\mu^* \theta}$$

$$= \frac{\mu_i}{\mu^*} + \frac{\varepsilon_i}{\mu^* \theta}$$

Thus, individuals will set μ_i so that

$$(11.6) \quad \frac{1}{\mu^*} EU'f' = V'$$

Note that now, μ varies with θ only through the effect of θ on $U'f'$. Thus

$$(11.7) \quad \frac{d\mu}{d\theta} = \frac{-E(U''f'^2 + U'f'')\varepsilon_i/\mu^2\theta^2}{V'' - E(U''f'^2 + U'f'')/\mu^2}$$

Thus, if $\sigma_\varepsilon^2 = 0$, μ is invariant to θ in contrast to the first best solution, where an increase in the productivity of the individual (θ) lead to an increase in his effort. Clearly, this relative performance criterion lacks the property of "flexibility" we noted earlier. (If the variance of ε is small, there is a small effect on effort; but it can either increase or decrease, depending on U'' .) By choosing f appropriately, one can presumably obtain any desired response, at least locally.)

12. Prizes that Influence Choice of Techniques

The choice of techniques when the reward system has a prize structure was initially studied in the context of credit rationing by Stiglitz and Weiss (1980). A loan from a bank is like a compensation scheme with a piece wise linear piece rate. Should the agent earn less than the interest payment, he goes bankrupt and is rewarded zero. If he has produced more than his interest payment, the agent may keep the surplus. A rise in the interest rate (quota level) results in 'riskier' [Rothschild and Stiglitz (1970)] strategies being adopted by the borrowers.

By analogy to the credit market, it is clear that a potentially tremendous advantage of competitive compensation schemes (i.e., contests or quotas) over piece rates is that they encourage entrepreneurs to disregard their natural risk aversion and adopt more profitable although riskier strategies. This benefit is even greater in fields like Research and Development where prizes (in the form of patents) encourage taking risks that can dramatically shorten the time to discovery.

The compensation schemes we have been considering simply do not have enough instruments to control separately both inputs (effort) and techniques.

13. Competition, Cooperation and Worker Satisfaction

The models presented in this paper have argued that there is a distinct role for competition--real competition, in the sense in which the word is ordinarily used, not the peculiar static sense in which much of neoclassical economics has come to use that term--in situations where there is imperfect information about the difficulties associated with different tasks, and in which it is prohibitively costly to observe inputs directly. We have argued that competitive systems have greater flexibility, greater adaptability to changes in the environment (θ) than do other forms of compensation.

Although competitive compensation schemes are used, their use appears to be more limited than the analysis of this paper would suggest.¹ Indeed, many of the considerations which seem to be central to personnel departments in choosing among compensation schemes seem to be omitted from the kind of analysis presented above. In this section, we want to briefly explore some of these missing elements, and to suggest a broader framework within which the questions of job-structure may be addressed.

First, it should be noted that there are often technological returns to cooperation, e.g. sharing of information. In the specification embodied in our model the technology is completely individualistic. Obviously, there are no incentives for cooperation in a competitive system; on the contrary, there are even incentives for destructive activities. In contrast, in the piece rate system there are at least no negative incentives for cooperation.

¹ We do observe compensation schemes that incorporate elements of prizes (promotions, patents) and piece rates. The study of mixed systems is very complicated and cumbersome; in general, such mixed systems are potentially superior to the polar schemes analyzed here.

Secondly, there may be incentives, within small groups, for collaboration within competitive systems: if the two or more contestants agree to limit their effort, their expected utility will be increased. Whether collusion will, in fact, occur, depends on the ease with which the contestants can collude and the availability of enforcement mechanisms. Thus, when the contestants are repeatedly playing against each other, and/or when they have frequent contact with each other, so social as well as economic sanctions may be employed, collusion is not an unlikely outcome.

There is, however, a more fundamental objection to the approach taken in this paper. We have assumed that individuals' choices among jobs can be described as if individuals simply evaluate the probability distribution of efforts and income, and choose that job which maximizes their expected utility.

It is, of course, well known that there are other motivations (both sociological and psychological) for individual behavior besides those which are usually encompassed within economic models of the kind of we have presented here. Individuals are concerned not only with the outcomes (the income they will receive, the level of effort they will have to put forth) but also with the processes (the social environment in which they will work, the amount of stimulation that they will receive on the job). For some purposes it may suffice to lump all of these together as "non-pecuniary characteristics of the job" and to treat "leisure" as a good, while "work" is a bad. But such an approach may subsume most of what is crucial in the determination of job and wage structures into a black box, about which, therefore, economists will have little to say. And such an approach seems to provide an inadequate description of our behavior, as well as that of most of our friends.

This broader perspective has several implications for the questions which we have been addressing.

First, individuals who "like cooperation" receive "disutility" not just from the variance in their income to which competition may give rise (indeed, it may be less than for other forms of payment, as we have noted), but from the competitive process itself, while for others, the process of competing may be positively enjoyable.

Secondly, in non-competitive production arrangements, considerable reliance may be placed on social incentives rather than on economic incentives. Individuals do not shirk because of the disapproval it generates.

For example, the nonconvexity problem with a small prize and a large number of players arises from each worker's desire to do no work and still reap the benefits of the losers' prize. Although the manager (principal) is assumed unable to monitor the agents' effort levels, it is very likely that workers can observe each other. While a worker may be reluctant to "rat" on a coworker who is doing only slightly less work, the cheating strategy involves a discrete reduction in effort. An employee who does not come into the office three out of five days a week may be dismissed for not pulling his own weight or ostracized by his fellow employees. If we can penalize cheaters rather than just giving them the losers' prize, then pure strategy equilibria will exist even for the smallest of prizes. The possibility of obtaining the first best solution gives each individual an incentive for extending disapproval to those who shirk (cf. Akerlof and Soskice's (1976) theory of sanctions).

Thirdly, just as different individuals have different attitudes about risk, so too will they have different attitudes towards competition and cooperation: some individuals will thrive on competition, others will dislike it. Thus, one should not look for an "ideal" compensation scheme; rather, there should be a

variety of compensation schemes, tailored to the different attitudes of different workers.

Fourthly, society may find the aftermath of competition, that is the presence of losers, to be unacceptable. An advantage to individualistic schemes is that everyone can have a high output, meet their quota, and be rewarded. By contrast, when we don't know where to set the quota and thus base rewards on relative performance, someone must come in last and be a loser. We suppose that their decline in use is a sign not of their ineffectiveness, but rather that our advanced society wishes to find incentive schemes that are less primitive and unsophisticated. The "backwardness" of stigmatizing losers has been gradually replaced by perhaps less effective but more acceptable schemes of positive rewards and positive reinforcements.

Competition works best when all the participants are similar. A difficulty with the implementation of penalties is that the losers are usually more than just unlucky, they are often not as able. In order for them to compete, they would have to work harder than the average worker. Worse, the presence of a sure loser destroys everyone's incentives to work hard. When the different relative abilities are known, handicapping (i.e., as in golf tournaments) can restore the competitive environment that arises in a "fair" contest.

These remarks, in turn, have several further implications. First, it may be important for firms to screen individuals, not only concerning their productive characteristics, but also concerning their "personality" (in order to determine for instance whether they are competitive, and will do well (be happy) in a competitive environment) and their social characteristics (e.g., to determine whether they are the kind of individual who is susceptible to "social" incentives).

Group homogeneity may be important for two reasons. We have suggested that social incentives play an important role in determining economic behavior, but for social incentives to be effective, the worker must respond to the social

pressure of his peers. But this in turn requires that the individual feel some affinity to the group. Thus, social incentives may be less effective in more socially heterogeneous groups.

Moreover, placing a competitive person or a shirker (free rider) within a cooperative productive group may have serious effects on the cooperative activities of the individuals within the group. The point is that individuals, within these production groups, are being asked not to pursue what may be narrowly (myopically) within their own interests on the grounds that it is in the group interest, and, in that sense, in their long run interest. When workers observe an individual within the group acting in his own interest, and facing no obvious consequences, then they too may imitate that kind of behavior. There is thus a kind of precariousness associated with the effectiveness of social incentives in heterogeneous societies, even though, when they can be made effective, they result in Pareto superior outcomes.

At the same time, there are also additional pressures for limiting the use of competitive schemes, particularly in environments where the economic organization is not subjected to competition itself. The choice of an incentive scheme is usually a matter left to managerial discretion, in particular, to those at the top of the managerial hierarchy. Even if the choice of managers were based on past performance, past performance itself is determined both by skill, effort, and luck. Those who have succeeded in becoming managers have had, on average, better than average luck. In particular, a significant fraction of them know that were they subjected to repeated competitive evaluation, they might lose their position. Since, at that juncture, they have little if anything to gain by subjecting themselves to this kind of competitive pressure, and much to lose, one would have thought there would be strong incentives not to employ a competitive structure.

When the economic unit is in competition with other economic units, so

its relative performance is constantly monitored, even if managers can reduce internal competition, they cannot eliminate the external competition (although they may try to do so, through collusion). Thus, if internal competition is an effective incentive device, then they may be "forced" to use a competitive incentive system. The presence of external competition allows, at the same time, the possibility of combining both cooperative and competitive behavior: individuals within the firm (product unit) cooperate together, with one of their goals being competition with production units engaged in similar economic activities.

12. Concluding Remarks

This paper is one of a series attempting to explore the role of competition in economic activity, not the stylized competition of conventional neo-classical analysis, but "real" competition. Here, we have focused on the role of competition in the internal organization of the firm, in providing incentives for workers to perform well, while at the same time limiting the amount of risk which they have to bear. In addition, the competitive system has the important property of adaptability (flexibility), entailing an automatic adjustment of equilibrium effort to changes in the economic environment. As a result, the competitive system may be superior to the two other forms of compensation schemes examined, the linear piece rate system and the relative performance scheme. At the same time, we noted [in the appendix] that there were circumstances where competition could become sufficiently ruthless as to completely eliminate all consumer surplus of the participants: competition can result in excessive effort. To its credit, the use of a single penalty with a large number of contestants can often approach the first best optimum.

The use of these competitive compensation schemes seems less widespread than their evident advantages would suggest. We suggested

this may be due to some important aspects of worker satisfaction which the traditional economic models ignore. These other considerations are probably less important in the analysis of competition between firms than in competition within the firm. These are questions which we hope to pursue in the sequel to this paper.

APPENDIXMIXED STRATEGY EQUILIBRIA

We have shown in the text that in the simple prize to the winner contest no symmetric pure strategy Nash equilibrium exists either as the number of contestants becomes large or as the variance (risk) due to epsilon vanishes. In both instances, the expected prize is sufficiently small that it isn't worth competing for. Thus, both agents would prefer to do no work and earn the loser's reward. This difficulty can be partially eliminated by combining performance standards with contests when ϵ has a finite support [Stiglitz (1981)]. Here we consider the mixed-strategy equilibrium.

Assume, as the extreme case, that $\sigma_{\epsilon}^2 = 0$. Clearly, any individual can, by increasing his level of effort slightly above that of his rival, assure his victory. Each contestant's reaction function is discontinuous, and no pure strategy equilibrium exists. But if $H(\mu)$ is the probability distribution of the maximum of the effort levels of the competing individuals, the expected utility of individual when pursuing effort level μ_1 , is

$$U(\bar{Y}-x) + \Delta U \cdot H(\mu_1) - V(\mu_1) = k$$

Clearly, if

$$k = U(\bar{Y}-x) - V(0)$$

$$H(\mu) = \frac{V(\mu) - V(0)}{\Delta U}$$

the individual is indifferent as to his level of effort. With N players

$$H(\mu) = F(\mu)^{N-1}$$

$$F(\mu) = \left[\frac{V(\mu) - V(0)}{\Delta U} \right]^{\frac{1}{N-1}}$$

$$(1) \quad \bar{Y} = \int_0^{\hat{\mu}} \mu f(\mu) d\mu = \int_0^{\hat{\mu}} [1-F(\mu)] d\mu$$

where the maximum level of effort $\hat{\mu}$ satisfies

$$(2) \quad V(\hat{\mu}) - V(0) = \Delta U.$$

The optimal feasible mixed strategy equilibrium is determined by the choice of x that maximizes expected utility, k .

$$U'(\bar{Y}-x) [\bar{Y}'-1] = 0.$$

Thus, we will look for the solution to $\bar{Y}' = 1$. From (1) and (2)

$$(3) \quad \frac{dY}{dx} = \frac{(\hat{\mu}-\bar{Y})S}{(N-1)\Delta U - (\hat{\mu}-\bar{Y})\Delta U'}$$

To provide a worked out example of the mixed strategy solution, we further assume

$$(i) \quad V(\mu) = \mu^2/2$$

$$(ii) \quad U(x) = x \text{ risk neutrality.}$$

We choose a risk neutral utility function for our analytic simplicity and because the deficiencies of the mixed strategy solutions will be most apparent when compared with the first best outcome that is achievable using a simple piece rate.

$$(2') \quad \frac{\hat{\mu}^2}{2} = 2x$$

$$(1') \quad \bar{Y} = \int_0^{\hat{\mu}} \left[1 - \left(\frac{\mu}{\hat{\mu}} \right)^{\frac{2}{N-1}} \right] d\mu = \frac{2}{N+1} \cdot \hat{\mu}$$

at x ,

$$(3') \quad (N-1)x = [\hat{\mu} - \bar{Y}]$$

Thus

$$\hat{\mu} = \frac{4}{N+1} ; x = \frac{4}{(N+1)^2} ; \text{ and } \bar{Y} = \frac{8}{(N+1)^2}$$

$$\text{Expected Utility: } EU = U(\bar{Y}-x) - V(0) = \bar{Y}-x = \frac{4}{(N+1)^2}$$

In the first best solution, workers choose μ to maximize

$$EU = \mu - \mu^2/2 \rightarrow \mu=1 \text{ and } EU = \frac{1}{2} .$$

We observe that with only 2 workers, the mixed strategy solution is able to come reasonably close ($\frac{4}{9} / \frac{1}{2}$) to the first best level of utility. But as the number of contestants increases, competition of this form is so ruthless that all consumer surplus is eliminated. Since workers know that once they become engaged in a competitive battle, all of their consumer surplus will be competed away, they will not sign contracts of this form (if there are contracts with positive consumer surplus available). When θ is variable, the terms of the contest can be set in such a way that ruthless competition prevails only for some values of θ , while for other (smaller) values of θ , a conventional contest occurs. Contracts entailing some piece rate compensation also easily eliminate such quandries; we shall not discuss these mixed strategy solutions further here (see Gilbert and Stiglitz(1979) for a more extended discussion in the context of patent races).

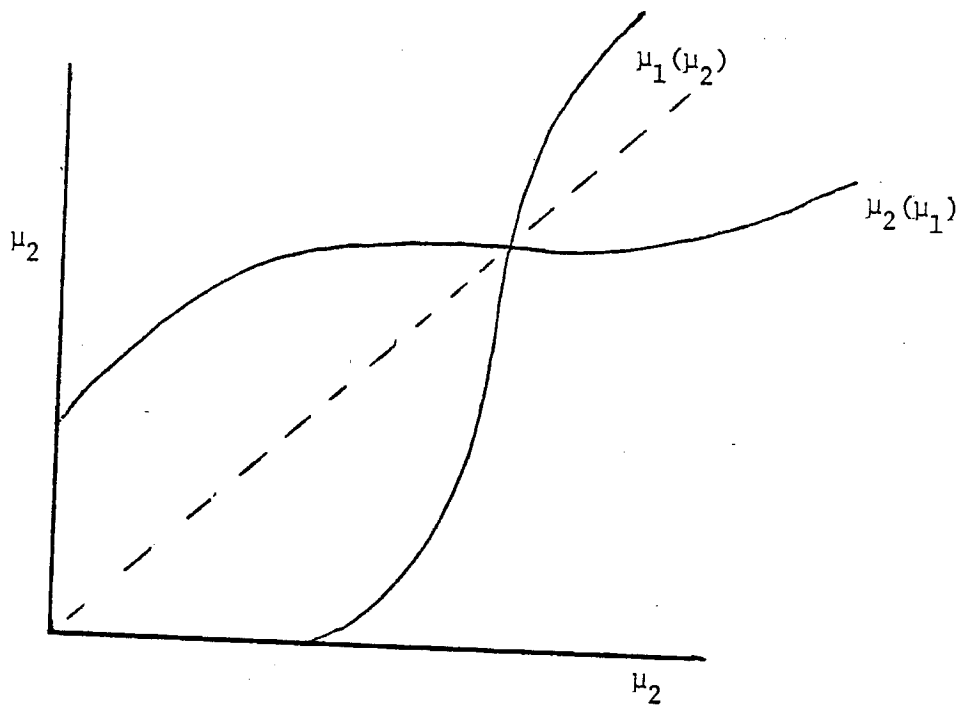


FIGURE 1. Symmetric Equilibrium

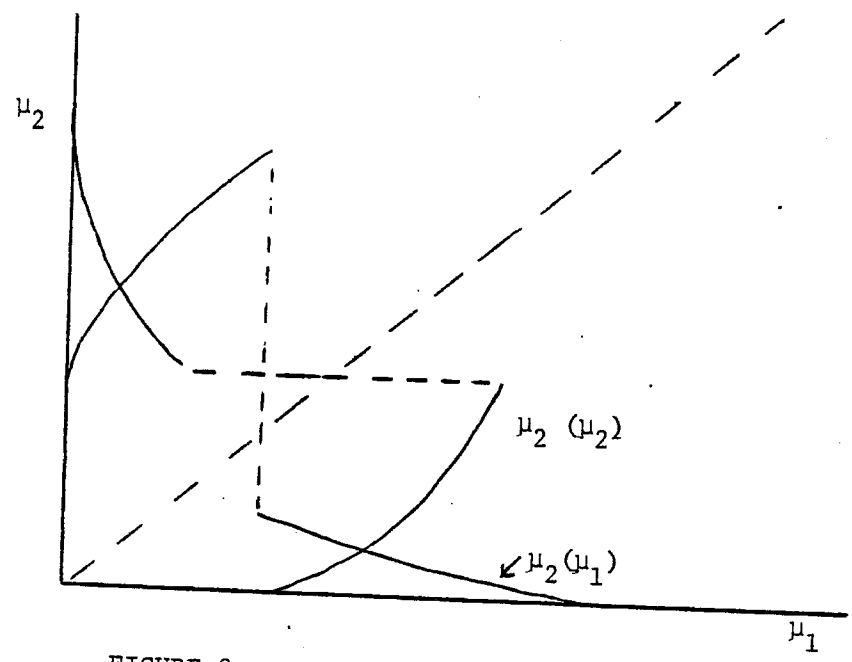


FIGURE 2. Asymmetric Equilibrium

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