

QUALITY AND PRICES

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Econometric Research Program
Research Memorandum No. 297

May 1982

Econometric Research Program
PRINCETON UNIVERSITY
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Princeton, New Jersey

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by

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In the absence of perfect information, prices play an important role in conveying information about quality. Although the phenomenon of judging quality by price is pervasive, and its importance long recognized (Scitovsky, Alcaly and Klevorick and Akerlof) there have been few attempts to develop a consistent and complete equilibrium theory (Stiglitz [1976], Nalebuff [1980], Wilson [1980], Weiss [1980]). This paper examines how the fact that prices may convey information about quality affects the behavior of a monopolist (monopsonist) and the equilibrium in a competitive market. As in other recent studies, competitive equilibrium with imperfect information differs markedly both from competitive equilibrium with perfect information and monopolistic markets with imperfect information.

There are five important results of our analysis:

1. In competitive equilibrium, even though all workers appear to the employer to be identical (before has has hired them) or all products appear to the consumer to be identical (before he has purchased them), prices will differ. The conventional law of a single price is replaced by a quality-price schedule. In one of the models we formulate, the wage of the job to which the worker applies conveys perfectly the information about his ability, while in the other models examined, the information is far from perfect.

* The authors are indebted to R. Lindsey and R. Arnott for helpful comments. Financial support from the National Science Foundation and the Rhodes Trust is gratefully acknowledged.

2. The competitive equilibrium which emerges is characterized by unemployment (see also Stiglitz [1976]). Unemployed workers will not cut their wages, as that would be interpreted as a signal that they are less productive. Lower wages would increase their probability of employment, but not enough to compensate for the lower wage.

3. The monopsonist may offer a wage distribution, but if he does, it will have only two wages. This is all that is required to enable him optimally to exploit the differences in reservation wages and abilities among his applicants.

4. With sequential entry of firms, the quality of applicants to firms may change through time. In a model with sequential entry, a second entrant that offers a higher wage than the first firm will obtain a quality of workers that is higher than that which the first firm would have obtained had it paid this higher wage. Gausch and Weiss [1980] have referred to this phenomenon as an "advantage to being late." This suggests a serious problem of inefficiency if new firms into a market delayed their entry as a result. We establish that in a competitive equilibrium in which workers anticipate the job offers to come and search optimally, there is, in fact, no advantage to being late.

5. Most of the analysis assumes that contracts are binding, but in the final section of the paper, we take this provision to be one of the endogenous features of market equilibrium. We establish that in our model (which assumes no firm specific training costs), equilibrium will never be characterized by all firms having binding contracts.

1. The Basic Model

To make our analysis as simple as possible, we assume that a firm has no information about any particular worker (other than that he is applying for a particular job). On the other hand, we shall assume that the firm has perfect statistical information; that is, we shall be concerned with equilibria in which the firm knows the (relevant aspects of the) probability distribution of the characteristics of the applicant pool.

We assume that each worker has a reservation wage (which can be thought of as his opportunity cost in self-employment). Workers differ in their productivity in a simple, multiplicative way: a worker with a productivity of "a" can do in an hour what it takes an individual of productivity "ã" a/\tilde{a} hours to do.¹ The average efficiency unit of a worker with reservation wage w is denoted by $a(w)$, and we assume workers with higher reservation wages are, on average, more productive, i.e. $a'(w) > 0$. The distribution of workers in the population with reservation wage w is $F(w)$, which, for simplicity, we assume is differentiable; we let

$$f(w) = F'(w)$$

denote the density of the reservation wages. Finally, we let $\bar{A}(w)$ denote the average efficiency units of all workers with reservation wage less than or equal to w :

¹ There is no concern here about comparative advantage, simply absolute advantage. Cf. Stiglitz [1975].

$$(1) \quad \bar{A}(w) = \frac{\int_0^w a(x) f(x) dx}{F(w)}$$

In Figure 1 we illustrate a possible shape of $A(w)$: initially, increases in wage result in rapid increases in the average quality of applicants, but eventually diminishing returns sets in.¹

If all workers (or a representative sample of workers) applied to any firm, and the firm sought simply to minimize its labor costs per efficiency unit,²

$$(2) \quad \min_{\{w\}} w/\bar{A}(w)$$

then it would pay a wage, w^* , such that

$$(3) \quad \bar{A}'(w^*) = \bar{A}/w^*$$

w^* is sometimes referred to as the efficiency wage, and is depicted in Figure 1 as the point of tangency of a line from the origin to the $A(w)$ curve.

w^* is the equilibrium wage in a competitive market only, however, under the restrictive assumption that the ability distribution facing any firm is identical to that facing the entire economy. This is a plausible assumption if:

¹ There is no a priori reason why the $A(w)$ function should have such a simple shape. See, for instance, Stiglitz [1976].

² The firm is, of course, concerned with maximizing profits. If it is unconstrained in the amount of labor it can have at any wage, then a necessary condition for profit maximization is the minimization of labor costs. See Stiglitz [1976].

(i) there are no costs to applying to any firm, so each individual applies to all firms offering a wage higher than his reservation wage; and

(ii) all firms announce their wage offers simultaneously.

In other cases, the set of applicants to one firm will depend on the job offers made by other firms. It is this interdependence which makes the analysis of equilibrium in the market so complicated.

In this paper, we focus our attention on a set of models in which there is sequential entry of firms; some firms make job offers before others.¹ In the first set of such models, we assume that contracts are binding: thus, there is a cost of accepting a lower wage job -- the lost opportunity of getting a higher paid job later. This obviously has an effect on the reservation wage of individuals, and this in turn has an effect on firm behavior. In the second set of models, we allow for the possibility of recontracting.

We begin our analysis by considering what would happen if there were two firms in the market, under the assumption that workers are completely myopic, i.e. ignored the possibility of future (higher) wage offers in their decision to apply for a job.

The first firm to enter the market chooses a wage, w_1 , to maximize $\bar{A}(w)/w$. His first order condition satisfies

$$\bar{A}'(w_1) \cdot w_1 - \bar{A}(w_1) = 0 .$$

¹ In the analysis, the sequential nature of job offers is an endogenous characteristic of the market; that is, all firms could make offers simultaneously but, under our assumptions, this is not an equilibrium. As the discussion below makes clear, there is a close relationship between these models, and those where there are positive search costs and simultaneous wage offers.

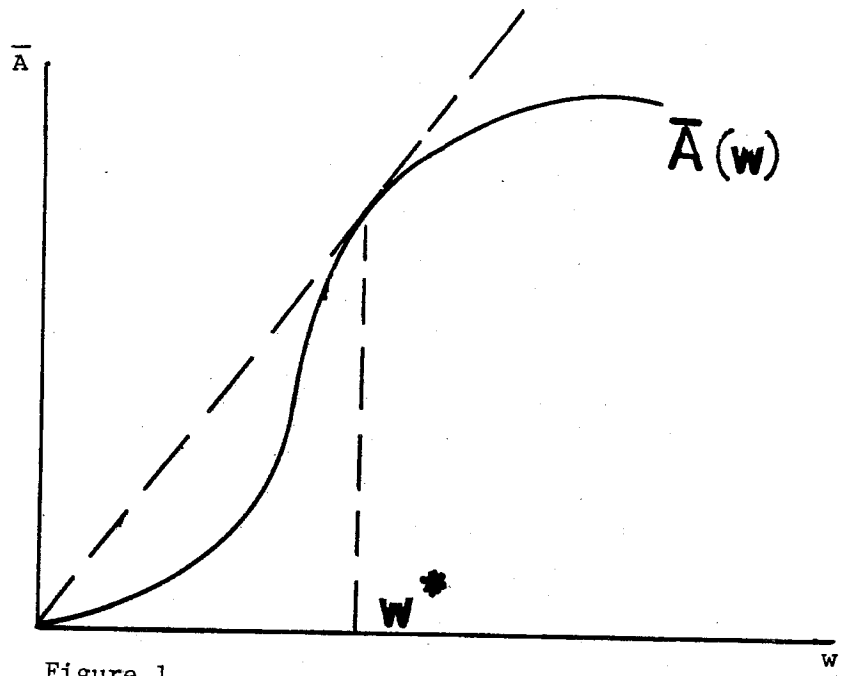


Figure 1.

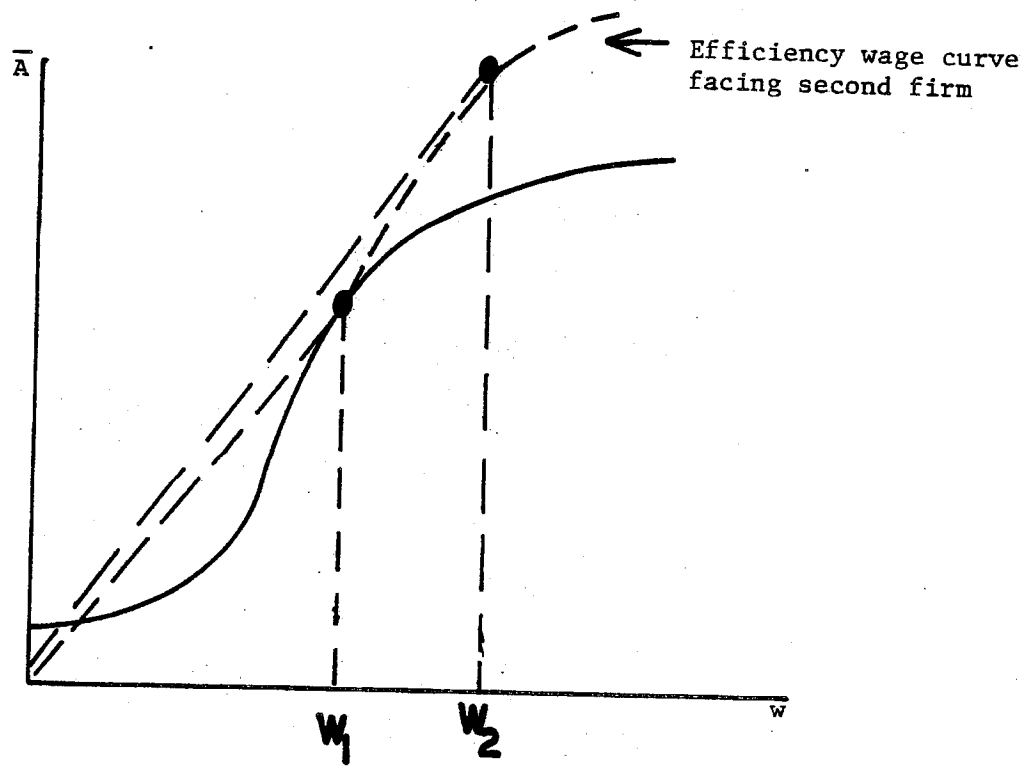


Figure 2.

Further define:

$h(w)$ job applicant's probability of employment at wage w ,

N^* firm's optimal labor input, measured in efficiency units.

The firm chooses $h(w_1)$ to satisfy

$$(4) \quad \bar{L}h(w_1)\bar{A}(w_1)F(w_1) = N^*$$

where \bar{L} is the total labor force. Equations (1), (3), and (4) determine the first firm's optimal choice of wage, w_1 , and each applicant's chance of employment, $h(w_1)$.

In a world with no recontracting and myopic workers, after the first firm has made its selection, a second firm has a greater incentive to enter the market. The second firm can actually achieve a higher efficiency units per wage ratio, and thus has an advantage of being late (Guasch and Weiss [1980]). The first firm has hired a random selection of workers with reservation wages less than w_1 . At wages less than w_1 , the second firm faces the same assortment of workers as the relative properties of the distribution are unchanged. However, at wages greater than w_1 , there are relatively fewer lower quality applicants (as some have been hired by the first firm) and thus the higher quality marginal applicants become relatively more important. This can easily be seen diagrammatically. After the first firm enters, the efficiency units/wage curve shifts up at the tangency, creating a new dominant wage [as in Figure 2], w_2 .

The second firm faces a new density of workers given by:

$$f_2(w) = \begin{cases} [1 - h(w_1)]f(w)/1-n & w \leq w_1 \\ f(w)/1-n & w > w_1 \end{cases}$$

where n is the proportion of the labor force hired by the first firm:

$$n = h(w_1)F(w_1) .$$

The second firm can offer a wage of $w_1 + \epsilon$ and achieve a better efficiency units/wage ratio. The new measure of average efficiency units, $\bar{A}_2(w)$,

takes into account the new density of workers.¹

$$(5) \quad \left. \frac{\partial (\bar{A}_2(w)/w)}{\partial w} \right|_{w_1} = \frac{\bar{A}(w_1) h(w_1)}{w_1^2 [1 - h(w_1)]} > 0$$

Because the first firm has hired some of the poorer quality workers, the second firm now finds it relatively cheaper to offer a higher wage and attract better workers.

Our analysis has not only established the advantages of being late but also demonstrated that no single wage (price) competitive equilibrium exists (at least under the postulated conditions). Any firm, recognizing the advantages of being late, would postpone entering. In order to develop an appropriate equilibrium concept, we must examine the weaknesses of our simple story.

There are two essentially different but both important problems with the model we have just presented. First, workers failed to anticipate the entry of the second firm. A sequence of rising wage offers should raise a job seeker's reservation wage. Thus reservation wages were not rational and search behavior was not endogenous to the system. Second, although the efficiency units/wage may be greater at $w_1 + \epsilon$, there may not be a sufficient number of applicants at that wage to meet the second firm's optimal demand for labor input. It is easy to imagine a case in which the first firm hires so much of the initial labor supply at w_1 that the second firm can only attract a miniscule number of workers at $w_1 + \epsilon$ and is thus constrained in its demand for labor.

The solution to the problem caused by sequential entry lies in incorporating rational expectations both on the part of the firms and workers into their equilibrium behavior. We proceed by first determining the competitive solution when the constraints on the acquisition of labor by later firms are never binding, and then examining the advantages

¹ At w_1 , $\bar{A}_2(w)$ is not differentiable. In equation (5) we are referring to the right derivative.

of being late in duopoly where they may be. This analysis is followed by a characterization of the monopoly solution.

2. The Competitive Solution

If there is to be a wage distribution in a competitive market, then it must be the case that the expected efficiency units per wage is constant across all wage offers. Competitive firms would only make multiple wage offers as a matter of indifference.

Let us hypothesize that our market operates in the following manner. Wages are called off starting at some bottom level and proceeding upwards. Workers apply for jobs by joining a queue. A certain number of applicants are hired, chosen randomly from those who applied. Employed workers have binding contracts with a no recontracting clause and thus they may not continue searching. Workers not selected join the next queue when the next higher salary is called off.

We show that there is, in fact, an equilibrium of this form (when all contracts are required to be binding).¹ A rational expectations equilibrium is characterized by a number of job offers at wage w , $N(w)$, and a set of beliefs about the probability of getting a job at wage w , $\hat{h}(w)$. An individual with those beliefs and an ability (self-employment reservation wage) \hat{w} who maximizes his expected utility, will have a reservation wage $R(\hat{w})$. Given the ability distribution $f(\hat{w})$, and the job distribution $N(w)$, this will generate an actual probability of getting a job at wage w of $h(w)$, and the average ability of those hired at wage w will be

¹ Later, we show that if this provision of the contract is also endogenous, contracts will not be binding, at least in our simple model.

$A(w)$.¹ A rational expectations equilibrium requires

- (i) Individuals' beliefs about the probability of getting a job at wage w are correct:

$$\hat{h}(w) = h(w)$$

- (ii) The labor costs per efficiency unit of all firms be the same

$$A(w) = kw \quad \text{for all } w \text{ for which } N(w) > 0$$

(and for all w for which $N(w) = 0$, the labor costs were a firm to make an offer must not be lower:

$$A(w) \leq kw \quad \text{for all } w \text{ for which } N(w) = 0.)$$

Although one's first reaction might be to suppose that wages offered are sequentially lower, this cannot be an equilibrium under our assumptions. The advantages to latecomers in our simple model only arose when salary offers increased. A firm that offers a lower wage than its predecessors faces the same quality/wage distribution. Thus, firms entering later are willing to offer higher wages. Their applicant pool will have relatively more high productivity workers as many of the low efficiency workers will have already been hired. Note that if wages are called off sequentially downwards, then everyone will apply for each job until the salary is below their fallback opportunities, \hat{w} . The job distribution has no effect on their behavior; in particular, there is no interdependence between the sequence of wages and the workers' reservation wages. With sequentially

¹ $A(w)$ is also defined where $N(w) = 0$ as the average ability of those that would apply to an infinitesimally small firm offering a wage of w .

rising wages, a worker's decision concerning when to enter the labor market depends on both his fallback opportunities and the distribution of salaries that are available.

Thus, given our earlier analysis, if there exists a competitive equilibrium with sequential entry and no recontracting, it must entail rising wages. We now provide a simple mathematical characterization of the equilibrium.

Workers are forced to trade-off between losing the chance for high salaries by joining low wage queues and increasing the probability of getting no job at all by postponing joining low wage queues. We assume workers are risk neutral. At the lowest wage job that they apply for, workers will be indifferent between being hired and being turned away. Their lowest acceptable salary is their expected salary.¹ To see this,

let

$R(\hat{w})$ reservation wage of type \hat{w} worker

$m(w,R)$ chance of employment at a wage between w and R of a worker who enters the job market at R .

The probability of getting a job at wage w is the probability of not having been hired until then, $1-m(w,R)$, times $h(w)$, the proportion of applicants at wage w who are hired². Thus $\partial m/\partial w = [1-m]h(w)$ and integration yields

$$(5') \quad m(w,R) = 1 - e^{-\int_R^w h(\bar{w}) d\bar{w}}$$

If $\rho(\hat{w},R)$ is the expected salary of a worker with self-employment \hat{w} , and reservation wage R , then

¹ Thus the reservation wage is independent of the employment probability at the reservation wage.

² Mathematically, not being hired at wage w is like not observing any blips in a poisson process that has a density h .

$$(6) \quad \rho(\hat{w}, R) = \int_R^{\infty} [1 - m(w, R)] h(w)w \, dw + \hat{w}[1 - m(\infty, R)]$$

A worker's expected salary is an average of the wages available, weighted by the probability of getting them.

The risk neutral individual sets R to maximize (6):

$$\frac{\partial \rho}{\partial R} = -h(R)R - \int_R^{\infty} \frac{dm(w, R)}{dR} h(w)w \, dw - \hat{w} \frac{dm(\infty, R)}{dR} = 0.$$

But from (5')

$$\frac{\partial m}{\partial R} = -(1 - m)h(R).$$

Hence if

$$\frac{\partial \rho}{\partial R} = h(R) \left\{ \int_R^{\infty} (1 - m(w, R)) h(w)w \, dw + \hat{w}[1 - m(\infty, R)] - R \right\}$$

$$(7) \quad = h(R) \{ \rho(\hat{w}, R) - R \} = 0,$$

$$(7') \quad R = \rho(\hat{w}, R)$$

Without ambiguity, we shall denote the reservation wage of an individual with ability \hat{w} (the solution to 7') as $\rho(\hat{w})$.

The optimal wage at which to enter is a worker's expected wage. At any lower wage he would rather not be hired, but starting at any higher wage implies that he forfeits acceptable opportunities.

The actual number of jobs of type w is $N(w)$. Thus, $N(w)/h(w)$, the number of workers seeking jobs of wage w , may be characterized by the sum of workers who are in the market but have not been hired by wage w . Let $\psi(w)$ denote the ability of the most able person applying at wage w . ψ is found by solving for the inverse of ρ in (7').

$$(8) \quad \frac{N(w)}{h(w)} = \int_0^{\psi(w)} [1 - m(w, \rho(\hat{w}))] f(\hat{w}) d\hat{w} = \int_0^w [1 - m(w, \hat{w})] f(\psi(\hat{w})) d\hat{w}$$

Firms offering wage w are interested in the average productivity of their employees, $\bar{A}(w)$; where

$$(9) \quad \bar{A}(w) = \frac{\int_0^{\psi(w)} [1 - m(w, \rho(\hat{w}))] f(\hat{w}) a(\hat{w}) d\hat{w}}{N(w)} \cdot \frac{h(w)}{N(w)} .$$

The criterion determining the competitive equilibrium is

$$(10) \quad \bar{A}(w) = kw .$$

The cost per efficiency unit of labor must be the same for all wages that are offered. Only if this condition is met will firms be indifferent between offering any of the wages in the distribution.¹ The marginal cost of offering higher wages is exactly offset by the higher quality.

In Appendix A we show how (10) can be converted into a simple non-linear second order differential equation. Standard theorems on the existence of solutions to second order differential equations provide sufficient conditions to ensure the existence of a competitive equilibrium. We illustrate the analyses with a simple example.

In our model, firms are unable to make any direct inferences concerning quality; still, in our competitive equilibrium each agent engages in fully rational optimizing behavior. The traditional problems associated with single price markets under uncertainty [Akerlof (1970)], such as the collapse of markets for lemons, are thus resolved. These problems stem in part from

¹ Because firms may offer a multiplicity of wages, they will not be constrained by an insufficient supply of labor (efficiency units). If a firm needs more than $N(w)$ then it will also hire employees at $w + \epsilon$, until the supply, $\int_w^{w+\delta} N(x) dx$, satisfies their demand.

employing the traditional assumptions of a single price, an assumption which in the present context is inappropriate. When workers' actions are correlated with their productivity, then it is possible to construct self-selection devices that take advantage of differential responses to uncertainty. The underlying response of a particular individual to wage distributions and unemployment may be unobservable; but all that the market is concerned with are the statistical consequences. In the face of uncertainty, those with more lucrative fallback opportunities are willing to take greater risks of unemployment in demanding higher wages.

This simple model has one important implication: although there may be unemployment in the competitive equilibrium, there will be no pressure on firms to lower their wage offers. The realization that firms attract different quality labor at different wages provides a micro-economic foundation that explains the macro-economic phenomena of downward sticky wages [Butler (1980), Stiglitz (1976, 1977b), Weiss (1980)].

3. Monopsony

Since a monopsonist has control over the whole wage distribution, it is not surprising that he chooses a distribution that is fundamentally different from that characterizing the competitive equilibrium. A monopsonist will offer only 1 or 2 wages.¹ There is very little use of the price system to encourage self-sorting on the part of workers.

¹ This can be viewed as a generalization of the take it or leave it strategy discussed by Riley and Zeckhauser [forthcoming]. See also Stiglitz [forthcoming].

The model in which wages are called off starting at the bottom and rising upwards may be characterized by two relevant parameters, (1) workers' expectations conditional on employment and (2) probability of employment. When a worker chooses to enter at wage R , he has determined his probability of employment, $m(\infty, R)$ and his expected salary conditional on being employed, denoted by s . Workers who enter at higher wages have a lower chance of being hired but have higher conditional wage expectations.

Since the expectations are correct, what a worker expects to earn when employed is the same as what the firm expects to pay. We may simplify the sequential wage offering process to a distribution at a single point of time.¹ Workers can apply for only a single job. Workers choose a wage (their expectation conditional on being hired) and an associated probability of employment, $h(w)$.

A prospective employee with fallback \hat{w} will maximize his expected income

$$(11) \quad \rho(\hat{w}) = s(\hat{w})h(s(\hat{w})) + \hat{w}[1 - h(s(\hat{w}))]$$

where $s(\hat{w})$ is the wage "chosen" by an individual with fallback wage \hat{w} . He will choose s to satisfy

$$(12) \quad h'(s(\hat{w}))[s(\hat{w}) - \hat{w}] + h(s(\hat{w})) = 0.$$

Equation (12) defines $s(\hat{w})$, the expected salary conditional on being employed that is based on the fallback opportunity, \hat{w} , and the employment prospects as determined by the monopsonist, $h(w)$. In determining $h(w)$

¹ That is, our earlier analysis could be similarly simplified to describe the equilibrium in a market in which individuals can apply to a single firm.

the monopsonist is restricted by $h'(w) < 0$. He cannot offer both a higher wage and a greater chance of employment as that would imply that no one would apply for the lower wage. The monopsonist's problem will be to minimize his wage bill subject to obtaining his desired level of efficiency units. His expected wage cost, c , is

$$(13) \quad c = \int_0^{\infty} h(s(\hat{w}))s(\hat{w})f(\hat{w})d\hat{w} = \int h[s - \hat{w} + \hat{w}]f\hat{w}d\hat{w}$$

$$= \int h(s - \hat{w})f(\hat{w})d\hat{w} + \int h\hat{w}f(\hat{w})d\hat{w}$$

Letting

$$u = h(s(\hat{w}))[s(\hat{w}) - \hat{w}], \quad \frac{du}{d\hat{w}} = (h'[s - \hat{w}] + h) \frac{ds}{d\hat{w}} - h = -h$$

(using (12)) and

$$dv = f(\hat{w})d\hat{w}$$

and integrating by parts, we obtain¹

$$(14) \quad c = h(s(\hat{w}))[s(\hat{w}) - \hat{w}]F(\hat{w}) \Big|_0^{\infty} - \int_0^{\infty} -h(s(\hat{w}))[\hat{w}f(\hat{w}) + F(\hat{w})]d\hat{w}$$

$$= \int_0^{\infty} h(s(\hat{w}))[\hat{w}f(\hat{w}) + F(\hat{w})]d\hat{w}$$

The firm's problem can thus be formulated as

$$(15) \quad \min_{\{h\}} \int_0^{\infty} h(s(\hat{w}))[\hat{w}f(\hat{w}) + F(\hat{w})]d\hat{w}$$

subject to the labor constraint

$$(16) \quad \int_0^{\infty} h(s(\hat{w})) a(\hat{w}) f(\hat{w})d\hat{w} \geq N$$

and the non-negativity constraint on h'

$$(17) \quad h' < 0.$$

¹ We make use of the fact that at the highest wages offered, the reservation wage must equal the fallback wage.

This can be formulated as a standard variational problem with Hamiltonian

$$(18) \quad H = hj + \gamma h'$$

with

$$(19) \quad - \frac{d\gamma}{dw} = j .$$

where

$$(20) \quad j = \hat{w}f(\hat{w}) + F(\hat{w}) - \lambda f(\hat{w})a(\hat{w})$$

and where λ is the Lagrange multiplier associated with constraint (16).

Thus,

$$(21) \quad - h' = \begin{cases} 0 & \text{if } \gamma < 0 \\ \infty & \text{if } \gamma > 0 \\ 0, \infty & \text{if } \gamma = 0 . \end{cases}$$

The linearity of the structure ensures that $h' < 0$ only at discrete points.¹ We know from (20) that for small values of w , $j < 0$. For there to be more than one value of w for which $h(w) > 0$ we must have from (19)

$$j = 0 \quad \text{whenever } \gamma = 0$$

(except at the highest wage). From (20), this requires

$$(22) \quad \lambda = \frac{\hat{w}}{a(\hat{w})} + \frac{f(\hat{w})}{f(\hat{w})a(\hat{w})}$$

¹ If $\gamma > 0$ over an interval, $-h' = \infty$ over the interval. The non-negative constraint on h' assures us then that h could only be positive at the end point of the interval.

The right hand side of (22) is a simple function of \hat{w} . (22) can readily be satisfied for a particular value \hat{w} . Indeed (22) can be thought of as determining λ , where \hat{w} is the lowest wage offered.

Given γ_0 , we can integrate (19) to solve for $\gamma(\hat{w})$. We depict a possible solution in Figure 3, where we have chosen γ_0 and λ so that $\gamma(w)$ is tangent to the horizontal axis at only one point. There exists the possibility of a second tangency as in Figure 2, by choosing γ_0 appropriately. A third tangency could only arise by fluke, and even if it did, there would be no loss in using only two wages. We now establish this:

Assume there were three distinct wages, w_1 , w_2 , and w_3 . Denote $h(w_i)$ by h_i .

The population is divided into three groups, those that apply to job 1, job 2, or job 3. For the individual who is indifferent between applying to job 1 or 2, with fall-back wage \hat{w}_1 ,

$$(23) \quad w_1 h_1 + \hat{w}_1 [1 - h_1] = w_2 h_2 + \hat{w}_1 [1 - h_2]$$

and similarly for the individual who is indifferent between applying to job 2 and 3:

$$(24) \quad w_2 h_2 + \hat{w}_2 [1 - h_2] = w_3 h_3 + \hat{w}_2 [1 - h_3]$$

Labor costs are

$$(25) \quad c = w_1 h_1 F(\hat{w}_1) + w_2 h_2 [F(\hat{w}_2) - F(\hat{w}_1)] + w_3 h_3 [F(\hat{w}_3) - F(\hat{w}_2)]$$

while effective labor supply

$$(26) \quad N = A(\hat{w}_1, 0)h_1 + A(\hat{w}_2, \hat{w}_1)h_2 + A(\hat{w}_3, \hat{w}_2)h_3$$

where

$$(27) \quad A(x, y) = \int_y^x a(w)f(w)dw$$

Figure 3

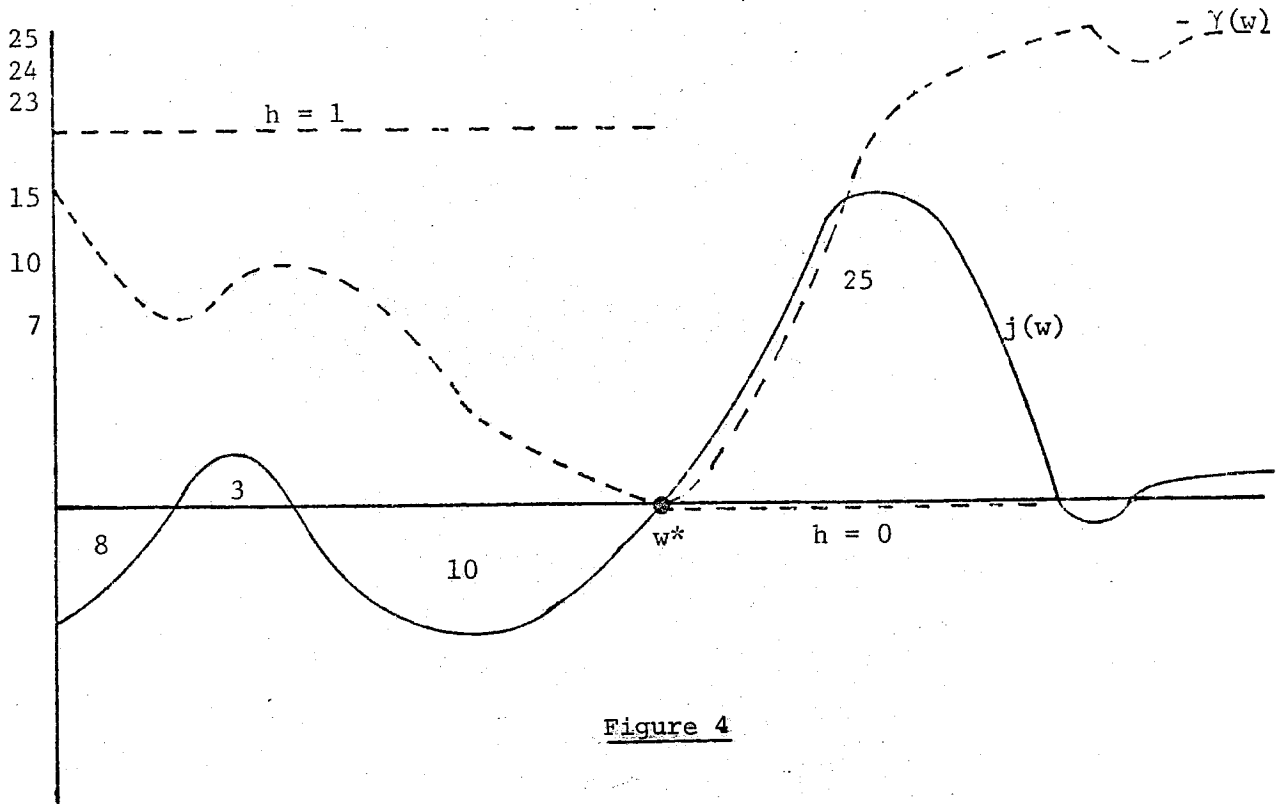
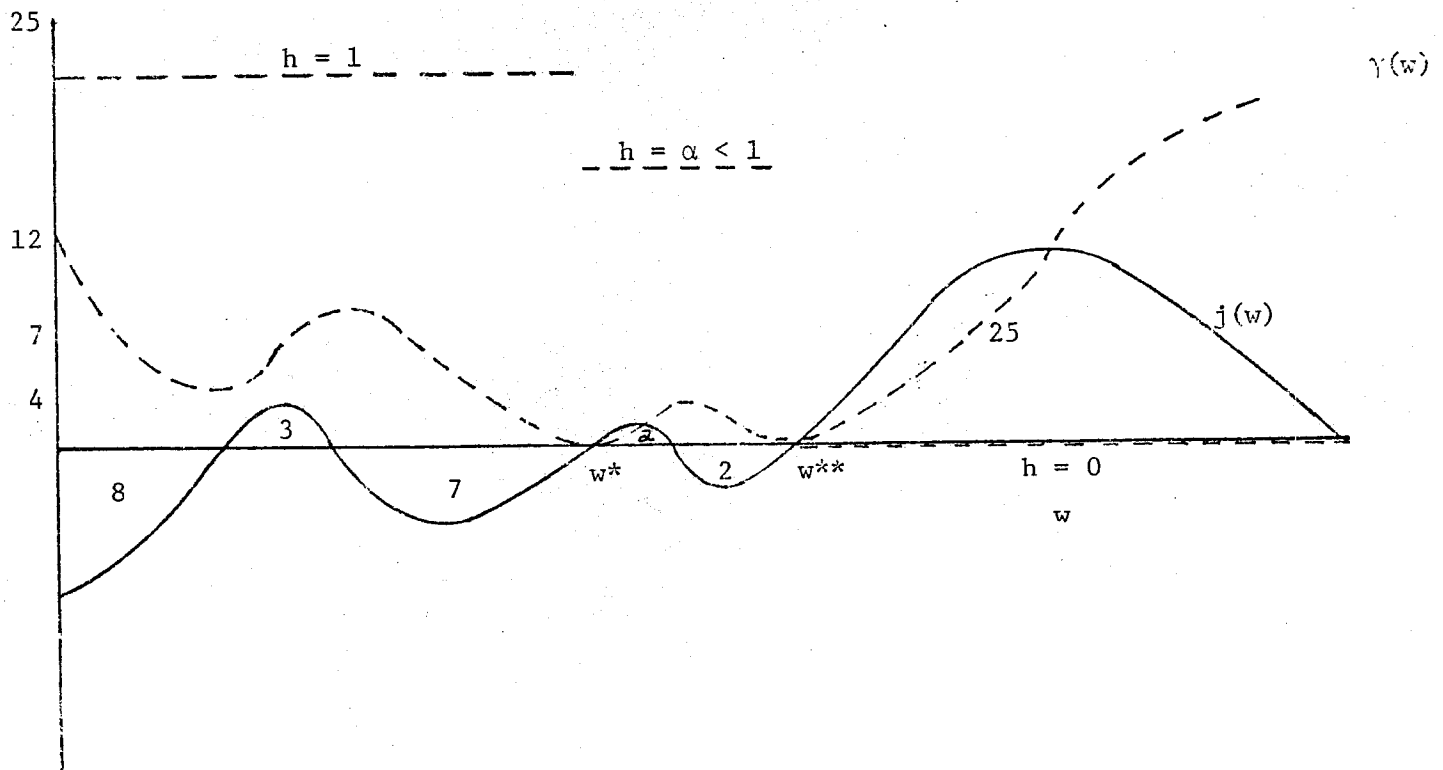


Figure 4



Assume we specify what groups are to apply to each job, i.e. $\hat{w}_1, \hat{w}_2, \hat{w}_3$, where \hat{w}_1 and \hat{w}_2 satisfy (23), (24) and $\hat{w}_3 = w_3$. (A person will apply for a job if and only if the wage he gets exceeds his opportunity cost.) Hence, we can solve (23) and (24) for $w_1 h_1$ and $w_2 h_2$ as linear functions of h_1, h_2 and h_3 :

$$(23') \quad w_1 h_1 = \hat{w}_1 (h_1 - h_2) + \hat{w}_3 h_3 + \hat{w}_2 (h_2 - h_3)$$

$$(24') \quad w_2 h_2 = \hat{w}_3 h_3 + \hat{w}_2 (h_2 - h_3)$$

Substituting into (25), we obtain

$$(28) \quad c = \hat{w}_1 (h_1 - h_2) F(\hat{w}_1) + \hat{w}_2 (h_2 - h_3) F(\hat{w}_2) + \hat{w}_3 h_3 F(\hat{w}_3)$$

We can solve (26) for h_3 as a function of h_1 and h_2 , and substitute the result into (28), to obtain a linear equation in h_1 and h_2 :

$$(29) \quad c = \{ \hat{w}_1 F(\hat{w}_1) - [\hat{w}_3 F(\hat{w}_3) - \hat{w}_2 F(\hat{w}_2)] \frac{A(\hat{w}_1, 0)}{A(\hat{w}_3, \hat{w}_2)} \} h_1$$

$$+ \{ \hat{w}_2 F(\hat{w}_2) - \hat{w}_1 F(\hat{w}_1) - [\hat{w}_3 F(\hat{w}_3) - \hat{w}_2 F(\hat{w}_2)] \cdot \frac{A(\hat{w}_2, \hat{w}_1)}{A(\hat{w}_3, \hat{w}_2)} \} h_2$$

$$+ \frac{[\hat{w}_3 F(\hat{w}_3) - \hat{w}_2 F(\hat{w}_2)] N}{A(\hat{w}_3, \hat{w}_2)}$$

Clearly, only by fluke will the coefficients of h_1 and h_2 be identical, in which case a three wage distribution will be just as good as any other; while if

the coefficients of h_1 and h_2 differ, the two wage distribution will be optimal.¹

¹ We have ignored the constraints $1 \geq h_1 \geq h_2 \geq h_3 \geq 0$. These may later be incorporated into the analysis, without altering the conclusions. Assume, for instance, that $h_1 = h_2$. Then from (23) $h_1 = h_2 \Rightarrow w_1 = w_2$, which immediately contradicts our original assumption of three distinct wages.

In general, one imagines that the main advantage of a wage distribution is that the monopsonist exploits the risk aversion and differential search costs of job hunters [Salop (1977)]. We have illustrated how a monopsonist can use a wage distribution to induce self-selection behavior based on quality even when workers are risk neutral and there are no search costs. Our quiet monopsonist may also choose to offer a single wage with certainty, taking fullest advantage of job seekers' willingness to trade off lower wages for greater employment prospects. Differences in opportunity cost will induce differences in observed behavior toward risk.

4. A Duopoly Model

These results are surprisingly different in the context of a duopoly model. The main distinction of a duopoly model is that each firm realizes that it has the power to affect expectations. This power can be exploited so as to give the second firm an advantage even greater than in our earlier analysis. There we noted that the second firm may benefit from the probability that the first firm has hired a number of the undesirable workers. In addition, however, the presence of the second firm lowers the quality of those who apply to the first. Since the optimal wage of the

second firm exceeds the optimal wage of the first, the individual who would have been indifferent to applying -- if there were no second firm -- now does not apply.

We will not provide here a complete characterization of the equilibrium, but will rather outline the structure of the duopoly model and illustrate our analysis with an example.

The essential features of the equilibrium arise from the interdependence of the profit functions of the two firms:

$$\pi^1 = \pi^1(w_1, N_1, w_2, N_2)$$

$$\pi^2 = \pi^2(w_2, N_2, w_1, N_1)$$

There are two important asymmetries in the structure.

- a) The first firm must move before the second.
- b) While increases in N_2 lower the profits of the first firm (since they will raise reservation wages), increases in N_1 raise the profits of the second firm.

Given the temporal structure of this problem, the natural equilibrium concept is that of von Stackleberg: The first firm announces a wage and employment policy, and the second firm optimizes against it. Finally, the first firm chooses $\{w_1, N_1\}$ to maximize his own profits, given the known reaction function of the second firm.

An Example

Both firms demand 100 efficiency units of labor.¹ The labor supply is such that there are 50 workers with self-employment wage \$10 and $a(10) = 10$ efficiency units, and 1 worker with a self-employment wage 0 and $a(0) = 3$ efficiency units. The two firms enter the market sequentially.

The only feasible single wage that the first firm could offer would be \$10. All 51 workers would apply and the firm would have an average productivity per wage, $\frac{\bar{A}(w)}{w} = \frac{503}{510} = .986$.² The firm would be willing to pay the lower efficiency unit (e.u.) worker up to 4.4 as that would leave $\frac{\bar{A}}{w}$ unchanged ($\frac{100}{97 + 4.4} = .986$). At the single wage, 10, each worker has a .1988 chance of employment. Thus the 3 e.u. worker must decide between a .1988 chance of 10 plus a .8012 chance of what the second firm will offer and 4.4 for certain.

The second firm would be willing to pay the 3 e.u. worker up to 3 before it no longer wished for him to apply. With a guarantee of 3 in the second period the 3 e.u. worker expects to make 4.4 if he applies for the job paying 10 in the first period. As a matter of indifference, he will take the first firm's offer of 4.4 for certain. In either case, the second firm can pay its 10 e.u. workers 10 (and its 3 e.u. worker 3 if he applies) and have $\frac{\bar{A}}{w} = 1$. The disadvantage of the low productivity worker is borne entirely by the first firm.

¹ Normally their demand for labor would be endogenous, but let us assume that the endogenous solution is indeed 100.

² The firm buys 97 efficiency units at \$1 /efficiency unit from the more productive workers.

5. Will There be Binding Contracts in Equilibrium?

In an earlier paper, "Prices and Queues," [Stiglitz (1976)], similar issues were discussed in a world where recontracting was permissible. In this section we show that recontracting arrangements indeed dominate no recontracting agreements. Firms would choose to pay lower wages and allow their workers freedom to accept higher paying employment if they find any. The institutions that prohibit or impose costs on recontracting must arise from forces outside of our model.¹

The simplest model within which to investigate these questions entails a slight simplification of our earlier analysis. We assume there are two periods of search. We first present the no-recontracting solution. In order to keep productivity/wage constant a worker of type \hat{w} must be induced to apply for the same salary job in both periods. Thus the higher fallback wage in the first period must be counterbalanced by a lower probability of employment.

We start by considering the decision made in the second period where workers of type \hat{w} (fallback self-employment wage) who have not found work choose w to maximize

$$v(\hat{w}) = wg(w) + \hat{w}(1 - g(w))$$

where $g(w)$ is the probability of finding a job paying w in the second period.

$$\frac{g'(w)}{g(w)} = \frac{-1}{w - \hat{w}}$$

The condition for a competitive equilibrium price distribution,

$$A(w) = a(\hat{w}(w)) = kw, \text{ implies } \hat{w}(w) = a^{-1}(kw). \text{ This defines } g(w):$$

¹ We have, for instance, ignored firm specific information and training costs.

$$\frac{g'(w)}{g(w)} = \frac{-1}{w - a^{-1}(kw)}$$

In the previous period, we replace \hat{w} by the "expected" fallback, $v(\hat{w})$. The distribution of employment probability in the first period is $h(w)$. The equilibrium condition below defines $h(w)$.¹

$$\frac{h'(w)}{h(w)} = \frac{-1}{w - v(\hat{w})} = \frac{-1}{(w - \hat{w})(1 - g(w))} = \frac{-1}{(1 - g(w))(w - a^{-1}(kw))}$$

The equilibrium without binding contracts is much more difficult to characterize. In the first period workers apply for the job with the highest expected income, i.e.

$$\max_{\{w\}} = h(w)v(w) + (1 - h(w))v(\hat{w})$$

where $v(x) = \max_w \{g(w)w + (1 - g(w))x\}$. If workers get a job, they need not keep it. $g(w)$ is the probability next period of finding a better job. Hence

$$\frac{h'(w)}{h(w)} = - \frac{v'(w)}{v(w) - v(a^{-1}(kw))}$$

Since $A(w) = a(\hat{w}(w)) = kw$ in the competitive equilibrium, we have substituted $a^{-1}(kw)$ for $\hat{w}(w)$ to define $h(w)$. To determine the second period probability of employment schedule, $g(w)$, we perform a similar calculation to obtain

$$\frac{g'(w)}{g(w)} = \frac{-1}{w - \bar{w}}$$

¹ If there were a third period, then the first period distribution would be

$$\frac{j'(w)}{j(w)} = \frac{-1}{w - \hat{w}} \frac{1}{1 - g(w)} \frac{1}{1 - h(w)}$$

This process may be extended indefinitely.

where \bar{w} is the fallback wage, either \hat{w} or the salary of the first period offer. Now the average ability of those applying for jobs the second period is given by

$$A_2(w) = [1 - h(\hat{w}^{-1}(\bar{w}(w)))]f(\bar{w}(w))a(\bar{w}(w)) \\ + h(\bar{w}(w))f(\hat{w}(\bar{w}(w)))k\bar{w}(w) / [(1 - h(\hat{w}^{-1}(\bar{w}(w))))f(\bar{w}(w)) + h(\bar{w}(w))f(\hat{w}(\bar{w}(w)))].$$

The workers applying for a salary w in the second period come from two groups. The first group are those who failed to obtain a job in the first period. A person with a self-employment fallback of \bar{w} will try for a job paying $\hat{w}^{-1}(\bar{w})$ in the first period and will fail with probability $[1 - h(\hat{w}^{-1}(\bar{w}))]$. The second group are those who were hired at \bar{w} . The equilibrium condition in the first period guarantees that $a(\hat{w}(\bar{w}(w))) = k\bar{w}$.

Fortunately, it is much simpler to consider the stability of these contractual arrangements. Imagine all firms are in the no-recontracting equilibrium. We will show that a firm would like to enter and offer a recontracting option. In the old system a worker with fall-back wage \hat{w} expects to earn $\bar{\rho}(\hat{w}) = h(w_1)w_1 + [1 - h(w_1)]v(\hat{w})$. In order to attract a worker of type w to the new firm which allows recontracting, the entrant would need to guarantee the same expected salary. With a $\mu(\bar{w})$ chance of employment at the recontracting firm, expected salary is

$$\rho(\hat{w}) = \mu(\bar{w}_1)v(\bar{w}_1) + (1 - \mu(\bar{w}_1))v(\hat{w})$$

There is thus a salary $\bar{w}_1 < w_1$ for which $v(\bar{w}_1) = w_1$, which will induce the same length of queues and the same quality of applicants.

A firm offering recontracting is able to use the second period's conditional expectations to raise its workers' expected salaries. Identical workers will be attracted to the lower wages at the recontracting firm. In particular a firm allowing recontracting could offer a wage below the minimum wage offered by a non-recontracting firm and obtain the same quality of labor. The converse is also true. If all firms offered recontracting contracts, a firm which announced it would not allow recontracting would find that at the same wages, it faced a shorter queue and obtained lower quality workers. Thus, the recontracting equilibrium is "stable."

The intuition behind these results may be seen most clearly in the case of two ability groups. The individual's indifference curves between wage, w , and probability of being hired, h , are depicted in Figure 5. The equilibrium is characterized by a pair $\{w_1, h_1\}$ $\{w_2, h_2\}$ such that only the high ability apply to the high wage firm, and the low ability apply to the low wage firm, (as is standard in self-selection equilibria). Now, allowing recontracting shifts both indifference curves to the left, by the amount $v^{-1}(w_1) - w_1$. Hence, not only can the low wage firm lower its wage and obtain low quality workers, but the self selection point is shifted to the left, so that the high wage firm can obtain the high quality labor at a much lower wage.

While the role of non-recontracting clauses has been centrally important to the first two sections, we have shown that its presence must arise from forces outside of the current framework.

6. Conclusions

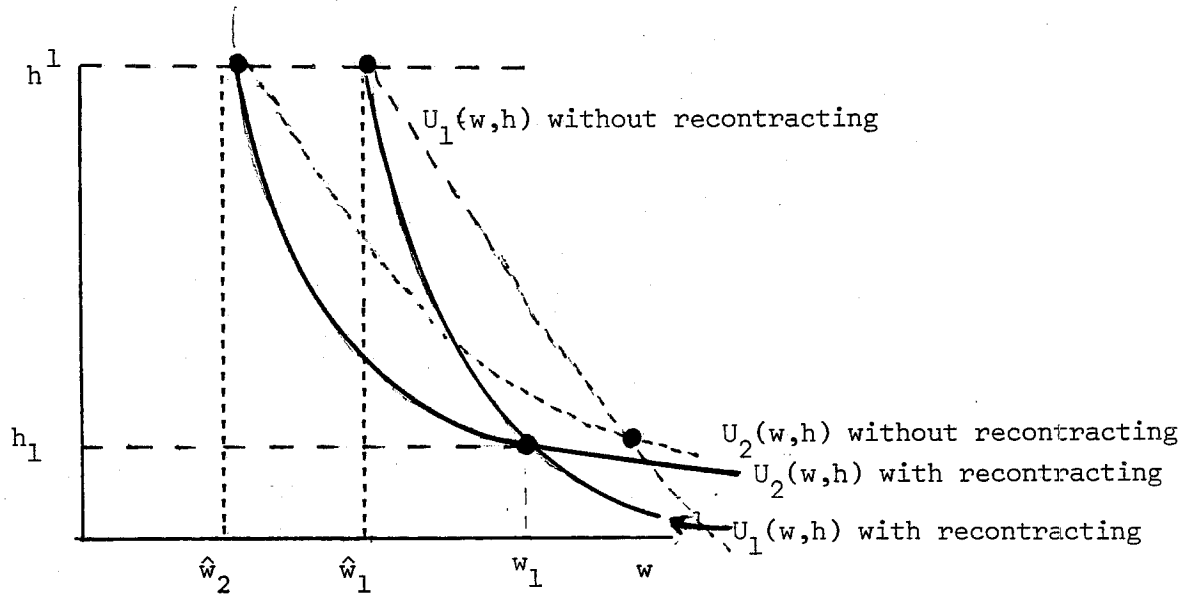
There are three important lessons to be learned from the kind of modelling we have attempted here.

First, we have seen how sensitive the nature of the equilibrium is to certain specific institutional and technological assumptions. If there are no search costs, and recontracting is allowed, all individuals will apply to all jobs exceeding their reservation wage, and the market equilibrium will be characterized by a simple efficiency wage (Stiglitz [1976]), but if either of these assumptions are violated, then there will not exist an equilibrium with a single wage; competitive equilibrium will be characterized by a wage distribution.

Secondly, we saw how at least one of the institutional features which we showed played a critical role in the structure of the equilibrium -- the provisions of the contract restricting future employment opportunities of the worker (the no-recontracting provision) -- could be analyzed within the context of the model itself. We showed that, at least in the simple model examined here, not all contracts would be binding.

Thirdly, our analysis suggests that great care needs to be taken in formulating appropriate equilibrium conditions in models with imperfect information. Some of the difficulties encountered in earlier studies of market equilibrium with imperfect information (including Akerlof's justly celebrated paper on the Theory of Lemons) arose from borrowing equilibrium notions which might have been appropriate to the perfect information context to these new situations. In the conventional model, market equilibrium is defined as having demand equalling supply (full employment) and a single wage (for individuals who otherwise appear to be identical). But in a market equilibrium with imperfect information, we have shown that there may be a wage distribution and unemployment.

Figure 5



APPENDIX

Characterization of the Equilibrium

In Section 2, we derived a simple equation characterizing the competitive equilibrium, when there was no recontracting. In this Appendix, we manipulate equations (8) - (10) to derive a second order non-linear differential equation with an associated boundary value condition which characterizes the competitive equilibrium. We first illustrate this technique by solving explicitly for the wage distribution for a specific example.

The choice functions $h(w)$ and $N(w)$ must be chosen to satisfy $\bar{A}(w) = kw$. The example below works through the tedious calculations to show $\bar{A}'(w) = k$ for a particular choice of $f(\cdot)$ and $a(\cdot)$. Let

$$f(w) = [1 + \frac{w}{4}] / 6 \quad 0 \leq w \leq 4$$

$$a(w) = \frac{5k}{3} [1 + \frac{w}{4}]^2 \quad 0 \leq w \leq 4$$

Claim

$$N(w) = \frac{1}{9} \quad 0 \leq w \leq 4$$

$$h(w) = \frac{1}{2w} \quad 0 \leq w \leq 4$$

Proof:

$$m(w, \hat{\rho}(w)) = 1 - e^{-\int_{\hat{\rho}(w)}^w \frac{1}{2x} dx} = 1 - (\frac{\hat{\rho}(w)}{w})^{1/2}$$

$$\hat{\rho}(w) = \int_{\hat{\rho}(w)}^4 (\frac{\hat{\rho}(w)}{x})^{1/2} \frac{1}{2x} x dx + \hat{w} [\frac{\hat{\rho}(w)}{4}]^{1/2} = [1 + \frac{\hat{w}}{4}]^2$$

$$\hat{\rho}^{-1}(\hat{w}) = 4[\hat{w} - 1]^{1/2}$$

$$f(\hat{\rho}^{-1}(\hat{w})) = \frac{\hat{w}^{1/2}}{6}$$

$$\bar{A}'(w) = \frac{[a(\rho^{-1}(w)) - \bar{A}(w)]}{1-m(\infty, w)} f(\rho^{-1}(w))h(w)/N(w)$$

$$\bar{A}'(w) = \frac{wk [5/3 - 1]}{\frac{w}{2}} \cdot \frac{w^{1/2}}{6} \cdot \frac{1}{2w} \cdot \frac{1}{1/9} = k$$

More generally, we can differentiate (10) logarithmically and use the equilibrium condition $\bar{A} = kw$ to obtain

$$(A1) \quad \int_0^{\psi(w)} [1-m(w, \rho(\hat{w}))] f(\hat{w}) d\hat{w} = \psi'(w) f(\psi(w)) \left[\frac{a(\psi(w))}{k} - w \right]$$

$$(A2) \quad k\psi'f(\psi) - h(w)\psi'f(\psi) [a(\psi) - kw] = [\psi''f(\psi) + f'(\psi)\psi'^2][a(\psi) - kw] \\ + \psi'f(\psi)[a'(\psi)\psi' - k]$$

and then divide by ψ' and use $\psi'' = -h(w)\psi'(w)$

$$(A2') \quad \psi'(w) = \frac{2f(\psi)k}{a'(\psi)f(\psi) + f'(\psi)a(\psi) - kwf'(\psi)}$$

We differentiate (A2') again to get (after dividing both sides by ψ')

$$(A3) \quad -h[a'f + f'a - kwf'] = 3kf' - \{a''f + 2a'f' + af'' - kwf''\} \psi'(w)$$

Now we substitute the value for w from (A2')

$$(A2'') \quad w = \frac{-2fk}{\psi'f'k} + \frac{a'f + f'a}{kf'}$$

Define

$$\gamma(\psi(w)) = [a''f + 2a'f' + af'' - f''a'f/f' - f''a]$$

Then

$$(A4) \quad -h[a'f + f'a - a'f - f'a] - \frac{hkf^2}{\psi'} = 3kf' - 2fkk''/f' - \gamma(\psi) - \psi'$$

Now we substitute

$$\rho'(\psi) = 1/\psi'(w)$$

and

$$\rho''(\psi) = h(w)\rho'(\psi)^2$$

to get (after dividing both sides by ψ')

$$(A5) \quad -\rho''(\psi) \cdot 2fk = [3kf' - 2fkk''/f'] \cdot \rho'(\psi) - \gamma(\psi)$$

and if this is evaluated at $y = \psi(w)$

$$(A5') \quad \rho''(y) \cdot 2f(y)k + \rho'(y) [3kf'(y) - 2f(y)kf''(y)/f'(y)] = \gamma(y)$$

where now everything is in terms of y .

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